Chapter 1

Design for Flexure

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1.1 Introduction

Design of reinforced concrete elements for flexure involves; i) sectional design and ii) member detailing. Sectional design includes the determination of cross-sectional geometry and the required longitudinal reinforcement as per Chapter 10 of ACI 318-05. Member detailing includes the determination of bar lengths, locations of cut-off points and detailing of reinforcement as governed by the development, splice and anchorage length requirements specified in chapter 12 of ACI 318-05. This Chapter of the Handbook deals with the sectional design of members for flexure on the basis of the Strength Design Method of ACI 318-05. The Strength Design Method requires the conditions of static equilibrium and strain compatibility across the depth of the section to be satisfied.

The following are the assumptions for Strength Design Method:

i. Strains in reinforcement and concrete are directly proportional to the distance from neutral axis. This implies that the variation of strains across the section is linear, and unknown values can be computed from the known values of strain through a linear relationship.
ii. Concrete sections are considered to have reached their flexural capacities when they develop 0.003 strain in the extreme compression fiber.
iii. Stress in reinforcement varies linearly with strain up to the specified yield strength. The stress remains constant beyond this point as strains continue increasing. This implies that the strain hardening of steel is ignored.
iv. Tensile strength of concrete is neglected.
v. Compressive stress distribution of concrete can be represented by the corresponding stress-strain relationship of concrete. This stress distribution may be simplified by a rectangular stress distribution as described in Fig. 1-1.

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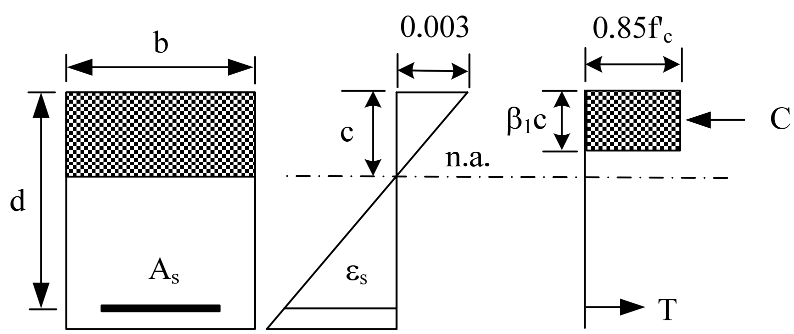


$$\beta_1 = 0.85 \quad for\ f_c' \le 4000\ psi$$

$$\beta_1 = 0.85 - 0.05\frac{f_c' - 4000}{1000} \ge 0.65 \quad for\ f'_c > 4000 psi$$

Fig. 1-1 Ultimate strain profile and corresponding rectangular stress distribution

1.2 Nominal and Design Flexural Strengths (M_n, and ϕM_n)

Nominal moment capacity M_n of a section is computed from internal forces at ultimate strain profile (when the extreme compressive fiber strain is equal to 0.003). Sections in flexure exhibit different modes of failure depending on the strain level in the extreme tension reinforcement. Tension-controlled sections have strains either equal to or in excess of 0.005 (Section 10.3.4 of ACI 318-05). Compression-controlled sections have strains equal to or less than the yield strain, which is equal to 0.002 for Grade 60 reinforcement (Section 10.3.3 of ACI 318-05). There exists a transition region between the tension-controlled and compression-controlled sections (Section 10.3.4 of ACI 318-05). Tension-controlled sections are desirable for their ductile behavior, which allows redistribution of stresses and sufficient warning against an imminent failure. It is always a good practice to design reinforced concrete elements to behave in a ductile manner, whenever possible. This can be ensured by limiting the amount of reinforcement such that the tension reinforcement yields prior to concrete crushing. Section 10.3.5 of ACI 318-05 limits the strain in extreme tension reinforcement to 0.004 or greater. The amount of reinforcement corresponding to this level of strain defines the maximum amount of tension reinforcement that balances compression concrete. The ACI Code requires a lower strength reduction factor (ϕ-factor) for sections in the transition zone to allow for increased safety in sections with reduced ductility. Figure 1-2 illustrates the variation of ϕ-factors with tensile strain in reinforcement for Grade 60 steel, and the corresponding strain profiles at ultimate.

The ACI 318-05 Code has adopted strength reduction factors that are compatible with ASCE7-02 load combinations, except for the tension controlled section for which the ϕ-factor is increased from 0.80 to 0.90. These ϕ-factors appear in Chapter 9 of ACI 318-05. The ϕ-factors used in earlier editions of ACI 318 and the corresponding load factors have been moved to Appendix C of ACI 318-05. The designer has the option of using either the ϕ-factors in the main body of the Code (Chapter 9) or those given in Appendix C, so long as ϕ-factors are used with the corresponding load factors. The basic design inequality remains the same, irrespective of which pair of ϕ and load factors is used:

Factored (ultimate) moment $\le$ Reduced (design) strength

$$M_u \le \phi M_n$$

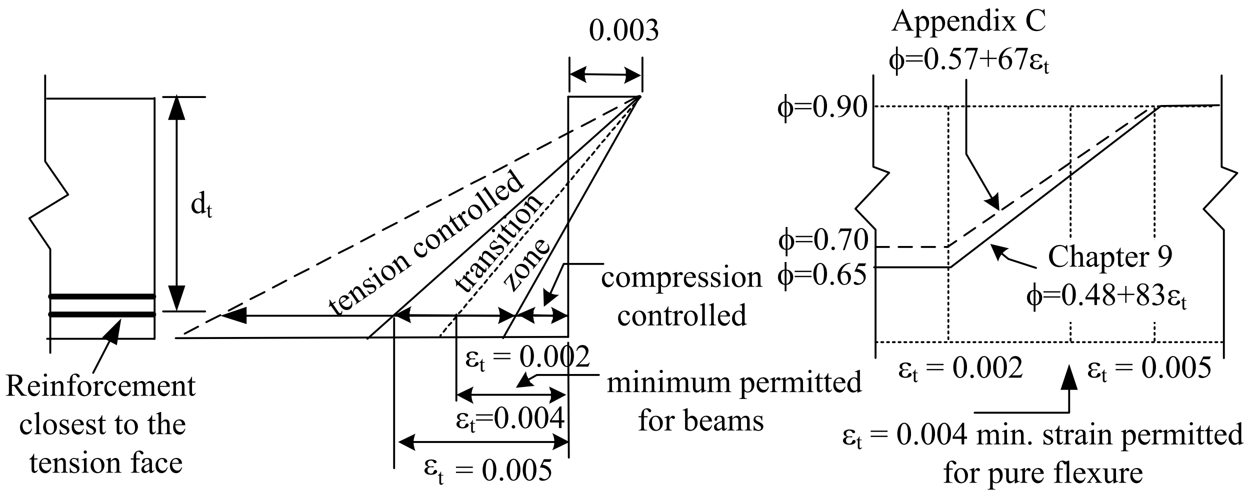


Fig. 1-2 Capacity reduction (ϕ) factors for Grade 60 reinforcement

1.2.1 Rectangular Sections with Tension Reinforcement

Nominal moment capacity of a rectangular section with tension reinforcement is computed from the internal force couple shown in Fig. 1-1. The required amount of reinforcement is computed from the equilibrium of forces. This computation becomes easier for code permitted sections where the tension steel yields prior to the compression concrete reaching its assumed failure strain of 0.003. Design aids **Flexure 1** through **Flexure 4** (included at the end of the chapter) were developed using this condition. Accordingly;

$$T = C \tag{1-1}$$

$$A_s f_y = 0.85 f'_c \, \beta_1 c b \tag{1-2}$$

$$\rho = \frac{0.85 f'_c \, \beta_1 c}{d f_y} \tag{1-3}$$

where;

$$\rho = \frac{A_s}{bd} \tag{1-4}$$

The c/d ratio in Eq. (1-3) can be written in terms of the steel strain ε_s illustrated in Fig. 1-1. For sections with single layer of tension reinforcement, d = d_t and $\varepsilon_s = \varepsilon_t$. The c/d ratio for this case becomes;

$$\frac{c}{d} = \frac{c}{d_t} = \frac{0.003}{0.003 + \varepsilon_t} \tag{1-5}$$

$$\rho = \frac{0.85 f_c' \beta_1}{f_y} \frac{0.003}{0.003 + \varepsilon_t} \tag{1-6}$$

Eq. (1-6) was used to generate the values for reinforcement ratio ρ (%) in **Flexure 1** through **Flexure 4** for sections with single layer of tension reinforcement. For other sections, where the centroid of tension reinforcement does not necessarily coincide with the centroid of extreme tension layer, the ρ values given in **Flexure 1** through **Flexure 4** should be multiplied by the ratio d_t/d.

The nominal moment capacity is computed from the internal force couple as illustrated below:

$$M_n = A_s f_y \left(d - \frac{\beta_1 c}{2}\right) \tag{1-7}$$

From Eq. (1-2);

$$\beta_1 c = \frac{A_s f_y}{0.85 f'_c\, b} \tag{1-8}$$

$$M_n = bd^2 \left[1 - \frac{\rho f_y}{1.7 f_c'}\right] \rho f_y \tag{1-9}$$

$$M_n = bd^2 K_n \tag{1-10}$$

Where;

$$K_n = \left[1 - \frac{\rho f_y}{1.7 f_c'}\right] \rho f_y \tag{1-11}$$

Flexure 1 through **Flexure 4** contains ϕK_n values computed by Eq. (1-11), where the ϕ-factor is obtained from Fig. 1-2 for selected values of ε_t listed in the design aids.

Design Examples 1 through 4 illustrate the application of **Flexure 1** to **Flexure 4**.

1.2.2 Rectangular Sections with Compression Reinforcement

Flexural members are designed for tension reinforcement. Any additional moment capacity required in the section is usually provided by increasing the section size or the amount of tension reinforcement. However, the cross-sectional dimensions in some applications may be limited by architectural or functional requirements, and the extra moment capacity may have to be provided by additional tension and compression reinforcement. The extra steel generates an internal force couple, adding to the sectional moment capacity without changing the ductility of the section. In such cases, the total moment capacity consists of two components; i) moment due to the tension reinforcement that balances the compression concrete, and ii) moment generated by the internal steel force couple consisting of compression reinforcement and equal amount of additional tension reinforcement, as illustrated in Fig. 1-3.

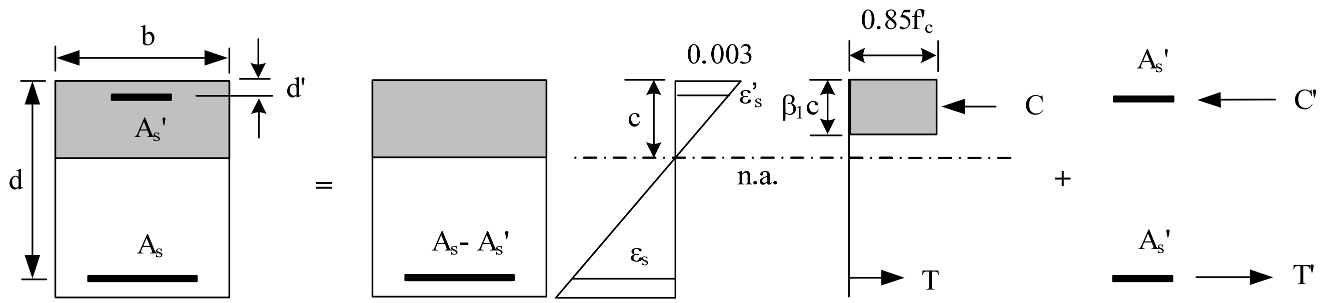


Fig. 1-3 A rectangular section with compression reinforcement

$$M_n = M_1 + M_2 \tag{1-12}$$

$$M_1 = K_n bd^2 \tag{1-13}$$

$$M_2 = A_s' f'_s (d - d') \tag{1-14}$$

Assuming f'$_s$ is equal to or greater than f_y;

$$M_2 = K_n' b d^2 \tag{1-15}$$

where; $$K_n' = \rho' f_y (1 - \frac{d'}{d}) \tag{1-16}$$

and $$\rho' = \frac{A_s'}{bd} \tag{1-17}$$

Since the steel couple does not involve a force in concrete, it does not affect the ductility of the section, i.e., adding more tension steel over and above the maximum permitted by the Code does not create an over-reinforced section, and is permissible by ACI 318-05 so long as an equal amount is placed in the compression zone. This approach is employed for design, as well as in generating **Flexure 5**, which provides the amount of compression reinforcement. The underlining assumption in computing the steel force couple is that the steel in compression is at or near yield, developing compressive stress equal to the tensile yield strength. While this assumption is true in most heavily reinforced sections since the compression reinforcement is near the extreme compression fiber with a strain of 0.003, especially for Grade 60 steel with 0.002 yield strain, it is possible to design sections with non-yielding compression reinforcement. The designer, in this case, has to adjust (increase) the amount of compression reinforcement in proportion to the ratio of yield strength to compression steel stress. The strain in compression steel ε'$_s$ can be computed from Fig. 1-3 as ε'$_s$ = ε_s (c-d')/(d-d'), once ε_s is determined from flexural design tables for sections with tension reinforcement (**Flexure 1** through **Flexure 4**) to assess if the compression steel is yielding.

The application of **Flexure 5** is illustrated in **Design Example 5**.

1.2.3 T-Sections

Most concrete slabs are cast monolithically with supporting beams, with portions of the slab participating in flexural resistance of the beams. The resulting one-way structural system has a T-section. The flange of a T-section is formed by the effective width of the slab, as defined in Section 8.10 of ACI 318-05, and also illustrated in **Flexure 6**. The rectangular beam forms the web of the T-section. T-sections may also be produced by the precast industry as single and double T's because of their superior performance in positive moment regions. A T-section provides increased area of compression concrete in the flange, where it is needed under positive bending, with reduced dead load resulting from the reduced area of tension concrete in the web.

The flange width in most T-sections is significantly wider than the web width. Therefore, the amount of tension reinforcement placed in the web can easily be equilibrated by a portion of the flange concrete in compression, placing the neutral axis in the flange. Therefore, most T-sections behave as rectangular sections, even though they have T geometry, and are designed using **Flexure 1 through Flexure 4** as rectangular sections with section widths equal to flange widths.

Rarely, the required amount of tension reinforcement in the web (or the applied moment) is high enough to bring the neutral axis below the flange, creating an additional compression zone in the web. In such a case, the section behaves as a T-section with total moment capacity consisting of components due to; i) compression concrete in the overhangs ($b-b_w$) and a portion of total tension steel balancing the overhangs, ρ_f and ii) the remaining tension steel, ρ_w balancing the web concrete. The condition for T-section behavior is expressed below:

$$M_u > \phi[0.85 f_c' b h_f (d - \frac{h_f}{2})] \qquad (1\text{-}18)$$

The components of moment for T-section behavior are illustrated in Fig. 1-4, and are expressed below.

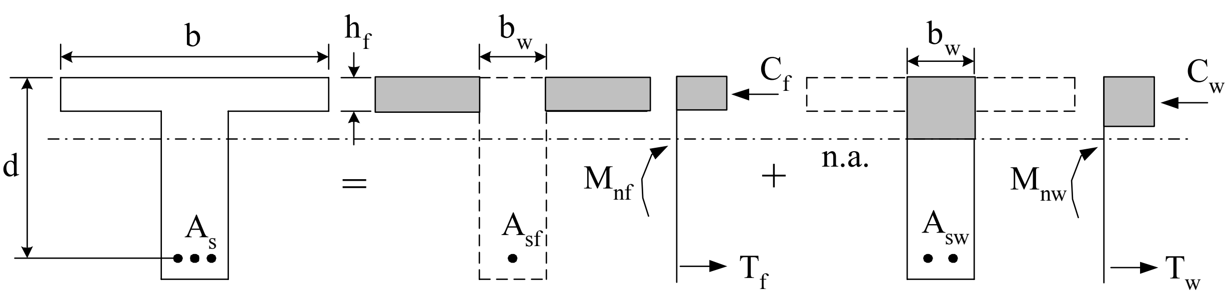


Fig. 1-4 T-section behavior

$$M_n = M_{nf} + M_{nw} \qquad (1\text{-}19)$$

$$M_{nf} = K_{nf}(b - b_w)d^2 \qquad (1\text{-}20)$$

$$M_{nw} = K_{nw} b_w d^2 \qquad (1\text{-}21)$$

and;

$$\rho_f = \frac{A_{sf}}{(b-b_w)d} \tag{1-22}$$

$$\rho_w = \frac{A_{sw}}{b_w d} \tag{1-23}$$

Moment components, M_{nf} and M_{nw} can be obtained from **Flexure 1 though Flexure 4** when the tables are entered with ρ_f and ρ_w values, respectively. For design, however, ρ_f needs to be found first and this can be done from the equilibrium of internal forces for the portion of total tension steel balancing the overhang concrete. This is illustrated below.

$$T_f = C_f \tag{1-24}$$

$$A_{sf} f_y = 0.85 f_c' h_f (b - b_w) \tag{1-25}$$

$$\rho_f = \frac{0.85 f_c'}{f_y} \frac{h_f}{d} \tag{1-26}$$

Eq. (1-26) was used to generate **Flexure 7 and Flexure 8**. **Flexure Example 6** through **Flexure Example 8** illustrate the use of **Flexure 7** and **Flexure 8**.

When T-beam flanges are in tension, part of the flexural tension reinforcement is required to be distributed over an effective area as illustrated in **Flexure 6** or a width equal to one-tenth the span, whichever is smaller (Sec. 10.6.6). This requirement is intended to control cracking that may result from widely spaced reinforcement. If one-tenth of the span is smaller than the effective width, *additional* reinforcement shall be provided in the outer portions of the flange to minimize wide cracks in these regions.

1.3 Minimum Flexural Reinforcement

Reinforced concrete sections that are larger than required for strength, for architectural and other functional reasons, may need to be protected by minimum amount of tension reinforcement against a brittle failure immediately after cracking. Reinforcement in a section becomes effective only after the cracking of concrete. However, if the area of reinforcement is too small to generate a sectional capacity that is less than the cracking moment, the section can not sustain its strength upon cracking. To safeguard against such brittle failures, ACI 318 requires a minimum area of tension reinforcement both in positive and negative moment regions (Sec. 10.5.1).

$$A_{s,\min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \geq 200 b_w d / f_y \tag{1-27}$$

The above requirement is indicated in **Flexure 1** through **Flexure 4** by a horizontal line above which the reinforcement ratio ρ is less than that for minimum reinforcement.

When the flange of a T-section is in tension, the minimum reinforcement required to have a sectional capacity that is above the cracking moment is approximately twice that required for rectangular sections. Therefore, Eq. (1-27) is used with b_w replaced by $2b_w$ or the width of the flange, whichever is smaller (Sec. 10.5.2). If the area of steel provided in every section of a member is high enough to provide at least one-third greater flexural capacity than required by analysis, then the minimum steel requirement need not apply (Sec. 10.5.3). This exception prevents the use of excessive reinforcement in very large members that have sufficient reinforcement.

For structural slabs and footings, minimum reinforcement is used for shrinkage and temperature-control (Sec. 10.5.4). The minimum area of such reinforcement is 0.0018 times the gross area of concrete for Grade 60 deformed bars (Sec. 7.12.2.1). Where higher grade reinforcement is used, with yield stress measured at 0.35% strain, the minimum reinforcement ratio is proportionately adjusted as $(0.0018 \times 60{,}000)/f_y$. The maximum spacing of shrinkage and temperature reinforcement is limited to three times the slab or footing thickness or 18 in, whichever is smaller (Sec. 10.5.4).

1.4 Placement of Reinforcement in Sections

Flexural reinforcement is placed in a section with due considerations given to the spacing of reinforcement, crack control and concrete cover. It is usually preferable to use sufficient number of small size bars, as opposed to fewer bars of larger size, while also respecting the spacing requirements.

1.4.1 Minimum Spacing of Longitudinal Reinforcement

Longitudinal reinforcement should be placed with sufficient spacing to allow proper placement of concrete. The minimum spacing requirement for beam reinforcement is shown in **Flexure 9**.

1.4.2 Concrete Protection for Reinforcement

Flexural reinforcement should be placed to maximize the lever arm between internal forces for increased moment capacity. This implies that the main longitudinal reinforcement should be placed as close to the concrete surface as possible. However, the reinforcement should be protected against corrosion and other aggressive environments by a sufficiently thick concrete cover (Sec. 7.7), as indicated in **Flexure 9**. The cover concrete should also satisfy the requirements for fire protection (Sec. 7.7.7).

1.4.3 Maximum Spacing of Flexural Reinforcement and Crack Control

Beams reinforced with few large size bars may experience cracking between the bars, even if the required area of tension reinforcement is provided and the sectional capacity is achieved. Crack widths in these members may exceed what is usually regarded as acceptable limits of cracking for various exposure conditions. ACI 318-05 specifies a maximum spacing limit "s" for reinforcement closest to the tension face. This limit is specified in Eq. (1-28) to ensure proper crack control.

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right) \tag{1-28}$$

where; c_c is the least distance from the surface of reinforcement to the tension face of concrete, and f_s is the service load stress in reinforcement. f_s can be computed from strain compatibility analysis under unfactored service loads. In lieu of this analysis, f_s may be taken as 2/3 f_y. Eq. (1-28) does not provide sufficient crack control for members subject to very aggressive exposure conditions or designed to be watertight. For such structures, special investigation is required (Sec. 10.6.5).

The maximum spacing of flexural reinforcement for one-way slabs and footings is limited to three times the slab or footing thickness or 18 in, whichever is smaller (Sec. 10.5.4).

1.4.4 Skin Reinforcement

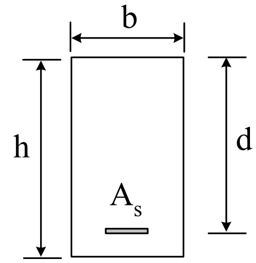
In deep flexural members, the crack control provided by the above expression may not be sufficient to control cracking near the mid-depth of the section, between the neutral axis and the tension concrete. For members with a depth h > 36 in, skin reinforcement with a maximum spacing of s, as defined in Eq. (1-28) and illustrated in **Flexure 10** is needed (Sec. 10.6.7). In this case, c_c is the least distance from the surface of the skin reinforcement to the side face. ACI 318 does not specify the area of steel required as skin reinforcement. However, research has indicated that bar sizes of No. 3 to No. 5 or welded wire reinforcement with a minimum area of 0.1 square inches per foot of depth provide sufficient crack control[2].

[2] Frosch, R.J., "Modeling and Control of Side Face Beam Cracking," ACI Structural Journal, V. 99, No.3, May-June 2002, pp. 376-385.

1.5 Flexure Examples

FLEXURE EXAMPLE 1 - Calculation of area of tension reinforcement for a rectangular tension controlled cross-section.

For a rectangular section subjected to a factored bending moment M_u, determine the required area of tension reinforcement for the dimensions given. Assume interior construction not exposed to weather.

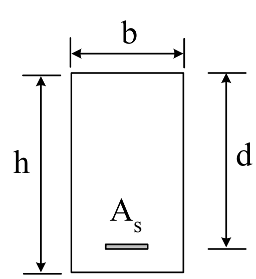


Given:
$M_u = 90$ kip-ft
$f_c' = 4{,}000$ psi
$f_y = 60{,}000$ psi
$b = 10$ in
$h = 20$ in

Procedure	Calculation	ACI 318-05 Section	Design Aid
Estimate "d" by allowing for clear cover, the radius of longitudinal reinforcement and diameter of stirrups.	Considering a minimum clear cover of 1.5 inches for interior exposure, allow 2.50 in to the centroid of main reinforcement $d = 20 - 2.50 = 17.50$ in	7.7.1	Flexure 9
Compute $\phi K_n = M_u \times 12{,}000 / (bd^2)$	$\phi K_n = 90\text{x}12{,}000/[10\text{x}(17.50)^2] = 353$ psi		
Select ρ from Flexure 1 Compute required area of steel; $A_s = \rho bd$ Determine the provided area of steel (For placement of reinforcement see Flexure Example 9)	For $\phi K_n = 353$ psi; $\rho = 0.70\%$ $A_s = \rho bd = 0.0070\text{x}10\text{x}17.5 = 1.22\ \text{in}^2$ Use 3 #6 $(A_s)_{prov.} = (3)(0.44) = 1.32\ \text{in}^2$ (Note: 3 # 6 can be placed within a 10 in. width). $(\rho)_{prov} = (1.32)/[(10)(17.5)] = 0.75\%$ <u>Note:</u> for $(\rho)_{prov} = 0.75\%$; $\varepsilon_t = 0.0163$ $\varepsilon_t = 0.0163 > 0.005$ "tension controlled" section and $\phi = 0.9$.	 7.6.1 3.3.2 10.3.4 9.3.2	Flexure 1 Flexure 9 Flexure 1

FLEXURE EXAMPLE 2 - Calculation of nominal flexural capacity of a rectangular beam subjected to positive bending.

For a rectangular section with specified tension reinforcement and geometry determine the nominal flexural capacity M_n.



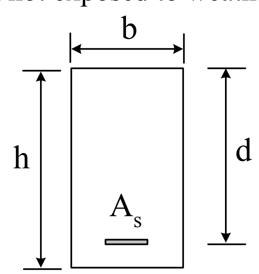
Given:
3 #6 Bars as bottom tension reinforcement
$f_c' = 4{,}000$ psi
$f_y = 60{,}000$ psi
$b = 10$ in
$d = 18$ in

Procedure	Calculation	ACI 318 2005 Section	Design Aid
Compute the area and percentage of steel provided	A_s = 3 x 0.44 = 1.32 in^2 $\rho = A_s/bd$ = 1.32/(10)(18) = 0.73%		
Select ϕK_n from Flexure 1 Compute $\phi M_n = \phi K_n\ bd^2$ /12,000 Select corresponding ϕ from Flexure 1 Compute $M_n = \phi M_n / \phi$	For ρ = 0.73%; ϕK_n = 370 psi ϕM_n = 370 x 10 x $(18)^2$ / 12,000 = 100 k-ft ϕ = 0.9 (ε_t =0.01675 > 0.005 "tension controlled") M_n = 100/0.9 = 111 k-ft	10.3.4 9.3.2	Flexure 1 Flexure 1

FLEXURE EXAMPLE 3 - Calculation of area of tension reinforcement for a rectangular cross section in the transition zone.

For a rectangular section subjected to a factored bending moment M_u, determine the required area of tension reinforcement for the dimensions given. Assume interior construction not exposed to weather.

<u>Given:</u>
M_u = 487 kip-ft
f_c' = 4,000 psi
f_y = 60,000 psi
b = 14 in
h = 26 in
1.0 in. maximum aggregate size



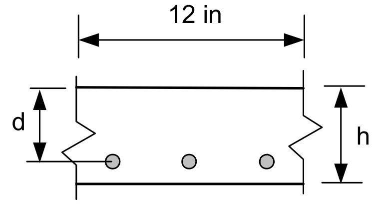
Procedure	Calculation	ACI 318 2005 Section	Design Aid
Estimate "d" by allowing for clear cover, the radius of longitudinal reinforcement and diameter of stirrups.	Considering a minimum clear cover of 1.5 in for interior exposure, allow 2.50 in. to the centroid of main reinforcement d = 26 - 2.50 = 23.50 in	7.7.1	Flexure 9
Compute $\phi K_n = M_u$ x 12,000 / (bd^2)	ϕK_n = 487x12,000/[14x$(23.50)^2$] = 756 psi		
Select ρ from Flexure 1 Compute $A_s = \rho bd$ Determine the area of steel provided	For ϕK_n = 756 psi; ρ = 1.63% $A_s = \rho bd$ = 0.0163x14 x 23.5 = 5.36 in^2 Try #8 bars; 5.36 /0.79 = 6.8 Need 7 #8 bars in a single layer. However 7 #8 bars can not be placed in a single layer within a 14 in. width without violating the spacing limits. Therefore, try placing them in double layers. Allow 3.5 in. from the extreme tension fiber to the centroid of double layers of reinforcement. Revise d = 26 – 3.5 = 22.5 in.	7.6.1 3.3.2	Flexure 1 Flexure 9

	ϕK_n = 487x12,000/[14x(22.50)^2] = 825 psi For ϕK_n = 825 psi; ρ = 1.98% $A_s = \rho bd$ = 0.0198x14 x 22.5 = 6.24 in^2 Try #8 bars; 6.24 /0.79 = 7.9 Select 8 #8 bars in two layers (4 # 8 in each layer). Note that 4 #8 bars can be placed within a 14 in. width. $(A_s)_{prov.}$= (8) (0.79) = 6.32 in^2 $(\rho)_{prov.}$ = 6.32 / [(14)(22.5)] = 0.020 For $(\rho)_{prov.}$ = 0.020 ϕK_n = 826 psi ε_t = 0.0042 <u>Note:</u> ε_t = 0.0042 < 0.005 "transition zone"; ϕ = 0.83 and $\phi K_n > M_u$	7.6.1 3.3.2 10.3.4 9.3.2	Flexure 1 Flexure 9 Flexure 1 Flexure 1

FLEXURE EXAMPLE 4 - Selection of slab thickness and area of flexural reinforcement.

For a slab subject to a factored bending moment M_u, determine the thickness h and required area of tension reinforcement. The slab has interior exposure.

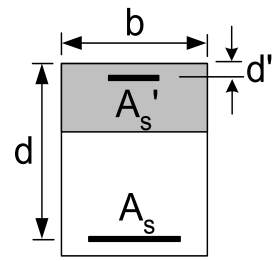
<u>**Given:**</u>
M_u = 11 kip-ft/ft
f_c' = 4,000 psi
f_y = 60,000 psi



Procedure	Calculation	ACI 318 2005 Section	Design Aid
Unless a certain slab thickness is desired, a trial thickness can be selected such that a section with good ductility, stiffness and bar placement characteristics is obtained. Try ρ = 50% of ρ at max. limit of tension controlled section.	ρ = 0.5(ρ at ε_t=0.005) = 0.5 x 0.018 = 0.0091 for ρ = 0.0091; ϕK_n = 453 psi $\phi K_n = M_u$ x 12,000 / (bd^2) $d^2 = M_u$ x 12,000/(ϕK_n b) d^2 = 11 x 12,000/(453 x 12) = 24.3 in^2 d = 5.0 in		Flexure 1
Select bar size and cover concrete. (For reinforcement placement see Flexure Example 10).	$A_s = \rho bd$ = 0.0091 (12) (5.0) = 0.55 in^2 #5 at 6 in. provides A_s = 0.62 in.^{2} O.K. Cover = 0.75 in	7.7.1	
Compute h with due considerations given to cover and bar radius. Note that the slab thickness must also satisfy deflection control. (For placement of reinforcement see Flexure Example 10)	h = d + d_b/2 + cover = 5.0 + 0.625/2 + 0.75 h = 6.1 in Use h = 6.5 in *<u>Note:</u>* The slab thickness should be checked to satisfy the requirements of Table 9.5(a) for deflection control.	9.5.2 and Table 9.5(a)	

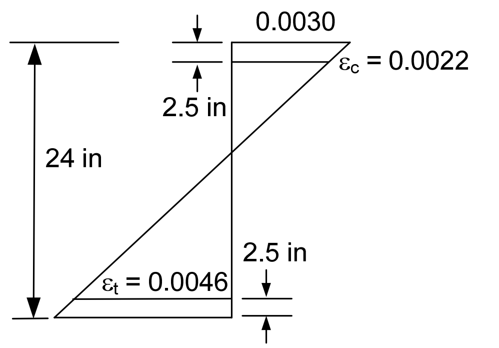
FLEXURE EXAMPLE 5 - Calculation of tension and compression reinforcement area for a rectangular beam section, subjected to positive bending.

For a rectangular section subjected to a factored positive moment M_u, determine the required area of tension and compression reinforcement for the dimensions given below.



Given:
M_u = 580 kip-ft
f_c' = 4,000 psi
f_y = 60,000 psi
b = 14 in
h = 24 in
d' = 2.5 in

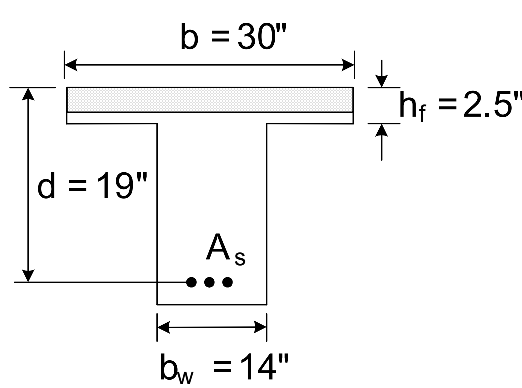
Procedure	Calculation	ACI 318 2005 Section	Design Aid
Estimate "d" by allowing for clear cover, the radius of longitudinal reinforcement and diameter of stirrups.	Considering a minimum clear cover of 1.5 in for interior exposure, allow 2.5 in. to the centroid of main reinforcement. d = 24 - 2.5 = 21.5 in	7.7.1	Flexure 9
Compute $\phi K_n = M_u$ x 12,000 / (bd^2)	ϕK_n = 580x12,000/[14x$(21.5)^2$] =1075 psi		
Select ρ from Flexure 1	ϕK_n = 1075 psi is outside the range of Flexure 1. This indicates that the amount of steel needed exceeds the maximum allowed if only tension steel were to be provided. Therefore, compression steel is needed.		Flexure 1
Compute (A_s-A_s') In this problem select a reinforcement ratio close to the maximum allowed to take advantage of the full capacity of compression concrete. Select ρ = 1.80 % (slightly below ρ_{max} = 2.06% so that when the actual bars are placed ρ_{max} is not exceeded). Compute moment to be resisted by	Select; ρ = 0.018 (ε_t = 0.005) A_s-A_s'= ρbd = 0.018 x 14 x 21.5 = 5.42 in^2 Try #8 bars; 5.42 / 0.79 = 6.9 Select 7 #8 bars for A_s-A_s' However, 7 # 8 can not be placed in a single layer. Try using double layers. Allow 3.5 in. from the extreme tension fiber to the centroid of double layers of # 8 bars. Revise d = 24 – 3.5 = 20.5 in A_s-A_s'= ρbd = 0.018 x 14 x 20.5 = 5.17 in^2 Try #8 bars; 5.17 / 0.79 = 6.5 Select 7 #8 bars for (A_s-A'$_s$) to be placed in double layers. A_s-A_s' = (7)(0.79) = 5.53 in.^{2} Corresponding ρ = 5.53/[(14)(20.5)] = 0.019 < ρ_{max} = 0.0206 O.K. For ρ = 0.019 ϕK_n = 823 psi; ε_t = 0.0046	7.6.1 3.3.2 10.3.4 9.3.2	Flexure 1 Flexure 9 Flexure 1

compression concrete and corresponding tension steel (A_s-A_s').	and ϕ = 0.87 $\phi M_n = \phi K_n$ b d^2 /12000 ϕM_n = 823x14 $(20.5)^2$ / 12,000 = 404 k-ft		
Compute moment to be resisted by the steel couple (with an equal tension and compression steel area of A_s')	ϕM_n' = M_u - ϕM_n ϕM_n' = 580 - 404 = 176 k-ft		
Compute A_s' Note: As' is determined from Flexure 5, which was developed based on the assumption that at ultimate the compression steel is at or near yield. The strain diagram shown indicates the yielding of compression steel ($f'_s = f'_y$). If the compression steel does not yield ($f'_s < f'_y$) then the area of compression steel used should be reduced by f'_s/f'_y. Determine the area of compression steel provided.	ϕK_n' = ϕM_n' x 12,000 / (bd^2) ϕK_n'= 176x12,000 /[14 x $(20.5)^2$] = 359 psi K_n' = 359/0.87 = 413 psi d'/d = 2.5/20.5 = 0.12; ρ' = 0.78% A_s' = ρ' bd = 0.0078 x 14 x 20.5 = 2.24 in^2  Use 3 #8 bars as compression reinforcement. $(A_s')_{prov.}$ = (3)(0.79) = 2.37 in^2		Flexure 5
Add equal area of steel to the bottom bars to facilitate steel force couple. Determine the total area of bottom reinforcement, A_s	Add 3 #8 bars to the bottom reinforcement. Total bottom reinforcement: 7 #8 + 3 # 8 = 10 # 8 to be provided in double layers (5 #8 in each layer). 5 #8 can be placed within a width of 14 in. A_s = 5.53 + 2.37 = 7.90 in^2 *Note:* For this section: ε_t = 0.00460 and ϕ = 0.87	7.6.1 3.3.2	Flexure 9

FLEXURE EXAMPLE 6 -   Calculation of tension reinforcement area for a T beam section subjected to positive bending, behaving as a rectangular section.

For a T section subjected to a factored bending moment M_u, determine the required area of tension reinforcement for the dimensions given.

Given:
M_u = 230 kip-ft
f_c' = 4,000 psi
f_y = 60,000 psi
b = 30 in
b_w = 14 in
d = 19 in
h_f = 2.5 in

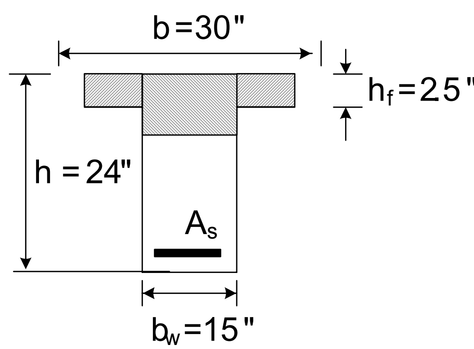


Procedure	Calculation	ACI 318 2005 Section	Design Aid
Assume tension controlled section (ϕ = 0.9). Determine if the section behaves as a T or a rectangular section. If $M_u > \phi[0.85f'_c\, b\, h_f\, (d-h_f/2)]$ T-section, otherwise rectangular section behavior.	$0.9\,[0.85f'_c b\, h_f\,(d - h_f/2)]$ = 0.9[0.85(4)(30)(2.5)(19-2.5/2)] = 4073 k-in = 340 k-ft > M_u = 230 k-ft Therefore, the neutral axis is within the flange and the section behaves as a rectangular section with width b = 30 in.		
Compute $\phi K_n = M_u$ x 12,000 / (bd^2)	ϕK_n = (230)(12,000)/[(30)$(19)^2$] = 255 psi		
Select ρ from Flexure 1 Compute $A_s = \rho bd$ Find provided area of steel. Read ε_t and ϕ from Flexure 1.	For ϕK_n = 255 psi; ρ = 0.50 % $A_s = \rho bd$ = 0.0050x30x19 = 2.85 in^2 Use 5 #7 with A_s = (5)(0.6) = 3.00 in^2 ρ = 3.00 / [(30)(19)] = 0.0053 For ρ = 0.0053; ε_t = 0.025 > 0.005 "tension controlled" section and ϕ = 0.9	 10.3.4 9.3.2	Flexure 1 Flexure 1

FLEXURE EXAMPLE 7 - Computation of the area of tension reinforcement for a T beam section, subjected to positive bending, behaving as a tension controlled T-section.

For a T section subjected to a factored bending moment M_u, determine the required area of tension reinforcement for the dimensions given. The beam has interior exposure.

<u>**Given:**</u>
M_u= 400 kip-ft
f_c' = 4,000 psi
f_y = 60,000 psi
b = 30 in
b_w =15 in
h = 24 in
h_f = 2.5 in
Max. aggregate size = 1.0 in.



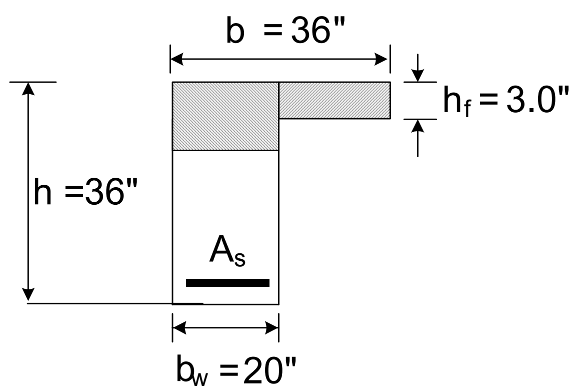
Procedure	Calculation	ACI 318 2005 Section	Design Aid
Estimate "d" by allowing for clear cover, the radius of longitudinal reinforcement and diameter of stirrups.	Considering a minimum clear cover of 1.5 in. for interior exposure, allow 2.5 in. to the centroid of longitudinal reinforcement. d = 24 - 2.5 = 21.5 in	7.7.1	Flexure 9
Assume tension controlled section and determine if the section behaves as a T section or a rectangular section. If $M_u > \phi[0.85f'_c\, b\, h_f\, (d-h_f/2)]$ T-section, otherwise rectangular section behavior.	$0.9\,[0.85f'_c b\, h_f\,(d - h_f/2)]$ = 0.9(0.85)(4)(30)(2.5)(21.5-2.5/2) = 4647 k-in = 387 k-ft < M_u = 400 k-ft Therefore, the neutral axis is below the flange and the section behaves as a T section.		

Compute the amount of steel that balances compression concrete in flange overhangs from Flexure 7.	$d/h_f = 21.5/2.5 = 8.6$ $\rho_f = 0.66\%$		Flexure 7
Find the amount of moment resisted by ρ_f from Flexure 1.	For $\rho_f = 0.66\%$ $\phi K_n = 334$ psi and $\phi = 0.9$ $\phi M_f = \phi K_n (b-b_w)d^2/12{,}000$ $= 334(30 - 15)(21.5)^2/12{,}000 = 193$ k-ft		Flexure 1
Determine the amount of steel needed to resist the remaining moment, that is to be resisted by the web; ρ_w	$\phi M_w = M_u - \phi M_f = 400 - 193 = 207$ k-ft $\phi K_n = \phi M_w \times 12{,}000 / [(b_w)(d)^2] = 358$ psi for $\phi K_n = 358$ psi; $\rho_w = 0.71\%$		Flexure 1
Compute the total area of tension reinforcement	$A_f = \rho_f (b-b_w)d = 0.0066(30\text{-}15)(21.5)$ $= 2.13$ in^2 $A_w = \rho_w b_w d = 0.0071(15)(21.5) = 2.29$ in^2 $A_s = A_f + A_w = 2.13 + 2.29 = 4.42$ in^2 Try using # 9 bars; 4.42/1.00 = 4.42 Use 5 #9 bars in a single layer $(A_s)_{prov.} = (5)(1.0) = 5.00$ in^2	7.6.1 3.3.2	Flexure 9
Note: The ϕ factor can be computed using the reinforcement ratio that balances web concrete.	Provided area of steel that balances web concrete: 5.00 – 2.13 = 2.87 in^2 $(\rho_w)_{prov.} = 2.87/(15)(21.5) = 0.0089$ This corresponds to $\varepsilon_t = 0.0132$ and $\phi = 0.9$ (tension controlled section)	10.3.4 9.3.2	Flexure 1

FLEXURE EXAMPLE 8 - Calculation of the area of tension reinforcement for an L beam section, subjected to positive bending, behaving as an L-section in the transition zone.

For an L section subjected to a factored bending moment M_u, determine the required area of tension reinforcement for the dimensions given. The beam has interior exposure.

Given:
$M_u = 1800$ kip-ft
$f_c' = 4{,}000$ psi
$f_y = 60{,}000$ psi
$b = 36$ in
$b_w = 20$ in
$h = 36$ in
$h_f = 3.0$ in
Max. aggregate size = 3/4 in.



Procedure	Calculation	ACI 318 2005 Section	Design Aid
Estimate "d" by allowing for clear cover, diameter of stirrups and the radius of longitudinal reinforcement.	Considering a minimum clear cover of 1.5 in. for interior exposure, allow 2.5 in. to the centroid of main reinforcement. d = 36 - 2.5 = 33.5 in	7.7.1	Flexure 9

Assume tension controlled section and determine if the section behaves as a T-section (in this case as an L-section) or a rectangular section. If $M_u > \phi\ [0.85f'_c\ b\ h_f\ (d\text{-}h_f/2)]$ T-section otherwise rectangular section behavior.	$0.9\ (0.85)f'_c b\ h_f\ (d - h_f/2)$ $= 0.9(0.85)(4)(36)(3.0)(33.5\text{-}3.0/2)$ = 10,575 k-in = 881 k-ft < M_u = 1800 k-ft Therefore, the neutral axis is below the flange and the section behaves as a T section.		
Compute the amount of steel that balances compression concrete in the flange overhang, from Flexure 7	$d/h_f = 33.5/3.0 = 11.2$ ρ_f = 0.51 %		Flexure 7
Find the amount of moment resisted by ρ_f from Flexure 1.	For ρ_f = 0.51 % ϕK_n = 261 psi (ϕ = 0.90) $\phi M_f = \phi K_n\ (b\text{-}b_w)d^2/12{,}000$ $= 261\ (36 – 20)(33.5)^2/12{,}000 = 391$ k-ft		Flexure 1
Determine the amount of steel required to resist the remaining moment. This additional moment is to be resisted by the web; ρ_w	$\phi M_w = M_u - \phi M_f = 1800 – 391 = 1409$ k-ft $\phi K_n = \phi M_w \times 12{,}000 / [(b_w)(d)^2]$ $\phi K_n = 1409\text{x}12{,}000/[(20)(33.5)^2] = 753$ psi for ϕK_n = 753 psi; ρ_w = 1.63 % Note: ϕ = 0.90 (tension controlled).		Flexure 1
Compute the total area of tension reinforcement.	$A_f = \rho_f\ (b\text{-}b_w)d = 0.0051(36\text{-}20)(33.5)$ $= 2.73\ in^2$ $A_w = \rho_w\ b_w d = 0.0163(20)(33.5) = 10.92\ in^2$ $A_s = A_f + A_w = 2.73 + 10.92 = 13.65\ in^2$ Select #9 bars; 14 - #9 bars are needed. 14 - #9 bars can not be placed in a single layer. Therefore, use double layers of reinforcement and revise the design.	7.6.1 3.3.2	Flexure 9
Recalculate the effective depth “d” and revise design. Assume cover of 3.5 in. to the centroid of double layers of reinforcement.	d = 36 – 3.5 = 32.5 in. Note: Reduced “d” will result in increased area of steel and the beam will continue behaving as a T-Beam (no need to check again).		
Compute the amount of steel that balances compression concrete in the flange overhang, from Flexure 7	$d/h_f = 32.5/3.0 = 10.8$ ρ_f = 0.53 %		Flexure 7
Find the amount of moment resisted by ρ_f from Flexure 1.	For ρ_f = 0.53 % ϕK_n = 271 psi (ϕ = 0.90) $\phi M_f = \phi K_n\ (b\text{-}b_w)d^2/12{,}000$ $= 271\ (36 – 20)(32.5)^2/12{,}000 = 382$ k-ft		Flexure 1
Determine the amount of steel required to resist the remaining moment. This additional moment is to be resisted by the web; ρ_w	$\phi M_w = M_u - \phi M_f = 1800 – 382 = 1418$ k-ft $\phi K_n = \phi M_w \times 12{,}000 / [(b_w)(d)^2]$ $\phi K_n = 1418\text{x}12{,}000/[(20)(32.5)^2] = 806$psi for ϕK_n = 806 psi; ρ_w = 1.77 % Note: ϕ = 0.90.		Flexure 1
Compute the total area of tension reinforcement required.	$A_f = \rho_f\ (b\text{-}b_w)d = 0.0053(36\text{-}20)(32.5)$ $= 2.76\ in^2$ $A_w = \rho_w\ b_w d = 0.0177(20)(32.5) = 11.51\ in^2$ $A_s = A_f + A_w = 2.76 + 11.51 = 14.27\ in^2$ Use 16 #9 bars in two layers (8 #9 in each layer, which can be placed within 20 in. width.	7.6.1 3.3.2	Flexure 9

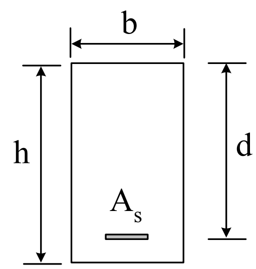
Ensure $\phi M_n \geq M_u$ based on provided reinforcement.	$A_w = A_s - A_f = 16.0 - 2.76 = 13.24$ in^2 Provided ρ_w that balances web concrete; $\rho_w = 13.24 / [(20)(32.5)] = 0.0204 = 2.04\%$ For $\rho_w = 2.04$ %; $\phi K_n = 827$ and $\phi = 0.82$ $\phi M_w = \phi\ K_n\ b_w d^2 / 12{,}000$ $= (827)(20)(32.5)^2 / 12{,}000 = 1456$ k-ft For the contribution of flange overhang $(0.90)K_n = 271$ psi (found earlier) $(0.82)K_n = 271\ (0.82/0.90) = 247$ psi $\phi M_f = \phi K_n\ (b-b_w)d^2/12{,}000$ $\phi M_f = (247)(36-20)(32.5)^2 / 12{,}000 = 348$k-ft $\phi M_n = \phi M_w + \phi M_f = 1456 + 348 = 1804$ k-ft $\phi M_n = 1804$ k-ft $> M_u = 1800$ k-ft O.K.		Flexure 1

FLEXURE EXAMPLE 9 - Placement of reinforcement in the rectangular beam section designed in Flexure Example 1.

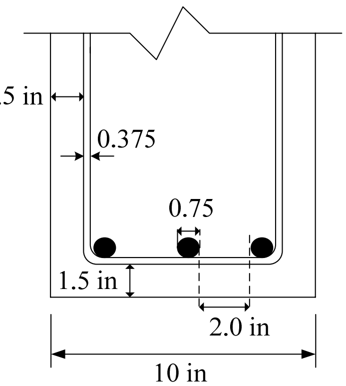
Select and place flexural beam reinforcement in the section provided below, with due considerations given to spacing and cover requirements.

<ins>**Given:**</ins>

$A_s = 1.22$ in^2
$b = 10$ in
$h = 20$ in
$f_y = 60{,}000$ psi
#3 stirrups
Max. aggregate size: 3/4 in
Interior exposure

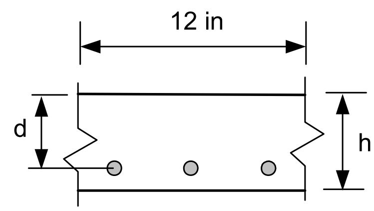


Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine bar size and number of bars.	Select # 6 bars; No. of bars = 1.22/0.44 = 2.8 Use 3 # 6 bars		Appendix A
Determine bar spacing.	Considering minimum clear cover of 1.5 inches on each side for interior exposure and allowing 2 stirrup bar diameters; $s = [10 - 2(1.5) - 2(0.375) - 3(0.75)] / 2 = 2.0$ in	7.7.1	Flexure 9 Appendix A
Check against minimum spacing	$(s)_{min} = \{d_b;\ 1\frac{1}{3}\ a_{max};\ 1\ \text{in}\}$ $(s)_{min} = \{0.75\ \text{in};\ 1\frac{1}{3}(3/4\ \text{in});\ 1\ \text{in}\} = 0.75$ in s = 2.0 in > 0.75 in O.K.	7.6	Flexure 9
Check against maximum spacing as governed by crack control	$(s)_{maz} = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right)$ $f_s = 2/3 f_y = 2/3\ (60{,}000) = 40{,}000$ psi	10.6.4	Eq.(1-28)

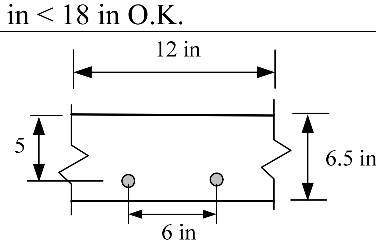
	$c_c = (1.5 + 0.375) = 1.875$ in $(s)_{max} = 15\ (1.0) – 2.5\ (1.875) = 10.3$ in s = 2.0 in < 12 in O.K.		
Final bar placement	Provide 3 # 6 as indicated below. 		Flexure 9

FLEXURE EXAMPLE 10 - Placement of reinforcement in the slab section designed in Example 4.

Select and place reinforcement in the 6 in slab shown below.

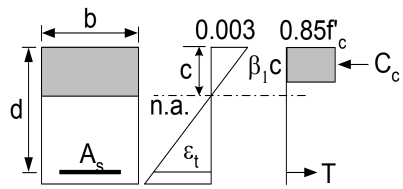


Given:
$A_s = 0.62$ in^2/ft
f_y = 60,000 psi
d = 5 in
h = 6.5 in

Procedure	Calculation	ACI 318 2005 Section	Design Aid
Determine bar size and number of bars for a one-foot slab width.	Select # 5 bars; No. of bars = 0.62/0.31= 2 Use 2 # 5 bars per foot of slab width.		Appendix A
Check for minimum area of reinforcement needed for temperature and shrinkage control. Note that the same minimum reinforcement must also be provided in the transverse direction.	For Grade 60 steel $A_{s,min} = 0.0018\ A_g$ $A_{s,min} = 0.0018\ (6.0)(12.0) = 0.13$ in^2 $A_s = 2 \times 0.31 = 0.62 > 0.13$ O.K.	7.12.2.1	
Check for maximum spacing of reinforcement.	2 # 5 bars per foot results in s = 6 in $(s)_{max}$ = 3h or 18 in, whichever is smaller $(s)_{max} = 3(6) = 18$ in s = 6 in < 18 in O.K.	10.5.4	
Final reinforcement placement	 Note clear cover = (6.5 – 5) – 0.625/2 = 1.2 in > ¾ in O.K.	7.7.1	Flexure 9

1.6 Flexure Design Aids

Flexure 1 - Flexural coefficients for rectangular beams with tension reinforcement, f_y = 60,000 psi



$\phi M_n \geq M_u$ $\phi M_n = \phi K_n \, b \, d^2 / 12000$ $\rho = A_s / b \, d$

where, M_n is in kip-ft; K_n is in psi; b and d are in inches

f_y = 60000 psi

f_c' (psi) :			**3000**		**4000**		**5000**		**6000**	
β_1 :			0.85		0.85		0.80		0.75	
ρ_{min} **:**			0.0033		0.0033		0.0035		0.0039	
ε_t	ϕ	$\phi_{App\ C}$	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**
0.20000	0.90	0.90	0.05	29	0.07	38	0.08	45	0.09	51
0.15000	0.90	0.90	0.07	38	0.09	51	0.11	60	0.13	67
0.10000	0.90	0.90	0.11	56	0.14	75	0.17	88	0.19	99
0.07500	0.90	0.90	0.14	74	0.19	98	0.22	116	0.25	130
0.05000	0.90	0.90	0.20	108	0.27	144	0.32	169	0.36	191
0.04000	0.90	0.90	0.25	132	0.34	176	0.40	208	0.44	234
0.03500	0.90	0.90	0.29	149	0.38	198	0.45	234	0.50	264
0.03000	0.90	0.90	0.33	170	0.44	227	0.52	268	0.58	302
0.02500	0.90	0.90	0.39	199	0.52	266	0.61	314	0.68	354
0.02000	0.90	0.90	0.47	240	0.63	320	0.74	378	0.83	427
0.01900	0.90	0.90	0.49	251	0.66	334	0.77	395	0.87	445
0.01800	0.90	0.90	0.52	262	0.69	349	0.81	412	0.91	465
0.01700	0.90	0.90	0.54	274	0.72	365	0.85	431	0.96	487
0.01600	0.90	0.90	0.57	287	0.76	383	0.89	453	1.01	511
0.01500	0.90	0.90	0.60	302	0.80	403	0.94	476	1.06	538
0.01400	0.90	0.90	0.64	318	0.85	425	1.00	502	1.13	567
0.01300	0.90	0.90	0.68	337	0.90	449	1.06	531	1.20	600
0.01250	0.90	0.90	0.70	347	0.93	462	1.10	546	1.23	618
0.01200	0.90	0.90	0.72	357	0.96	476	1.13	563	1.28	637
0.01150	0.90	0.90	0.75	368	1.00	491	1.17	581	1.32	657
0.01100	0.90	0.90	0.77	380	1.03	507	1.21	600	1.37	678
0.01050	0.90	0.90	0.80	393	1.07	523	1.26	620	1.42	701
0.01000	0.90	0.90	0.83	406	1.11	541	1.31	641	1.47	726
0.00950	0.90	0.90	0.87	420	1.16	561	1.36	664	1.53	752
0.00900	0.90	0.90	0.90	436	1.20	581	1.42	689	1.59	780

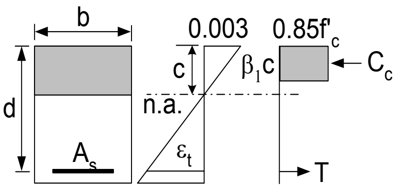
Flexure 1 - (Cont'd)

f_y = 60000 psi

f_c' (psi) :			3000		4000		5000		6000	
β_1 :			0.85		0.85		0.80		0.75	
ρ_{min} :			0.0033		0.0033		0.0035		0.0039	
ε_t	ϕ	$\phi_{App\ C}$	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)
0.00870	0.90	0.90	0.93	446	1.24	594	1.45	704	1.63	798
0.00840	0.90	0.90	0.95	456	1.27	608	1.49	720	1.68	817
0.00810	0.90	0.90	0.98	467	1.30	622	1.53	738	1.72	836
0.00770	0.90	0.90	1.01	482	1.35	642	1.59	762	1.79	864
0.00740	0.90	0.90	1.04	494	1.39	658	1.63	781	1.84	886
0.00710	0.90	0.90	1.07	506	1.43	675	1.68	801	1.89	909
0.00680	0.90	0.90	1.11	519	1.47	693	1.73	822	1.95	933
0.00650	0.90	0.90	1.14	533	1.52	711	1.79	844	2.01	958
0.00620	0.90	0.90	1.18	548	1.57	731	1.85	868	2.08	985
0.00590	0.90	0.90	1.22	563	1.62	751	1.91	892	2.15	1014
0.00560	0.90	0.90	1.26	580	1.68	773	1.98	918	2.22	1044
0.00530	0.90	0.90	1.31	597	1.74	796	2.05	946	2.30	1076
0.00500	0.90	0.90	1.35	615	1.81	820	2.13	975	2.39	1109
0.00480	0.88	0.89	1.39	616	1.85	821	2.18	977	2.45	1112
0.00460	0.87	0.87	1.43	617	1.90	823	2.24	979	2.52	1115
0.00440	0.85	0.86	1.46	618	1.95	824	2.30	982	2.58	1118
0.00430	0.84	0.85	1.48	619	1.98	825	2.33	983	2.62	1119
0.00420	0.83	0.85	1.51	619	2.01	826	2.36	984	2.66	1121
0.00410	0.82	0.84	1.53	620	2.04	827	2.39	985	2.69	1122
0.00400	0.82	0.83	1.55	620	2.06	827	2.43	986	2.73	1124

Notes: - The vaues of ρ above the rule are less than ρ_{min}.
- ϕK_n values are based on ϕ factors specified in Chapter 9 of ACI 318-05. When App. C values of ϕ are used, ϕK_n values in the transition zone may be up to 2.4% higher (more conservative)

Flexure 2 - Flexural coefficients for rectangular beams with tension reinforcement, f_y = 60,000 psi



$\phi M_n \geq M_u$ $\qquad \phi M_n = \phi K_n \, b \, d^2 / 12000$ $\qquad \rho = A_s / b\,d$

Where, M_n is in kip-ft; b and d are in inches

f_y = 60000 psi

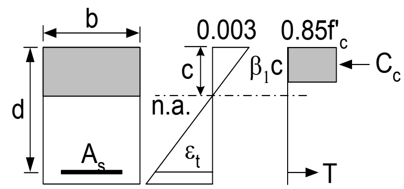
f_c' (psi) :			**7000**		**8000**		**9000**		**10000**	
β_1 :			0.7		0.65		0.65		0.65	
ρ_{min} **:**			0.0042		0.0045		0.0047		0.0050	
ε_t	ϕ	ϕ_{Appc}	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**
0.20000	0.90	0.90	0.10	55	0.11	59	0.12	66	0.14	73
0.15000	0.90	0.90	0.14	73	0.14	78	0.16	87	0.18	97
0.10000	0.90	0.90	0.20	108	0.21	115	0.24	129	0.27	143
0.07500	0.90	0.90	0.27	142	0.28	151	0.32	170	0.35	189
0.05000	0.90	0.90	0.39	208	0.42	221	0.47	249	0.52	276
0.04000	0.90	0.90	0.48	255	0.51	271	0.58	305	0.64	339
0.03500	0.90	0.90	0.55	288	0.58	306	0.65	344	0.73	382
0.03000	0.90	0.90	0.63	330	0.67	351	0.75	395	0.84	439
0.02500	0.90	0.90	0.74	387	0.79	411	0.89	463	0.99	514
0.02000	0.90	0.90	0.91	467	0.96	497	1.08	559	1.20	621
0.01900	0.90	0.90	0.95	487	1.00	518	1.13	583	1.26	648
0.01800	0.90	0.90	0.99	509	1.05	542	1.18	610	1.32	677
0.01700	0.90	0.90	1.04	533	1.11	568	1.24	639	1.38	710
0.01600	0.90	0.90	1.10	559	1.16	596	1.31	670	1.45	745
0.01500	0.90	0.90	1.16	588	1.23	627	1.38	705	1.53	784
0.01400	0.90	0.90	1.23	621	1.30	662	1.46	744	1.63	827
0.01300	0.90	0.90	1.30	657	1.38	700	1.55	788	1.73	876
0.01250	0.90	0.90	1.34	676	1.43	722	1.60	812	1.78	902
0.01200	0.90	0.90	1.39	697	1.47	744	1.66	837	1.84	930
0.01150	0.90	0.90	1.44	719	1.52	768	1.71	864	1.91	960
0.01100	0.90	0.90	1.49	743	1.58	793	1.78	892	1.97	991
0.01050	0.90	0.90	1.54	768	1.64	820	1.84	923	2.05	1025
0.01000	0.90	0.90	1.60	795	1.70	849	1.91	955	2.13	1061
0.00950	0.90	0.90	1.67	824	1.77	880	1.99	990	2.21	1100
0.00900	0.90	0.90	1.74	855	1.84	914	2.07	1028	2.30	1142

Flexure 2 - Cont'd

f_y = 60000 psi										
f_c' (psi) :			7000		8000		9000		10000	
β_1 :			0.7		0.65		0.65		0.65	
ρ_{min} :			0.0042		0.0045		0.0047		0.0050	
ε_t	ϕ	$\phi_{App\ C}$	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)
0.00870	0.90	0.90	1.78	875	1.89	935	2.13	1052	2.36	1169
0.00840	0.90	0.90	1.83	896	1.94	957	2.18	1077	2.42	1197
0.00810	0.90	0.90	1.88	917	1.99	981	2.24	1103	2.49	1226
0.00770	0.90	0.90	1.95	948	2.07	1014	2.32	1140	2.58	1267
0.00740	0.90	0.90	2.00	972	2.13	1040	2.39	1170	2.66	1300
0.00710	0.90	0.90	2.06	998	2.19	1068	2.46	1201	2.74	1334
0.00680	0.90	0.90	2.13	1025	2.26	1097	2.54	1234	2.82	1371
0.00650	0.90	0.90	2.19	1053	2.33	1127	2.62	1268	2.91	1409
0.00620	0.90	0.90	2.26	1083	2.40	1160	2.70	1305	3.00	1450
0.00590	0.90	0.90	2.34	1114	2.48	1194	2.79	1343	3.10	1493
0.00560	0.90	0.90	2.42	1148	2.57	1230	2.89	1384	3.21	1538
0.00530	0.90	0.90	2.51	1183	2.66	1269	3.00	1428	3.33	1586
0.00500	0.90	0.90	2.60	1221	2.76	1310	3.11	1474	3.45	1637
0.00480	0.88	0.89	2.67	1225	2.83	1314	3.19	1478	3.54	1642
0.00460	0.87	0.87	2.74	1228	2.91	1318	3.27	1483	3.63	1648
0.00440	0.85	0.86	2.81	1232	2.99	1322	3.36	1488	3.73	1653
0.00430	0.84	0.85	2.85	1233	3.03	1325	3.41	1490	3.78	1656
0.00420	0.83	0.85	2.89	1235	3.07	1327	3.45	1493	3.84	1659
0.00410	0.82	0.84	2.93	1237	3.11	1329	3.50	1495	3.89	1661
0.00400	0.82	0.83	2.98	1239	3.16	1332	3.55	1498	3.95	1664

Notes: - The vaues of ρ above the rule are less than ρ_{min}.
- ϕK_n values are based on ϕ factors specified in Chapter 9 of ACI 318-05. When App. C values of ϕ are used, ϕK_n values in the transition zone may be up to 2.4% higher (more conservative)

Flexure 3 - Flexural coefficients for rectangular beams with tension reinforcement, f_y = 75,000 psi



$\phi M_n \geq M_u$ $\phi M_n = \phi K_n\, b\, d^2 / 12000$ $\rho = A_s / b\, d$

where, M_n is in kip-ft; b and d are in inches

f_y = 75000 psi

f_c' (psi) :			**3000**		**4000**		**5000**		**6000**	
β_1 :			0.85		0.85		0.80		0.75	
ρ_{min} **:**			0.0027		0.0027		0.0028		0.0031	
ε_t	ϕ	$\phi_{App\ C}$	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**
0.15000	0.90	0.90	0.06	38	0.08	51	0.09	60	0.10	67
0.10000	0.90	0.90	0.08	56	0.11	75	0.13	88	0.15	99
0.07500	0.90	0.90	0.11	74	0.15	98	0.17	116	0.20	130
0.05000	0.90	0.90	0.16	108	0.22	144	0.26	169	0.29	191
0.04000	0.90	0.90	0.20	132	0.27	176	0.32	208	0.36	234
0.03500	0.90	0.90	0.23	149	0.30	198	0.36	234	0.40	264
0.03000	0.90	0.90	0.26	170	0.35	227	0.41	268	0.46	302
0.02500	0.90	0.90	0.31	199	0.41	266	0.49	314	0.55	354
0.02000	0.90	0.90	0.38	240	0.50	320	0.59	378	0.67	427
0.01900	0.90	0.90	0.39	251	0.53	334	0.62	395	0.70	445
0.01800	0.90	0.90	0.41	262	0.55	349	0.65	412	0.73	465
0.01700	0.90	0.90	0.43	274	0.58	365	0.68	431	0.77	487
0.01600	0.90	0.90	0.46	287	0.61	383	0.72	453	0.81	511
0.01500	0.90	0.90	0.48	302	0.64	403	0.76	476	0.85	538
0.01400	0.90	0.90	0.51	318	0.68	425	0.80	502	0.90	567
0.01300	0.90	0.90	0.54	337	0.72	449	0.85	531	0.96	600
0.01250	0.90	0.90	0.56	347	0.75	462	0.88	546	0.99	618
0.01200	0.90	0.90	0.58	357	0.77	476	0.91	563	1.02	637
0.01150	0.90	0.90	0.60	368	0.80	491	0.94	581	1.06	657
0.01100	0.90	0.90	0.62	380	0.83	507	0.97	600	1.09	678
0.01050	0.90	0.90	0.64	393	0.86	523	1.01	620	1.13	701
0.01000	0.90	0.90	0.67	406	0.89	541	1.05	641	1.18	726
0.00950	0.90	0.90	0.69	420	0.92	561	1.09	664	1.22	752
0.00900	0.90	0.90	0.72	436	0.96	581	1.13	689	1.28	780

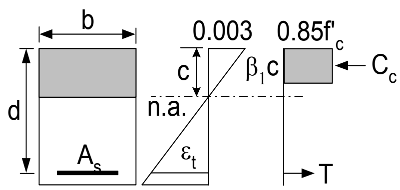
Flexure 3 - Cont'd

f_y = 75000 psi

f_c' (psi) :			3000		4000		5000		6000	
β_1 :			0.85		0.85		0.80		0.75	
ρ_{min} :			0.0027		0.0027		0.0028		0.0031	
ε_t	ϕ	$\phi_{App\ C}$	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)
0.00870	0.90	0.90	0.74	446	0.99	594	1.16	704	1.31	798
0.00840	0.90	0.90	0.76	456	1.01	608	1.19	720	1.34	817
0.00810	0.90	0.90	0.78	467	1.04	622	1.23	738	1.38	836
0.00770	0.90	0.90	0.81	482	1.08	642	1.27	762	1.43	864
0.00740	0.90	0.90	0.83	494	1.11	658	1.31	781	1.47	886
0.00710	0.90	0.90	0.86	506	1.14	675	1.35	801	1.51	909
0.00680	0.90	0.90	0.88	519	1.18	693	1.39	822	1.56	933
0.00650	0.90	0.90	0.91	533	1.22	711	1.43	844	1.61	958
0.00620	0.90	0.90	0.94	548	1.26	731	1.48	868	1.66	985
0.00590	0.90	0.90	0.97	563	1.30	751	1.53	892	1.72	1014
0.00560	0.90	0.90	1.01	580	1.34	773	1.58	918	1.78	1044
0.00530	0.90	0.90	1.04	597	1.39	796	1.64	946	1.84	1076
0.00500	0.90	0.90	1.08	615	1.45	820	1.70	975	1.91	1109
0.00480	0.88	0.89	1.11	616	1.48	821	1.74	977	1.96	1112
0.00460	0.87	0.87	1.14	617	1.52	823	1.79	979	2.01	1115
0.00440	0.85	0.86	1.17	618	1.56	824	1.84	982	2.07	1118
0.00430	0.84	0.85	1.19	619	1.58	825	1.86	983	2.10	1119
0.00420	0.83	0.85	1.20	619	1.61	826	1.89	984	2.13	1121
0.00410	0.82	0.84	1.22	620	1.63	827	1.92	985	2.15	1122
0.00400	0.82	0.83	1.24	620	1.65	827	1.94	986	2.19	1124

Notes: - The vaues of ρ above the rule are less than ρ_{min}.
- ϕK_n values are based on ϕ factors specified in Chapter 9 of ACI 318-05. When App. C values of ϕ are used, ϕK_n values in the transition zone may be up to 2.4% higher (more conservative)

Flexure 4 - Flexural coefficients for rectangular beams with tension reinforcement, f_y = 75,000 psi



$\phi M_n \geq M_u$ $\phi M_n = \phi K_n\, b\, d^2 / 12000$ $\rho = A_s / b\, d$

where, M_n is in kip-ft; b and d are in inches

f_y = 75000 psi

f_c' (psi) :			**7000**		**8000**		**9000**		**10000**	
β_1 :			0.7		0.65		0.65		0.65	
ρ_{min} :			0.0033		0.0036		0.0038		0.0040	
ε_t	ϕ	**ϕ_{App} c**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**	ρ(%)	**ϕK_n (psi)**
0.20000	0.90	0.90	0.08	55	0.09	59	0.10	66	0.11	73
0.15000	0.90	0.90	0.11	73	0.12	78	0.13	87	0.14	97
0.10000	0.90	0.90	0.16	108	0.17	115	0.19	129	0.21	143
0.07500	0.90	0.90	0.21	142	0.23	151	0.26	170	0.28	189
0.05000	0.90	0.90	0.31	208	0.33	221	0.38	249	0.42	276
0.04000	0.90	0.90	0.39	255	0.41	271	0.46	305	0.51	339
0.03500	0.90	0.90	0.44	288	0.47	306	0.52	344	0.58	382
0.03000	0.90	0.90	0.50	330	0.54	351	0.60	395	0.67	439
0.02500	0.90	0.90	0.60	387	0.63	411	0.71	463	0.79	514
0.02000	0.90	0.90	0.72	467	0.77	497	0.86	559	0.96	621
0.01900	0.90	0.90	0.76	487	0.80	518	0.90	583	1.00	648
0.01800	0.90	0.90	0.79	509	0.84	542	0.95	610	1.05	677
0.01700	0.90	0.90	0.83	533	0.88	568	0.99	639	1.11	710
0.01600	0.90	0.90	0.88	559	0.93	596	1.05	670	1.16	745
0.01500	0.90	0.90	0.93	588	0.98	627	1.11	705	1.23	784
0.01400	0.90	0.90	0.98	621	1.04	662	1.17	744	1.30	827
0.01300	0.90	0.90	1.04	657	1.11	700	1.24	788	1.38	876
0.01250	0.90	0.90	1.07	676	1.14	722	1.28	812	1.43	902
0.01200	0.90	0.90	1.11	697	1.18	744	1.33	837	1.47	930
0.01150	0.90	0.90	1.15	719	1.22	768	1.37	864	1.52	960
0.01100	0.90	0.90	1.19	743	1.26	793	1.42	892	1.58	991
0.01050	0.90	0.90	1.23	768	1.31	820	1.47	923	1.64	1025
0.01000	0.90	0.90	1.28	795	1.36	849	1.53	955	1.70	1061
0.00950	0.90	0.90	1.33	824	1.41	880	1.59	990	1.77	1100
0.00900	0.90	0.90	1.39	855	1.47	914	1.66	1028	1.84	1142

Flexure 4 – Cont'd

f_y = 75000 psi

f_c' (psi) :			7000		8000		9000		10000	
β_1 :			0.7		0.65		0.65		0.65	
ρ_{min} :			0.0033		0.0036		0.0038		0.0040	
ε_t	ϕ	ϕ_{AppC}	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)	ρ(%)	ϕK_n (psi)
0.00870	0.90	0.90	1.42	875	1.51	935	1.70	1052	1.89	1169
0.00840	0.90	0.90	1.46	896	1.55	957	1.74	1077	1.94	1197
0.00810	0.90	0.90	1.50	917	1.59	981	1.79	1103	1.99	1226
0.00770	0.90	0.90	1.56	948	1.65	1014	1.86	1140	2.07	1267
0.00740	0.90	0.90	1.60	972	1.70	1040	1.91	1170	2.13	1300
0.00710	0.90	0.90	1.65	998	1.75	1068	1.97	1201	2.19	1334
0.00680	0.90	0.90	1.70	1025	1.80	1097	2.03	1234	2.26	1371
0.00650	0.90	0.90	1.75	1053	1.86	1127	2.09	1268	2.33	1409
0.00620	0.90	0.90	1.81	1083	1.92	1160	2.16	1305	2.40	1450
0.00590	0.90	0.90	1.87	1114	1.99	1194	2.23	1343	2.48	1493
0.00560	0.90	0.90	1.94	1148	2.06	1230	2.31	1384	2.57	1538
0.00530	0.90	0.90	2.01	1183	2.13	1269	2.40	1428	2.66	1586
0.00500	0.90	0.90	2.08	1221	2.21	1310	2.49	1474	2.76	1637
0.00480	0.88	0.89	2.14	1225	2.27	1314	2.55	1478	2.83	1642
0.00460	0.87	0.87	2.19	1228	2.33	1318	2.62	1483	2.91	1648
0.00440	0.85	0.86	2.25	1232	2.39	1322	2.69	1488	2.99	1653
0.00430	0.84	0.85	2.28	1233	2.42	1325	2.72	1490	3.03	1656
0.00420	0.83	0.85	2.31	1235	2.46	1327	2.76	1493	3.07	1659
0.00410	0.82	0.84	2.35	1237	2.49	1329	2.80	1495	3.11	1661
0.00400	0.82	0.83	2.38	1239	2.53	1332	2.84	1498	3.16	1664

Notes: - The vaues of ρ above the rule are less than ρ_{min}.
- ϕK_n values are based on ϕ factors specified in Chapter 9 of ACI 318-05. When App. C values of ϕ are used, ϕK_n values in the transition zone may be up to 2.4% higher (more conservative)

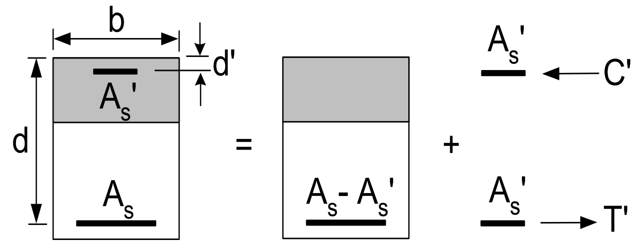
Flexure 5 - Reinforcement ratio (ρ') for compression reinforcement

$\phi M_n + \phi M_n' \geq M_u$

$\phi M_n = \phi K_n \, b \, d^2 / 12000$

$\rho = (A_s - A_s') / b \, d$

(From Flexure 1 through 4)



$\phi M_n' = \phi K_n' \, b \, d^2 / 12000$

$\rho' = A_s' / b \, d$

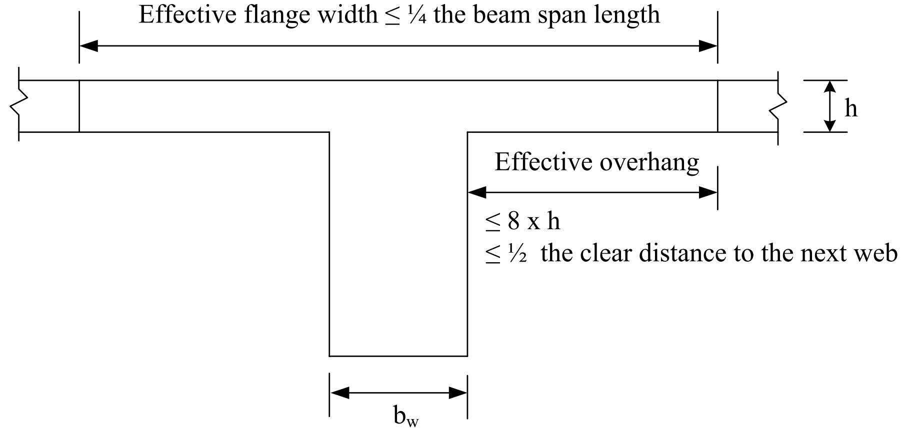
where, M_n' is in kip-ft; K_n' is in psi; b and d are in inches

f_y	60000 (psi)						75000 (psi)					
d/d'	0.02	0.06	0.1	0.14	0.18	0.22	0.02	0.06	0.1	0.14	0.18	0.22
K_n' (psi)	ρ' (%)						ρ' (%)					
20	0.03	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03
40	0.07	0.07	0.07	0.08	0.08	0.09	0.05	0.06	0.06	0.06	0.07	0.07
60	0.10	0.11	0.11	0.12	0.12	0.13	0.08	0.09	0.09	0.09	0.10	0.10
80	0.14	0.14	0.15	0.16	0.16	0.17	0.11	0.11	0.12	0.12	0.13	0.14
100	0.17	0.18	0.19	0.19	0.20	0.21	0.14	0.14	0.15	0.16	0.16	0.17
120	0.20	0.21	0.22	0.23	0.24	0.26	0.16	0.17	0.18	0.19	0.20	0.21
140	0.24	0.25	0.26	0.27	0.28	0.30	0.19	0.20	0.21	0.22	0.23	0.24
160	0.27	0.28	0.30	0.31	0.33	0.34	0.22	0.23	0.24	0.25	0.26	0.27
180	0.31	0.32	0.33	0.35	0.37	0.38	0.24	0.26	0.27	0.28	0.29	0.31
200	0.34	0.35	0.37	0.39	0.41	0.43	0.27	0.28	0.30	0.31	0.33	0.34
220	0.37	0.39	0.41	0.43	0.45	0.47	0.30	0.31	0.33	0.34	0.36	0.38
240	0.41	0.43	0.44	0.47	0.49	0.51	0.33	0.34	0.36	0.37	0.39	0.41
260	0.44	0.46	0.48	0.50	0.53	0.56	0.35	0.37	0.39	0.40	0.42	0.44
280	0.48	0.50	0.52	0.54	0.57	0.60	0.38	0.40	0.41	0.43	0.46	0.48
300	0.51	0.53	0.56	0.58	0.61	0.64	0.41	0.43	0.44	0.47	0.49	0.51
320	0.54	0.57	0.59	0.62	0.65	0.68	0.44	0.45	0.47	0.50	0.52	0.55
340	0.58	0.60	0.63	0.66	0.69	0.73	0.46	0.48	0.50	0.53	0.55	0.58
360	0.61	0.64	0.67	0.70	0.73	0.77	0.49	0.51	0.53	0.56	0.59	0.62
380	0.65	0.67	0.70	0.74	0.77	0.81	0.52	0.54	0.56	0.59	0.62	0.65
400	0.68	0.71	0.74	0.78	0.81	0.85	0.54	0.57	0.59	0.62	0.65	0.68
420	0.71	0.74	0.78	0.81	0.85	0.90	0.57	0.60	0.62	0.65	0.68	0.72
440	0.75	0.78	0.81	0.85	0.89	0.94	0.60	0.62	0.65	0.68	0.72	0.75
460	0.78	0.82	0.85	0.89	0.93	0.98	0.63	0.65	0.68	0.71	0.75	0.79
480	0.82	0.85	0.89	0.93	0.98	1.03	0.65	0.68	0.71	0.74	0.78	0.82

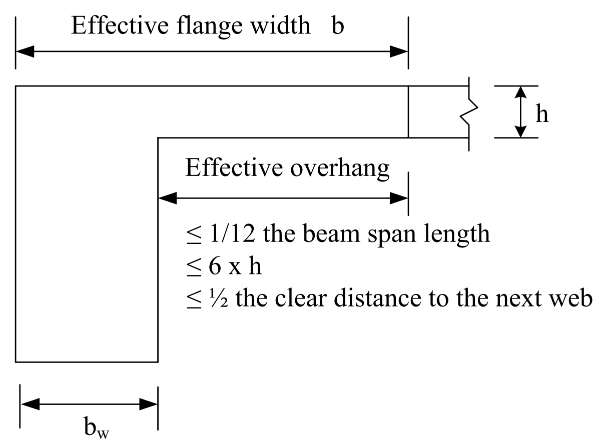
Flexure 5 - Cont'd

f_y	60000 (psi)						75000 (psi)					
d/d'	0.02	0.06	0.1	0.14	0.18	0.22	0.02	0.06	0.1	0.14	0.18	0.22
K_n' (psi)	ρ' (%)						ρ' (%)					
500	0.85	0.89	0.93	0.97	1.02	1.07	0.68	0.71	0.74	0.78	0.81	0.85
520	0.88	0.92	0.96	1.01	1.06	1.11	0.71	0.74	0.77	0.81	0.85	0.89
540	0.92	0.96	1.00	1.05	1.10	1.15	0.73	0.77	0.80	0.84	0.88	0.92
560	0.95	0.99	1.04	1.09	1.14	1.20	0.76	0.79	0.83	0.87	0.91	0.96
580	0.99	1.03	1.07	1.12	1.18	1.24	0.79	0.82	0.86	0.90	0.94	0.99
600	1.02	1.06	1.11	1.16	1.22	1.28	0.82	0.85	0.89	0.93	0.98	1.03
620	1.05	1.10	1.15	1.20	1.26	1.32	0.84	0.88	0.92	0.96	1.01	1.06
640	1.09	1.13	1.19	1.24	1.30	1.37	0.87	0.91	0.95	0.99	1.04	1.09
660	1.12	1.17	1.22	1.28	1.34	1.41	0.90	0.94	0.98	1.02	1.07	1.13
680	1.16	1.21	1.26	1.32	1.38	1.45	0.93	0.96	1.01	1.05	1.11	1.16
700	1.19	1.24	1.30	1.36	1.42	1.50	0.95	0.99	1.04	1.09	1.14	1.20
720	1.22	1.28	1.33	1.40	1.46	1.54	0.98	1.02	1.07	1.12	1.17	1.23
740	1.26	1.31	1.37	1.43	1.50	1.58	1.01	1.05	1.10	1.15	1.20	1.26
760	1.29	1.35	1.41	1.47	1.54	1.62	1.03	1.08	1.13	1.18	1.24	1.30
780	1.33	1.38	1.44	1.51	1.59	1.67	1.06	1.11	1.16	1.21	1.27	1.33
800	1.36	1.42	1.48	1.55	1.63	1.71	1.09	1.13	1.19	1.24	1.30	1.37
820	1.39	1.45	1.52	1.59	1.67	1.75	1.12	1.16	1.21	1.27	1.33	1.40
840	1.43	1.49	1.56	1.63	1.71	1.79	1.14	1.19	1.24	1.30	1.37	1.44
860	1.46	1.52	1.59	1.67	1.75	1.84	1.17	1.22	1.27	1.33	1.40	1.47

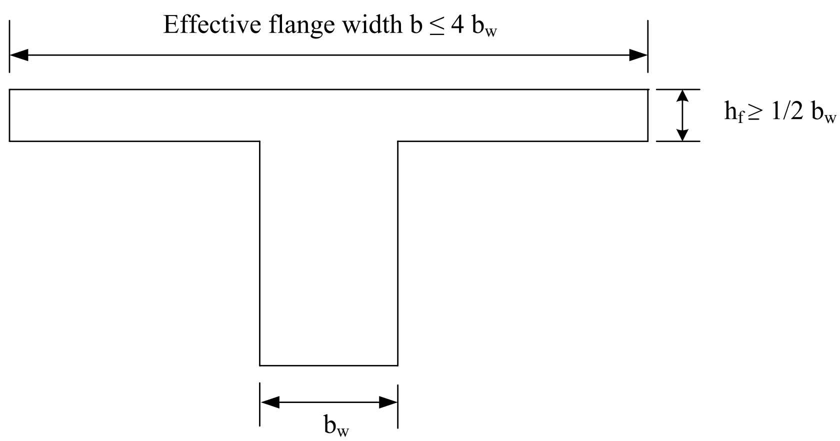
Flexure 6 - T-Beam construction and definition of effective flange width



T-sections resulting from monolotically built slabs and beams



L-sections resulting from monolotically built slabs and beams



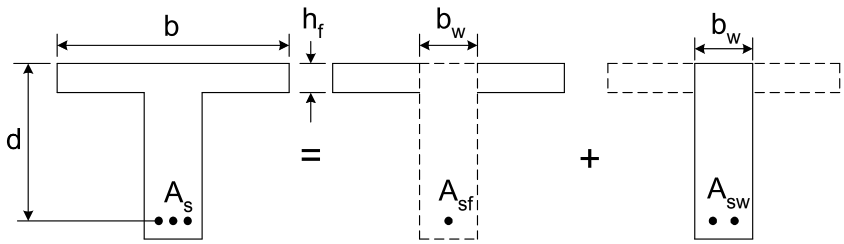
Isolated Precast T-Beams cast to have a T-shape

Flexure 7 - Reinforcement ratio ρ_f(%) balancing concrete in overhang(s) in T or L beams; f_y = 60,000 psi

$\phi M_{nf} + \phi M_{nw} \geq M_u$

$\rho_f = A_{sf}/[(b-b_w)d]$

$\rho_w = A_{sw}/(b_w d)$



$A_s = A_{sf} + A_{sw}$

Use Flexure 1 or 2 with ρ_f and $(b-b_w)$ to find ϕM_{nf} Use Flexure 1 or 2 with ρ_w and b_w to find ϕM_{nw}

f_y = 60000 psi

f_c' (psi) :	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9000**	**10000**
d/h_f	**ρ_f (%)**							
2	2.13	2.83	3.54	4.25	4.96	5.67	6.38	7.08
3	1.42	1.89	2.36	2.83	3.31	3.78	4.25	4.72
4	1.06	1.42	1.77	2.13	2.48	2.83	3.19	3.54
5	0.85	1.13	1.42	1.70	1.98	2.27	2.55	2.83
6	0.71	0.94	1.18	1.42	1.65	1.89	2.13	2.36
7	0.61	0.81	1.01	1.21	1.42	1.62	1.82	2.02
8	0.53	0.71	0.89	1.06	1.24	1.42	1.59	1.77
9	0.47	0.63	0.79	0.94	1.10	1.26	1.42	1.57
10	0.43	0.57	0.71	0.85	0.99	1.13	1.28	1.42
11	0.39	0.52	0.64	0.77	0.90	1.03	1.16	1.29
12	0.35	0.47	0.59	0.71	0.83	0.94	1.06	1.18
13	0.33	0.44	0.54	0.65	0.76	0.87	0.98	1.09
14	0.30	0.40	0.51	0.61	0.71	0.81	0.91	1.01
15	0.28	0.38	0.47	0.57	0.66	0.76	0.85	0.94
16	0.27	0.35	0.44	0.53	0.62	0.71	0.80	0.89
17	0.25	0.33	0.42	0.50	0.58	0.67	0.75	0.83
18	0.24	0.31	0.39	0.47	0.55	0.63	0.71	0.79
19	0.22	0.30	0.37	0.45	0.52	0.60	0.67	0.75
20	0.21	0.28	0.35	0.43	0.50	0.57	0.64	0.71
21	0.20	0.27	0.34	0.40	0.47	0.54	0.61	0.67
22	0.19	0.26	0.32	0.39	0.45	0.52	0.58	0.64
23	0.18	0.25	0.31	0.37	0.43	0.49	0.55	0.62
24	0.18	0.24	0.30	0.35	0.41	0.47	0.53	0.59

Flexure 7 - Cont'd

f_y = 60000 psi

f_c' (psi) :	3000	4000	5000	6000	7000	8000	9000	10000
d/h_f	ρ_f (%)							
25	0.17	0.23	0.28	0.34	0.40	0.45	0.51	0.57
26	0.16	0.22	0.27	0.33	0.38	0.44	0.49	0.54
27	0.16	0.21	0.26	0.31	0.37	0.42	0.47	0.52
28	0.15	0.20	0.25	0.30	0.35	0.40	0.46	0.51
29	0.15	0.20	0.24	0.29	0.34	0.39	0.44	0.49
30	0.14	0.19	0.24	0.28	0.33	0.38	0.43	0.47
31	0.14	0.18	0.23	0.27	0.32	0.37	0.41	0.46
32	0.13	0.18	0.22	0.27	0.31	0.35	0.40	0.44
33	0.13	0.17	0.21	0.26	0.30	0.34	0.39	0.43
34	0.13	0.17	0.21	0.25	0.29	0.33	0.38	0.42
35	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
36	0.12	0.16	0.20	0.24	0.28	0.31	0.35	0.39
37	0.11	0.15	0.19	0.23	0.27	0.31	0.34	0.38
38	0.11	0.15	0.19	0.22	0.26	0.30	0.34	0.37
39	0.11	0.15	0.18	0.22	0.25	0.29	0.33	0.36
40	0.11	0.14	0.18	0.21	0.25	0.28	0.32	0.35

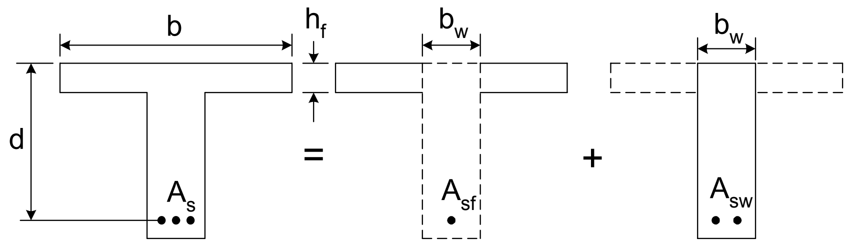
Flexure 8 - Reinforcement ratio ρ_f(%) balancing concrete in overhang(s) in T or L beams; f_y = 75,000 psi

$\phi M_{nf} + \phi M_{nw} \geq M_u$

$\rho_f = A_{sf}/[(b-b_w)d]$

$\rho_w = A_{sw}/(b_w d)$

$A_s = A_{sf} + A_{sw}$



Use Flexure 3 or 4 with ρ_f and $(b-b_w)$ to find ϕM_{nf} | Use Flexure 3 or 4 with ρ_w and b_w to find ϕM_{nw}

f_y = 75000 psi

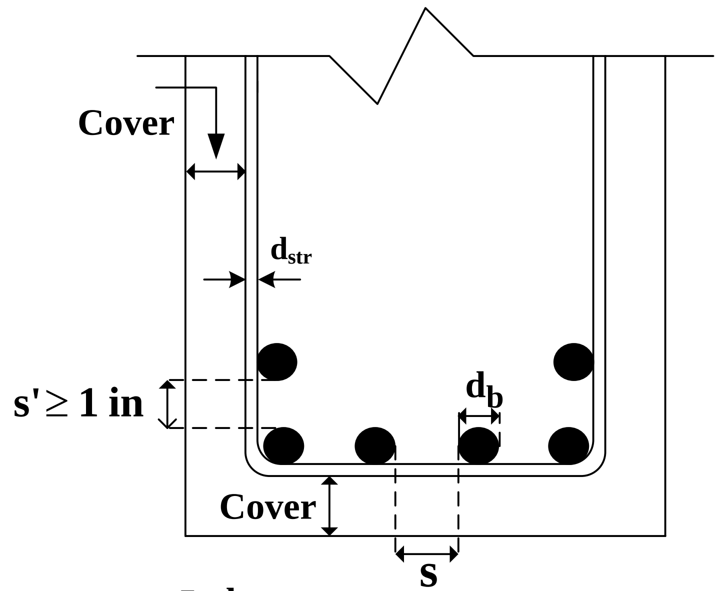
f_c' (psi) :	3000	4000	5000	6000	7000	8000	9000	10000
d/h_f	ρ_f (%)							
2	1.70	2.27	2.83	3.40	3.97	4.53	5.10	5.67
3	1.13	1.51	1.89	2.27	2.64	3.02	3.40	3.78
4	0.85	1.13	1.42	1.70	1.98	2.27	2.55	2.83
5	0.68	0.91	1.13	1.36	1.59	1.81	2.04	2.27
6	0.57	0.76	0.94	1.13	1.32	1.51	1.70	1.89
7	0.49	0.65	0.81	0.97	1.13	1.30	1.46	1.62
8	0.43	0.57	0.71	0.85	0.99	1.13	1.28	1.42
9	0.38	0.50	0.63	0.76	0.88	1.01	1.13	1.26
10	0.34	0.45	0.57	0.68	0.79	0.91	1.02	1.13
11	0.31	0.41	0.52	0.62	0.72	0.82	0.93	1.03
12	0.28	0.38	0.47	0.57	0.66	0.76	0.85	0.94
13	0.26	0.35	0.44	0.52	0.61	0.70	0.78	0.87
14	0.24	0.32	0.40	0.49	0.57	0.65	0.73	0.81
15	0.23	0.30	0.38	0.45	0.53	0.60	0.68	0.76
16	0.21	0.28	0.35	0.43	0.50	0.57	0.64	0.71
17	0.20	0.27	0.33	0.40	0.47	0.53	0.60	0.67
18	0.19	0.25	0.31	0.38	0.44	0.50	0.57	0.63
19	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60
20	0.17	0.23	0.28	0.34	0.40	0.45	0.51	0.57

Flexure 8 - Cont'd

f_y = 75000 psi

f_c' (psi) :	3000	4000	5000	6000	7000	8000	9000	10000
d/h_f	ρ_f (%)							
21	0.16	0.22	0.27	0.32	0.38	0.43	0.49	0.54
22	0.15	0.21	0.26	0.31	0.36	0.41	0.46	0.52
23	0.15	0.20	0.25	0.30	0.34	0.39	0.44	0.49
24	0.14	0.19	0.24	0.28	0.33	0.38	0.43	0.47
25	0.14	0.18	0.23	0.27	0.32	0.36	0.41	0.45
26	0.13	0.17	0.22	0.26	0.31	0.35	0.39	0.44
27	0.13	0.17	0.21	0.25	0.29	0.34	0.38	0.42
28	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
29	0.12	0.16	0.20	0.23	0.27	0.31	0.35	0.39
30	0.11	0.15	0.19	0.23	0.26	0.30	0.34	0.38
31	0.11	0.15	0.18	0.22	0.26	0.29	0.33	0.37
32	0.11	0.14	0.18	0.21	0.25	0.28	0.32	0.35
33	0.10	0.14	0.17	0.21	0.24	0.27	0.31	0.34
34	0.10	0.13	0.17	0.20	0.23	0.27	0.30	0.33
35	0.10	0.13	0.16	0.19	0.23	0.26	0.29	0.32
36	0.09	0.13	0.16	0.19	0.22	0.25	0.28	0.31
37	0.09	0.12	0.15	0.18	0.21	0.25	0.28	0.31
38	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
39	0.09	0.12	0.15	0.17	0.20	0.23	0.26	0.29
40	0.09	0.11	0.14	0.17	0.20	0.23	0.26	0.28

Flexure - 9 Bar spacing and cover requirements



$$s \geq \begin{cases} d_b \\ 1\frac{1}{3}\ a_{max} \\ 1 \text{ in} \end{cases}$$

a_{max} = Max. aggregate size

Minimum Cover for protection of reinforcement (Section 7.7.1)

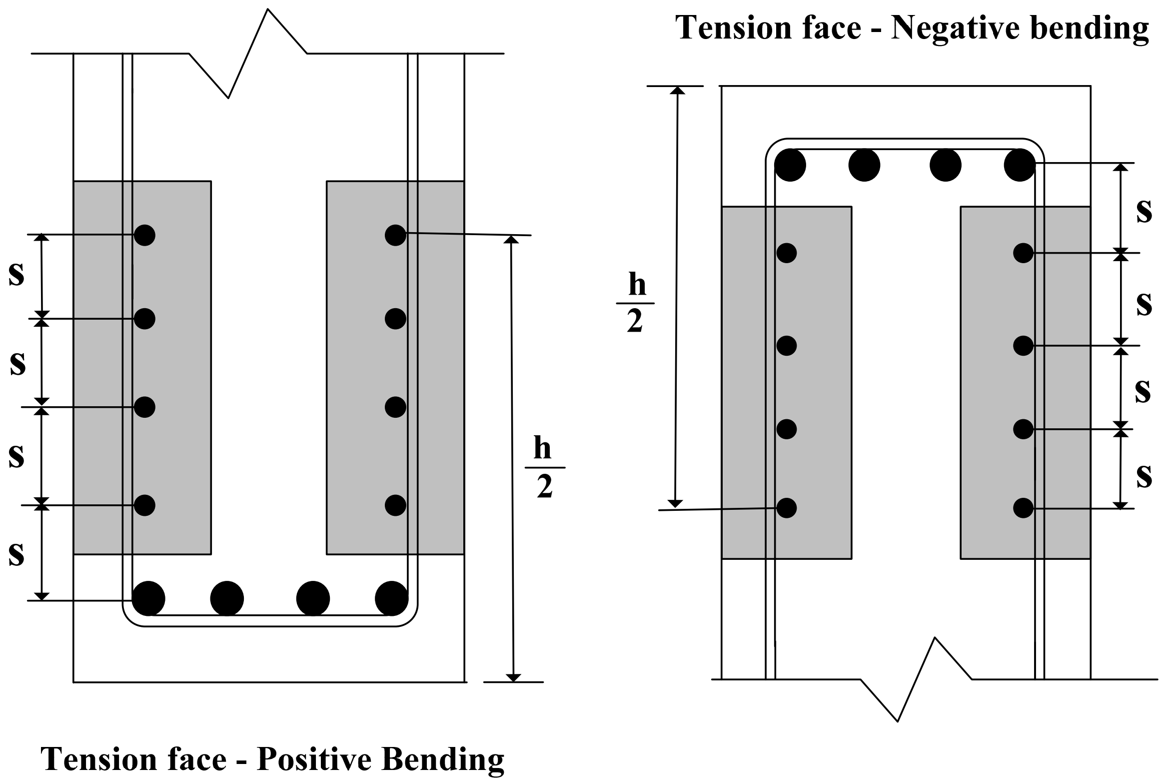
Not exposed to weather or in contact with ground	
Beams and columns	1 ½ in
Slabs, walls, joists with No. 11 and smaller bars	¾ in
Slabs, walls, joists with No. 14 and 18 bars	1 ½ in
Exposed to earth or weather	
Members with No. 5 and smaller bars	1 ½ in
Members with No. 6 through 18 bars	2 in
Cast against and permanently exposed to earth	3 in

Notes: i) The minimum cover is measured from the concrete surface to the outermost surface of stirrups; or to the outermost surface of main bars if more than one layer is used without stirrups.

ii) In corrosive environments or other severe exposure conditions, the amount of cover shall be suitably increased (Section 7.7.5).

ii) The minimum cover shall also satisfy the fire protection requirement (Section 7.7.7).

Flexure 10 - Skin Reinforcement



Chapter 2

Design for Shear

By Richard W. Furlong

2.1 Introduction

Shear is the term assigned to forces that act perpendicular to the longitudinal axis of structural elements. Shear forces on beams are largest at the supports, and the shear force at any distance *x* from a support decreases by the amount of load between the support and the distance *x*. Under uniform loading, the slope of the shear diagram equals the magnitude of the unit uniform load. Shear forces exist only with bending forces. Concrete beams are expected to crack in flexure, with such cracks forming perpendicular to longitudinal tension reinforcement, i.e., perpendicular also to a free edge. Principal tension stresses change direction from horizontal at the longitudinal reinforcement to 45^o at the neutral axis and vertical at the location of maximum compression stress. Consequently, cracks in concrete tend to "point" toward the region of maximum compression stress as indicated by the cracks shown in Fig. 4.1. Axial compression force plus bending makes the area of compressed concrete larger than without axial force.

ACI 318-05 permits the evaluation of shear capacity for most beams to be taken as the combination of strength from concrete without shear reinforcement V_c plus the strength V_s provided by shear reinforcement. Shear strength of a slab that resists flexural forces in two orthogonal directions around a column (flat plates, footings and pile caps), is evaluated as the shear strength of a prism located at a distance of half the slab depth *d* from the faces of the column.

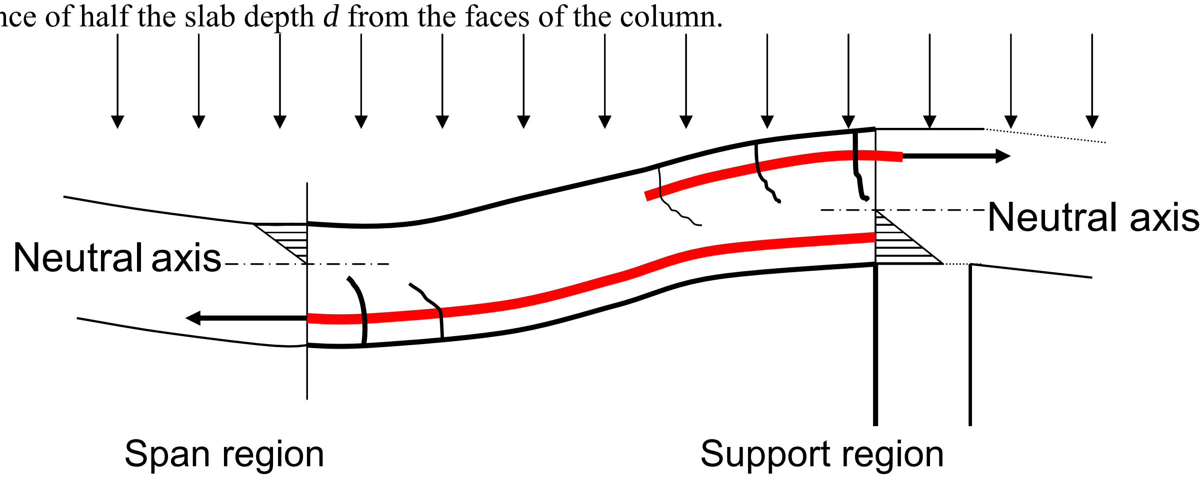


Fig. 4.1 – Reinforced concrete beam in bending

2.2 Shear strength of beams

Equation (11-3) of ACI 318-05, Section 11.3.1.1 permits the shear strength V_c of a beam without shear reinforcement to be taken as the product of an index limit stress of $2\sqrt{f_c'}$ times a nominal area $b_w d$. With f_c' expressed in lb/in^2 units and beam dimensions in inches, nominal shear strength $V_c = 2\sqrt{f_c'}b_w d$ in units of lb. Shear reinforcement is not required for slabs, which can be considered as very wide beams. If the width of a beam is more than twice the thickness h of the beam, ACI 318-05, Section 11.5.6.1(c) exempts such beams from the requirement of shear reinforcement as long as the shear capacity of the concrete is greater than the required shear force. A more complex method for determining V_c is given in ACI 318-05, Section 11.3.2.1. The method is demonstrated in SHEAR EXAMPLE 2. A special type of ribbed floor slab known as a joist system can be constructed without any shear reinforcement in the joist ribs. Joist system relative dimensions, slab thickness, rib width and spacing between ribs are specified in ACI 318-05, Section 8.11.

A diagonal crack that might result in shear failure, as suggested in Fig.2.2, can form no closer to the face of the support than the distance d from the face of the support. Consequently, Section 11.1.3.1 of ACI 318-05 permits the maximum required value of shear V_u to be determined at a distance d from the face of such a support when the support provides compression resistance at the face of the beam opposite the loading face. If loads had been suspended from the bottom of the beam, or if the support were no deeper than the beam itself, maximum required shear must be taken as the shear at the face of the support.

The most common form of shear reinforcement is composed of a set of bars bent into U-shaped stirrups as indicated by the vertical bars in Fig. 2.2. The stirrups act as tension hangers with concrete performing as compression struts.

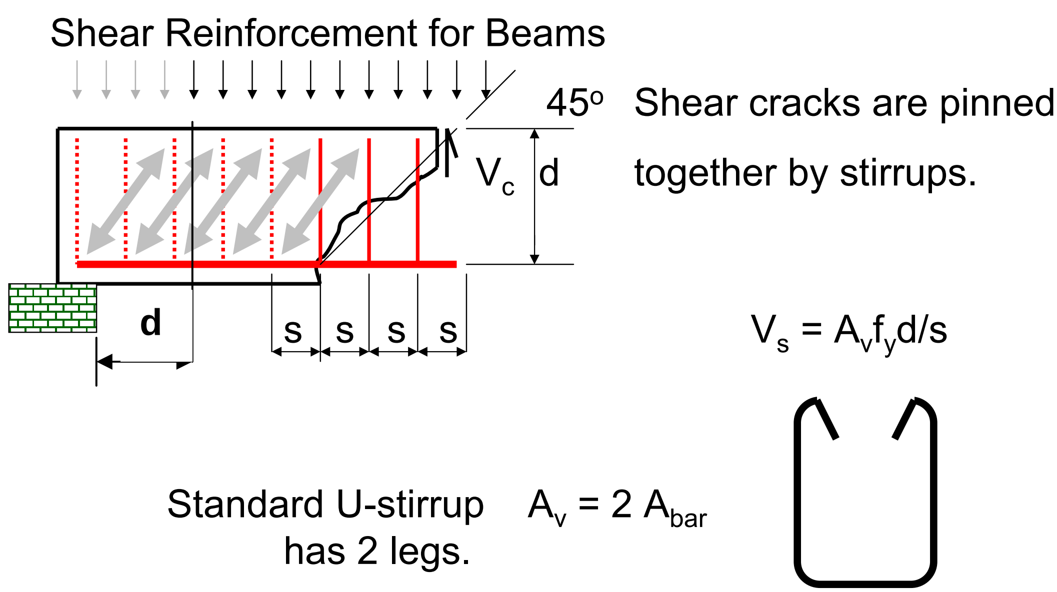


Fig. 2.2 – Shear reinforcement

Each vertical leg of a stirrup has a tension capacity equal to its yield strength, and the most common stirrup has 2 vertical legs. The shear capacity of vertical stirrups is the tension strength of one stirrup times the number of stirrups that interrupt potential cracks on a 45-degree angle from the tension steel. Thus, $V_s = A_v f_y d/s$. A U-stirrup has an area A_v = 2(area of one stirrup leg). Shear capacity at any location along a beam $V_n = V_c$ plus V_s.

2.3 Designing stirrup reinforcement for beams

Shear reinforcement A_v must provide the strength required in addition to the strength of concrete V_c. Thus, the required amount of $A_v = (V_n - V_c)/(f_{yt}d/s)$. The strength reduction factor ϕ for shear is 0.75. ϕV_n must be greater than V_u. When the quantities A_v, f_y and d are known, stirrup spacing s can be computed as

$$s = (\phi A_v f_{yt} d) / (V_u - \phi V_c) \tag{2.1}$$

ACI 318, Section 11.5.6.1 requires the placement of shear reinforcement in all beams for which the required strength is more than half the value of ϕV_c. The full development of a critical shear crack between stirrups is prevented by ACI 318-05, Section 11.5.5, which sets the maximum spacing of stirrups at $d/2$ when $V_u < 6\phi V_c$, but maximum spacing is $d/4$ when $V_u > 6\phi V_c$. Concrete cannot act effectively as compression struts if the required amount of V_s exceeds $8V_c = 8b_w d\sqrt{f_c'}$ regardless of shear reinforcement. Thus, a beam section must be made larger if $V_n > 10b_w d\sqrt{f_c'}$.

A graph given in design aid **SHEAR 1** displays limits of nominal shear stress values of $V_n/(b_w d)$ for concrete strength f_c' from 3000 psi to 10,000 psi. The graph is not intended for precise evaluation of member capacity, as precise strength values are given in other design aids. Rather, the graphs clearly show stress ranges for which design requirements change. No shear reinforcement (stirrups) are required if $V_n/(b_w d)$ is less than $1.0\sqrt{f_c'}$. The capacity V_c of concrete in sections reinforced for shear is $2.0b_w d\sqrt{f_c'}$. The strength of stirrups can be added to the concrete strength V_c to determine the total strength of a section. Required stirrups must be spaced no more than $d/2$ apart where $V_n/(b_w d) < 6.0\sqrt{f_c'}$. Where $V_n/(b_w d) > 6.0\sqrt{f_c'}$, maximum stirrup spacing becomes $d/4$. The compressive strut capacity of concrete is reached if $V_n/(b_w d) = 10.0\sqrt{f_c'}$. Additional stirrups cannot increase section shear strength, as the concrete strength is considered exhausted when $V_n/(b_w d) > 10\sqrt{f_c'}$.

Design aid **SHEAR 2** consists of 3 tables that may be used to determine shear capacity for rectangular sections of width b or b_w from 10 in to 32 in and thickness h from 10 in to 48 in. It is assumed that depth d is 2.5 inches less than thickness for $h < 30$ in, but that larger longitudinal bars would make $d \approx h - 3$ in for deeper beams.

Table 2a gives values $K_{fc} = \sqrt{(f_c'/4000)}$ to be used as modifiers of K_{vc} when members are made with concrete strength different from $f_c' = 4000$ psi. In conjunction with required stirrups, the nominal shear strength of concrete $V_c = K_{fc}K_{vc}$.

Table 2b contains values K_{vs} for determining nominal stirrup capacity $V_s = K_{vs}(A_v/s)$.

Table 2c gives values K_{vc} in kips. K_{vc} is the shear strength of concrete when required stirrups areused in members made with $f_c' = 4000$ psi concrete.

The nominal strength of a rectangular section is the sum of concrete strength V_c and reinforcement strength V_s to give $V_n = K_{fc}K_{vc} + K_{vs}(A_v/s)$.

SHEAR 3 is a design aid for use if Grade 60 stirrups larger than #5 are to be used, and sections must be deep enough for tension strength bar development of larger stirrups or closed ties. Required thickness of section values are tabulated for concrete strengths from 3000 psi to 10,000 psi and for #6, #7 and #8 stirrups. It should be noted that ACI 318–05, Section 11.5.2 limits the yield strength of reinforcing bar stirrups to no more than 60,000 psi.

ACI 318-05, Section 11.5.6.3 sets lower limits on the amount of shear reinforcement used when such reinforcement is required for strength. These limits are intended to prevent stirrups from yielding upon

the formation of a shear crack. The limit amount of A_v must exceed $50b_w s/f_y > 0.75\sqrt{f_c'}b_w s/f_y$. The second quantity governs when f_c' is greater than 4444 lb/in^2.

The design of shear reinforcement includes the selection of stirrup size and the spacing of stirrups along the beam. Design aids **SHEAR 4.1** and **SHEAR 4.2** give strength values V_s of #3 U stirrups and #4 U stirrups (two vertical legs) as shear reinforcement tabulated for depth values d from 8 in to 40 in and stirrup spacing s from 2 in to maximum permitted spacing s = d/2. Each table also lists the maximum section width for which each stirrup size may be used without violating the required minimum amount of shear reinforcement. SHEAR 4.1 applies for Grade 40 stirrups, and SHEAR 4.2 applies for Grade 60 stirrups.

2.4 Shear strength of two-way slabs

Loads applied to a relatively small area of slabs create shear stress perpendicular to the edge(s) of the area of load application. Columns that support flat plate slabs and columns that are supported by footings are the most common examples. ACI 318-05, Section 11.12.2.1 provides expressions for determining shear strength in such conditions for which shear failure is assumed to occur near the face(s) of the columns. Failure is assumed to occur on the face(s) of a prism located at a distance of d/2 from each column face. The perimeter b_o of the prism multiplied by the slab depth d is taken as the area of the failure surface.

Three expressions are given for computing a critical stress on the failure surface. A coefficient $\alpha_s = 40$ for interior columns, $\alpha_s = 30$ for edge columns and $\alpha_s = 20$ for corner columns is used to accommodate columns located along the perimeter of slabs. The critical (failure) stress may be taken as the least value of either $4\sqrt{f_c'}$, $(2 + 4/\beta)\sqrt{f_c'}$, or $(\alpha_s d/b_o + 2)\sqrt{f_c'}$. The quantity β is the ratio of long side to short side of the column. The first expression governs for centrally loaded footings and for interior columns unless the ratio β exceeds 2 or the quantity $40d/b_o$ is less than 2. Shear strength at edge columns and corner columns that support flat plates must be adequate not only for the direct force at the column but also for additional shear forces associated with moment transfer at such columns.

Diagrams for the prism at slab sections for columns are shown with SHEAR EXAMPLES 5, 7 and 8.

Design aid **SHEAR 5.1** gives shear strength values of two-way slabs at columns as limited by potential failure around the column perimeter.

Table 5.1a gives values of K1 as a function of slab d and column size b and h.

Table 5.1b gives values of the shear stress factor K2 as a function of the ratio β_c between the longer side and the shorter side of rectangular column sections.

Table 5.1c gives values of nominal strength V_c as a function of the product K1K2 and the nominal compressive strength of slab concrete f_c'.

Design aid **SHEAR 5.2** is similar to SHEAR 5.1 for determining slab shear capacity at round columns. For circular columns, there is no influence of an aspect ratio as for rectangular columns, and the design aid is less complex.

Table 5.2a gives, for slab d and column diameter h, values of a shape parameter K3 in sq in units.

Table 5.2b gives, for K3 and slab concrete f_c', the value of nominal shear capacity V_c in kip units.

2.5 Shear strength with torsion plus flexural shear

Torsion or twisting of a beam creates shear stress that is greatest at the perimeter of sections. The shear stress due to torsion adds to flexural shear stress on one vertical face, but it subtracts from flexural shear on the opposite vertical face. Shear stress due to torsion is negligibly small near the center of sections. ACI 318-05, Section 11.6 provides empirical expressions for torsion strength. It is assumed that significant torsion stress occurs only around the perimeter of sections, and no torsion resistance is attributed to concrete. The definitions of section properties are displayed in Fig. 2.3.

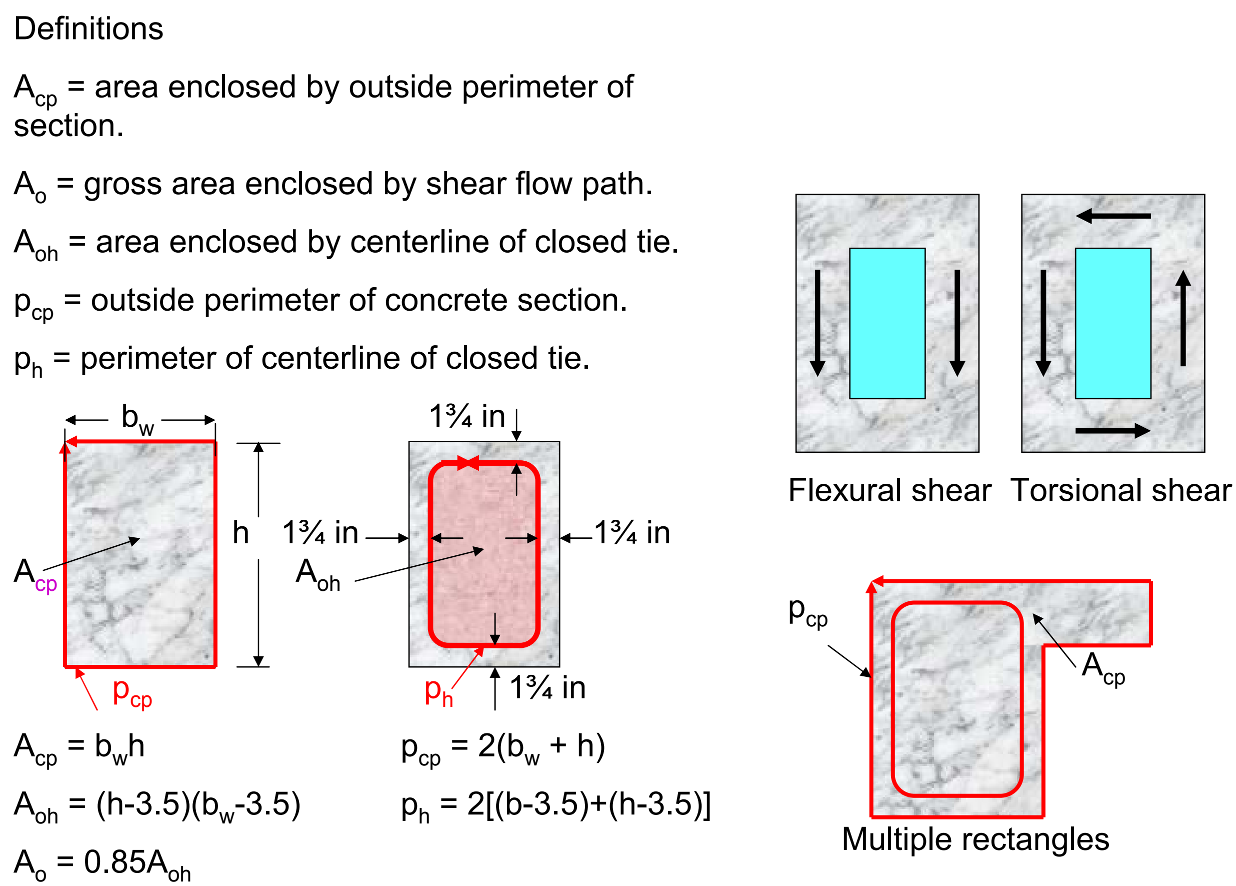


Fig. 2.3 – Torsion strength definitions of section properties

Concrete beams properly reinforced for torsion display considerable ductility, continuing to twist without failure after reinforcement yields. Consequently, ACI 318-05, Section 11.6.2.2 permits design for torsion in indeterminate beams to be made for the torsion force that causes cracking. A member is determinate if torsion forces can be determined from the equations of statics without considering compatibility relationships in the structural analysis. A member is indeterminate if torsion forces must be estimated with consideration of compatibility conditions, i.e., there exists more than one load path for resisting torsion. The illustrations in Fig. 2.4 show two conditions of a spandrel beam supporting a brick ledge. The determinate beam in the upper sketch must transfer to columns allof the eccentric load on the ledge only through the twisting resistance (torsion) of the beam. In contrast, the indeterminate beam in the lower sketch supports a slab that extends outward to receive the eccentric load on the ledge. The eccentric load can be transferred to columns both by torsion of the beam and by flexure of the cantilevered slab.

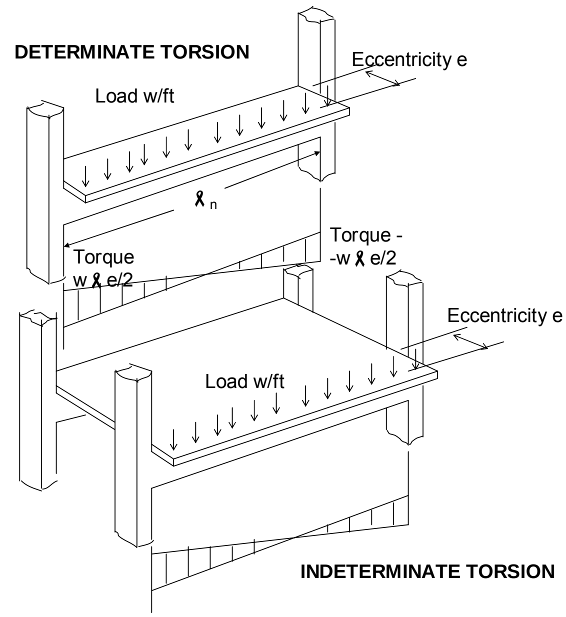


Fig. 2,4 – Determinate torsion versus Indeterminate torsion

Cracking torque T_{cr} is to be computed without consideration of torsion reinforcement.

$$T_{cr} = 4\sqrt{f_c'}(A_{cp})^2 / p_{cp} \tag{2.2}$$

Torques smaller than one-quarter of the cracking torque T_{cr} will not cause any structurally significant reduction in either the flexural or shear strength and can be ignored. An upper limit to the torque resistance of concrete functioning as compression struts is taken from ACI 318-05, Eq. (11-18) as:

$$T_{max} = 17(A_{oh})^2 \sqrt{f_c'} / p_h \, . \tag{2.3}$$

Torsion reinforcement requires both closed ties and longitudinal bars that are located in the periphery of the section. With torsion cracks assumed at an angle θ from the axis of the member, torsion strength from closed ties is computed as

$$T_n = (2A_oA_tf_{yt} \cot \theta)/ \mathrm{s} \tag{2.4}$$

The angle θ must be greater than 30 degrees and less than 60 degrees. A value $\theta = 45^{o}$ has been used for design aids in this chapter. The size of solid concrete sections must be large enough to resist both flexural shear V_u and torsion shear T_u within the upper limits established for each. ACI 318-05, Eq. (11-18) gives

$$\sqrt{[V_u /(b_wd)]^2 + [T_up_h /(1.7A_{oh}^2\,]^2} \le \phi[\, V_c/(b_wd) + 8\sqrt{f_c'}\,] \, . \tag{2.5}$$

In addition, ACI 318, Eq (11-22) requires that longitudinal bars with an area A_ℓ be placed around the periphery of sections.

$$A_\ell = A_tp_h / \mathrm{s} \, . \tag{2.6}$$

Longitudinal spacing of transverse closed ties must be no greater than p_h /8 or 12 in. The spacing between longitudinal bars in the periphery of sections must be no greater than 12 in. Where torsion reinforcement is required, the area of 2 legs of closed tie (A_v + $2A_t$) must be greater than $0.75(b_w s/f_{yt})\sqrt{f_c'}$ but be not less than $50b_w s/f_y$.

Design aid **SHEAR 6.1** displays critical values of torsion strength for rectangular sections made with concrete strength f_c' = 4000 psi. If concrete strength f_c' is different from 4000 psi, the correction factor K_{fc} from SHEAR 2, Table 2a must be multiplied by torque values T_n from Table 6.1a and T_{cr} from Table 6.1b.

Table 6.1a displays values of K_t, the maximum torque ${}_{lim}T_n$ a section can resist as a function of section thickness *h* and width *b*. It is assumed that the distance from section surface to the center of closed ties is 1.75 in.

Table 6.1b displays values K_{tcr} of torque T_{cr} that will cause sections to crack as a function of section dimensions *b* and *h*.

Design aid **SHEAR 6.2** can be used to determine the torsion strength of closed ties. Numbers K_{ts} for width b and thickness *h* listed in the charts are multiplied by the ratio between tie area A_t and tie spacing *s* in order to compute the nominal torque T_s resisted by closed ties. The distance from section surface to tie centerline is taken to be 1.75 in.

Table 6.2a applies for Grade 40 ties. Table 6.2b applies for Grade 60 ties.

2.6 Deep beams

The definition of deep beams is found in ACI 318-05 Section 11.8.1. Deep beams have a span-to-depth ratio not greater than 4 or a concentrated force applied to one face within a distance less than 2*d* from the supported opposite face. If a non-linear analysis is not used for deep beams, the beams can be designed by the strut-and-tie method given in Appendix A of ACI 318-05. Shear reinforcement must include both horizontal bars and vertical bars. Beams more than 8 in thick must have two reinforcement grids, one in each face. A maximum shear limit V_n, and minimum shear reinforcement values A_v for vertical bars and A_{vh} for horizontal bars for deep beams are given in Fig. 2.5.

Deep beams may be designed using Appendix A (Strut & Tie model).

$V_n < (10\sqrt{f_c'})(b_w d)$.

$A_v > 0.0025 b_w s$ with $s<d/5$ or 12 in

$A_{vh} > 0.0015 b_w s_2$ with $s_2<d/5$ or 12 in

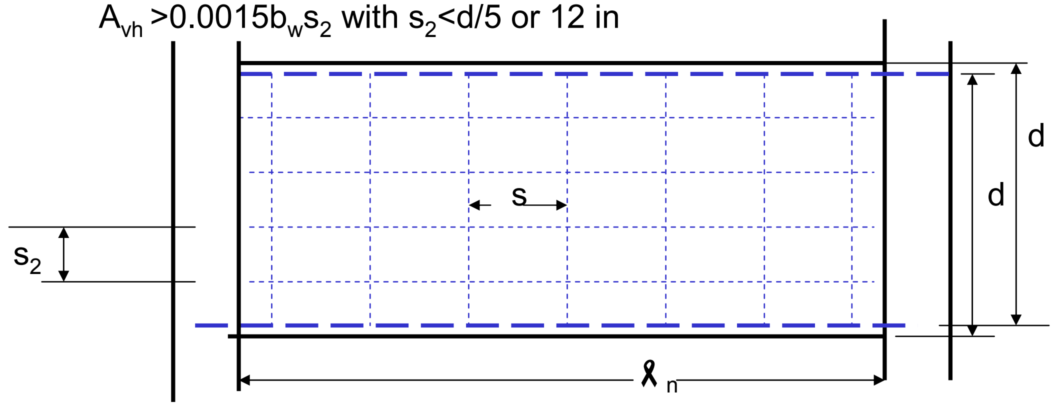


Fig. 2.5 – Deep beam limits in ACI 318-05, Section 11.8

ACI 318-05, Appendix A presents rules for analysis of forces on a truss composed of nodal points connected by concrete compression struts and reinforcing bar tension members. Diagonal concrete compression struts may cross lines of vertical (tension strut) reinforcement, and the angle between any reinforcement and the axis of the diagonal compression strut cannot be less than 25 degrees. Concrete strut area A_{cs} has a width b and a thickness A_{cs}/b that may be considered to increase at a rate equal to the distance along the strut from the node to the center of the strut. A nodal point at which the 3 force components act toward the joint is termed a CCC joint. If two nodal point forces act toward the joint and one force is (tension) away from the joint, the nodal point is designated as CCT. Nodal points with two tensile forces and one compression force is designated CTT, and if all force components act away from the node, the designation becomes TTT. A prismatic strut has the same thickness throughout its length, and a strut wider at the center than at the ends of its length is called a bottle-shaped strut. ACI 318, Section A.3 specifies the nominal strength F_n of compression struts without longitudinal reinforcement. Two coefficients, β_s for strut shape and β_n for nature of nodal points are used. For struts of uniform cross section in which strut area A_{cs} can be taken as the same as the nodal bearing area A_{nn}, then $A_{nn} = A_{cs}$ and

$$F_n = \beta_n f_{cs} A_{cs} = 0.85 \beta_n \beta_s f_c' A_{cs}\,. \qquad (2.7)$$

for which $\beta_s = 1$ for a strut of uniform cross section.
$\beta_s = 0.75$ for a bottle-shaped strut.
$\beta_s = 0.40$ for a strut that could be required to resist tension.
$\beta_s = 0.60$ for all other cases
$\beta_n = 1$ for struts at CCC nodal points
$\beta_n = 0.80$ for struts at CCT nodal points
$\beta_n = 0.60$ for struts at CTT nodal points.

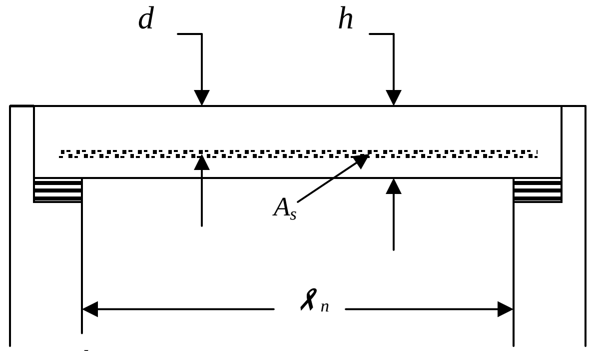
The capacity of prismatic (constant size) concrete struts can be based on strength at its nodal points. The capacity reduction factor for shear, $\phi = 0.75$, must be applied to computed values of nominal strength. Concrete compressed struts must be "confined" laterally by reinforcement with a density that satisfies the minimum reinforcement relationship of Equation (A4) from ACI318-05, Section A3.3.1

$$\sum A_{vi}(\sin \alpha_i)/(bs_i) \geq 0.003 \qquad (2.8)$$

The subscripted index i refers to the 2 directions, horizontal and vertical, for the sum of shear reinforcement densities. The angle α is the angle between the diagonal and the direction of tension reinforcement, and α must be greater than 25 degrees and less than 65 degrees. Minimum requirements for placement of shear reinforcement specified in ACI 318, Section 11.8.4 and Section 11.8.5 will satisfy Eq. (2.8).

Design aid **SHEAR 7** gives solutions to Equation (2.8) for angles γ between a vertical line and the compression strut, with $\gamma = 25^\circ$ to 65° in increments of 15°. In each chart, solutions to Eq. (A4) are tabulated for bars #3 to #6, and the product of beam width b and bar spacing s. For a given angle γ the sum of values for vertical bars and for horizontal bars must be at least 0.003. The sine of a vertical angle γ applies for vertical bars, and the cosine of γ applies for horizontal bars. Reinforcement limits specified in ACI 318-05, Sections 11.8.4 and 11.8.5 limit the maximum product of width and spacing permitted for any tie bar area. Each table shows the value $(As_i \sin \gamma)/(bs_i)$ when the bar size and spacing limit is reached.

SHEAR EXAMPLE 1 – Determine stirrups required for simply supported beam



Determine the required shear V_n for which this beam should be designed. Use the simplified method ACI 318-05 Section 11.3.1.1 to determine the strength ϕV_c with normal weight concrete. If stirrups are needed, specify a spacing from face of support to the #3 U stirrups that may be required.

Given:

Live load = 1.5 k/ft
Superimposed Dead load = 1.4 k/ft
ℓ_n = 20.0 ft
f_c' = 3000 psi
Stirrups are Grade 60 (f_y = 60,000 psi)

b_w = 14 in
d = 19.5 in (taken as h – 2.5 in)
h = 22 in

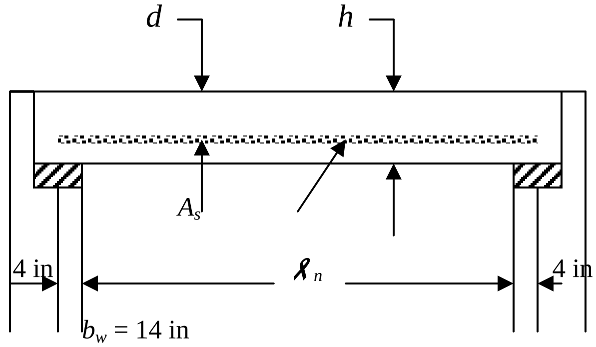
A_s = 3.16 sq in (4 #8 longitudinal bars)

ACI 318-05 Section	Procedure	Calculation	Design Aid
9.2.1	Step 1 - Determine factored (required) load w_u. Compute beam weight Compute total dead load = beam self weight. + superimposed DL Compute $w_u = 1.2D + 1.6L$	Self weight = 14in(22in)(0.15k/ft^3)/144in^2/ft^2 = 0.32 k/ft DL = 0.32 + 1.40 = 1.72 k/ft w_u = 1.2(1.72) + 1.6(1.50) = 4.47 k/ft	
11.1.2.1	Step 2 – Determine V_u at distance d from face of support. Compute $V_u = w_u(\ell_n/2 – \text{d})$	V_u = (4.47k/ft)(20.0ft/2 – 19.5in/12in/ft) = 37.4 k	
11.2.1.1	Step 3 – Determine the strength of concrete in shear V_c using the simplified method. Compute $V_c = 2(\sqrt{f_c'})\, b_w d$	$V_c = 2(\sqrt{3000\text{psi}})$14in(19.5in) = 29,900 lbs = 29.9 k	
	Alternate procedure using Design Aids with f_c' find $\phi V_c = \phi K_{fc} K_{vc}$	For f_c' = 3000psi, K_{fc} = 0.866 For for b=14in & h=22in, K_{vc} = 34.5 k	SHEAR 2 Table 2a Table 2c
9.2.2.3	Compute $V_c = K_{fc}K_{vc}$	V_c = (0.866)34.5k = 29.9 k	
11.5.5.1	Step 4 – If $V_u > 0.5\phi V_c$, stirrups are req'd. Compute $0.5\phi V_c$ Compare V_u and $0.5\phi V_c$	$0.5\phi V_c$ = 0.5(0.75)22.5k = 11.2 k V_u = 37.4k > 11.2k, stirrups are required	
11.1.1	Step 5 – Compute $_{max}V_s = V_u/\phi - V_c$	$_{max}V_s$ = 37.4k/0.75 – 29.9k = 20.0 k	
11.5.6.9	Step 6 – Note that section is large enough if $V_s < 4V_c$ Section size is adequate	$4V_c$ = 4(29.9k) = 119.6 k > V_s = 20 k	

SHEAR EXAMPLE 1 - Continued

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.5.6.2	Step 7 – Determine stirrup spacing for maximum V_s = 20 k		
	Compute A_v for #3 U stirrup	Two legs give $A_v = 2(0.11in^2) = 0.22\ in^2$	
	Compute $s = A_v f_y d/V_s$	$s = (0.22in^2)(60k/in^2)(19.5in)/20k = 12.9$ in	
11.5.4.1	Maximum spacing = $d/2$	max. s = 19.5in/2 = 9.75in, use s = 10 in	
	Alternate procedure using Design Aid		SHEAR 4.2
	#3 Grade 60 stirrups and $s = d/2$ = 10 in	Shear strength V_s = 26 k	Table 4.2a
11.5.5.3	Since max b_w=26 in and b_w =14 in,	$A_v f_y > 50 b_w s$	
11.5.5.1	Step 8 – Determine position beyond which no stirrups are required. No stirrups req'd if $V_u < ½\phi V_c$. With zero shear at midspan, the distance z from mid-span to $V_n = ½V_c$ becomes $z = ½\phi V_c/w_u$	z = 0.5(0.75)29.9k/4.47k/ft = 2.51 ft	
	Stirrups are required in the space (10.0 ft - 2.51 ft) = 7.49 ft from face of each support. Compute in inches	7.49 ft = 7.49ft(12in/ft) = 90 in	
	Begin with a half space = 5 in, and compute n = number of stirrup spaces required	n = (90in – 5in)/10 = 8.5 in Use 9 spaces.	
	Use 10 #3 U stirrups spaced	5 in, 9 @ 10 in from each support.	

SHEAR EXAMPLE 2 – Determine beam shear strength of concrete by method of ACI 318-05, Section 11.3.2.1



Use the detailed method of ACI 318-05, Section 11.3.2.1 to determine the value of ϕV_c attributable to normal weight concrete using the detailed method to determine the strength ϕV_c Assume normal weight concrete is used.

Given:

w_u = 4.47 kips/ft
ℓ_n = 20.0 ft
f_c' = 3000 psi
Stirrups are Grade 60 (f_y = 60,000 psi)

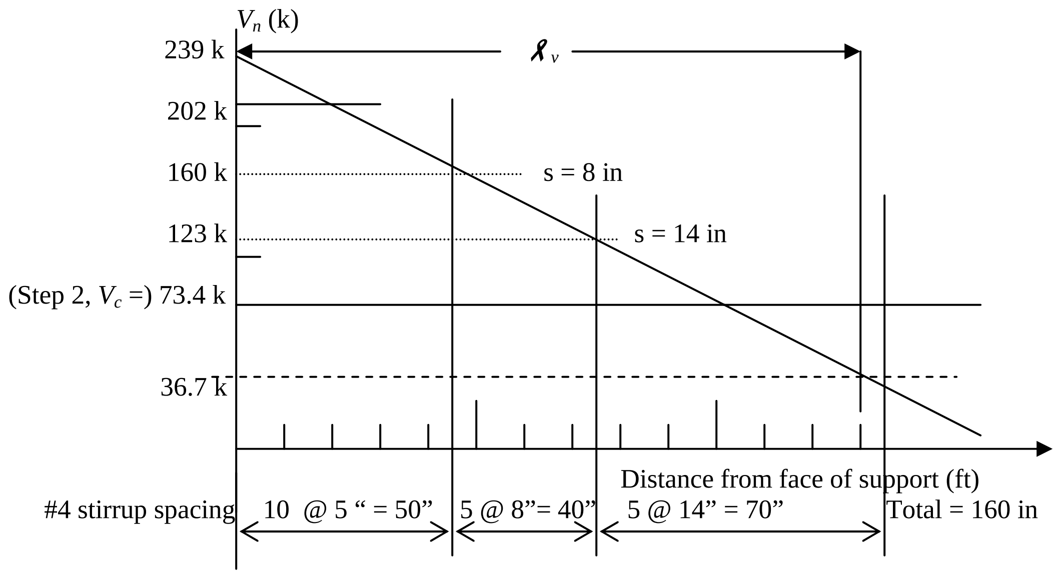
b_w = 14 in
d = 19.5 in (h – 2.5 in)
h = 22 in
A_s = 3.16 sq in (4 #8 longitudinal bars)

ACI 318-02 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1 – Calculate the moment M_u at d from the face of support. Distance d from the face of support is d + 4 in from column centerline. Compute $\ell = \ell_n$ + column thickness $M_u = (w_u \ell/2)(d+4) - w_u(d+4)^2/24$	d + 4in = 19.5in + 4in = 23.5 in ℓ = 20.0ft + 2(4in)/12in/ft = 20.67 ft M_u = 4.47k/ft(20.67ft)(19.5in+4in)/2 - 4.47k/ft(23.5ft)^2/24 = 983 in-k	
	Step 2 – Compute $\rho_w = A_s/(b_w d)$	ρ_w = 3.16in^2/(14in x 19.5in) = 0.012	
	Step 3 – Compute at d from support $V_u = w_u(\ell/2 - 21.5/12)$	V_u = 4.47k/ft(20.67ft/2-23.5in/12in/ft)= 37.4 k	
	Step 4 – Compute $\rho_w V_u d/M_u$	$\rho_w V_u d/M_u$ = 0.012(37.4k)23.5in/983in-k = 0.011	
11.3.2.1	Step 5 – Compute $\phi V_c/(b_w d)$ $= \phi[1.9\sqrt{f_c'} + 2500\rho_w V_u d/M_u]$ Compute $\phi V_c = 98.7(b_w d)$	= 0.75[1.9√3000psi+2500(0.010] = 98.7 lb/in^2 ϕV_c = 98.7lb/in^2(14in x 19.5in) = 26,900 lbs = 26.9 k	

Compare with SHEAR EXAMPLE 1 for which ϕV_c = 22.5 k
Frequently the more complex calculation for V_c will indicate values 10% t0 15% higher than those from the simpler procedure.

SHEAR EXAMPLE 3 – Vertical U-stirrups for beam with triangular shear diagram

Determine the size and spacing of stirrups for a beam if b_w = 20 in, d = 29 in (h = 32 in), f_y = 60,000 psi, f_c' = 4000 psi, V_u = 177 k, w_u = 11.6 k/ft. Assume normal weight concrete is used.



ACI 318-05 Section	Procedure	Calculation	Design Aid
9.3.2.3 11.1.3.1	Step 1 – Determine $V_n = {}_{max}V_u/\phi$ (ϕ = 0.75) Compute w_u/ϕ Compute V_u/ϕ at d from face of support. At d from face, $V_u/\phi = V_u/\phi - (w_u/\phi)_u d$ V_n must exceed 199 k at support.	V_u/ϕ = 179 /(0.75) = 239 k w_u/ϕ = 11.6 /(0.75) = 15.5 k/ft V_u/ϕ at d = 239 – 15.5(29/12) = 202 k	
11.3.1.1	Step 2 – Determine $V_c = K_{fc}K_{vc}$ For b = 20 in and h = 32 in Show this V_c line on graph above	with f_c'=4000 psi K_{fc} = 1 K_{vc} = 73.4 k V_c = (1)(73.4k) = 73.4 k	SHEAR 2 Table 2a Table 2c
11.5.5.1	Step 3 – Compute distance ℓ_v over which stirrups are required, $\ell_v = \dfrac{(V_n - 0.5V_c)}{(w_u/\phi)}$	$\ell_v = \dfrac{239k - 0.5(73.4k)}{15.5k/ft}$ = 13.0 ft = 156 in	
  11.5.6.2 11.5.4.3	Step 4 – Select stirrup size for max V_s Compute max $V_s = (V_u$ at $d - V_c)$ Read stirrup spacing for d = 29 in and V_n = 126 k Select #4 stirrups, and use 5-in spacing from face of support Note, if $V_s > 2V_c$, s must be < d/4 = 7.25 in Since s = 5 in is < 7.25 in, spacing is OK	 max V_s = 202 – 73.4 = 128.6 kips With #3 stirrups, s must be < 3 in With #4 stirrups, s can be 5 in compute $2V_c$ = 2(73.4) = 147 k	 SHEAR 4.2 Table 4.2a Table 4.2b
11.5.4.1 11.1.1	Step 5 – Determine V_n with maximum stirrup spacing of s =14 in for d = 29 in $V_n = V_c + V_s$ Show this line on graph above	 V_s = 48 k V_n = 73.4k + 49.5k = 123 k	SHEAR 4.2 Table 4.2b

SHEAR EXAMPLE 3 –Vertical U-stirrups for beam with triangular shear diagram (continued)

ACI 318-05 Section	Procedure	Calculation	Design Aid
	Step 6 – Stirrup spacing can be selected for convenience of placement. With s = 5 in, compute $V_{n5} = V_c + V_{s5}$ With s = 8 in, compute $V_{n8} = V_c + V_{s8}$ Construct these lines on graph above	 V_{n5} = 73.4k + 129.5k = 203 k V_{n8} = 73.4k + 87k = 160 k	 SHEAR 4.2 Table 4.2b Table 4.2c
	Step 7 -Determine distances from face of support to point at which each selected spacing is adequate. Use graph to see that strength is adequate at each position for which spacing changes. 10 spaces @ 5 in = 50 in V_n = 203 k Plus 5 spaces @ 8 in = 40 in V_n = 160 k Plus 5 spaces @ 14 in = 70 in V_n = 123 k	 50 + 40 = 90 in 90 + 70 = 160 in > ℓ_v	

Editor's Note:

Generally, beams of the same material and section dimensions will be used in continuous frames. Each end of the various spans will have a triangular shear diagram that differs from other shear diagrams. The same chart constructed for the section selected can be used with the other shear diagrams simply by sketching the diagram superimposed on the chart already prepared. New sets of spacings can be read such that the strength associated with a spacing exceeds the ordinate on the diagram of required shear strength $V_n = V_u / \phi$.

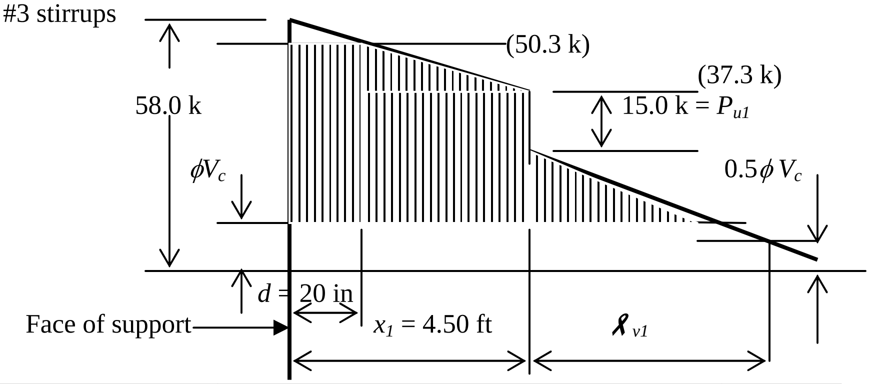
SHEAR EXAMPLE 4 – Vertical U-stirrups for beam with trapezoidal and triangular shear diagram

Determine the required spacing of vertical #3 stirrups for the shear diagram shown.

Given: Normal weight concrete

b = 13 in	V_u = 58.0 kips
d = 20 in	w_u = 4.6 k/ft
f_c' = 4000 psi	x_1 = 4.50 ft
f_y = 60,000 psi	P_{u1} = 15.0 kips

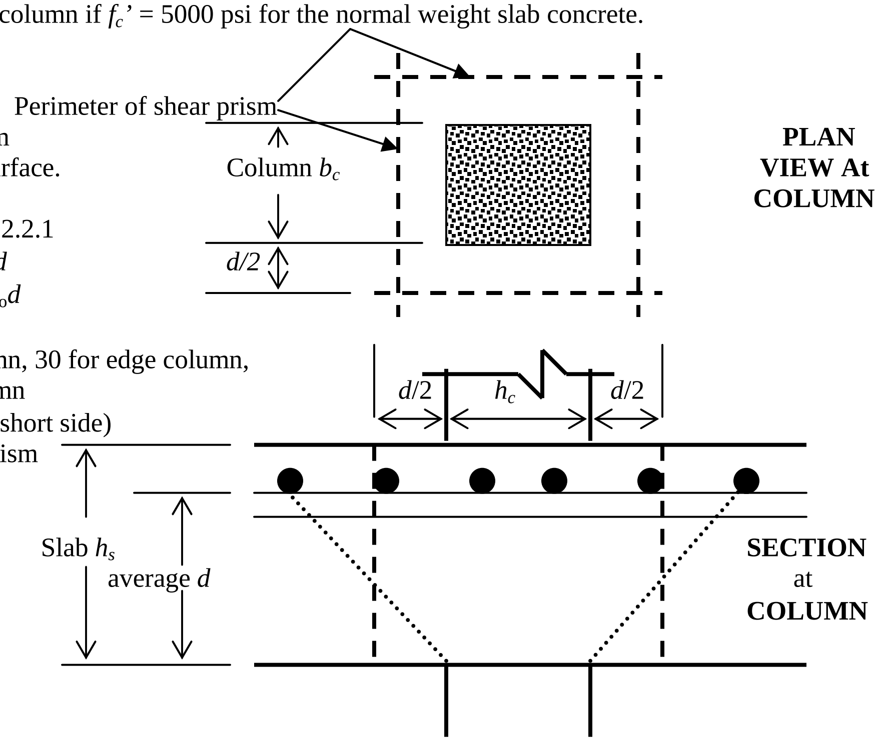
ϕ = 0.75 for shear



ACI 318-05 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1 – Determine at d from face of support the value of maxV_u = $V_u - w_u\, d/12$	maxV_u = 58.0k – 4.6k/ft(20in)/12in/ft = 50.3 k	
11.3.1.1 9.3.2.5	Step 2 – Determine the value $\phi V_c = 2\phi(\sqrt{f_c'})bd$ OR use Design Aid for b=13in & h=22.5in, K_{vc} = 32.9 k	ϕV_c=2(0.75)(√4000lb/in^2)13in(20in)= 24,700lb $\phi V_c = 0.75K_{fc}K_{vc}$= 0.75(1.00)32.9k = 24.7 k	 SHEAR 2 Table 2c
	Step 3 – Determine required V_u each side of P_{u1} Left of P_{u1}, $V_u = V_u - w_u\, x_1$ Right of P_{u1}, change in $V_u = P_{u1}$	 Left P_{u1}, V_u = 58.0k – 4.6k/ft(4.5ft) = 37.3 k Right P_{u1} ,V_u = 37.3k – 15k = 22.3 k	
11.1.1 11.5.6.2	Step 4 – Determine spacing s_1 required for #3 U-stirrups at face of support. A_v = 2(0.11) = 0.22 sq in $s_1 = \phi A_v f_y d/(\max V_u - \phi V_c)$	 s_1=(0.75)0.22in^2(60.0k/in^2)20in/(50.3k-24.7k) = 7.7 in	
11.5.4.1 11.5.6.2 11.1.1	Step 5 – Since maximum spacing $s = d/2$ with d = 10 in, determine value of $\phi V_u = \phi V_c + \phi A_v f_y d/s$ OR – Use Design Aid for V_s when s = 10 in $\phi V_u = \phi V_c + \phi V_s$	max spacing smax = 20in/2 = 10 in ϕV_u=24.7k+0.75(0.22in^2)60k/in^2(20in)/10in = 44.5 k V_s = 26 k for #3 stirrups @ 10 in spacing ϕV_u = 24.7k + 0.75(26k) = 44.2 kips	 SHEAR 4.2
	Step 6 – Determine distance x from face of support to point at which V_u = 44.5 kips. x = (*Change in shear*)/w_u	 x = (58.0k – 44.5k)/4.6k/ft = 2.93 ft = 35 in	
11.5.4.1	Step 7 – Determine distance ℓ_{v1} , distance beyond x_1 at which no stirrups are required. Find $\ell_{v1} = (V_u - V_c/2)/w_u$ Compute $x_1 + \ell_{v1}$ Conclude: use s = 7 in until ϕV_u < 44.5 k and use s = 10 in until $\phi V_u < 0.5\phi V_c$	 ℓ_{v1} = (22.3k – 24.7k/2)/4.6k/ft = 2.16 ft $x_1 + \ell_{v1}$ = 4.50ft + 2.16ft = 6.76 ft = 81 in 5 spaces @ 7 in (35 in) 5 spaces @ 10 in (50 in) 85 in > 81 in OK	

SHEAR EXAMPLE 5 – Determination of perimeter shear strength at an interior column supporting a flat slab (α_s = 40)

Determine the shear capacity V_n of a 10-in thick two-way slab based on perimeter shear strength at an interior 16-in x 24-in rectangular column if f_c' = 5000 psi for the normal weight slab concrete.



Mechanism of shear prism defining shear strength surface.

ACI 318-05, Section 11.12.2.1

$V_n = (2 + 4/\beta_c)(\sqrt{f_c'})b_o d$

or $V_n = (2 + \alpha_s/b_o)(\sqrt{f_c'})b_o d$

$\leq 4 (\sqrt{f_c'})b_o d$

α_s = 40 for interior column, 30 for edge column, 20 for corner column

β_c = (col long side) / (col short side)

b_o = perimeter of shear prism = $2(b_c + d + h_c + d)$

Since $(2 + \alpha_s/b_o) \leq 4$
For interior column,
$b_c + h_c \leq 80$

[Note that the uniform load acting within the shear perimeter of the prism does not contribute to the magnitude of required load V_c. The area within the shear perimeter is negligibly small with respect to the area of a flat plate around an interior column, usually only one to two percent. In contrast for footings, the area within the shear perimeter may be 15% or more of the bearing area of the footing. Computation of V_c for footing "slabs" must reflect that influence.]

ACI 318-05 Section	Procedure	Calculation	Design Aid
7.7.1c	Step 1 – Estimate d keeping clear cover 0.75 in $d = h_s$ - 0.75 – bar diameter	d = 10in – 0.75in – 0.75$_{est}$in ≈ 8.5 in	
11.12.2.1	Step 2 – Use $b_o = 2(b_c + d + h_c + d)$	b_o = 2(16in + 8.5in + 24in + 8.5) = 114 in	
	Step 3 - Compute $\beta_c = h_c / b_c$ Since β_c <2, Compute $V_n = 4(\sqrt{f_c'})b_o d$	β_c = 24in /16in = 1.50 $V_n = 4(\sqrt{5000 \text{lb/in}^2})$114in(8.5in) = 274,000 lbs = 274 k	
	ALTERNATE METHOD with Design Aid		
	Step 1 – Compute $b_c + h_c$ Use $d \approx 8.5$ in	16in + 24in = 40 in Interpolate K1 = (3.58ksi + 4.18ksi)/2 = 3.88 ksi	SHEAR 5.1 Table 5.1a
	Step 2 – Compute $\beta_c = h_c / b_c$ Since b_c < 2,	β_c = 24in/16in = 1.5 K2 = 1	SHEAR 5.1 Table 5.1b
	Step 3 – With f_c' = 5000 psi, and K1*K2 = 1(3.88ksi), interpolate	V_n = 212k+(3.88ksi-3.00ksi)(283k–212k) = 275 kips	SHEAR 5.1 Table 5.1c

SHEAR EXAMPLE 6 – Thickness required for perimeter shear strength of a flat slab at an interior rectangular column

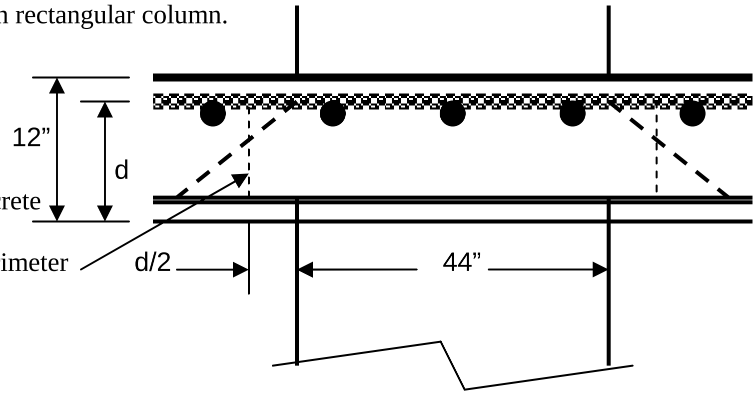
Given: f_c' = 5000 psi See SHEAR EXAMPLE 5 for diagram of shear perimeter and Code clauses.
h_c = 24 in
b_c = 16 in
V_u = 178 k
Assume normal weight concrete

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.12.2.1 9.3.2.3	Step 1 – Set up expression for ϕV_c $\phi V_c = 4\phi(\sqrt{f_c'})b_o d$ $= 4\phi(\sqrt{f_c'})2(h_c + d + b_c + d)d$	ϕV_c =4(0.75)($\sqrt{5000}$psi)2(24in+d+16in+d)d in = (424.3lb/in^2)[(40in)$d$in + 2d^2in^2)	
9.1.1	Step 2 – Equate V_u to ϕV_c and solve for d	178k(1000 lb/k) = 424.3lb/in^2(40d + 2d^2)in^2 419.5 in^2 = (40d + 2d^2)in^2 209.8 + 100 = 100 + 20d + d^2 d = ($\sqrt{309.8}$) –10 = 17.6 – 10 = 7.6 in	
7.7.1	Step 3 – Allow for 0.75 in clear cover of tension bars to make $h_s = d$ +0.75 + bar diameter (estimated)	h_s = 7.6in + 0.75in + 0.625in = 8.98 in	
ALTERNATE METHOD using Design Aid			SHEAR 5.1
9.3.2.1	Step 1 – Compute minimum $V_n = V_u/\phi$ and compute $(h_c + b_c)$	V_n = 178k/0.75 = 237 k $(h_c + b_c)$ = (24in + 16in) = 40 in	
11.12.1.2	Step 2 – With f_c' = 5000 psi and V_n = 237 k Use $(h_c + b_c)$ = 40 in, interpolate K1K2	237 k is between V_n = 212 k and V_n = 283 k K1K2 = 3.00ksi+1.00ksi$\frac{(237k-212k)}{(283k-212k)}$=3.35 ksi	Table 5.1c
11.12.2.1	Step 3 – Compute $\beta_c = h_c/b_c$	β_c = 24in/16in = 1.5 < 2, so K2 = 1 and K1 = K1K2 ksi /K2 = 3.35ksi	Table 5.1b
	Step 4 – Table 5.1a with $(h_c + b_c)$ = 40 and K1 = 3.35 ksi Interpolate for d	d =7in + (8in-7in)(3.35ksi–3.02ksi)/(3.58ksi–3.02ksi) d = 7 in + 0.59 in = 7.59 in	Table 5.1a
7.7.1	Step 5 – Allow for 0.75 in clear cover of tension bars to make $h_s = d$ +0.75 + bar diameter (estimated)	h_s = 7.59in + 0.75in + 0.625in = 9.0 in slab	

SHEAR EXAMPLE 7 – Determination of perimeter shear strength at an interior rectangular column supporting a flat slab (β_c > 4)

Determine the shear capacity V_n of a 12-in thick two-way slab based on perimeter shear strength at an interior 12in x 44-in rectangular column.

Given: f_c' = 4000 psi
h_c = 44 in
b_c = 12 in
Interior column, α_s = 40
Assume normal weight concrete



Edge of shear perimeter

ACI 318-05 Section	Procedure	Calculation	Design Aid
7.7.1c	Step 1 Estimate d keeping clear cover 0.75 in $d \approx h_s$ – 0.75in – bar thickness (est. 1 in)	$d \approx$ 12in – 0.75in – 1.0in, use d = 10.2 in	
11.12.2.1	Step 2 - Compute $b_c + h_c$ With d = 10.2, find K1 from Table 5.1a	$b_c + h_c$ = 12in + 44in = 56 in K1 = 6.08ksi + (7.68ksi–6.08ksi)(10.2in-10in)/(12in-10in) = 6.24 ksi	SHEAR 5.1 Table 5.1a
11.12.2.1	Step 3 – Compute $\beta_c = h_c / b_c$ With β_c = 3.67, interpolate for K2	β_c = 44in/12in = 3.67 K2 = 0.778 + (0.763-0.778)(3.67-3.60)/(3.8-3.6)= 0.773	SHEAR 5.1 Table 5.1b
	Step 4 – Compute K1(K2) With K1K2 = 4.82 and f_c' = 4000 find V_n	K1K2 = 6.24ksi(0.773) = 4.82 ksi V_n = 253k + (316k-253k)(4.82ksi-4.00ksi) = 305 k	SHEAR 5.1 Table 5.1c

ALTERNATE METHOD – Compute strength directly using ACI 318-05, Eqn. (11-33)

7.7.1c	Step 1: Estimate d keeping clear cover 0.75 in $d \approx h_s$ – 0.75 – estimate of bar thickness	$d \approx$ 12in – 0.75in – 1.0in, use d 10.2 in	
11.12.1.2	Step 2 – Compute $b_o = 2(b_c + d + h_c + d)$	b_o = 2(12in+10.2in+44in+10.2in)= 153 in	
	Step 3 – Compute $\beta_c = h_c/b_c$	β_c = 44in / 12 in = 3.67	
11.12.2.1	Step 4 – Compute $V_n = (2 + 4/\beta_c)(\sqrt{f_c'})b_o d$	V_n = (2+4/3.67)($\sqrt{4000}$lb/in^2)153in(10.2in) = 305,000 lbs = 305 k	

SHEAR EXAMPLE 8 – Determine required thickness of a footing to satisfy perimeter shear strength at a rectangular column

Given: P_u = 262 k
Column $b_c = h_c$ = 16 in
Footing size = 7 ft x 7 ft

Footing size = 7 ft x 7 ft
f_c' = 3000 psi normal weight concrete
f_y = 60,000 psi

ACI 318-05 Section	Procedure	Calculation	Design Aid
	Step 1 – Determine net bearing pressure under factored load P_u $f_{br} = P_u$ / (footing area)	f_{br} = 262k /(7.0ft x7.0ft)=5.35 k/ft^2	
	Step 2 –Express $V_u = f_{br}$(footing area–prism area) $= f_{br}[7.0 \times 7.0 - (16+d)^2/144)]$	V_u=5.35k/ft^2[49ft^2–(16in+$d$in)^2/144in^2/ft^2] = 252.6k–(1.188k/in)d –(0.0372k/in^2)d^2	
11.12.2.1 9.3.2.3	Step 3 – Express $\phi V_c = \phi[4(\sqrt{f_c'})b_o d]$ $= \phi[4(\sqrt{f_c'})4(16 + d)d]$/1000lb/k	ϕV_c = 0.75[4($\sqrt{3000}$lb/in^2) 4(16+d)in($d$in)]/1000lb/k = [0.657k/in^2(16d + d^2)in^2]	
	Step 4 – Equate $V_u = \phi V_c$ and solve for d	(252.6 - 1.188d - 0.0372d^2)k = 0.657(16d + d^2)k $0.694d^2 + 11.70d = 252.6$ $d^2 + 16.86d + 8.43^2 = 364.0 + 71.1$ $d = (\sqrt{435.1}) - 8.43$ = 12.4 in	
7.7.1	Step 5 – Allow 4 in clear cover below steel plus one bottom bar diameter to make $h \approx d + 4$	Use footing h = 12.4 + 4 = Make h = 17 in	
ALTERNATE METHOD using Design Aid SHEAR 5.1			
	Step 1 – Determine net bearing pressure under factored load P_u $f_{br} = P_u$ / (footing area)	f_{br} = 262k / (7.0ft x 7.0ft) = 5.35 k/sq ft	
  9.3.2.3	Step 2- Estimate that bearing area of shear prism is 10% of footing area Compute $V_u = f_{br}$ (1 - 0.10) A_{ftg} Compute $V_n = V_u / \phi$	V_u = 5.35k/ft^2(0.90)7.0ft(7.0ft) = 236 k V_n = 236/0.75 = 315 k	
11.12.2.1	Step 3 – Find K1K2 with V_n =315 k and f_c' = 3000 psi Note that since h_c/b_c < 2, K2 = 1. Thus,	K1K2=5.00ksi + $(6.0\text{ksi}-5.0\text{ksi})\frac{(315\text{k}-274\text{k})}{(329\text{k}-274\text{k})}$ K1K2 = 5.75 ksi K1 = 5.75 ksi	Table 5.1c Table 5.1b
11.12.2.1	Step 4 – Compute $h_c + b_c$ With K1 = 5.75 ksi and $h_c + b_c$ = 32 in, find d	$h_c + b_c$ = 16in + 16in = 32in d = 12in + $(14\text{in}-12\text{in})\frac{(5.75\text{ksi}-5.38\text{ksi})}{(6.72\text{ksi}-5.38\text{ksi})}$ d = 12.6 in As above, make footing h = 17 in	Table 5.1a

In ALTERNATE METHOD, Check assumed Step 2 proportion of shear prism area to footing area..

%footing area = $100[(d + b_c)/12]^2 / (A_{ftg})$ = 100[(16in+12.6in)/(12in/ft)]^2/(7ft x 7ft) = 11.6% (estimate was 10%)

V_u should have been (1 – 0.116)262 = 232 k instead of estimated 236 k

SHEAR EXAMPLE 9 – Determination capacity of a flat slab based on required perimeter shear strength at an interior round column

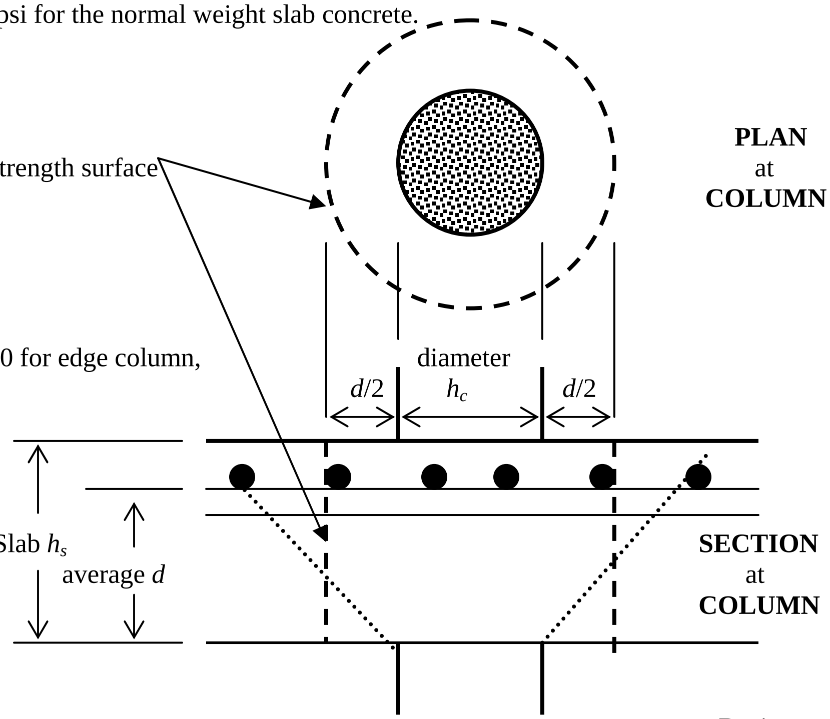
Determine the shear capacity V_n of a 9-in thick two-way slab based on perimeter shear strength at an interior circular column of 18-in diameter if f_c' = 5000 psi for the normal weight slab concrete.

Mechanism of shear prism representing perimeter shear strength surface.

$V_n = 4(\alpha_s d/b_o + 2)(\sqrt{f_c'})\, b_o d$
$\quad \le 4(\sqrt{f_c'}) b_o d$

α_s = 40 for interior column, 30 for edge column, 20 for corner column

b_o = perimeter of shear prism
$= \pi(h_c + d)$



ACI 318-05 Section	Procedure	Calculation	Design Aid
7.7.1c	Step 1 – Estimate d keeping clear cover 0.75in $d = h_s$ - 0.75 – bar diameter (estimate #6 bar)	d = 9in – 0.75in – 0.75in ≈ 7.5 in	
11.12.2.1 9.3.2.3	Step 2 – Express $V_u = \phi V_n = 4\phi(\sqrt{f_c'}) b_o d$ $= 4\phi(\sqrt{f_c'})\pi(h_c + d)d$ Solve for V_u	V_u=4(0.75)(√5000lb/in2)π(18in+ 7.5in)7.5in = 127,000 lb = 127 k	
	ALTERNATE METHOD with Design Aid SHEAR 5.2		SHEAR 5.2
	Step 1 – Estimate $d \approx 7.5$ in as above		
11.12.1.2 &	Step 2 – Find K3 with h_c=18in and d=7.5in	$K3 = 2199in^2 + (2614in^2 - 2199in^2)\dfrac{(7.5in - 7.0in)}{(8.0in - 7.0in)}$ $= 2406\ in^2$	Table 5.2a
11.12.2.1	Step 3 – Find V_n with f_c' = 5000 psi, and K3 = 2406 in^2,	$V_n = 141k + \dfrac{(2406in^2 - 2000in^2)}{(3000in^2 - 2000in^2)}(212k - 141k)$ = 170 k	Table 5.2b
9.3.2.3	Step 4 - Compute $V_u = \phi V_n$	$V_u = \phi V_n$ = (0.75)170 k = 127 k	

SHEAR EXAMPLE 10 – Determine thickness required for a flat slab based on required perimeter shear strength at an interior round column

Determine the thickness required for a two-way slab to resist a shear force of 152 kips, based on perimeter shear strength at an interior circular column of 20-in diameter if f_c' = 4000 psi for the normal weight slab concrete.

Mechanism of shear prism
that represents perimeter shear strength surface
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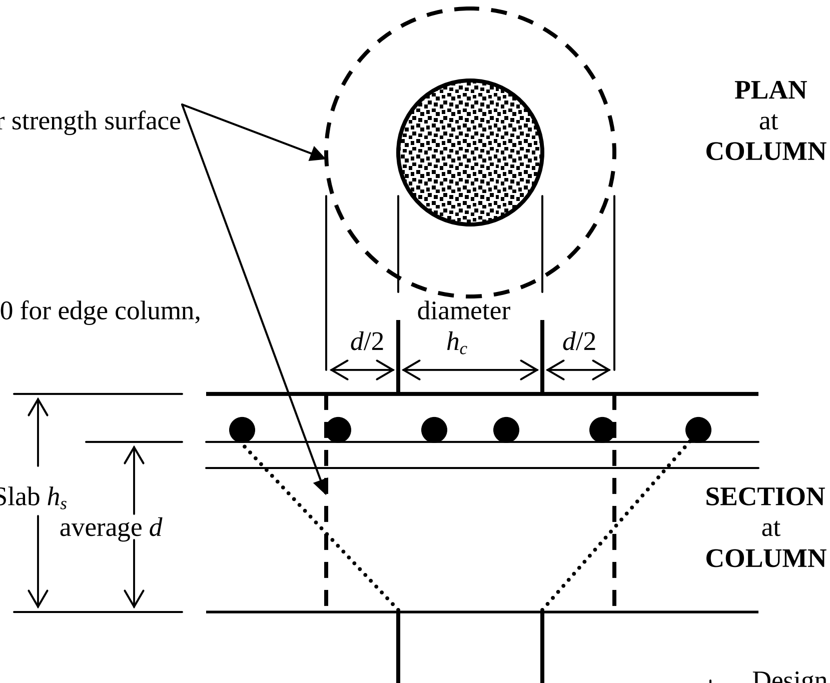


$V_n = 4(a_s d/b_o + 2)(\sqrt{f_c'})\, b_o d$
$\quad \le 4(\sqrt{f_c'}) b_o d$
α_s = 40 for interior column, 30 for edge column,
20 for corner column

b_o = perimeter of shear prism
$= \pi(h_c + d)$

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.12.1.2 11.12.2.1 9.3.2.3	Step 1 – Set up equation, $\phi V_n = 4\phi(\sqrt{f_c'})\,\pi(h_c + d)d$	$\phi V_n = 4(0.75)(\sqrt{4000\text{lb/in}^2})\pi\dfrac{(20\text{in}+d\text{in})}{1000\text{ lb/k}}$ $= 0.596\text{k/in}^2(20d + d^2)\text{in}^2$	
11.12.2.1	Step 2 – Equate V_u to ϕV_n and solve for d.	$152\text{k} = 0.596\text{k/in}^2(20d + d^2)\text{in}^2$ $= (11.92\text{k/in})d + (0.596\text{k/in}^2)d^2$ $255\text{in}^2 = (20\text{in})d + 1.0\, d^2$ $[0.5(20\text{in})]^2 + 255\text{in}^2 = (d - 10\text{in})^2$ $d = \sqrt{(355\text{in}^2)} - 10\text{in} = 18.84\text{in} - 10.0\text{in}$ $= 8.84$ in	
7.7.1c	Step – 3 Make h_s deep enough for 0.75-in concrete cover plus diameter of top bars. Estimate #7 bars.	$h_s \approx 8.84\text{in} + 0.75\text{in} + 0.88\text{in}$ Use $h_s \approx 10.5$ in	
	ALTERNATE METHOD with Design Aid		SHEAR 5.2
9.3.2.3 11.12.1.2	Step 1 – Compute $V_n = V_u/\phi$ With f_c' = 4000 psi and V_n = 203 k, obtain K3	$V_n = 152\text{k}/(0.75) = 203$ k $K3 = 3000\text{in}^2 + (4000\text{in}^2 - 3000\text{in}^2)\dfrac{(203\text{k}-190\text{k})}{(253\text{k}-190\text{k})}$ $K3 = 3206\text{ in}^2$	Table 5.2b
11.12.2.1 7.7.1c	Step 2 – With h_c = 20 in and K3 = 3206 in^2 with 0.75 in cover plus bottom bar diameter	Find $d = 8.0\text{in} + (9\text{in}-8\text{in})\dfrac{(3206\text{in}^2 - 2815\text{in}^2)}{(3280\text{in}^2 - 2815\text{in}^2)}$ $d = 8.84$ in Use $h_s \approx 10.5$ in	Table 5.2a

SHEAR EXAMPLE 11 – Determine thickness of a square footing to satisfy perimeter shear strength under a circular column.

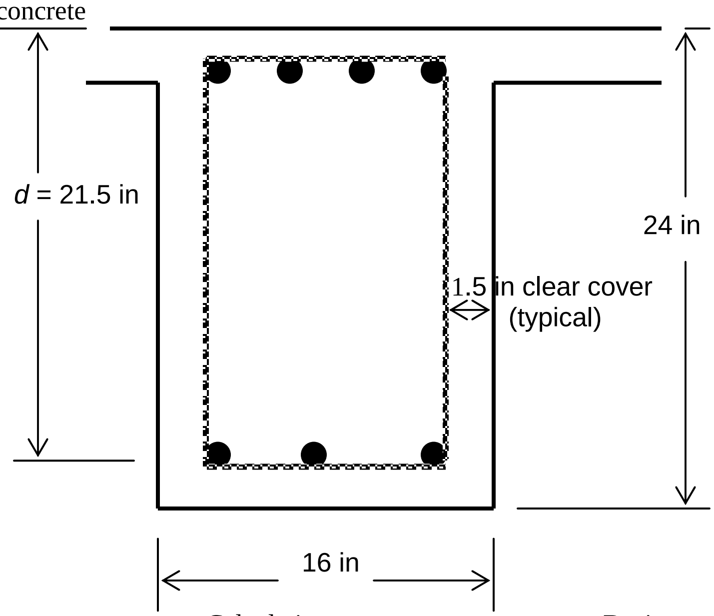
Given: P_u = 262 kips — Footing size = 7 ft by 7 ft with normal weight concrete
f_c' = 3000 psi — Column diameter = 18 in
Grade 60 reinforcement

ACI 318-05 Section	Procedure	Calculation	Design Aid
	Step 1 – Compute net bearing pressure Under P_u $f_{net} = P_u / A_{ftg}$	f_{net} = 262k /(7.0ft x 7.0ft) = 5.35 k/ft^2	
11.12.2.1 11.12.1.2 9.3.2.3	Step 2 – Express $\phi V_c = \phi[4(\sqrt{f_c'})\, b_o d]$ $\phi V_c = \phi[4(\sqrt{f_c'})\pi(h_c + d)d]$	ϕV_c =(0.75)[4($\sqrt{3000}$lb/in^2)π(18in+d)d]/1000lb/k = 0.5162k/in^2(18d + d^2)in^2	
	Step 3 – Express $V_u = f_{net}(A_{ftg} - A_{prism})$	V_u =5.35k/ft2[7ft(7ft)–(π/4)(18in+d)^2/(12in/ft)^2] = 5.35[49 – 1.77 - 0.196d – 0.00545d^2] = 5.35[47.23 - 0.196d – 0.0055d^2] kips	
	Step 4 – Equate $\phi V_c = V_u$ and solve for d	0.5162(18d + d^2) = 5.35[47.23–0.196d–0.0055d^2] 18d + d^2 = 489.7 – 2.031d - 0.0570d^2 1.057d^2 + 20.31d = 489.7 d^2 + 19.21d + 9.61^2 = 463.3 + 92.35 d = $\sqrt{555.6}$ – 9.61 = 13.96 in	
7.7.1a	Step 5 – Allow for 3 in clear cover plus a bottom bar diameter to make footing	h_c = 13.96in + 3in + 0.88in = use 18-in footing	
ALTERNATE METHOD using Design Aid			SHEAR 5.2
	Step 1 – Estimate that area beneath shear prism will be 10% of area beneath footing	estimate V_u = (1.0 – 0.10)262 = 236 kips	
11.12.2.1 11.12.1.2 9.3.2.3	Step 2 – Find K3 with f_c' = 3000 psi and $V_n = V_u / \phi$ = 236/0.75 = 314 kips	K3 =5000in^2+(6000in^2-5000in^2)$\frac{(314\text{k}-274\text{k})}{(329\text{k}-274\text{k})}$ K3 = 5747 in^2	Table 5.2b
	Step 3 – For K3 = 5747 in^2 and col h_c = 18 in, Find footing d	ftg d = 14.0in + 2in$\frac{(5745-5630)\text{in}^2}{(6836-5630)\text{in}^2}$ = 14.1 in	Table 5.2a
7.7.1a	Step 4 – Allow for 3 in clear cover plus one bar diameter to make	h_f = 14.1in + 3in + .88in = use 18 in thick	
	Step 4 – Check Step 1 estimate, A_{prism}/A_{ftg} Since 0.115 > 10% 14.1 in for d may be higher than needed, but conclude that it is OK to use 18-in thick footing.	A_{prism}/A_{ftg} = [(π/4)(18+14.2)^2/144in^2/ft^2]/49ft^2 = 0.115	

SHEAR EXAMPLE 12 – Determine closed ties required for the beam shown to resist flexural shear and determinate torque

Given: f_c' = 5000 psi with normal weight concrete
Grade 60 reinforcement

V_u = 61 kips
T_u = 53 kip-ft determinate



ACI 318-05 Section	Procedure	Calculation	Design Aid
11.0	Step 1 - Determine section properties for torsion, allowing 0.25 in as radius of ties $A_{cp} = b_w h$ $A_{oh} = (b_w - 3.5)(h - 3.5)$ $A_o = 0.85A_{oh}$ $p_{cp} = 2(b_w + h)$ $p_h = 2(b_w - 3.5 + h - 3.5)$	A_{cp} = 16in(24in) = 384 in^2 A_{oh} = (16in – 3.5in)(24in - 3.5in) = 256 in^2 A_o = 0.85(256in) = 218 in^2 p_{cp} = 2(16in + 24in) = 80 in p_h = 2(16in – 3.5in + 24in - 3.5in) = 66 in	
 9.3.2.3 11.6.1	Step 2 – Compute cracking torsion T_{cr} $T_{cr} = 4\phi(\sqrt{f_c'})A_{cp}^2/p_{cp}$ Compute threshold torsion = $0.25T_{cr}$ Since T_u = 29 k-ft > 8.2 k-ft, ties for torsion are required.	 T_{cr} = 4(0.75)($\sqrt{5000}$lb/in^2)(384in)^2/80 = 391,000 in lb T_{cr} =391,000in lb/12000in k/in lb= 32.6 k-ft Threshold torsion = 0.25(32.6 k-ft)=8.2 k-ft	
11.6.3.1	Step 3 – Is section large enough? Compute $f_v = V_u/(b_w d)$ Compute $f_{vt} = T_u p_h/(1.7A_{oh}^2)$ Compute limit = $\phi[2\sqrt{f_c'} + 8\sqrt{f_c'})/1000$ Is $\sqrt{[f_v^2 + f_{vt}^2]}$ < Limit Therefore, section is large enough	 V_u = 61k/(16in x 21.5in) = 0.177 k/in^2 f_{vt} = 53ft-k(12in/ft)66in/[1.7(256in^2)] =0.377 k/in^2 Limit=0.75[2+8}($\sqrt{5000}$lb/in^2)1000lb/k = 0.530 k/in^2 $\sqrt{}$ [(0.177^2 + 0.377^2] = 0.416 < limit 0.53	
11.5.6.2 11.6.3.6 11.6.5.2	Step 4 Compute $A_v/s = [V_u - 2\phi\sqrt{f_c'}(b_w d)]/(\phi f_y d)$ Compute $A_t/s = T_u/[2\phi A_o f_y \cot\theta]$ Compute $(A_v/s + 2A_t/s)$ Use #4 ties for which $((A_v + 2A_t)/s$= 0.40 in, and compute $s = 0.40/(A_v/s + 2A_t/s)$ Is 0.75($\sqrt{f_c'}$)b_w/f_y < $(A_v/s + 2A_t/s)$	A_v/s=61k–[2(0.75)($\sqrt{5000}$k/in^2)(16in)21.5in] / 1000lb/in^2[0.75(60k/in^2)21.5in] =[61-36.4]/967.5 = 0.0253 in^2/in A_t/s = 53ft k (12in/ft) / [2(0.75)218in^2(60k/in^2)cot 45] = 0.324 in $(A_v/s+2A_t/s)$= 0.0253+2(0.0324)= 0.0900 in s = 0.40/(0.0900) = 4.44 in Use 4 in 0.75($\sqrt{5000}$)16/60,000 = 0.0141 < 0.0900	 YES

SHEAR EXAMPLE 12 – continued

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.6.3.7 11.6.5.3	Step 5 – Compute $A_\ell = (A_t/s)(p_h\cot^2 45$ Is $A_{\ell,min} = 5(\sqrt{f_c'})A_{cp}/f_y < A_\ell$?	A_ℓ=(0.20in^2/in/4.0in)66in(1.00)= 3.30 in^2 $A_{\ell,min}$= 5(√5000lb/in^2)384in^2/60,000lb/in^2 = 2.26 in^2 < 3.30 in^2 YES	
	In 8 positions, use #6 for bottom corners and bottom center and use #6 at mid-height in each vertical face. Excess area from flexural bars in top of section is adequate to replace the 3 #6 bars of A_ℓ in top of section.		
ALTERNATE METHOD using design aid			
11.2.1.1 11.5.6.2 11.6.2.2a 11.6.3.1 11.6.3.6	Step 1 – Look up parameters for f_c' = 5000 psi, Grade 60 reinforcement, b_w = 16 in, h = 24 in	$K_{fc}K_{vc}$ = (1.118)43.5k = 48.6 k K_{vs} = 1290 ksi $K_{fc}K_t$ = (1.118)89.1k-ft = 99.6 k-ft $K_{fc}K_{tcr}$ = (1.118)38.9k-ft = 43.5 k-ft K_{ts} = 1089 k-ft/in	SHEAR 2 Table 2a Table 2b SHEAR 6.1a SHEAR 6.1b SHEAR 6.2b
11.6.1a	Step 2 – If T_u = 53>0.25$K_{fc}K_{tcr}$, ties are required	53k-ft > 0.25(43.5k-ft) = 10.9 k-ft Therefore, ties are required.	
	Step 3 – Section is large enough if $\sqrt{[(V_u/(5\phi K_{fc}K_{vc})]^2 + [T_u/(\phi K_{fc}K_t)]^2} < 1$	$\sqrt{\{61k/[5(0.75)48.6k]\}^2 + \{53k\text{-}ft/[0.75(99.6k\text{-}ft)]\}^2}$ $=\sqrt{\{0.335\}^2 + \{0.710\}^2}$ = 0.785 < 1 Therefore, section is large enough.	
	Step 4 – Compute $(A_v/s+2A_t/s)$ = $(V_u/\phi - K_{fc}K_{vc})/K_{vs} + T_u/(\phi K_{ts})$ Compute $s < 0.40/(A_v/s+2A_t/s)$	$(A_v/s+2A_t/s)$=(61k/0.75-48.6k)/1290k/in^2 +53.0k-ft/[(0.75)1089k-ft] = 0.0254 + 0.0649 = 0.0903 in^2/in One #4 tie provides A_v = 0.40 in^2/in s < 0.40/0.0903 = 4.4 in. Use 4 in spacing	
11.6.3.7 11.6.5.3	Step 5 – Compute $A_\ell = (A_t/s)(p_h\cot^2 45)$ Is $A_{\ell,min} = 5(\sqrt{f_c'})A_{cp}/f_y < A_\ell$?	A_ℓ= (0.20/4)66in(1.00) = 3.30 in^2 $A_{\ell,min}$= 5(√5000lb/in^2)384in^2/60,000lb/in^2 = 2.26 in^2 < 3.30 in^2 YES	
11.6.6.2	In 8 positions, use #6 for bottom corners and bottom center and use #6 at mid-height in each vertical face. Excess area from flexural bars in top of section is adequate to replace the 3 #6 bars of A_ℓ in top of section.		

SHEAR EXAMPLE 13 – Determine closed ties required for the beam of Example 12 to resist flexural shear and indeterminate torque

Given: Use the same data as that for SHEAR EXAMPLE 12, except that the required torsion estimate of 51 k-ft is based on an indeterminate analysis, not an equilibrium requirement.

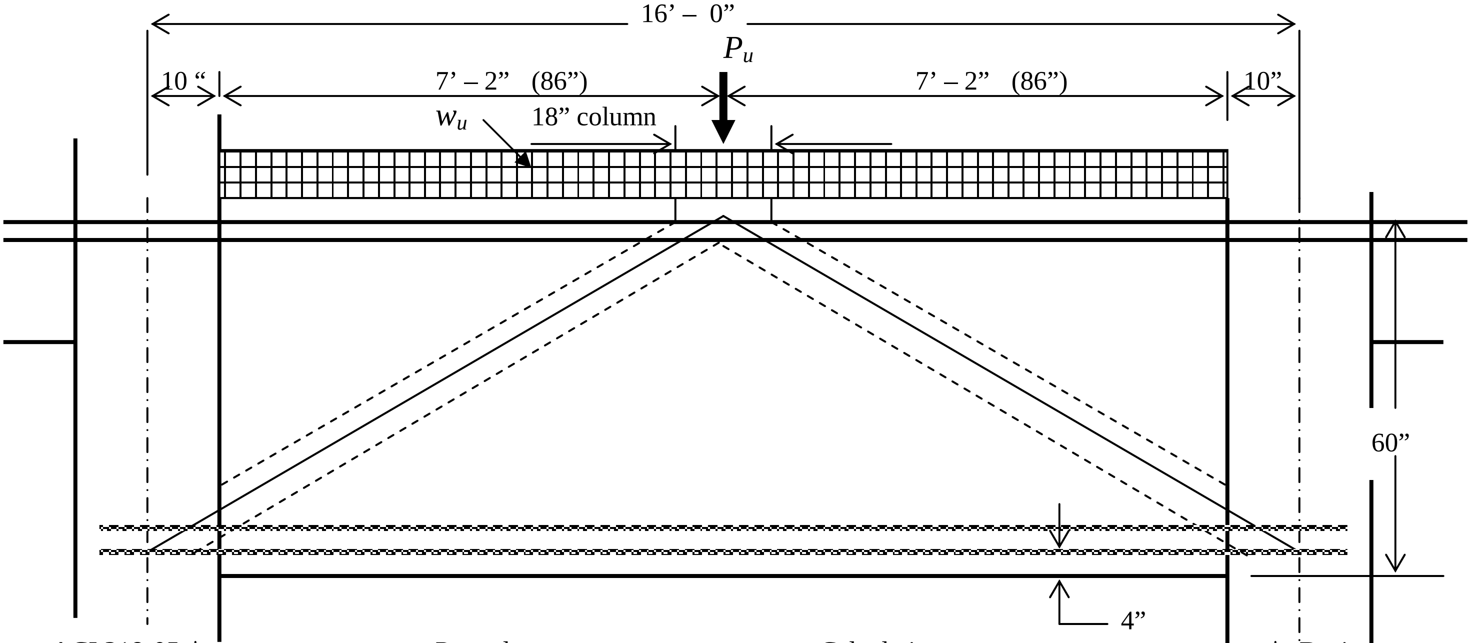
f_c' = 5000 psi	b_w = 16 in	V_u = 61 k
f_y = 60,000 psi	h = 24 in	T_u = 53 k-ft (based on indeterminate analysis)

Assume normal weight concrete

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.2.1.1 11.5.6.2 11.6.2.2a 11.6.3.1 11.6.3.6	Step 1 – Look up parameters for f_c' = 5000 psi, Grade 60 reinforcement, b_w = 16 in, h = 24 in	$K_{fc}K_{vc}$ = (1.118)43.5 = 48.6 k K_{vs} = 1290 ksi $K_{fc}K_t$ = (1.118)89.1 = 99.6 k-ft $K_{fc}K_{tcr}$ = (1.118)38.9 = 43.5 k-ft K_{ts} = 1089 k-ft/in	SHEAR 2 - Table2a & 2c SHEAR 2-2b SHEAR 6.1a SHEAR 6.1b SHEAR 6.2b
11.6.2.2a	Step 2 – If indeterminate $T_u > K_{fc}K_{tcr}$, $K_{fc}K_{tcr}$ can be used as T_u.	$K_{fc}K_{tcr}$ = 43.5 k-ft, which is > 53 k-ft Use T_u = 43.5 k-ft	
11.6.1a	Step 3 – If T_u=43.5>0.25$K_{fc}K_{tcr}$, ties are required	43.5k-ft > 0.25(43.5k-ft) = 10.9 k-ft so ties are required	
	Step 4 – Section is large enough if $\sqrt{[(V_u/(5\phi K_{fc}K_{vc})]^2 + [T_u/(\phi K_{fc}K_t)]^2} < 1$	$\sqrt{\{61k/[5(0.75)48.6k]\}^2 + \{43.5k\text{-}ft/[0.75(99.6k\text{-}ft)]\}^2}$ = = $\sqrt{\{0.335\}^2 + \{0.582\}^2}$ = 0.671 < 1 Therefore section is large enough.	
	Step 4 – Compute $(A_v/s+2A_t/s)$ = $(V_u/\phi - K_{fc}K_{vc})/K_{vs} + T_u/(\phi K_{ts})$ #4 ties provide 0.40 sq in/in Compute $s < 0.40/(A_v/s+2A_t/s)$	= (61k/0.75-48.6k)/1290k/in^2 + 43.5k-ft/(0.75)1089k-ft/in] = 0.0254 + 0.0533 = 0.0787 sq in/in s < 0.40/0.0787 = 5.1 in. Use 5 in spacing	
11.6.3.7 11.6.5.3	Step 5 – Compute $A_\ell = (A_t/s)(p_h \cot^2 45)$ Is $A_\ell > A_{\ell,min} = 5(\sqrt{f_c'})A_{cp}/f_y - p_h A_t/s$	A_ℓ = (0.0533/5.1)66in(1.00) = 1.76 sq in $A_{\ell,min}$ = 5($\sqrt{5000}$lb/in^2)384in^2/60,000lb/in^2 = 2.26 in^2 > 1.76 in^2 YES	
11.6.6.2	In 6 positions, use #5 in bottom corners and center and #5 in each vertical face. Excess flexural bars in top are adequate for the 3 #5 component of A_ℓ in top of section.		

SHEAR EXAMPLE 14 – Deep transfer beam design by strut-and-tie model

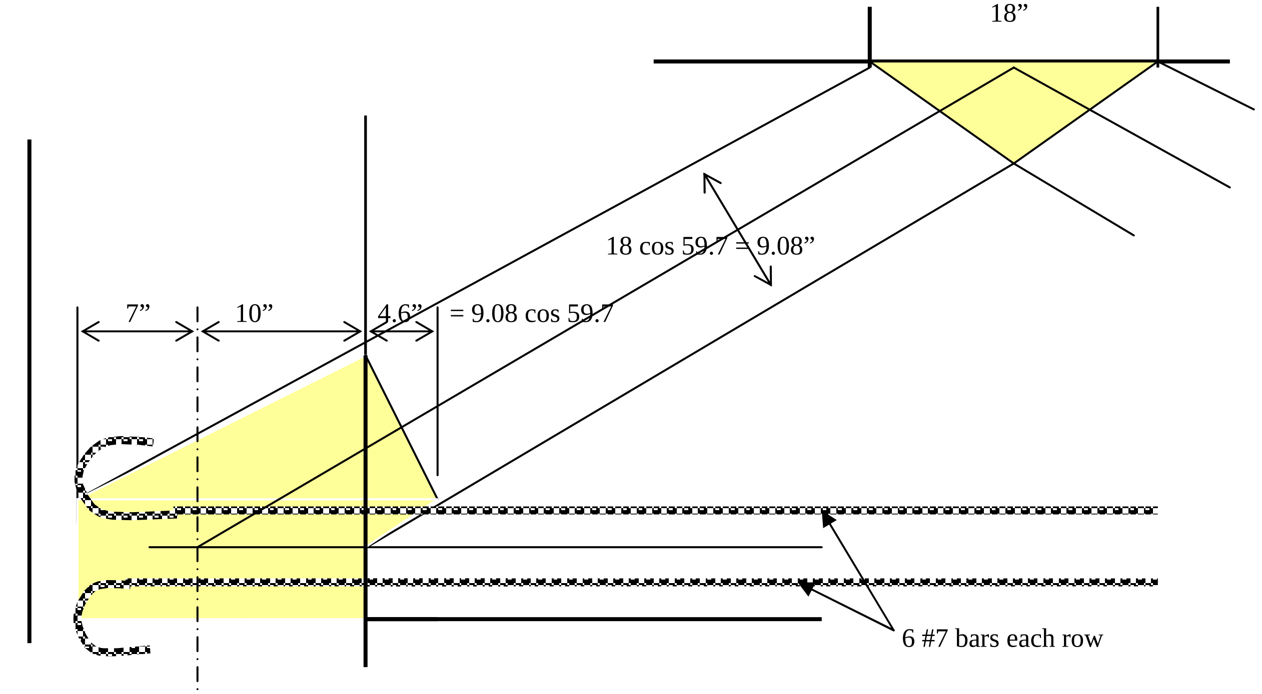
Given: P_u = 318 k. f_c' = 4000 psi normal weight concrete b_w = 14 in
w_u = 6.4 k/ft Grade 60 reinforcement



ACI 318-05 Reference	Procedure	Calculation	Design Aid
11.8.1 11.8.2 A.2.1	Step 1 – This is a deep beam if $\ell_n/d < 4$ This is a deep beam. Use truss as sketched. Compute strut angle $\gamma = \tan^{-1}$ (96in/56in)	ℓ_n/d = 2(86in)/(60in – 4in) = 3.07 < 4 $\gamma = \tan^{-1}$ (1.714) = 59.7°	
A.2.1 A.2.2	Step 2 – Determine strut forces. Compute concentrated load, including $F_v = P_u + 7.17w_u$ Diagonal strut force $F_u = 0.5F_v/\cos\gamma$ Tension strut force $T_u = 0.5F_v \tan\gamma$ Maximum $V_u = 0.5\ F_v$	F_v = 318 + 7.17ft(6.4k/ft) = 364 k F_u = 0.5(364k) /cos 59.7 = 361 k T_u = 0.5(364k) tan 59.7 = 311 k Maximum V_u = 0.5(364k) = 182 k	
11.8.3 9.3.2.6	Step 3 – Compute $10\sqrt{f_c'}(b_w d)$ Compute V_u/ϕ Since $10\sqrt{f_c'}(b_w d) > V_u/\phi$, section is adequate	$10\sqrt{f_c'}(b_w d)$=10($\sqrt{4000\text{lb/in}^2}$)14in(56in)=496,000 lb V_u/ϕ = 182k/0.75 = 243 k = 243, 000 lb	
11.8.4 & 11.8.5	Step 4 – Observe spacing limit s and s_1 < d/5 Using s = 10 in, Compute min $A_v = 0.0025b_w s$ Try 2 #4 vertical bars at 10-in spacing Compute min $A_{vh} = 0.0015b_w s_1$ Try 2 #4 horizontal bars at 11-in spacing	limit s and s_1 = 56in/ 5 = 11 in min A_v = 0.0025(16in)10in = 0.400 sq in min A_{vh} = 0.0015(16in)11in = 0.264 sq in	
A.3.2.1 A.3.2 A.2.6 A.3.1	Step 5 – Consider strut of uniform width (β_s = 1) Compute $f_{cu} = 0.85\beta_s f_c'$ Compute $F_{ns} = F_u/\phi$ Compute strut area $A_c = F_{ns}/f_{cu}$ Compute strut width $w_s = A_c/b_w$	 f_{cu} = 0.85(1.0)4000psi = 3400 psi = 3.4 ksi F_{ns} = 361k/0.75 = 481 k A_c = 481k / 3.4k/in^2 = 141 sq in w_s = 141in^{2}k/ 16in = 8.81 in	
A.4.1	Step 6 – Tension tie $A_{st} > T_u/(\phi f_y)$ Use 12 #7 bars hooked at columns	A_{st} > 311k/[0.75(60k/in^2)] = 6.91 sq in	

SHEAR EXAMPLE 14 – Deep transfer beam design by strut-and-tie model **(continued)**

Nodal zone at supported column



Nodal zone at supporting column

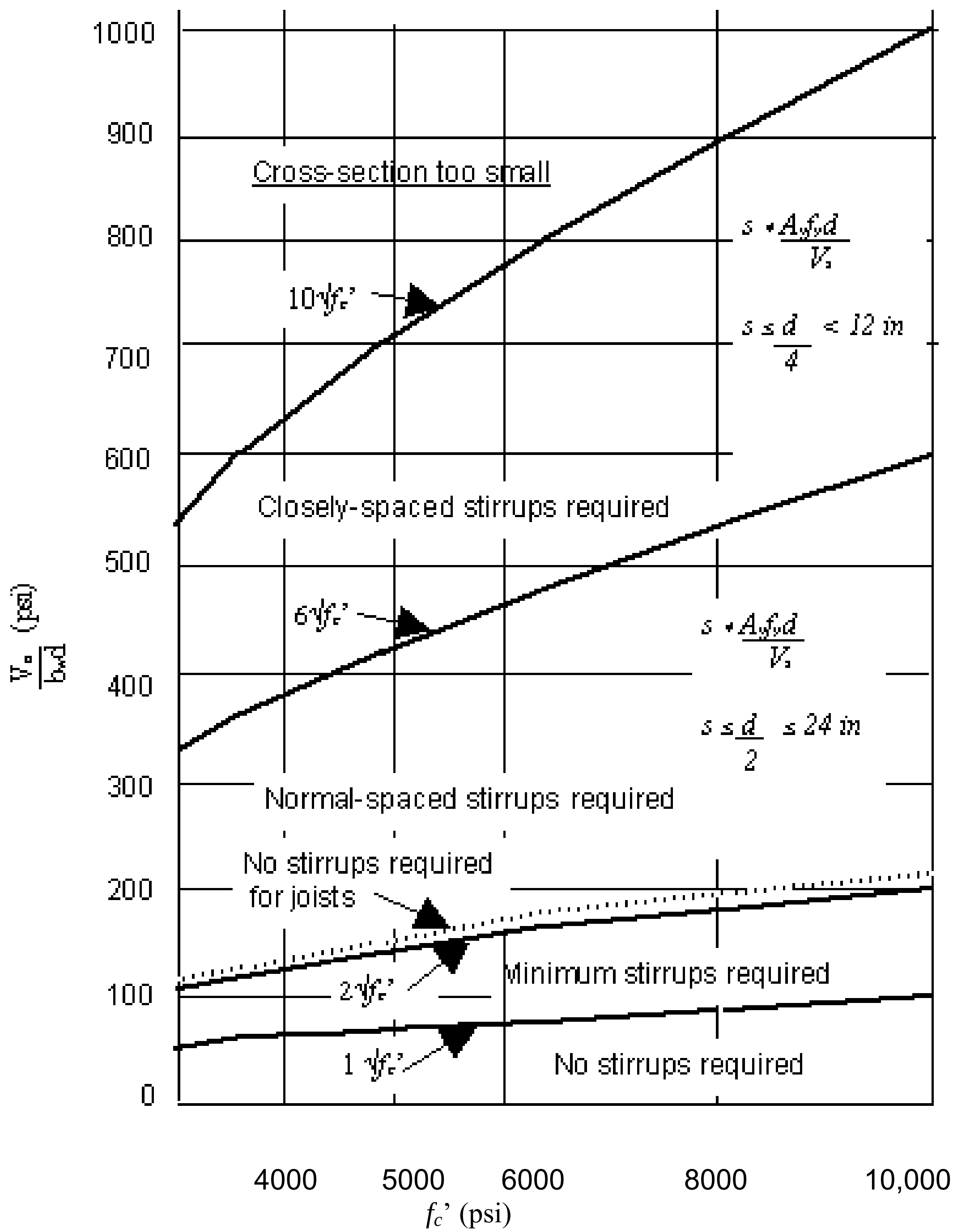
ACI 318-05 Reference	Procedure	Calculation	Design Aid
A.5.1 A.3.2.1	Step 7 – Check width of strut from C-C-C node at base of supported column. w_s = 18 cos59.7 Since w_s = 8.81 in < 9.08 in available, strut will be of uniform cross section area.	strut width = 18 cos59.7 = 9.08 in available	
A.4.3.2 REINFORCE-	Step 8 – Is nodal zone at supporting column long enough to develop # 7 bars ? 18 in are available, 60 ksi can be developed.	For #7 hooked bars, l_{hb} = 16.6 in	 MENT 18.1
A.3.3.1	Step 9 – Check Eq (A-4) $\Sigma A_{vi} \sin \gamma_{vi} /(b_w s_i)$ >0.003 Vertical: $A_v \sin \gamma_v /(b_w s)$ = Horizontal: $A_{vh} \sin (90-\gamma_v)/(b_w s_1)$ = Alternate, using Design Aid For $\gamma_v = 65^o$, $b_w s$ = 160, with #4 bars For $\gamma_v = 55^o$, $b_w s$ = 160, #4 bars Interpolate for $\gamma_v = 59.7^o$ For $\gamma_h = 25^o$, $b_w s_1$ = 176, #4 bars For $\gamma_h = 35^o$, $b_w s_1$ = 176, #4 bars Interpolate for $\gamma_h = 30.3^o$	 2(0.20) sin 59.7 /[16(10)] = 0.00216 2(0.20) sin 30.3 /[16(11)}= 0.00115 $\Sigma A_{vi} \sin \gamma_{vi} /(b_w s_i)$ = 0.00331 OK $A_v \sin \gamma_v /(b_w s)$ = 0.00227 $A_v \sin \gamma_v /(b_w s)$ = 0.00205 $A_v \sin \gamma_v /(b_w s)$ = 0.00216 $A_{vh} \sin \gamma_h /(b_w s_1)$ = 0.00099 $A_{vh} \sin \gamma_h /(b_w s_1)$ = 0.00134 $A_v \sin \gamma_v /(b_w s)$ = 0.00117 $\Sigma A_{vi} \sin \gamma_{vi} /(b_w s_i)$ = 0.00333 OK	 SHEAR 7 SHEAR 7 SHEAR 7 SHEAR 7

SHEAR 1 – Section limits based on required nominal shear stress = $V_n = V_u / (b_w d)$

Reference: ACI 318-02, Sections 11.11.1, 11.3.1.1, 11.5.4, 11.5.6.2, 11.2.6.8,.and 8.11.8

Section 11.2.1.1 states that when f_{ct} is specified for lightweight concrete, substitute $f_{ct}/6.7$ for $\sqrt{f_c'}$, but $f_{ct}/6.7$ must be $\leq \sqrt{f_c'}$.

$$V_n \geq \frac{V_u}{\phi}$$



SHEAR 2 - Shear strength coefficients K_{fc}, K_{vc} and K_{vs}

Reference: ACI 318-05, Sections 11.2.1.1 and 11.5.6.2

$V_{cn} = 2(\sqrt{f_c'})b_w d = 126K_{fc}$ $V_{sn} = A_v f_y d/s = A_v K_{vs}/s$ $V_n = V_{cn} + V_{sn}$ $b_w = b$

$K_{fc} = \sqrt{(f_c'/4000)}$ $K_{vs} = f_y\, d$ (kips) $K_{vc} = (2\sqrt{4000})b_w d/1000$ (kips)

Section 11.2.1.1 states when f_{ct} is specified for lightweight concrete, substitute $f_{ct}/6.7$ for $\sqrt{f_c'}$, but keep $f_{ct}/6.7 \# \sqrt{f_c'}$.

Table 2a Values K_{tc} for various values f_c'

f_c' psi	3000	4000	5000	6000	8000	10000
K_{fc}	0.866	1.000	1.118	1.225	0.707	1.581

For Table 2b and Table 2c,
d = h - 2.5 if $h < 30$
d = h - 3.0 if $h > 30$

Table 2b

Values K_{vs} (k/in)

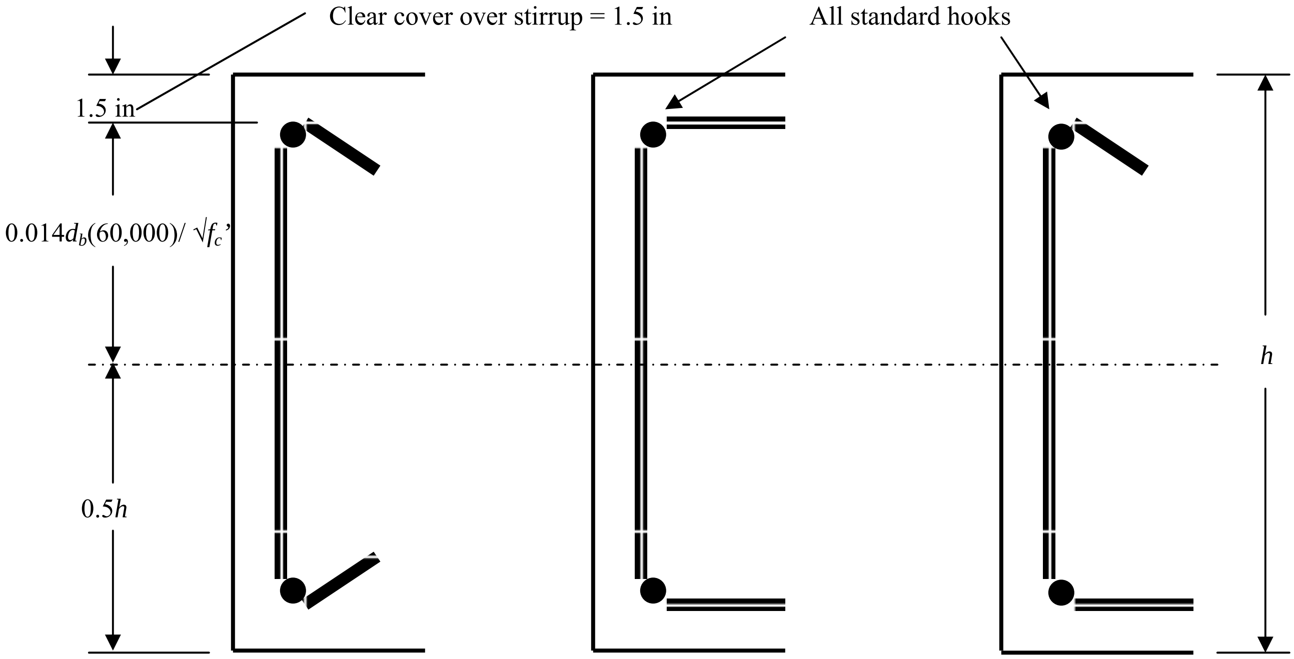
Beam h (ksi)	f_y(ksi) 40	60
10	300	450
12	380	570
14	460	690
16	540	810
18	620	930
20	700	1050
22	780	1170
24	860	1290
26	940	1410
28	1020	1530
30	1100	1620
32	1160	1740
34	1240	1860
36	1320	1980
38	1400	2100
40	1480	2220
42	1560	2340
44	1640	2460
46	1720	2580
48	1800	2700

Table 2c Values K_{vc} (kips)

Beam h (in) \ b (in)	10	12	14	16	18	20	22	24	26	28	30	32
10	9.5	11.4	13.3	15.2	17.1	19.0	20.9	22.8	24.7	26.6	28.5	30.4
12	12.0	14.4	16.8	19.2	21.6	24.0	26.4	28.8	31.2	33.6	36.1	38.5
14	14.5	17.5	20.4	23.3	26.2	29.1	32.0	34.9	37.8	40.7	43.6	46.6
16	17.1	20.5	23.9	27.3	30.7	34.2	37.6	41.0	44.4	47.8	51.2	54.6
18	19.6	23.5	27.5	31.4	35.3	39.2	43.1	47.1	51.0	54.9	58.8	62.7
20	22.1	26.6	31.0	35.4	39.8	44.3	48.7	53.1	57.6	62.0	66.4	70.8
22	24.7	29.6	34.5	39.5	44.4	49.3	54.3	59.2	64.1	69.1	74.0	78.9
24	27.2	32.6	38.1	43.5	49.0	54.4	59.8	65.3	70.7	76.2	81.6	87.0
26	29.7	35.7	41.6	47.6	53.5	59.5	65.4	71.3	77.3	83.2	89.2	95.1
28	32.3	38.7	45.2	51.6	58.1	64.5	71.0	77.4	83.9	90.3	96.8	103.2
30	34.2	41.0	47.8	54.6	61.5	68.3	75.1	82.0	88.8	95.6	102.5	109.3
32	36.7	44.0	51.4	58.7	66.0	73.4	80.7	88.0	95.4	102.7	110.1	117.4
34	39.2	47.1	54.9	62.7	70.6	78.4	86.3	94.1	102.0	109.8	117.6	125.5
36	41.7	50.1	58.4	66.8	75.1	83.5	91.8	100.2	108.5	116.9	125.2	133.6
38	44.3	53.1	62.0	70.8	79.7	88.6	97.4	106.3	115.1	124.0	132.8	141.7
40	46.8	56.2	65.5	74.9	84.2	93.6	103.0	112.3	121.7	131.1	140.4	149.8
42	49.3	59.2	69.1	78.9	88.8	98.7	108.5	118.4	128.3	138.1	148.0	157.9
44	51.9	62.2	72.6	83.0	93.4	103.7	114.1	124.5	134.8	145.2	155.6	166.0
46	54.4	65.3	76.2	87.0	97.9	108.8	119.7	130.5	141.4	152.3	163.2	174.1
48	56.9	68.3	79.7	91.1	102.5	113.9	125.2	136.6	148.0	159.4	170.8	182.2

SHEAR 3 – Minimum beam height to provide development length required for #6, #7 and #8 Grade 60 stirrups

Reference: ACI 318-05, Section 12.13.2.1 and Section 12.13.2.2
ACI 318-05, Section 11.5.2 states "Design yield strength of shear reinforcement (bars) shall not exceed 60,000 psi, …."



Minimum beam height $h = 2[0.014d_b(60{,}000)/\sqrt{f_c'} + 1.5]$ in inches

Minimum beam height h (in)			
Stirrup size / Concrete f_c'	#6	#7	#8
3000	26.0	29.8	33.7
4000	22.9	26.2	29.6
5000	20.8	23.8	26.8
6000	19.3	22.0	24.7
8000	17.1	19.4	21.8
10000	15.6	17.7	19.8

*Values shown in the table are for 1.5 in clear cover over stirrups. For cover greater than 1.5 in, add 2(cover-1.5) to tabulated values.

Example: Determine whether a beam 24 in high (h = 24 in) with 5000 psi concrete will provide sufficient development length for #6 Grade 60 vertical stirrups.

Solution: For #6 stirrups, minimum beam height reads 20.8 in for beams with 1.5-in clear cover over stirrups. Since h = 24 in, the beam is deep enough.

SHEAR 4.1 – Shear strength V_{sn} with Grade 40 stirrups

Reference: ACI 318-05, Sections 11.5.5.3 and 11.5.6.2

$V_s = V_n - V_c = A_v f_y(d/s)$ ${}_{Maximum}b_w = A_v f_y/(50s)$ if $f_c' < 4440$ psi

${}_{Maximum}b_w = 4A_v f_y/(3s\sqrt{f_c'})$ if $f_c' > 4440$ psi

TABLE 4.1a

Stirrup size	spa. s (in) / depth d (in)	Values of V_s (kips) 2	3	4	5	6	7	8	9	10	11	12	14	16	18	20
	8	35	23	18												
	10	44	29	22	18											
	12	53	35	26	21	18										
	14	62	41	31	25	21	18									
	16	70	47	35	28	23	20	18								
	18	79	53	40	32	26	23	20	18							
	20	88	59	44	35	29	25	22	20	18						
	22	97	65	48	39	32	28	24	22	19	18					
#3	**24**	106	70	53	42	35	30	26	23	21	19	18				
stirrups	**26**	114	76	57	46	38	33	29	25	23	21	19				
	28	123	82	62	49	41	35	31	27	25	22	21	18			
	30	132	88	66	53	44	38	33	29	26	24	22	19			
	32	141	94	70	56	47	40	35	31	28	26	23	20	18		
	34	150	100	75	60	50	43	37	33	30	27	25	21	19		
	36	158	106	79	63	53	45	40	35	32	29	26	23	20	18	
	38	167	111	84	67	56	48	42	37	33	30	28	24	21	19	
	40	176	117	88	70	59	50	44	39	35	32	29	25	22	20	18
Maximum b_w (in) for f_c'<4440 psi		**88**	**59**	**44**	**35**	**29**	**25**	**22**	**20**	**18**	**16**	**15**	**13**	**11**	**10**	**9**

Above the lines spacings are > d/2

TABLE 4.1b

Stirrup size	spa. s (in) / depth d (in)	Values of V_s (kips) 2	3	4	5	6	7	8	9	10	11	12	14	16	18	20
	8	64	43	32												
	10	80	53	40	32											
	12	96	64	48	38	32										
	14	112	75	56	45	37	32									
	16	128	85	64	51	43	37	32								
	18	144	96	72	58	48	41	36	32							
	20	160	107	80	64	53	46	40	36	32						
	22	176	117	88	70	59	50	44	39	35	32					
#4	**24**	192	128	96	77	64	55	48	43	38	35	32				
stirrups	**26**	208	139	104	83	69	59	52	46	42	38	35				
	28	224	149	112	90	75	64	56	50	45	41	37	32			
	30	240	160	120	96	80	69	60	53	48	44	40	34			
	32	256	171	128	102	85	73	64	57	51	47	43	37	32		
	34	272	181	136	109	91	78	68	60	54	49	45	39	34		
	36	288	192	144	115	96	82	72	64	58	52	48	41	36	32	
	38	304	203	152	122	101	87	76	68	61	55	51	43	38	34	
	40	320	213	160	128	107	91	80	71	64	58	53	46	40	36	32
Maximum b_w (in) for f_c'<4440 psi		**160**	**107**	**80**	**64**	**53**	**46**	**40**	**36**	**32**	**29**	**27**	**23**	**20**	**18**	**16**

Above the lines spacings are > d/2

SHEAR 4.2 – Shear strength V_{sn} with Grade 60 stirrups

Reference: ACI 318-05, Sections 11.5.5.3 and 11.5.6.2

$V_s = V_n - V_c = A_v f_y(d/s)$ $\qquad$ ${}_{Maximum}b_w = A_v f_y/(50s)$ if $f_c' < 4440$ psi

${}_{Maximum}b_w = 4A_v f_y/(3s\sqrt{f_c'})$ if $f_c' > 4440$ psi

TABLE 4.2a

		Values of V_s (kips)														
Stirrup size	spa. s (in)	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**14**	**16**	**18**	**20**
	depth d (in)															
	8	53	35	26												
	10	66	44	33	26						**Above the lines**					
	12	79	53	40	32	26						**spacings are > d/2**				
	14	92	62	46	37	31	26									
	16	106	70	53	42	35	30	26								
	18	119	79	59	48	40	34	30	26							
	20	132	88	66	53	44	38	33	29	26						
	22	145	97	73	58	48	41	36	32	29	26					
#3	**24**	158	106	79	63	53	45	40	35	32	29	26				
stirrups	**26**	172	114	86	69	57	49	43	38	34	31	29				
	28	185	123	92	74	62	53	46	41	37	34	31	26			
	30	198	132	99	79	66	57	50	44	40	36	33	28			
	32	211	141	106	84	70	60	53	47	42	38	35	30	26		
	34	224	150	112	90	75	64	56	50	45	41	37	32	28		
	36	238	158	119	95	79	68	59	53	48	43	40	34	30	26	
	38	251	167	125	100	84	72	63	56	50	46	42	36	31	28	
	40	264	176	132	106	88	75	66	59	53	48	44	38	33	29	26
Maximum b_w (in) for f_c'<4440 psi		**132**	**88**	**66**	**53**	**44**	**38**	**33**	**29**	**26**	**24**	**22**	**19**	**17**	**15**	**13**

TABLE 4.2b

		Values of V_s (kips)														
Stirrup size	spa. s (in)	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**14**	**16**	**18**	**20**
	depth d (in)															
	8	96	64	48												
	10	120	80	60	48						**Above the lines**					
	12	144	96	72	58	48						**spacings are > d/2**				
	14	168	112	84	67	56	48									
	16	192	128	96	77	64	55	48								
	18	216	144	108	86	72	62	54	48							
	20	240	160	120	96	80	69	60	53	48						
	22	264	176	132	106	88	75	66	59	53	48					
#4	**24**	288	192	144	115	96	82	72	64	58	52	48				
stirrups	**26**	312	208	156	125	104	89	78	69	62	57	52				
	28	336	224	168	134	112	96	84	75	67	61	56	48			
	30	360	240	180	144	120	103	90	80	72	65	60	51			
	32	384	256	192	154	128	110	96	85	77	70	64	55	48		
	34	408	272	204	163	136	117	102	91	82	74	68	58	51		
	36	432	288	216	173	144	123	108	96	86	79	72	62	54	48	
	38	456	304	228	182	152	130	114	101	91	83	76	65	57	51	
	40	480	320	240	192	160	137	120	107	96	87	80	69	60	53	48
Maximum b_w (in) for f_c'<4440 psi		**240**	**160**	**120**	**96**	**80**	**69**	**60**	**53**	**48**	**44**	**40**	**34**	**30**	**27**	**24**

SHEAR 5.1 – Shear capacity of slabs based on perimeter shear at interior rectangular columns ($\alpha_s = 40$)

Reference: ACI 318-05, Sections 11.12.1.2 and 11.12.2.1

$V_c = (\mathbf{K1})(\mathbf{K2})\sqrt{f_c'} \geq V_u/\phi$ K1 = $8(b+h+2d)d/1000$ K2 = $(2 + 4/\beta_c)/4$

$V_c = (2 + 4/\beta_c)(\sqrt{f_c'})b_o d$ (11-33) $\beta_c = \dfrac{\text{longer dimension of column section}}{\text{shorter dimension of column section}}$

$V_c = (2 + a_s d/b_o)(\sqrt{f_c'})b_o d$ (11-34)

$V_c = 4(\sqrt{f_c'})b_o d$ (11-35)

Note: Eq. (11-35) governs if $8d > b + h$, or if $40d/b_o \geq 2$, or if $\beta_c < 2$

TABLE 5.1a Values K1 (ksi)

d (in) / *b* + *h* (in)	3	4	5	6	7	8	9	10	12	14	16	18	20
16	0.53	0.77	1.04	1.34	1.68	2.05	2.45	2.88	3.84	4.93	6.14	7.49	8.96
20	0.62	0.90	1.20	1.54	1.90	2.30	2.74	3.20	4.22	5.38	6.66	8.06	9.60
24	0.72	1.02	1.36	1.73	2.13	2.56	3.02	3.52	4.61	5.82	7.17	8.64	10.24
28		1.15	1.52	1.92	2.35	2.82	3.31	3.84	4.99	6.27	7.68	9.22	10.88
32		1.28	1.68	2.11	2.58	3.07	3.60	4.16	5.38	6.72	8.19	9.79	11.52
36			1.84	2.30	2.80	3.33	3.89	4.48	5.76	7.17	8.70	10.37	12.16
40			2.00	2.50	3.02	3.58	4.18	4.80	6.14	7.62	9.22	10.94	12.80
44				2.69	3.25	3.84	4.46	5.12	6.53	8.06	9.73	11.52	13.44
48				2.88	3.47	4.10	4.75	5.44	6.91	8.51	10.24	12.10	14.08
52					3.70	4.35	5.04	5.76	7.30	8.96	10.75	12.67	14.72
56					3.92	4.61	5.33	6.08	7.68	9.41	11.26	13.25	15.36
60						4.86	5.62	6.40	8.06	9.86	11.78	13.82	16.00
64						5.12	5.90	6.72	8.45	10.30	12.29	14.40	16.64
68							6.19	7.04	8.83	10.75	12.80	14.98	17.28
72							6.48	7.36	9.22	11.20	13.31	15.55	17.92
76								7.68	9.60	11.65	13.82	16.13	18.56
80								8.00	9.98	12.10	14.34	16.70	19.20

TABLE 5.1b Values K2

β_c	≤ 2	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.5	5.0
K2	1.000	0.955	0.917	0.885	0.857	0.833	0.813	0.794	0.778	0.763	0.750	0.722	0.700

TABLE 5.1c Values V_c (kips)

K1*K2 (ksi) / f_c' (psi)	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	10.00	12.00	16.00	20.00
3000	55	110	164	219	274	329	383	438	548	657	876	1095
4000	63	126	190	253	316	379	443	506	632	759	1012	1265
5000	71	141	212	283	354	424	495	566	707	849	1131	1414
6000	77	155	232	310	387	465	542	620	775	930	1239	1549
8000	89	179	268	358	447	537	626	716	894	1073	1431	1789
10000	100	200	300	400	500	600	700	800	1000	1200	1600	2000

SHEAR 5.2 – Shear Capacity of Slabs Based on Perimeter Shear on Interior Round Columns

Reference: ACI 318-05, Sections 11.12.1.2 and 11.12.2.1

$$V_n = V_c = (\text{K3})\ \sqrt{f_c'}\quad \text{(kips)}$$

with h_c = column diameter (in) d = slab depth (in)

K3 = $4\pi d(d + h_c)$ if $h_c < 5.37d$

K3 = $2\pi d(h_c + 7.37d)$ if $h_c > 5.37d$ for which Table 5.2a values are in italics

TABLE 5.2a Values K3 (in^2)

d (in)	3	4	5	6	7	8	9	10	12	14	16	18	20
Col h (in)													
8	415	603	817	1056	1319	1608	1923	2262	3016	3870	4825	5881	7037
10	490	704	942	1206	1495	1810	2149	2513	3317	4222	5227	6333	7540
12	565	804	1068	1357	1671	2011	2375	2765	3619	4574	5630	6786	8042
14	641	905	1194	1508	1847	2212	2601	3016	3921	4926	6032	7238	8545
16	716	1005	1319	1659	2023	2413	2827	3267	4222	5278	6434	7690	9048
18	*756*	1106	1445	1810	2199	2614	3054	3518	4524	5630	6836	8143	9550
20	*794*	1206	1571	1960	2375	2815	3280	3770	4825	5981	7238	8595	10053
22	*831*	*1294*	1696	2111	2551	3016	3506	4021	5127	6333	7640	9048	10555
24	*869*	*1344*	1822	2262	2727	3217	3732	4272	5429	6685	8042	9500	11058
26	*907*	*1394*	1948	2413	2903	3418	3958	4524	5730	7037	8444	9952	11561
28	*945*	*1445*	*2037*	2563	3079	3619	4184	4775	6032	7389	8846	10405	12063
30	*982*	*1495*	*2100*	2714	3255	3820	4411	5026	6333	7741	9249	10857	12566
32	*1020*	*1545*	*2163*	2865	3431	4021	4637	5278	6635	8093	9651	11309	13069
34	*1058*	*1595*	*2226*	*2949*	3606	4222	4863	5529	6936	8444	10053	11762	13571
36	*1095*	*1646*	*2289*	*3024*	3781	4421	5087	5778	7235	8792	10450	12208	14067
38	*1133*	*1696*	*2351*	*3100*	*3940*	4624	5315	6032	7540	9148	10857	12667	14577
40	*1171*	*1746*	*2414*	*3175*	*4028*	4825	5542	6283	7841	9500	11259	13119	15079

TABLE 5.2b Values of V_n (kips)

Fc' (psi)	3000	4000	5000	6000	8000	10000
K3 (sq.in)						
700	38	44	49	54	63	70
1000	55	63	71	77	89	100
2000	110	127	141	155	179	200
3000	164	190	212	232	268	300
4000	219	253	283	310	358	400
5000	274	316	354	387	447	500
6000	329	380	424	465	537	600
7000	383	443	495	542	626	700
8000	438	506	566	620	716	800
9000	493	569	636	697	805	900
10000	548	633	707	775	894	1000
12000	657	759	849	930	1073	1200
14000	767	886	990	1084	1252	1400
16000	876	1012	1131	1239	1431	1600
18000	986	1139	1273	1394	1610	1800
20000	1095	1265	1414	1549	1789	2000
22000	1205	1392	1556	1704	1968	2200
24000	1314	1518	1697	1859	2147	2400
26000	1424	1645	1838	2014	2325	2600
28000	1534	1771	1980	2169	2504	2800

SHEAR 6.1 – Shear and Torsion Coefficients K_t and K_{tcr}

Reference: ACI 318-05, Sections 11.6.2.2a and 11.6.3.1

Use SHEAR 2, Table 2a for values of K_{fc}

Limit $T_n = K_{fc}K_t = 17(\sqrt{f_c'})(A^2_{oh})/p_h$ Cracking $T_{cr} = K_{fc}K_{tcr} = 4(\sqrt{f_c'})(A^2_{cp})/p_{cp}$

with $A_{oh} = (b-3.5)(h-3.5)$ $p_h = 2(b + h - 7)$ $p_{cp} = 2(b + h)$

Table 6.1a Values K_t (k-ft)

Beam *b* (in)	10	12	14	16	18	20	22	24	26	28	30
Beam *h* (in)											
10	6.2	9.1	12.3	15.6	19.0	22.4	25.9	29.5	33.0	36.7	40.3
12	9.1	13.8	18.8	24.1	29.6	35.2	41.0	46.9	52.9	58.9	64.9
14	12.3	18.8	25.9	33.6	41.5	49.8	58.3	67.0	75.8	84.7	93.7
16	15.6	24.1	33.6	43.8	54.5	65.7	77.3	89.1	101.3	113.6	126.0
18	19.0	29.6	41.5	54.5	68.3	82.7	97.7	113.1	128.9	145.0	161.3
20	22.4	35.2	49.8	65.7	82.7	100.6	119.3	138.5	158.3	178.6	199.2
22	25.9	41.0	58.3	77.3	97.7	119.3	141.8	165.2	189.3	214.0	239.3
24	29.5	46.9	67.0	89.1	113.1	138.5	165.2	193.0	221.7	251.1	281.3
26	33.0	52.9	75.8	101.3	128.9	158.3	189.3	221.7	255.2	289.7	325.0
28	36.7	58.9	84.7	113.6	145.0	178.6	214.0	251.1	289.7	329.4	370.3
30	40.3	64.9	93.7	126.0	161.3	199.2	239.3	281.3	325.0	370.3	416.9
32	43.9	71.1	102.9	138.7	177.9	220.2	265.0	312.1	361.2	412.1	464.6
34	47.6	77.2	112.1	151.4	194.7	241.4	291.1	343.4	398.1	454.8	513.4
36	51.3	83.4	121.3	164.3	211.7	262.9	317.6	375.2	435.6	498.3	563.2
38	54.9	89.6	130.6	177.3	228.8	284.7	344.3	407.4	473.6	542.5	613.9
40	58.6	95.8	140.0	190.3	246.1	306.6	371.4	440.0	512.1	587.3	665.3

Table 6.1b Values K_{tcr} (k-ft)

Beam *b* (in)	10	12	14	16	18	20	22	24	26	28	30
Beam *h* (in)											
10	5.3	6.9	8.6	10.4	12.2	14.1	15.9	17.9	19.8	23.3	23.7
12	6.9	9.1	11.4	13.9	16.4	19.0	21.6	24.3	27.0	31.9	32.5
14	8.6	11.4	14.5	17.6	20.9	24.3	27.8	31.3	34.9	41.4	42.3
16	10.4	13.9	17.6	21.6	25.7	30.0	34.4	38.9	43.4	51.6	52.8
18	12.2	16.4	20.9	25.7	30.7	35.9	41.3	46.8	52.5	62.4	64.0
20	14.1	19.0	24.3	30.0	35.9	42.2	48.6	55.2	62.0	73.9	75.9
22	15.9	21.6	27.8	34.4	41.3	48.6	56.1	63.9	71.8	85.8	88.3
24	17.9	24.3	31.3	38.9	46.8	55.2	63.9	72.9	82.1	98.2	101.2
26	19.8	27.0	34.9	43.4	52.5	62.0	71.8	82.1	92.6	111.0	114.5
28	21.7	29.7	38.6	48.1	58.2	68.9	80.0	91.5	103.4	124.1	128.2
30	23.7	32.5	42.3	52.8	64.0	75.9	88.3	101.2	114.5	137.5	142.3
32	25.7	35.3	46.0	57.6	69.9	83.0	96.7	111.0	125.8	151.3	156.7
34	27.7	38.1	49.8	62.4	75.9	90.3	105.3	121.0	137.3	165.3	171.3
36	29.7	41.0	53.5	67.2	82.0	97.6	114.0	131.1	148.9	179.5	186.3
38	31.7	43.8	57.4	72.2	88.1	105.0	122.8	141.4	160.8	193.9	201.4
40	33.7	46.7	61.2	77.1	94.2	112.4	131.6	151.8	172.7	208.6	216.8

SHEAR 6.2 – Shear and Torsion Coefficients K_{ts}

Reference: ACI 318-05, Section 11.6.3.6

$$T_n = (2A_oA_tf_y / s)\cot\theta = K_{ts}(A_t/s) \text{ k-ft}$$

with $A_o = 0.85(h\text{-}3.5)(b\text{-}3.5)$ and $\theta = 45$ degrees

TABLE 6.2a Values K_{ts} (ft-k/in) with Grade 40 ties

Beam *h* (in) \ Beam *b* (in)	10	12	14	16	18	20	22	24	26	28	30
10	120	157	193	230	267	304	341	377	414	451	488
12	157	205	253	301	349	397	445	494	542	590	638
14	193	253	312	372	431	491	550	610	669	729	788
16	230	301	372	443	513	584	655	726	797	868	938
18	267	349	431	513	596	678	760	842	924	1006	1089
20	304	397	491	584	678	771	865	958	1052	1145	1239
22	341	445	550	655	760	865	970	1074	1179	1284	1389
24	377	494	610	726	842	958	1074	1191	1307	1423	1539
26	414	542	669	797	924	1052	1179	1307	1434	1562	1689
28	451	590	729	868	1006	1145	1284	1423	1562	1701	1839
30	488	638	788	938	1089	1239	1389	1539	1689	1839	1989
32	525	686	848	1009	1171	1332	1494	1655	1817	1978	2140
34	562	734	907	1080	1253	1426	1599	1771	1944	2117	2290
36	598	783	967	1151	1335	1519	1703	1887	2072	2256	2440
38	635	831	1026	1222	1417	1613	1808	2004	2199	2395	2590
40	672	879	1086	1293	1499	1706	1913	2120	2327	2533	2740

TABLE 6.2b Values K_{ts} (ft-k/in) with Grade 60 ties

Beam *h* (in) \ Beam *b* (in)	10	12	14	16	18	20	22	24	26	28	30
10	180	235	290	345	401	456	511	566	622	677	732
12	235	307	379	452	524	596	668	741	813	885	957
14	290	379	469	558	647	736	826	915	1004	1093	1183
16	345	452	558	664	770	877	983	1089	1195	1302	1408
18	401	524	647	770	894	1017	1140	1263	1387	1510	1633
20	456	596	736	877	1017	1157	1297	1438	1578	1718	1858
22	511	668	826	983	1140	1297	1455	1612	1769	1926	2084
24	566	741	915	1089	1263	1438	1612	1786	1960	2135	2309
26	622	813	1004	1195	1387	1578	1769	1960	2152	2343	2534
28	677	885	1093	1302	1510	1718	1926	2135	2343	2551	2759
30	732	957	1183	1408	1633	1858	2084	2309	2534	2759	2985
32	787	1030	1272	1514	1756	1999	2241	2483	2725	2968	3210
34	843	1102	1361	1620	1880	2139	2398	2657	2917	3176	3435
36	898	1174	1450	1727	2003	2279	2555	2832	3108	3384	3660
38	953	1246	1540	1833	2126	2419	2713	3006	3299	3592	3886
40	1008	1319	1629	1939	2249	2560	2870	3180	3490	3801	4111

SHEAR 7 – Horizontal and vertical shear reinforcement for strut and tie method

Reference:

ACI 318-05 Section A.3.3.1 requires $\sum[(As_i \sin \gamma_i) / (bs_i)] \geq 0.003$ γ is strut angle with reinforcement
Section 11.8.4 requires $A_v > 0.0025\ bs_1$ A_v = area of vertical bars at spacing s_1
Section 11.8.5 requires $A_h > 0.0015\ bs_2$ A_h = area of horizontal bars at spacing s_2

Values of $\sum[(As_i \sin \gamma_i) / (bs_i)]$

Strut Angle γ with vertical = 25°

Vertical	2#3	2#4	2#5	2#6	Horizontal	2#3	2#4	2#5	2#6
Av	0.22	0.40	0.62	0.88	*Av*	0.22	0.40	0.62	0.88
bs_1(sq in)					bs_2(sq in)				
50	0.00186	0.00338	0.00524	0.00744	50	0.00399	0.00725	0.01124	0.01595
100	max bs_1	0.00169	0.00262	0.00372	100	0.00199	0.00363	0.00562	0.00798
150	= 88	0.00113	0.00175	0.00248	150	max bs_2	0.00242	0.00375	0.00532
200		max bs_1	0.00131	0.00186	200	=147	0.00181	0.00281	0.00399
250		= 160	max bs_1	0.00149	250		0.00145	0.00225	0.00319
300			= 248	0.00124	300		max bs_2	0.00187	0.00266
350	0.00106 at max bs_1			0.00106	350		=267	0.00161	0.00228
400				Max bs_1	400			0.00140	0.00199
				= 352	500	.00136@	max bs_2	max bs_2	0.00160
					600			=413	max 586

Strut Angle γ with vertical = 35°

Vertical	2#3	2#4	2#5	2#6	Horizontal	2#3	2#4	2#5	2#6
Av	0.22	0.40	0.62	0.88	*Av*	0.22	0.40	0.62	0.88
bs_1(sq in)					bs_2(sq in)				
50	0.00252	0.00459	0.00711	0.01009	50	0.00360	0.00655	0.01016	0.01442
100	max bs_1	0.00229	0.00356	0.00505	100	0.00180	0.00328	0.00508	0.00721
150	= 88	0.00153	0.00237	0.00336	150	max bs_2	0.00218	0.00339	0.00481
200		max bs_1	0.00178	0.00252	200	= 147	0.00164	0.00254	0.00360
250		= 160	max bs_1	0.00202	250		0.00131	0.00203	0.00288
300			= 248	0.00168	300		max bs_2	0.00169	0.00240
350	0.00143 at max bs_1			0.00144	350		= 267	0.00145	0.00206
400				Max bs_1	400			0.00127	0.00180
				= 352	500	00122@	max bs_2	max bs_2	0.00144
					600			= 413	max 586

Strut Angle γ with vertical = 45°

Vertical	2#3	2#4	2#5	2#6	Horizontal	2#3	2#4	2#5	2#6
Av	0.22	0.40	0.62	0.88	*Av*	0.22	0.40	0.62	0.88
bs_1(sq in)					bs_2(sq in)				
50	0.00252	0.00459	0.00711	0.01009	50	0.00360	0.00655	0.01016	0.01442
100	max bs_1	0.00229	0.00356	0.00505	100	0.00180	0.00328	0.00508	0.00721
150	= 88	0.00153	0.00237	0.00336	150	max bs_2	0.00218	0.00339	0.00481
200		Max bs_1	0.00178	0.00252	200	= 147	0.00164	0.00254	0.00360
250		= 160	max bs_1	0.00202	250		0.00131	0.00203	0.00288
300			= 248	0.00168	300		max bs_2	0.00169	0.00240
350	0.00177 at max bs_1			0.00144	350		=267	0.00145	0.00206
400				Max bs_1	400			0.00127	0.00180
				= 352	500	0.00106 at	max bs_2	max bs_2	0.00144
					600			= 413	max 586

SHEAR 7 – Horizontal and vertical shear reinforcement for strut and tie method
(continued)

Values of $\sum[(As_i \sin \gamma_i) / (bs_i)]$

Strut Angle γ with vertical = 55°

Vertical A_v bs_1(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88	Horizontal A_v bs_2(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88
50	0.00360	0.00655	0.01016	0.01442	50	0.002524	0.004588	0.007112	0.010094
100	max bs_1	0.00328	0.00508	0.00721	100	0.001262	0.002294	0.003556	0.005047
150	= 88	0.00218	0.00339	0.00481	150	max bs_2	0.001529	0.002371	0.003365
200		max bs_1	0.00254	0.00360	200	= 147	0.001147	0.001778	0.002524
250		= 153	max bs_1	0.00288	250		0.000918	0.001422	0.002019
300			= 248	0.00240	300		max bs_2	0.001185	0.001682
350	0.00205 at	max bs_1		0.00206	350		= 267	0.001016	0.001442
400				max bs_1	400			0.000889	0.001262
				= 352	500	0.00086 @	max bs_2	max bs_2	0.001009
					600			= 413	max 586

Strut Angle γ with vertical = 65°

Vertical A_v bs_1(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88	Horizontal A_v bs_2(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88
50	0.00399	0.00725	0.01124	0.01595	50	0.00041	0.00338	0.00524	0.007438
100	max bs_1	0.00363	0.00562	0.00798	100	0.00021	0.00169	0.00262	0.00372
150	= 88	0.00242	0.00375	0.00532	150	max bs_2	0.00113	0.00175	0.00248
200		max bs_1	0.00281	0.00399	200	= 147	0.00085	0.00131	0.00186
250		= 153	max bs_1	0.00319	250		0.00068	0.00105	0.00149
300			= 248	0.00266	300		max bs_2	0.00087	0.00124
350	0.00227 at	max bs_1		0.00228	350		= 267	0.00075	0.00106
400				max bs_1	400			0.00066	0.00093
				= 352	500	0.00063 @	max bs_2	max bs_2	0.00074
					600			= 413	max 586

Chapter 3

Short Column Design

By Noel. J. Everard[1] and Mohsen A. Issa[2]

3.1 Introduction

The majority of reinforced concrete columns are subjected to primary stresses caused by flexure, axial force, and shear. Secondary stresses associated with deformations are usually very small in most columns used in practice. These columns are referred to as "short columns." Short columns are designed using the interaction diagrams presented in this chapter. The capacity of a short column is the same as the capacity of its section under primary stresses, irrespective of its length.
Long columns, columns with small cross-sectional dimensions, and columns with little end restraints may develop secondary stresses associated with column deformations, especially if they are not braced laterally. These columns are referred to as "slender columns". Fig. 3-1 illustrates secondary moments generated in a slender column by P-δ effect. Consequently, slender columns resist lower axial loads than short columns having the same cross-section. This is illustrated in Fig. 3-1. Failure of a slender column is initiated either by the material failure of a section, or instability of the column as a member, depending on the level of slenderness. The latter is known as column buckling. Design of slender columns is discussed in Chapter 4.

The classification of a column as a “short column” or a “slender column” is made on the basis of its “Slenderness Ratio,” defined below.

Slenderness Ratio: $k\ell_u / r$

where, ℓ_u is unsupported column length; k is effective length factor reflecting end restraint and lateral bracing conditions of a column; and r is the radius of gyration reflecting the size and shape of a column cross-section. A detailed discussion of the parameters involved in establishing the slenderness ratio is presented in Chapter 4. Columns with slenderness ratios less than those specified in Secs. 10.12.2 and 10.13.2 for non-sway and sway frames, respectively, are designed as short columns using this chapter.

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Non-sway frames are frames that are braced against sidesway by shear walls or other stiffening members. They are also referred to as "braced frames." Sway frames are frames that are free to translate laterally so that secondary bending moments are induced due to P-δ effects. They are also referred to as "unbraced frames." The following are the limiting slenderness ratios for short column behavior:

Non-sway frames: $$\frac{k\ell_u}{r} \leq 34 - 12(M_1/M_2) \tag{3.1}$$

Sway frames: $$\frac{k\ell_u}{r} \leq 22 \tag{3.2}$$

Where the term $[34 - 12(M_1/M_2)] \leq 40$ and the ratio M_1/M_2 is positive if the member is bent in single curvature and negative if bent in double curvature.

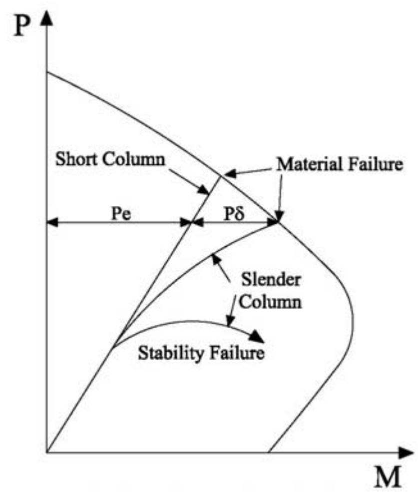

Fig. 3-1 Failure Modes in Short and Slender Columns

3.2 Column Sectional Capacity

In short columns the column capacity is directly obtained from column sectional capacity. The theory that has been presented in Section 1.2 of Chapter 1 for flexural sections, also applies to reinforced concrete column sections. However, column sections are subjected to flexure in combination with axial forces (axial compression and tension). Therefore, the equilibrium of internal forces changes, resulting in significantly different flexural capacities and behavioral modes depending on the level of accompanying axial load. Fig. 3-2 illustrates a typical column section subjected to combined bending and axial compression. As can be seen, different combinations of moment and accompanying axial force result in different column capacities and corresponding strain profiles, while also affecting the failure modes, i.e., tension or compression controlled behavior. The combination of bending moment and axial force that result in a column capacity is best presented by "column interaction diagrams." Interaction diagrams are constructed by computing moment and axial force capacities, as shown below, for different strain profiles.

$$P_n = C_c + C_{s1} + C_{s2} - T_s \tag{3-3}$$

$$M_n = C_c x_2 + C_{s1} x_1 + T_s x_3 \tag{3-4}$$

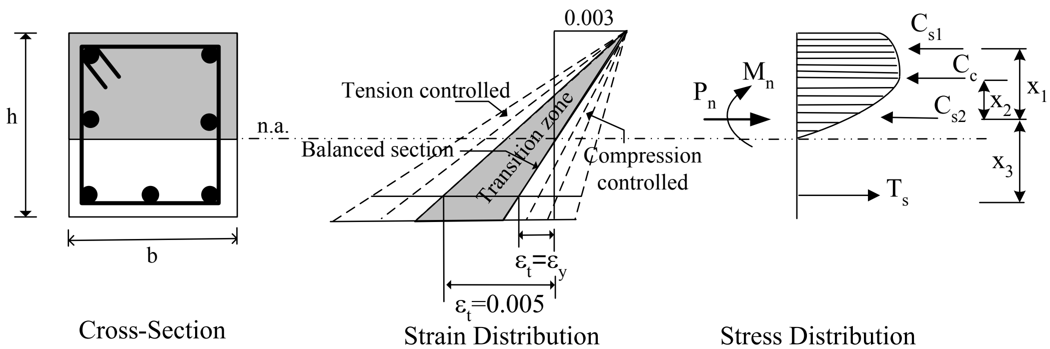

Fig. 3-2 Analysis of a column section

3.2.1 Column Interaction Diagrams

The column axial load - bending moment interaction diagrams included herein (**Columns 3.1.1** through **Columns 3.24.4**) conform fully to the provisions of ACI 318-05. The equations that were used to generate data for plotting the interaction diagrams were originally developed for ACI Special Publication SP-7[3]. In addition, complete derivations of the equations for square and circular columns having the steel arranged in a circle have been published in ACI Concrete International[4]. The original interaction diagrams that were contained in SP-7 were subsequently published in Special Publication SP-17A[5].

The related equations were derived considering the reinforcing steel to be represented as follows:

(a) For rectangular and square columns having steel bars placed on the end faces only, the reinforcement was assumed to consist of two equal thin strips parallel to the compression face of the section.
(b) For rectangular and square columns having steel bars equally distributed along all four faces of the section, the reinforcement was considered to consist of a thin rectangular or square tube.
(c) For square and circular sections having steel bars arranged in a circle, the reinforcement was considered to consist of a thin circular tube.

The interaction diagrams were developed using the rectangular stress block, specified in ACI 318-05 (Sec. 10.2.7). In all cases, for reinforcement that exists within the compressed portion of the depth perpendicular to the compression face of the concrete (a = βc), the compression stress in the steel was reduced by 0.85 $f_c^/$ to account for the concrete area that is displaced by the reinforcing bars within the compression stress block.

The interaction diagrams were plotted in non-dimensional form. The vertical coordinate $[K_n = P_n/(f_c^/ A_g)]$ represents the non-dimensional form of the nominal axial load capacity of the

[3] Everard and Cohen. "Ultimate Strength Design of Reinforced Concrete Columns," ACI Special Publication SP-7, 1964, pp. 152-182.
[4] Everard, N.J., "Axial Load-Moment Interaction for Cross-Sections Having Longitudinal Reinforcement Arranged in a Circle", ACI Structural Journal, Vol. 94, No. 6, November-December, 1997, pp. 695-699.
[5] ACI Committee 340, "Ultimate Strength Design Handbook, Volume 2, Columns, ACI Special Publication 17-A, American Concrete Institute, Detroit, MI, 1970, 226 pages.

section. The horizontal coordinate [$R_n = M_n /(f_c^{/} A_g h)$] represents the non-dimensional nominal bending moment capacity of the section. The non-dimensional forms were used so that the interaction diagrams could be used equally well with any system of units (i.e. SI or inch-pound units). The strength reduction factor (ϕ) was considered to be 1.0 so that the nominal values contained in the interaction diagrams could be used with any set of ϕ factors, since ACI 318-05 contains different ϕ factors in Chapter 9, Chapter 20 and Appendix "C".

It is important to point out that the ϕ factors that are provided in Chapter 9 of ACI 318-05 are based on the strain values in the tension reinforcement farthest from the compression face of a member, or at the centroid of the tension reinforcement. Code Section 9.3.2 references Sections 10.3.3 and 10.3.4 where the strain values for tension control and compression control are defined.

It should be note that the eccentricity ratios ($e/h = M/P$), sometimes included as diagonal lines on interaction diagrams, are not included in the interaction diagrams. Using that variable as a coordinate with either K_n or R_n could lead to inaccuracies because at the lower ends of the diagrams the e/h lines converge rapidly. However, straight lines for the tension steel stress ratios f_s / f_y have been plotted for assistance in designing splices in the reinforcement. Further, the ratio $f_s / f_y = 1.0$ represents steel strain $\varepsilon_y = f_y / E_s$, which is the boundary point for the ϕ factor for compression control, and the beginning of the transition zone for linear increase of the ϕ factor to that for tension control.

In order to provide a means of interpolation for the ϕ factor, other strain lines were plotted. The strain line for $\varepsilon_t = 0.005$, the beginning of the zone for tension control has been plotted on all diagrams. For steel yield strength 60.0 ksi, the intermediate strain line for $\varepsilon_t = 0.035$ has been plotted. For Steel yield strength 75.0 ksi, the intermediate strain line for $\varepsilon_t = 0.038$ has been plotted. It should be noted that all strains refer to those in the reinforcing bar or bars farthest from the compression face of the section. Discussions and tables related to the strength reduction factors are contained in two publications in Concrete International[6,7].

In order to point to designs that are prohibited by ACI 318-05, Section 10.3.5, strain lines for $\varepsilon_t = 0.004$ have also been plotted. Designs that fall within the confines of the lines for $\varepsilon_t = 0.004$ and K_n less than 0.10 are not permitted by ACI 318-05. This includes tension axial loads, with K_n negative. Tension axial loads are not included in the interaction diagrams. However, the interaction diagram lines for tension axial loads are very nearly linear from $K_n = 0.0$ to $R_n = 0.0$ with [$K_n = A_{st} f_y /(f_c^{/} A_g)$]. This is discussed in the next section.

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6 Everard, N. J., "Designing With ACI 318-02 Strength Reduction Factors", Concrete International, August, 2002, Vol. 24, No. 8, pp 91-93.

7 Everard, N. J., "Strain Related Strength Reduction Factors (ϕ) According to ACI 318-02, Concrete International, August, 2002, Vol. 34, No. 8, pp. 91-93.

Straight lines for $K_{\max}$ are also provided on each interaction diagram. Here, $K_{\max}$ refers to the maximum permissible nominal axial load on a column that is laterally reinforced with ties conforming to ACI 318-05 Section 7.10.5. Defining K_0 as the theoretical axial compression capacity of a member with $R_n = 0.0$, $K_{\max} = 0.80K_0$, or, considering ACI 318-05 Eq. (10-2), without the ϕ factor,

$$P_{n,\max} = 0.8\,[\,0.85 f_c^{/}(A_g - A_{st}) + f_y A_{st}\,] \qquad (3\text{-}5)$$

Then,

$$K_{\max} = P_{\max} / f_c^{/} A_g \qquad (3\text{-}6)$$

For columns with spirals conforming with ACI 318-05 Section 7.10.4, values of $K_{\max}$ from the interaction diagrams are to be multiplied by 0.85/0.80 ratio.

The number of longitudinal reinforcing bars that may be contained is not limited to the number shown in the illustrations on the interaction diagrams. They only illustrate the type of reinforcement patterns. However, for circular and square columns with steel arranged in a circle, and for rectangular or square columns with steel equally distributed along all four faces, it is a good practice to use at least 8 bars (and preferably at least 12 bars). Although side steel was assumed to be 50 percent of the total steel for columns having longitudinal steel equally distributed along all four faces, reasonably accurate and conservative designs result when the side steel consists of only 30 percent of the total steel. The maximum number of bars that may be used in any column cross section is limited by the maximum allowable steel ratio of 0.08, and the conditions of cover and spacing between bars.

3.2.2 Flexure with Tension Axial Load

Many studies concerning flexure with tension axial load show that the interaction diagram for tension axial load and flexure is very nearly linear between R_o and the tension axial load value K_{nt}, as is shown in Fig. 3-3. Here, R_0 is the value of R_n for $K_n = 0.0$, and $K_{nt} = A_{st} f_y / (f_c^{/} A_g)$

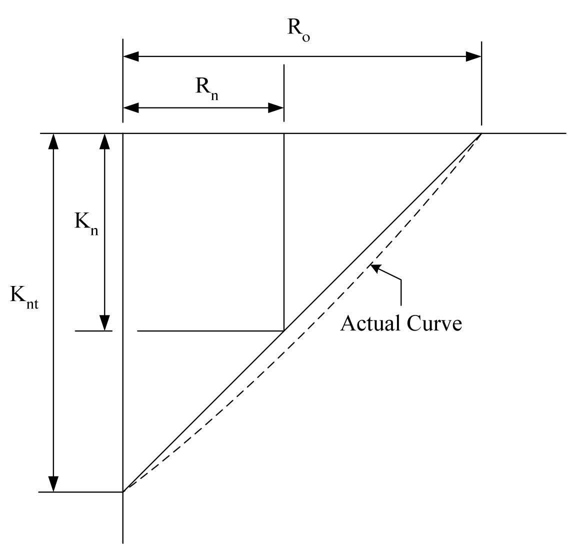

Fig. 3.3 Flexure with axial tension

Design values for flexure with tension axial load can be obtained using the equations:

$$K_n = K_{nt}\left[1.0 - R_n/R_0\right] \quad (3\text{-}7)$$

$$R_n = R_o\left[1.0 - K_n/K_{nt}\right] \quad (3\text{-}8)$$

Also, the tension side interaction diagram can be plotted as a straight line using R_0 and K_{nt}, as is shown in Fig. 3.3.

3.3 Columns Subjected to Biaxial Bending

Most columns are subjected to significant bending in one direction, while subjected to relatively small bending moments in the orthogonal direction. These columns are designed by using the interaction diagrams discussed in the preceding section for uniaxial bending and if required checked for the adequacy of capacity in the orthogonal direction. However, some columns, as in the case of corner columns, are subjected to equally significant bending moments in two orthogonal directions. These columns may have to be designed for biaxial bending.

A circular column subjected to moments about two axes may be designed as a uniaxial column acted upon by the resultant moment;

$$M_u = \sqrt{M_{ux}^2 + M_{uy}^2} \geq \varphi M_n = \sqrt{M_{nx}^2 + M_{ny}^2} \quad (3\text{-}9)$$

For the design of rectangular columns subjected to moments about two axes, this handbook provides design aids for two methods: 1) The Reciprocal Load ($1/P_i$) Method suggested by Bresler[8], and 2) The Load Contour Method developed by Parme, Nieves, and Gouwens[9]. The Reciprocal Load Method is more convenient for making an analysis of a trial section. The Load Contour Method is more suitable for selecting a column cross section. Both of these methods use the concept of a failure surface to reflect the interaction of three variables, the nominal axial load P_n and the nominal biaxial bending moments M_{nx} and M_{ny}, which in combination will cause failure strain at the extreme compression fiber. In other words, the failure surface reflects the strength of short compression members subject to biaxial bending and compression. The bending axes, eccentricities and biaxial moments are illustrated in Fig. 3.4.

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[8] Bresler, Boris. "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending," ACI Journal Proceedings, V. 57, No.11, Nov. 1960, pp. 481-490.
[9] Parme, A.L. Nieves, J. M. and Gouwens, A. "Capacity of Reinforced Rectangular Columns Subjected to Biaxial Bending." ACI Journal Proceedings, V. 63, No. 9, Sept. 1966, pp.911-923.

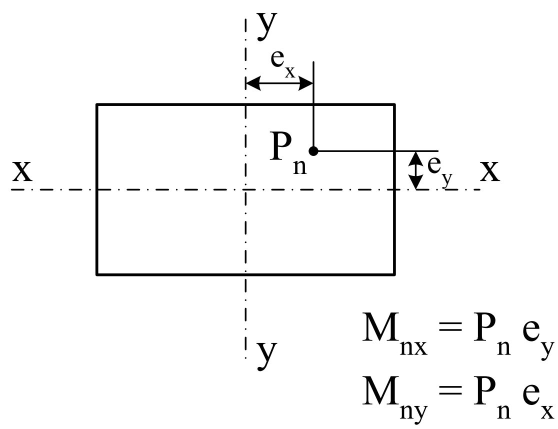


Fig. 3.4 Notations used for column sections subjected to biaxial bending

A failure surface S_1 may be represented by variables P_n, e_x, and e_y, as in Fig. 3.5, or it may be represented by surface S_2 represented by variables P_n, M_{nx}, and M_{ny} as shown in Fig. 3.6. Note that S_1 is a single curvature surface having no discontinuity at the balance point, whereas S_2 has such a discontinuity. (When biaxial bending exists together with a nominal axial force smaller than the lesser of P_b or 0.1 f'_c A_g, it is sufficiently accurate and conservative to ignore the axial force and design the section for bending only.)

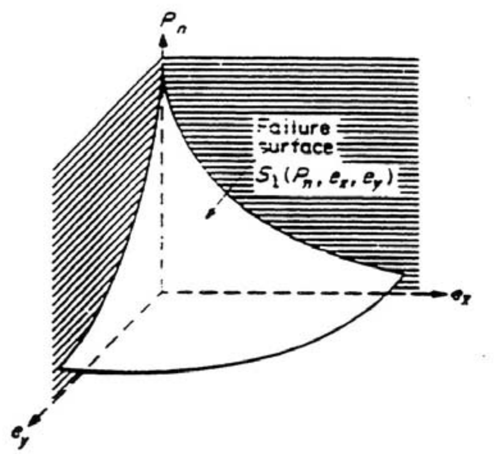


Fig. 3.5 Failure surface S_1

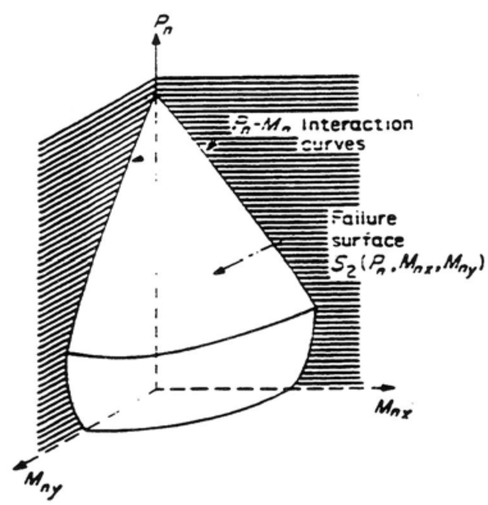


Fig. 3.6 Failure surface S_2

3.3.1 Reciprocal Load Method

In the reciprocal load method, the surface S_1 is inverted by plotting $1/P_n$ as the vertical axis, giving the surface S_3, shown in Fig. 3.7. As Fig. 3.8 shows, a true point ($1/P_{n1}$, e_{xA}, e_{yB}) on this reciprocal failure surface may be approximated by a point ($1/P_{ni}$, e_{xA}, e_{yB}) on a plane S'_3 passing through Points A, B, and C. Each point on the true surface is approximated by a different plane; that is, the entire failure surface is defined by an infinite number of planes.

Point A represents the nominal axial load strength P_{ny} when the load has an eccentricity of e_{xA} with $e_y = 0$. Point B represents the nominal axial load strength P_{nx} when the load has an eccentricity of e_{yB} with $e_x = 0$. Point C is based on the axial capacity P_o with zero eccentricity. The equation of the plane passing through the three points is;

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o} \tag{3-10}$$

Where:

P_{ni}: approximation of nominal axial load strength at eccentricities e_x and e_y

P_{nx}: nominal axial load strength for eccentricity e_y along the y-axis only (x-axis is axis of bending)

P_{ny}: nominal axial load strength for eccentricity e_x along the x-axis only (y-axis is axis of bending)

P_o: nominal axial load strength for zero eccentricity

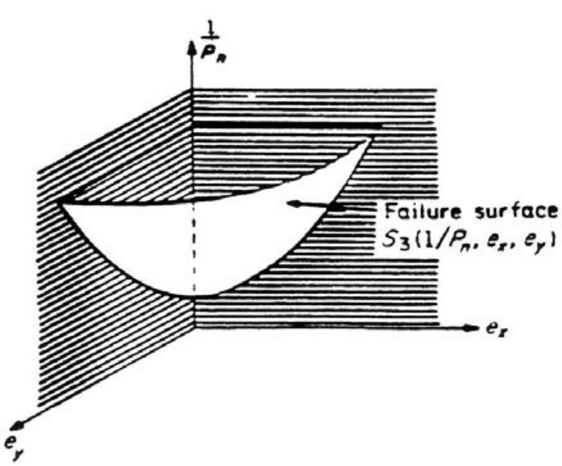


Fig. 3.7 Failure surface S3,, which is reciprocal of surface S1

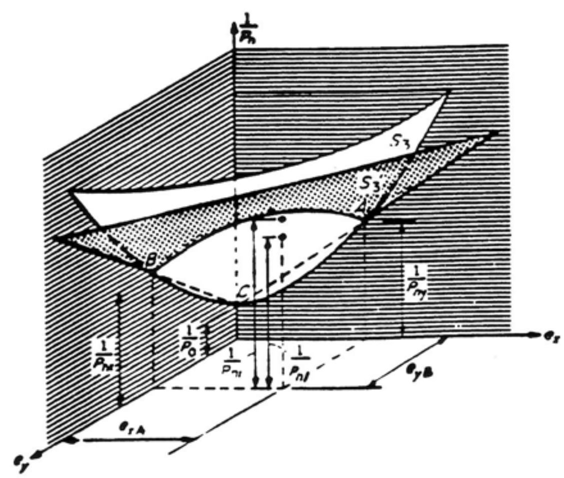


Fig. 3.8 Graphical representation of Reciprocal Load Method

For design purposes, when ϕ is constant, the $1/P_{ni}$ equation given in Eq. 3.9 may be used. The variable $K_n = P_n / (f'_c A_g)$ can be used directly in the reciprocal equation, as follows:

$$\frac{1}{K_{ni}} = \frac{1}{K_{nx}} + \frac{1}{K_{ny}} - \frac{1}{K_o} \tag{3-11}$$

Where, the values of K refer to the corresponding values of P_n as defined above. Once a preliminary cross section with an estimated steel ratio ρ_g has been selected, the actual values of R_{nx} and R_{ny} are calculated using the actual bending moments about the cross section X and Y axes, respectively. The corresponding values of K_{nx} and K_{ny} are obtained from the interaction diagrams presented in this Chapter as the intersection of appropriate R_n value and the assumed steel ratio curve for ρ_g. Then, the

value of the theoretical compression axial load capacity K_o is obtained at the intersection of the steel ratio curve and the vertical axis for zero R_n.

3.3.2 Load Contour Method

The load contour method uses the failure surface S_2 (Fig. 3.6) and works with a load contour defined by a plane at a constant value of P_n, as illustrated in Fig. 3.9. The load contour defining the relationship between M_{nx} and M_{ny} for a constant P_n may be expressed nondimensionally as follows:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\alpha} = 1 \tag{3-12}$$

For design, if each term is multiplied by ϕ, the equation will be unchanged. Thus M_{ux}, M_{uy}, M_{ox}, and M_{oy}, which should correspond to ϕM_{nx}, ϕM_{ny}, ϕM_{nox} , and ϕM_{noy}, respectively, may be used instead of the original expressions. This is done in the remainder of this section. To simplify the equation (for application), a point on the nondimensional diagram Fig. 3.10 is defined such that the biaxial moment capacities M_{nx} and M_{ny} at this point are in the same ratio as the uniaxial moment capacities M_{ox} and M_{oy}; thus

$$\frac{M_{nx}}{M_{ny}} = \frac{M_{ox}}{M_{oy}} \tag{3-12}$$

$$\text{or;} \qquad M_{nx} = \beta M_{ox} \qquad \text{and} \qquad M_{ny} = \beta M_{oy} \tag{3-13}$$

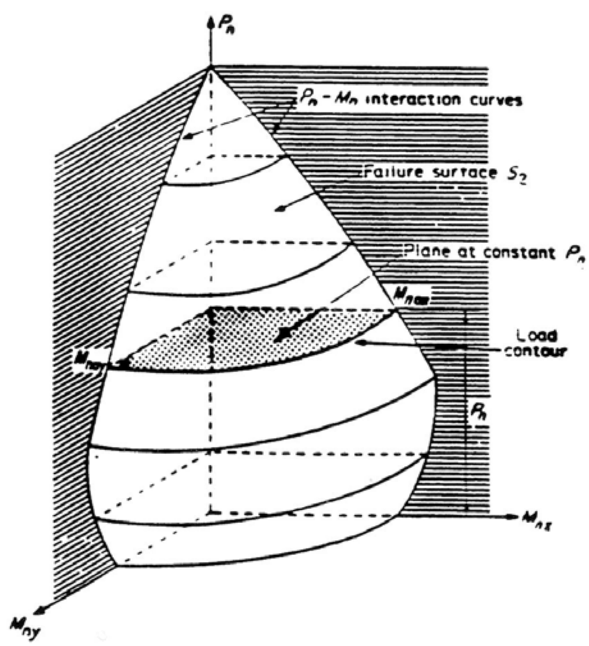


Fig. 3.10 Load contour for constant P_n on failure surface

In physical sense, the ratio β is the constant portion of the uniaxial moment capacities which may be permitted to act simultaneously on the column section. The actual value of β depends on the ration P_n/P_{og} as well as properties of the material and cross section. However, the usual range is between 0.55 and 0.70. An average value of. = 0.65 is suggested for design. The actual values of β are available from **Columns 3.25**.

The load contour equation given above (Eq. 3-10) may be written in terms of β, as shown below:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\log 0.5/\log\beta} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\log 0.5/\log\beta} = 1 \tag{3-14}$$

A plot of the Eq. 3-12 appears as **Columns 3.26**. This design aid is used for analysis. Entering with M_{nx}/M_{ox} and the value of β from **Columns 3.25**, one can find permissible M_{ny}/M_{oy}. The relationship using β may be better visualized by examining Fig. 3.10. The true relationship between Points A, B, and C is a curve; however, it may be approximated by straight lines for design purposes. The load contour equations as straight line approximation are:

i) For $\dfrac{M_{ny}}{M_{nx}} \geq \dfrac{M_{oy}}{M_{ox}}$

$$M_{oy} = M_{ny} + M_{nx}\left(\frac{M_{oy}}{M_{ox}}\right)\left(\frac{1-\beta}{\beta}\right) \tag{3-13}$$

ii) For $\dfrac{M_{ny}}{M_{nx}} \leq \dfrac{M_{oy}}{M_{ox}}$

$$M_{ox} = M_{nx} + M_{ny}\left(\frac{M_{ox}}{M_{oy}}\right)\left(\frac{1-\beta}{\beta}\right) \tag{3-14}$$

For rectangular sections with reinforcement equally distributed on all four faces, the above equations can be approximated by;

$$M_{oy} = M_{ny} + M_{nx}\left(\frac{b}{h}\right)\left(\frac{1-\beta}{\beta}\right) \tag{3-15}$$

For $\dfrac{M_{ny}}{M_{nx}} \leq \dfrac{M_{oy}}{M_{ox}}$ or $\dfrac{M_{ny}}{M_{nx}} \leq \dfrac{b}{h}$

where b and h are dimensions of the rectangular column section parallel to x and y axes, respectively. Using the straight line approximation equations, the design problem can be attacked by converting the nominal moments into equivalent uniaxial moment capacities M_{ox} or M_{oy}. This is accomplished by;

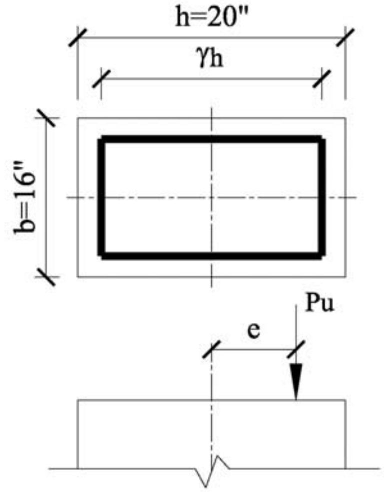
(a) assuming a value for b/h
(b) estimating the value of β as 0.65
(c) calculating the approximate equivalent uniaxial bending moment using the appropriate one of the above two equations
(d) choosing the trial section and reinforcement using the methods for uniaxial bending and axial load.

The section chosen should then be verified using either the load contour or the reciprocal load method.

3.4 Columns Examples

COLUMNS EXAMPLE 1 - Required area of steel for a rectangular tied column with bars on four faces (slenderness ratio found to be below critical value)

For a rectangular tied column with bars equally distributed along four faces, find area of steel.



Given: Loading

P_u= 560 kip and M_u= 3920 kip-in.
Assume φ = 0.70 or,
Nominal axial load P_n = 560/0.70 = 800 kip
Nominal moment M_n = 3920/0.70 = 5600 kip-in.

Materials

Compressive strength of concrete $f_c' = 4$ ksi
Yield strength of reinforcement f_y = 60 ksi
Nominal maximum size of aggregate is 1 in.

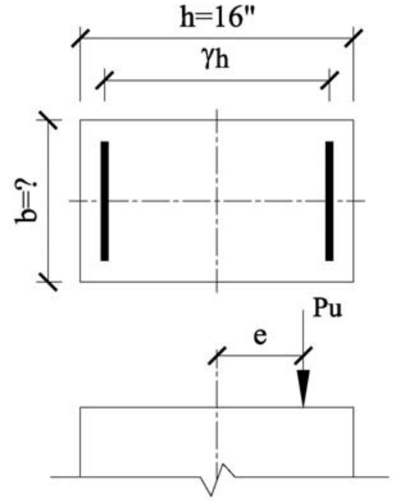
Design conditions

Short column braced against sidesway.

Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine column section size.	Given: h = 20 in. b = 16 in.		
Determine reinforcement ration ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement. A) Compute $K_n = \frac{P_n}{f_c' A_g}$ B) Compute $R_n = \frac{M_n}{f_c' A_g h}$ C) Estimate $\gamma \approx \frac{h - 5}{h}$ D) Determine the appropriate interaction diagram(s) E) Read ρ_g for k_n and R_n values from appropriate interaction diagrams F) Compute required A_{st} from $A_{st}=\rho_g A_g$	P_n= 800 kip M_n = 5600 kip-in. h = 20 in. b = 16 in. A_g = b x h = 20 x 16 = 320 in.^{2} $K_n = \frac{800}{(4)(320)} = 0.625$ $R_n = \frac{5600}{(4)(320)(20)} = 0.22$ $\gamma \approx \frac{20 - 5}{20} = 0.75$ For a rectangular tied column with bars along four faces, $f_c' = 4$ ksi, f_y = 60 ksi, and an estimated γ of 0.75, use R4-60.7 and R4-60.8. For k_n= 0.625 and R_n= 0.22 Read ρ_g = 0.041 for γ = 0.7 and ρ_g = 0.039 for γ = 0.8 Interpolating; ρ_g = 0.040 for γ = 0.75 Required A_{st} = 0.040× 320 in.^{2} = 12.8 in^2	10.2 10.3	Columns 3.2.2 (R4-60.7) and 3.2.3 (R4-60.8)

COLUMNS EXAMPLE 2 - For a specified reinforcement ratio, selection of a column section size for a rectangular tied column with bars on end faces only

For minimum longitudinal reinforcement (ρ_g= 0.01) and column section dimension h = 16 in., select the column dimension b for a rectangular tied column with bars on end faces only.



Given: Loading

P_u= 660 kips and M_u= 2790 kip-in.
Assume φ = 0.70 or,
Nominal axial load P_n = 660/0.70= 943 kips
Nominal moment M_n = 4200/0.70= 3986 kip-in.

Materials

Compressive strength of concrete $f_c^{/}$ = 4 ksi
Yield strength of reinforcement f_y = 60 ksi
Nominal maximum size of aggregate is 1 in.

Design conditions

Slenderness effects may be neglected because $k\ell_u/h$ is known to be below critical value

Procedure	Calculation			ACI 318-05 Section	Design Aid
Determine trial column dimension b corresponding to known values of variables on appropriate interaction diagram(s).	P_n= 943 kips, M_n = 3986 kip-in. h = 16 in. $f_c^{/}$ = 4 ksi, f_y= 60 ksi ρ_g = 0.01				
A) Assume a series of trial column sizes b, in inches; and compute A_g=$b\times h$, in.^{2}	24 384	26 416	28 448		
B) Compute $K_n = \frac{P_n}{f_c^{'} A_g}$	$\frac{943}{(4)(384)}$ $= 0.61$	$\frac{943}{(4)(416)}$ $= 0.57$	$\frac{943}{(4)(448)}$ $= 0.53$		
C) Compute $R_n = \frac{M_n}{f_c^{'} A_g h}$	$\frac{3986}{(4)(384)(16)}$ $= 0.16$	$\frac{3986}{(4)(416)(16)}$ $= 0.14$	$\frac{3986}{(4)(448)(16)}$ $= 0.14$		
D) Estimate $\gamma \approx \frac{h - 5}{h}$	0.7	0.7	0.7		
D) Determine the appropriate interaction diagram(s)	For a rectangular tied column with bars along four faces, $f_c^{/}$ = 4 ksi, f_y = 60 ksi, and an estimated γ of 0.70, use Interaction Diagram L4-60.7				
E) Read ρ_g for k_n and R_n values For γ = 0.7, select dimension corresponding to ρ_g nearest desired value of ρ_g = 0.01	0.018 Therefore, try a 16 x 28-in. column	0.014	0.011	10.2 10.3	Columns 3.8.2 (L4-60.7)

COLUMNS EXAMPLE 3 - Selection of reinforcement for a square spiral column (slenderness ratio is below critical value)

For the square spiral column section shown, select reinforcement.

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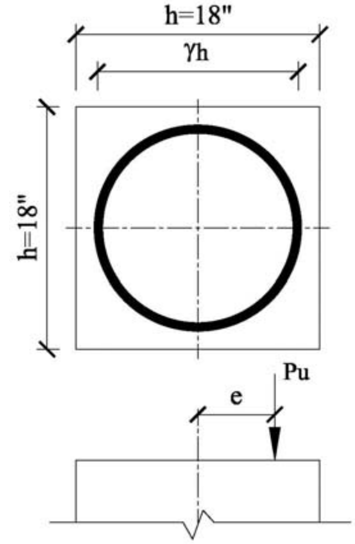


Given: Loading

P_u= 660 kips and M_u= 2640 kip-in.
Assume φ = 0.70 or,
Nominal axial load P_n = 660/0.70= 943 kips
Nominal moment M_n = 2640/0.70= 3771 kip-in.

Materials

Compressive strength of concrete $f_c^{/}$ = 4 ksi
Yield strength of reinforcement f_y = 60 ksi
Nominal maximum size of aggregate is 1 in.

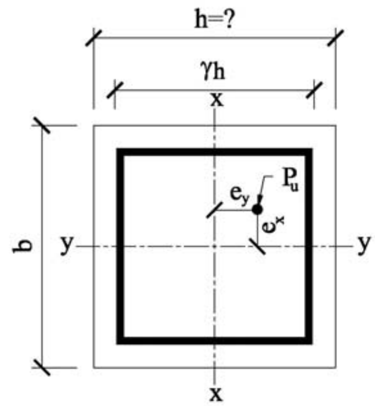
Design conditions

Column section size h = b = 18 in
Slenderness effects may be neglected because
k ℓ_u/h is known to be below critical value

Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine reinforcement ration ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement.	P_n= 943 kips M_n = 3771 kip-in. h = 18 in. b = 18 in. $A_g = b \times h = 18 \times 18 = 324$ in.^{2}		
A) Compute $K_n = \frac{P_n}{f_c^{'} A_g}$	$K_n = \frac{943}{(4)(324)} = 0.73$		
B) Compute $R_n = \frac{M_n}{f_c^{'} A_g h}$	$R_n = \frac{3771}{(4)(320)(18)} = 0.16$		
C) Estimate $\gamma \approx \frac{h - 5}{h}$	$\gamma \approx \frac{18 - 5}{18} = 0.72$		
D) Determine the appropriate interaction diagram(s)	For a square spiral column, $f_c^{/}$ = 4 ksi, f_y = 60 ksi, and an estimated γ of 0.72, use Interaction Diagram S4-60.7 and S4-60.8		
E) Read ρ_g for k_n and R_n values.	For k_n= 0.73 and R_n= 0.16 and, γ = 0.70: ρ_g = 0.035 γ = 0.80: ρ_g = 0.031 for γ = 0.72: ρ_g = 0.034 $A_{st} = 0.034 \times 320$ in.^{2} = 12.8 in^2	10.2 10.3	Columns 3.20.2 (S4-60.7) and 3.20.3 (S4-60.8)

COLUMNS EXAMPLE 4 - Design of square column section subject to biaxial bending using resultant moment

Select column section size and reinforcement for a square column with $\rho_g \le 0.04$ and bars equally distributed along four faces, subject to biaxial bending.



Given: Loading

P_u= 193 kip, M_{ux}= 1917 kip-in., and M_{uy}= 769 kip-in.

Assume φ = 0.65 or,

Nominal axial load P_n = 193/0.65= 297 kips

Nominal moment about x-axis M_{nx} = 1917/0.65= 2949 kip-in.

Nominal moment about y-axis M_{ny} = 769/0.65= 1183 kip-in.

Materials

Compressive strength of concrete $f_c^/ = 5$ ksi

Yield strength of reinforcement f_y = 60 ksi

Nominal maximum size of aggregate is 1 in.

Procedure	Calculation	ACI 318-05 Section	Design Aid
Assume load contour curve at constant P_n is an ellipse, and determine resultant moment M_{nx} from $M_{nr} = \sqrt{M_{nx}^2 + M_{ny}^2}$	For a square column: h=b $M_{nr} = \sqrt{(2949)^2 + (1183)^2} = 3177$ kip-in.		
A) Assume a series of trial column sizes h, in inches.	14 16 18		
B) Compute A_g=h^2, in.^{2}	196 256 324		
C) Compute $K_n = \dfrac{P_n}{f_c' A_g}$	$\dfrac{297}{(5)(196)} = 0.30$ $\dfrac{297}{(5)(256)} = 0.23$ $\dfrac{297}{(5)(324)} = 0.18$		
D) Compute $R_n = \dfrac{M_n}{f_c' A_g h}$	$\dfrac{3177}{(5)(196)(14)} = 0.23$ $\dfrac{3177}{(5)(256)(16)} = 0.16$ $\dfrac{3177}{(5)(324)(18)} = 0.11$		
E) Estimate $\gamma \approx \dfrac{h - 5}{h}$	0.64 0.69 0.72		
F) Determine the appropriate interaction diagram(s)	For a rectangular tied column with $f_c^/ = 5$ ksi, f_y = 60 ksi. Use Interaction Diagrams R5-60.6, R5-60.7, and R5-60.8.		Columns 3.3.1 (R5-60.6), 3.3.2 (R5-60.7), and 3.3.3 (R5-60.8)
E) Read ρ_g for R_n and k_n values , For γ = 0.60, For γ = 0.70, and For γ = 0.80 Interpolating for γ in step E	0.064 0.030 0.012 0.048 0.026 0.011 0.058 0.026 0.012 Therefore, try h = 15 in.		

Determine reinforcement ration ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement.	$A_g = h^2 = (15)^2 = 225$ in.^{2} $P_n = 297$ kip $M_{nr} = 3177$ kip-in.		
A) Compute $K_n = \dfrac{P_n}{f_c' A_g}$	$K_n = \dfrac{297}{(5)(225)} = 0.264$		
B) Compute $R_n = \dfrac{M_n}{f_c' A_g h}$	$R_n = \dfrac{3177}{(5)(225)(15)} = 0.188$		
C) Estimate $\gamma \approx \dfrac{h - 5}{h}$	$\gamma \approx \dfrac{15 - 5}{15} = 0.67$		
D) Determine the appropriate interaction diagram(s)	For a rectangular tied column with $f_c' = 5$ ksi, $f_y = 60$ ksi, and $\gamma = 0.67$. Use Interaction R5-60.6 and R5-60.7.	10.2 10.3	
E) Read ρ_g for k_n and R_n values from appropriate interaction diagrams	For $k_n = 0.264$, $R_n = 0.188$, and $\gamma = 0.60$: $\rho_g = 0.043$ $\gamma = 0.70$: $\rho_g = 0.034$ for $\gamma = 0.67$: $\rho_g = 0.037$		Columns 3.3.1 (R5-60.6) and 3.3.2 (R5-60.7)
F) Compute required A_{st} from $A_{st} = \rho_g A_g$ and add about 15 percent for skew bending	Required $A_{st} = 0.037 \times 225$ in.^{2} $= 8.26$ in^2 Use $A_{st} \approx 9.50$ in.^{2}		

COLUMNS EXAMPLE 5 - Design of circular spiral column section subject to very small design moment

For a circular spiral column, select column section diameter h and choose reinforcement. Use relatively high proportion of longitudinal steel (i.e., $\rho_g = 0.04$). Note that $k \ell_u/h$ is known to be below critical value.

.

Given: Loading

P_u= 940 kips and M_u= 480 kip-in.
Assume $\varphi = 0.70$ or,
Nominal axial load P_n = 940/0.70= 1343 kips
Nominal moment M_n = 480/0.70=686 kip-in.

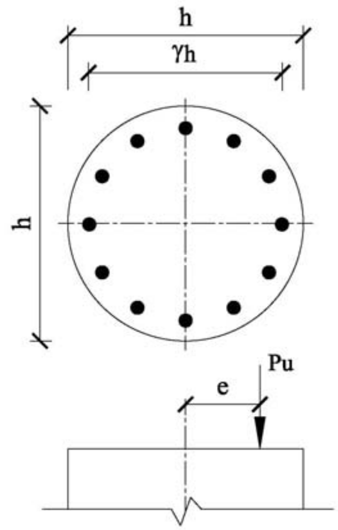
.

Materials

Compressive strength of concrete $f_c^{/} = 5$ ksi
Yield strength of reinforcement $f_y = 60$ ksi
Nominal maximum size of aggregate is 1 in.

Design condition

Slenderness effects may be neglected because
$k \ell P_u/h$ is known to be below critical value



Procedure	**Calculation**	**ACI 318-05 Section**	**Design Aid**
Determine trial column dimension b corresponding to known values of variables on appropriate interaction diagram(s).	P_n= 1343 kips, M_n = 686 kip-in. $f_c^{/} = 5$ ksi $f_y = 60$ ksi $\rho_g = 0.04$		
A) Assume a series of trial column sizes b, in inches; and compute $A_g=\pi(h/2)^2$, in.^{2}	12 16 20 113 201 314		
B) Compute $R_n = \dfrac{M_n}{f_c^{'} A_g h}$	$\dfrac{686}{(5)(113)(12)} = 0.101$ $\dfrac{686}{(5)(201)(16)} = 0.043$ $\dfrac{686}{(5)(314)(20)} = 0.021$		
C) Estimate $\gamma \approx \dfrac{h - 5}{h}$	0.64 0.69 0.72		
D) Determine the appropriate interaction diagram(s)	For a circular column with $f_c^{/} = 5$ ksi, $f_y = 60$ ksi. Use Interaction Diagrams C5-60.6, C5-60.7, C5-60.7 and C5-60.8.		Columns 3.15.1 (C5-60.6), 3.15.2 (C5-60.7), and 3.15.3 (C5-60.8)
E) Read R_n and ρ_g values , after interpolation	0.90 1.14 1.23 1.25 0.90 1.14 1.24		
F) Compute $A_g = \dfrac{P_n}{f_c^{/} k_n}$, in.^{2}	298 236 217		
G) Compute $h = 2\sqrt{\dfrac{A_g}{\pi}}$, in.	19.5 17.3 16.6 Therefore, try 17 in. diameter column		

Determine reinforcement ration ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement. A) Compute $K_n = \frac{P_n}{f_c^{'} A_g}$ B) Compute $R_n = \frac{M_n}{f_c^{'} A_g h}$ C) Estimate $\gamma \approx \frac{h - 5}{h}$ D) Determine the appropriate interaction diagram(s) E) Read ρ_g for k_n and R_n values from appropriate interaction diagrams	$Ag = \pi \left(\frac{17}{2}\right)^2 = 227\, in.^2$ $K_n = \frac{1343}{(5)(227)} = 1.18$ $R_n = \frac{686}{(5)(227)(17)} = 0.0356$ $\gamma \approx \frac{17 - 5}{17} = 0.71$ For a circular column with $f_c^{/} = 5$ ksi and $f_y = 60$ ksi. Use Interaction C5-60.7. For $k_n = 1.18$, $R_n = 0.0356$, and $\gamma = 0.71$: $\rho_g = 0.040$	Columns	Columns 3.15.2 (C5-60.7)
F) Compute required A_{st} from $A_{st} = \rho_g A_g$	Required $A_{st} = 0.040 \times 227$ in.^{2} $= 9.08$ in^2		

Chapter 4

Design of Slender Columns

By Murat Saatcioglu[1]

4.1 Introduction

The majority of reinforced concrete columns in practice are subjected to very little secondary stresses associated with column deformations. These columns are designed as short columns using the column interaction diagrams presented in Chapter 3. Rarely, when the column height is longer than typical story height and/or the column section is small relative to column height, secondary stresses become significant, especially if end restraints are small and/or the columns are not braced against side sway. These columns are designed as "slender columns." Fig. 3.1 eloquently illustrates the secondary moments generated in a slender column by P-Δ effects. Slender columns resist lower axial loads than short columns having the same cross-section. Therefore, the slenderness effect must be considered in design, over and above the sectional capacity considerations incorporated in the interaction diagrams. The significance of slenderness effect is expressed through *slenderness ratio*.

4.2 Slenderness Ratio

The degree of slenderness in a column is expressed in terms of "slenderness ratio," defined below:

Slenderness Ratio: $k\ell_u / r$

where, ℓ_u is unsupported column length; k is effective length factor reflecting the end restraint and lateral bracing conditions of a column; and *r* is the radius of gyration, reflecting the size and shape of a column cross-section.

4.2.1 Unsupported Length, ℓ_u

The unsupported length ℓ_u of a column is measured as the clear distance between the underside of the beam, slab, or column capital above, and the top of the beam or slab below. The unsupported length of a column may be different in two orthogonal directions depending on the supporting elements in

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respective directions. Figure 4.1 provides examples of different support conditions and corresponding unsupported lengths (ℓ_u). Each coordinate and subscript "x" and "y" in the figure indicates the plane of the frame in which the stability of column is investigated.

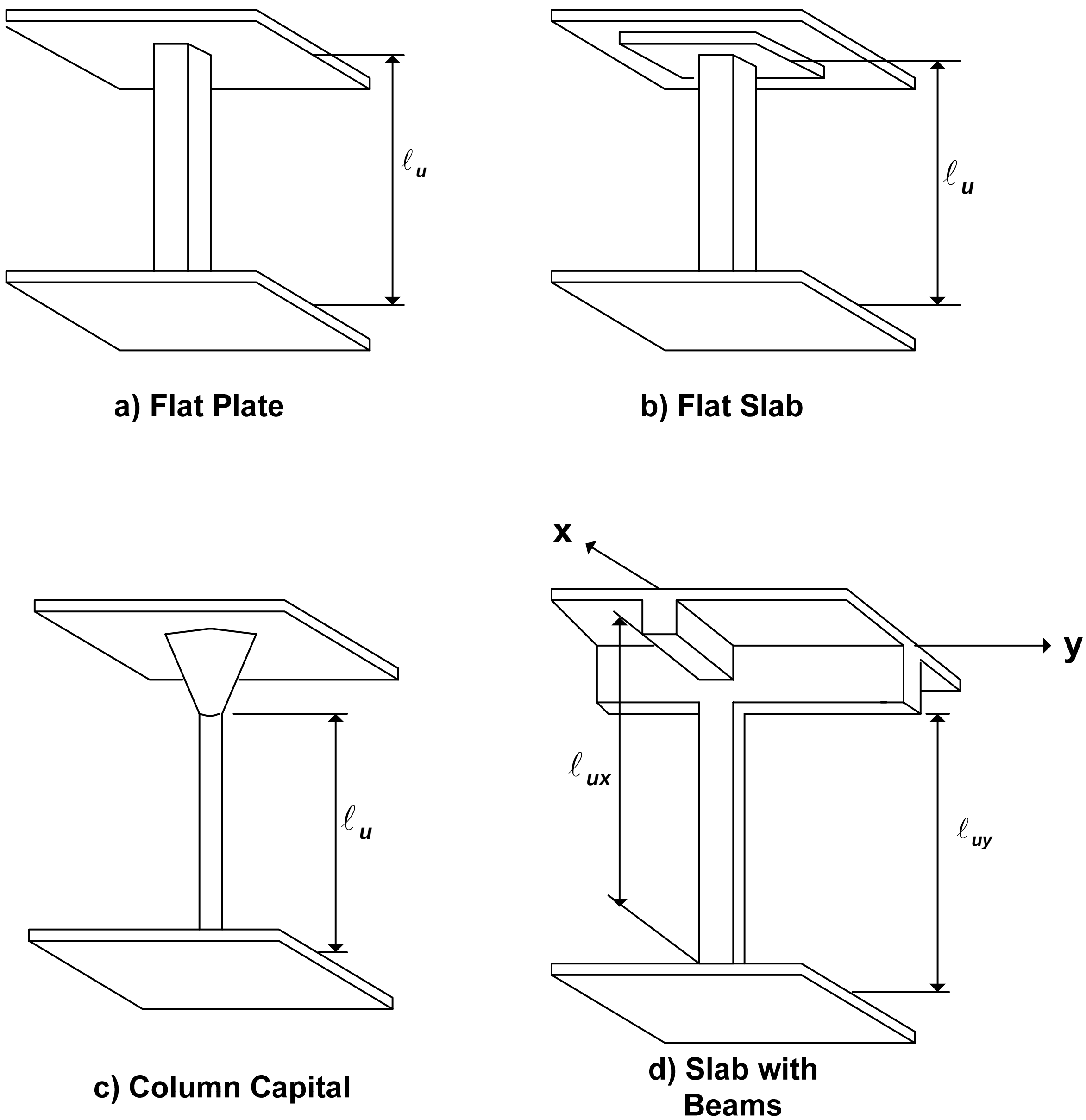


Fig. 4.1 Unsupported column length, ℓ_u

4.2.2 Effective Length Factor, k

The effective length factor k reflects the end restraint (support) and lateral bracing conditions of a column relative to a pin-ended and laterally braced "reference column." The reference column, shown in Fig. 4.2(a), follows a half sine wave when it buckles, and is assigned a *k* factor of 1.0. Therefore, the effective length $k\ell_u$ for this column is equal to the unsupported column length ℓ_u. A column with fully

restrained end conditions develops the deflected shape illustrated in Fig. 4.2(b). The portion of the column between the points of contraflexure follows a half sine wave, the same deflected shape as that of the reference column. This segment is equal to 50% of the unsupported column length ℓ_u. Therefore, the effective length factor k for this case is equal to 0.5. Effective length factors for columns with idealized supports can be determined from Fig. 4.2. It may be of interest to note that *k* varies between 0.5 and 1.0 for laterally braced columns, and 1.0 and ∞ for unbraced columns. A discussion of lateral bracing is provided in Sec. 4.3 to establish whether a given column can be considered to be as part of a sway or a non-sway frame.

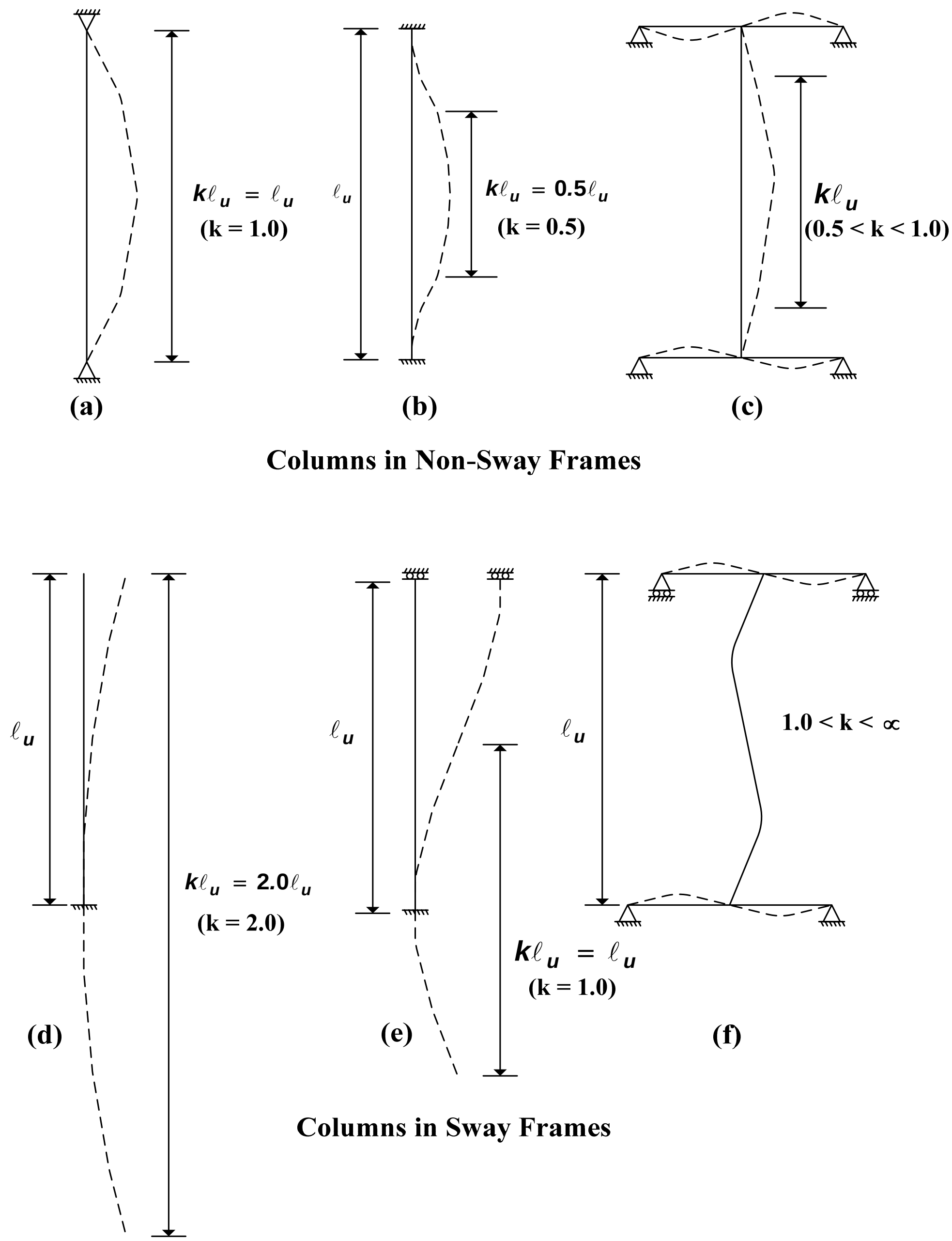


Fig. 4.2 Effective Length Factor *k* for Columns

Most columns have end restraints that are neither perfectly hinged nor fully fixed. The degree of end restraint depends on the stiffness of adjoining beams relative to that of the columns. Jackson and Moreland alignment charts, given in **Slender Columns 4.1** and **4.2** can be used to determine the effective length factor *k* for different values of relative stiffnesses at column ends. The stiffness ratios ψ_A and ψ_B used in **Slender Columns 4.1** and **4.2** should reflect concrete cracking, and the effects of sustained loading. Beams and slabs are flexure dominant members and may crack significantly more than columns which are compression members. The reduced stiffness values recommended by ACI 318-05 are given in **Slender Columns 4.3**, and should be used in determining *k*. Alternatively, **Slender Columns 4.4** may be used to establish conservative values of *k* for braced columns[2].

4.2.3 Radius of Gyration, r

The radius of gyration introduces the effects of cross-sectional size and shape to slenderness. For the same cross-sectional area, a section with higher moment of inertia produces a more stable column with a lower slenderness ratio. The radius of gyration r is defined below.

$$r = \sqrt{\frac{I}{A}} \tag{4-1}$$

It is permissible to use the approximations of $r = 0.3h$ for square and rectangular sections, and $r = 0.25h$ for circular sections, where "h" is the overall sectional dimension in the direction stability is being considered. This is shown in Fig. 4.3.

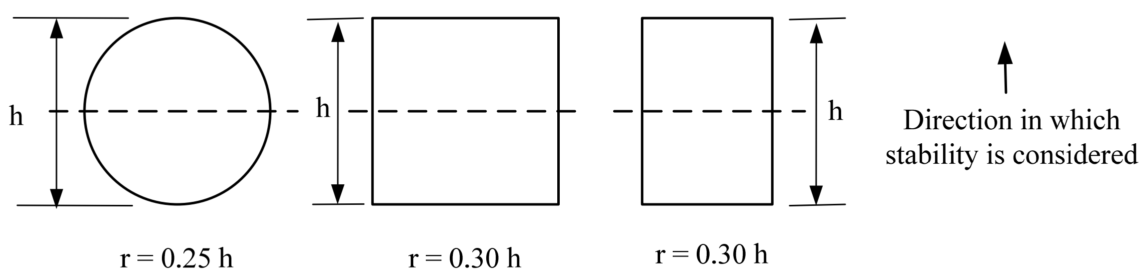


Fig. 4.3 Radius of gyration for circular, square and rectangular sections

4.3 Lateral Bracing and Designation of Frames as Non-Sway

A frame is considered to be "non-sway" if it is sufficiently braced by lateral bracing elements like structural walls. Otherwise, it may be designated as a "sway" frame. Frames that provide lateral resistance only by columns are considered to be sway frames. Structural walls that appear in the form of elevator shafts, stairwells, partial building enclosures or simply used as interior stiffening elements provide substantial drift control and lateral bracing. In most cases, even a few structural walls may be sufficient to brace a multi-storey multi-bay building. The designer can usually determine whether the frame is non-sway or sway by inspecting the floor plan. Frames with lateral bracing elements, where the total lateral stiffness of the bracing elements provides at least six times the summation of the stiffnesses of all the columns, may be classified as non-sway. ACI 318-05 permits columns to be designed as part of a non-sway frame if the increase in column end moments due to second-order

[2] "Concrete Design Handbook," Cement Association of Canada, third edition, 60 Queen Street, Ottawa, ON., Canada, K1P 5Y7, 2005.

effects does not exceed 5% of the first-order end moments (Sec. 10.11.4.1). Alternatively, Section 10.11.4.2 of ACI 318-05 defines a stability index "Q" (given in Eq. 4.2), where, $Q \leq 0.05$ indicates a non-sway column.

$$Q = \frac{\sum P_u \Delta_o}{V_{us} \ell_c} \tag{4.2}$$

Where, $\sum P_u$ is total factored axial load acting on all the columns in a story, V_{us} is total factored story shear, Δ_o is lateral story drift (deflection of the top of the story relative to the bottom of that story) due to V_{us}. The story drift Δ_o should be computed using the modified EI values given in **Slender Columns 4.3** with β_d defined as the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story. If Q exceeds approximately 0.2, the structure may have to be stiffened laterally to provide overall structural stability.

4.4 Design of Slender Columns

Design of a slender column should be based on a second-order analysis which incorporates member curvature and lateral drift effects, as well as material non-linearity and sustained load effects. An alternative approach is specified in ACI 318-05 for columns with slenderness ratios not exceeding 100. This approach is commonly referred to as the "Moment Magnification Method," and is based on magnifying the end moments to account for secondary stresses. The application of this procedure is outlined in the following sections.

4.4.1 Slender Columns in Non-Sway Frames

Slenderness effects may be neglected for columns in *non-sway frames* if the following inequality is satisfied:

$$\frac{k\ell_u}{r} \leq 34 - 12(M_1 / M_2) \tag{4-3}$$

Where $$(34 - 12M_1 / M_2) \leq 40 \tag{4-4}$$

M_1/M_2 is the ratio of smaller to larger end moments. This ratio is negative value when the column is bent in double curvature and positive when it is bent in single curvature. Fig. 4.4 illustrates columns in double and single curvatures. Columns in non-sway frames are more stable when they bend in double curvature, with smaller secondary effects, as compared to bending in single curvature. This is reflected in Eq. (4-3) through the sign of M_1/M_2 ratio. For negative values of this ratio the limit of slenderness in Eq. (4-3) increases, allowing a wider range of columns to be treated as short columns.

Slender columns in non-sway frames are designed for factored axial force P_u and amplified moment M_c. The amplified moment is obtained by magnifying the larger of the two end moments M_2 to account for member curvature and resulting secondary moments between the supports, while the supports are braced against sidesway. If M_c computed for the curvature effect between the ends is smaller than the larger end moment M_2, the design is carried out for M_2.

$$M_c = \delta_{ns} M_2 \tag{4-5}$$

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0 \tag{4-6}$$

The critical column load, P_c (Euler buckling load) is;

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \tag{4-7}$$

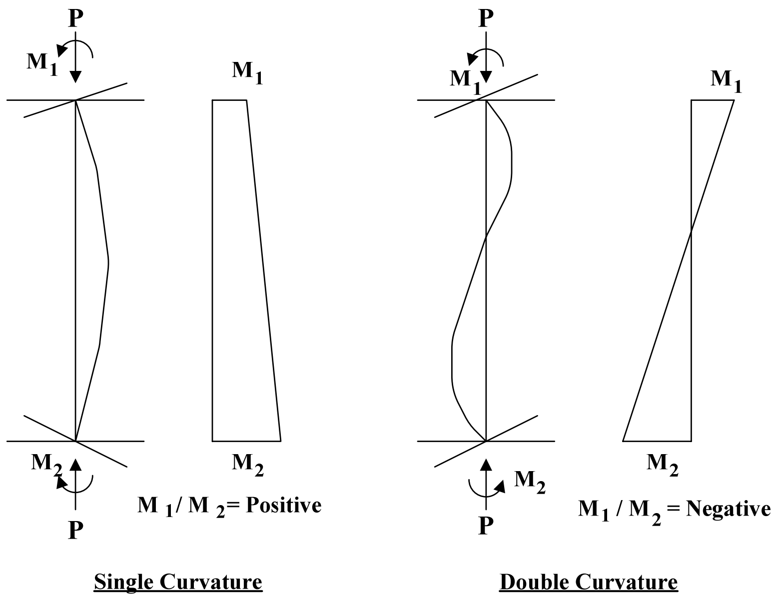


Fig. 4.4 Columns in Single and Double Curvature

EI in Eq. (4-7) is computed either with due considerations given to the presence of reinforcement in the section, as specified in Eq. (4-8), or approximately using Eq. (4-9).

$$EI = \frac{0.2E_cI_g + E_sI_{se}}{1 + \beta_d} \tag{4-8}$$

where β_d is the ratio of the maximum factored axial dead load to the total factored axial load. The moment of inertia of reinforcement about the cross-sectional centroid (I_{se}) can be computed using **Slender Columns 4.5**.

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \tag{4-9}$$

Note that Eq. (4-9) can be simplified further by assuming $\beta_d = 0.6$, in which case the equation becomes; $EI = 0.25E_cI_g$.

Coefficient C_m is equal to 1.0 for members with transverse loads between the supports. For the more common case of columns without transverse loads between the supports;

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \tag{4-10}$$

Where, M_1/M_2 is positive if the column is bent in single curvature.

When the maximum factored end moment M_2 is smaller than the minimum permissible design moment $M_{2,min}$, specified in Eq. (4-11), the magnification applies to $M_{2,min}$.

$$M_{2,min} \geq P_u(0.6 + 0.03h) \tag{4-11}$$

where h is the cross-sectional dimension in inches in the direction of the eccentricity of load. For columns for which $M_{2,min}$ is higher than M_2, the values of C_m, in Eq. (4-10) should either be taken 1.0 or determined based on the computed ratio of end moments (M_1/M_2). Once the amplified moment M_c is obtained, the designer can use the appropriate interaction diagrams given in Chapter 3 to determine the required percentage of longitudinal reinforcement.

4.4.2 Slender Columns in Sway Frames

Columns in sway frames are designed for the factored axial load P_u and the combination of factored gravity load moments and magnified sway moments. This is specified below, and illustrated in Fig. 4.5.

$$M_1 = M_{1ns} + \delta_s M_{1s} \tag{4-12}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \tag{4-13}$$

where, M_{1ns} and M_{2ns} are end moments due to factored gravity loads; and M_{1s} and M_{2s} are sway moments normally caused by factored lateral loads. All of these moments can be obtained from a first-order elastic frame analysis. Magnified sway moments $\delta_s M_{1s}$ and $\delta_s M_{2s}$ are obtained either from a second order frame analysis, with member flexural rigidity as specified in **Slender Columnss 4.3**, or by magnifying the end moments by sway magnification factor δ_s. The sway magnification factor is calculated either as given in Eq. (4-14) or Eq. (4-15).

$$\delta_s M_s = \frac{M_s}{1 - \dfrac{\sum P_u}{0.75\sum P_c}} \geq M_s \tag{4-14}$$

$$\delta_s M_s = \frac{M_s}{1 - Q} \geq M_s \tag{4-15}$$

However, if δ_s computed by Eq. (4-15) exceeds 1.5, $\delta_s M_s$ shall be calculated either through second order analysis or using Eq. (4-14).

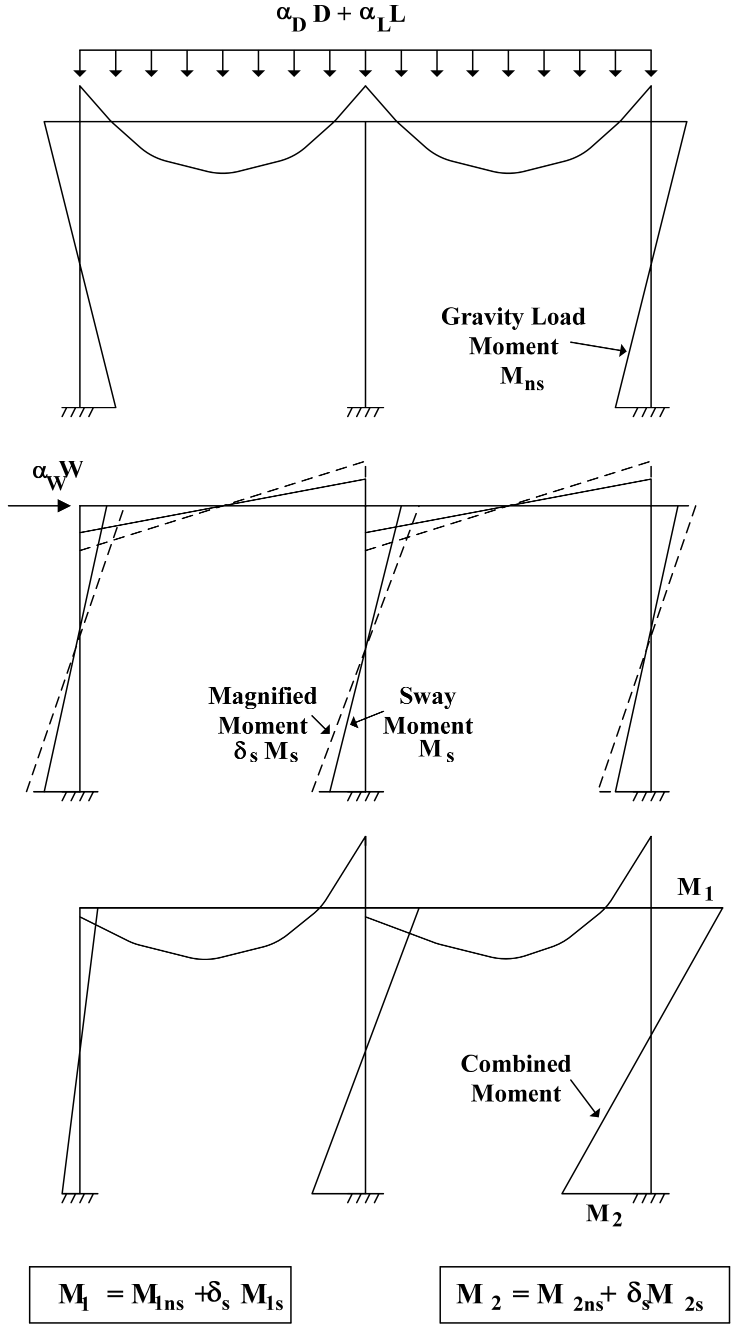


Fig. 4.5 Design moments in sway frames

In a sway frame, all the columns of a given story participate in the sway mechanism, and play roles in the stability of individual columns. Therefore, Eq. (4-14) includes $\sum P_u$ and $\sum P_c$ which give the summations of factored axial loads and critical loads for all the columns in the story, respectively. The critical column load P_c can be computed using Eqs. (4-7) through (4-9) with the effective length factor k computed for unbraced columns (for sway frames) and β_d as the ratio of the maximum factored

sustained shear within the story to the maximum total factored shear in the story. Eq. (4-14) provides an average δ_s for all the columns in a story. Therefore, it yields acceptable results if all the columns in a story undergo the same story drift. When significant torsion is anticipated under lateral loading, a second order analysis is recommended for finding the amplified sway moment, $\delta_s M_s$.

The magnification of moments through Eq. (4-15) is applicable only if the sway magnification factor δ_s does not exceed 1.5. If it does, then either the second-order analysis or Eq. (4-14) should be employed (Sec. 10.13.4.2).

The sidesway magnification discussed above is intended to amplify the end moments associated with lateral drift. Although the amplified end moment is commonly the critical moment for most sway columns, columns with high slenderness ratios may experience higher amplification of moments between the ends (rather than at the ends) because of the curvature of the column along the column height. This is assumed to occur when the inequality given in Eq. (4-16) is satisfied.

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\dfrac{P_u}{f'_c\, A_g}}} \tag{4-16}$$

The magnification of moment due to the curvature of column between the ends is similar to that for braced columns in non-sway frames. Therefore, if Eq. (4-16) is satisfied for a column, then the column should be designed for factored axial force P_u and magnified design moment (M_c) computed using Eqs. (4-5) and (4-6), with M_1 and M_2 computed from Eqs. (4-12) and (4-13).

Sometimes columns of a sway frame may buckle under gravity loads alone, without the effects of lateral loading. In this case one of the gravity load combinations may govern the stability of columns. The reduction of EI under sustained gravity loads may be another factor contributing to the stability of sway columns under gravity loads. Therefore, ACI 318-05 requires an additional check to safeguard against column buckling in sway frames under gravity loads alone (Sec. 10.13.6). Accordingly, the strength and stability of structure is reconsidered depending on the method of amplification used for sway moments. If a second order analysis was conducted to find $\delta_s M_{2s}$, two additional analyses are necessary using the reduced stiffness values given in **Slender Columns 4.3** with β_d taken as the ratio of the factored sustained axial dead load to total factored axial load. First, a second-order analysis is conducted under combined factored gravity loads and lateral loads equal to 0.5% of the gravity loads. Second, a first-order analysis is conducted under the same loading condition. The ratio of lateral drift obtained by the second-order analysis to that obtained by the first-order analysis is required to be limited to 2.5. If the sway moment was amplified by computing the sway magnification factor given in Eq. 4.14, as opposed to conducting second order analysis or using Eq. (4-15), then δ_s computed by using the gravity loads ($\sum P_u$ and $\sum P_c$ corresponding to the factored dead and live loads) is required to be positive and less than or equal to 2.5 to ensure the stability of the column. If the sway moment was amplified using Eq. (4-15), then the value of Q computed using $\sum P_u$ for factored dead and live loads should not exceed 0.60.

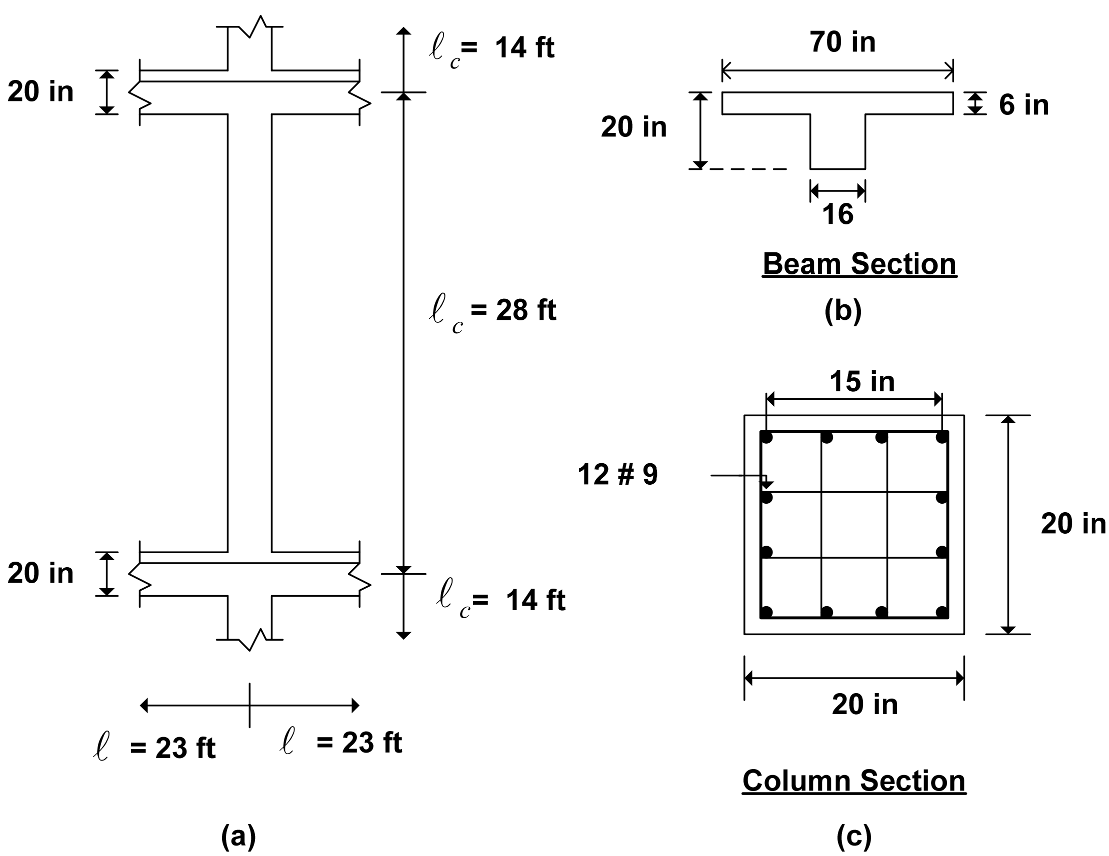
4.5 Slender Column Design Examples

SLENDER COLUMN EXAMPLE 1 - Design of an interior column braced against sidesway.

Consider a 10-story office building, laterally braced against sidesway by an elevator shaft (Q is computed to be much less than 0.05). The building has an atrium opening at the second floor level with a two-story high column in the opening to be designed. Design the column for the unfactored design forces given below, obtained from a first-order analysis. The framing beams are 16 in wide and 20 in deep with 23 ft (canter-to-centre) spans. The beam depth includes a slab thickness of 6 in. The story height is 14 ft (column height is 28 ft). It is assumed that the bracing elements provide full resistance to lateral forces and the columns only resist the gravity loads. Start the design with an initial column size of 20 in square. f'_c = 6,000 psi for all beams and columns; f_y = 60,000 psi.

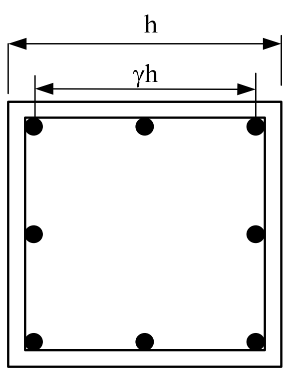
Unfactored Loads	Dead Load	Live Load
Axial load:	520 k	410 k
Top moment:	-1018 k-in	-620 k-in
Bottom moment:	-848 k-in	-540 k-in

Note: Moments are positive if counterclockwise at column ends. The column is bent in double curvature.



Slender Column Example 1

Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine factored design forces: *Note:* M_1 is the lower and M_2 is the higher end moment.	i) U = 1.4D $P_u = 1.4\ P_D = 1.4\ (520) = 728$ k $M_2 = 1.4\ M_{D2} = 1.4\ (1018) = 1425$ k-in $M_1 = 1.4\ M_{D1} = 1.4\ (848) = 1187$ k-in ii) U = 1.2 D + 1.6 L $P_u = 1.2\ P_D + 1.6\ P_L = 1.2\ (520) + 1.6\ (410) = 1280$ k $M_2 = 1.2\ M_{D2} + 1.6\ M_{L2} = 1.2\ (1018) + 1.6\ (620) = 2214$ k-in $M_1 = 1.2\ M_{D1} + 1.6\ M_{L1} = 1.2\ (848) + 1.6\ (540) = 1882$ k-in *Note:* Load Combination (ii) governs the design.	9.2	
Calculate slenderness ratio $k\ell_u / r$ i) Find unsupported column length ii) Find the radius of gyration iii) Find effective length factor "k." This requires the calculation of stiffness ratios at the ends. First find beam and column stiffnesses. Read k from **Slender Columns 4.1**	$\ell_u = 28 – 20/12 = 26.3$ ft $r = 0.3\ h = 0.3\ (20) = 6$ in $(I_g)_{beam} = (I_g)_{T\text{-}beam} = 19{,}527\ in^4$ $(I_g)_{column} = bh^3/12 = (20)(20)^3/12 = 13{,}333\ in^4$ Cracked (reduced) EI values: $(EI)_{beam} = (1{,}545)(19{,}527) = 30x10^6$ k-in^2 $(EI)_{col} = (3{,}091)(13{,}333) = 41x10^6$ k.in^2 $(EI/\ell)_{beam} = (30x10^6) / (23x12) = 109x10^3$ k-in for both left and right beams $(EI/\ell_c)_{col} = (41x10^6 / (28x12) = 122x10^3$ k-in for the atrium column to be designed. $(EI/\ell_c)_{col} = (41x10^6) / (14x12) = 244x10^3$ k-in for columns above and below $\Psi = (\Sigma EI/\ell_c)_{col} / (\Sigma EI/\ell)_{beam}$ $\Psi = [(EI/\ell_c)_{col,\ above} + (EI/\ell_c)_{col,\ below}] / [(EI/\ell)_{beam,\ left} + (EI/\ell)_{beam,\ right}]$ $\Psi_A = (244 + 122)x10^3 /(109 + 109)x10^3$ $\Psi_A = 1.7 = \Psi_B$ for $\Psi_B = \Psi_A = 1.7$; select k = 0.83 from **Slender Columns 4.1** (Note that **Slender Columns 4.4** gives a	 10.11.1	Figure 4.1 Slender Cols. 4.3 Slender Cols. 4.1 Slender Col. 4.4

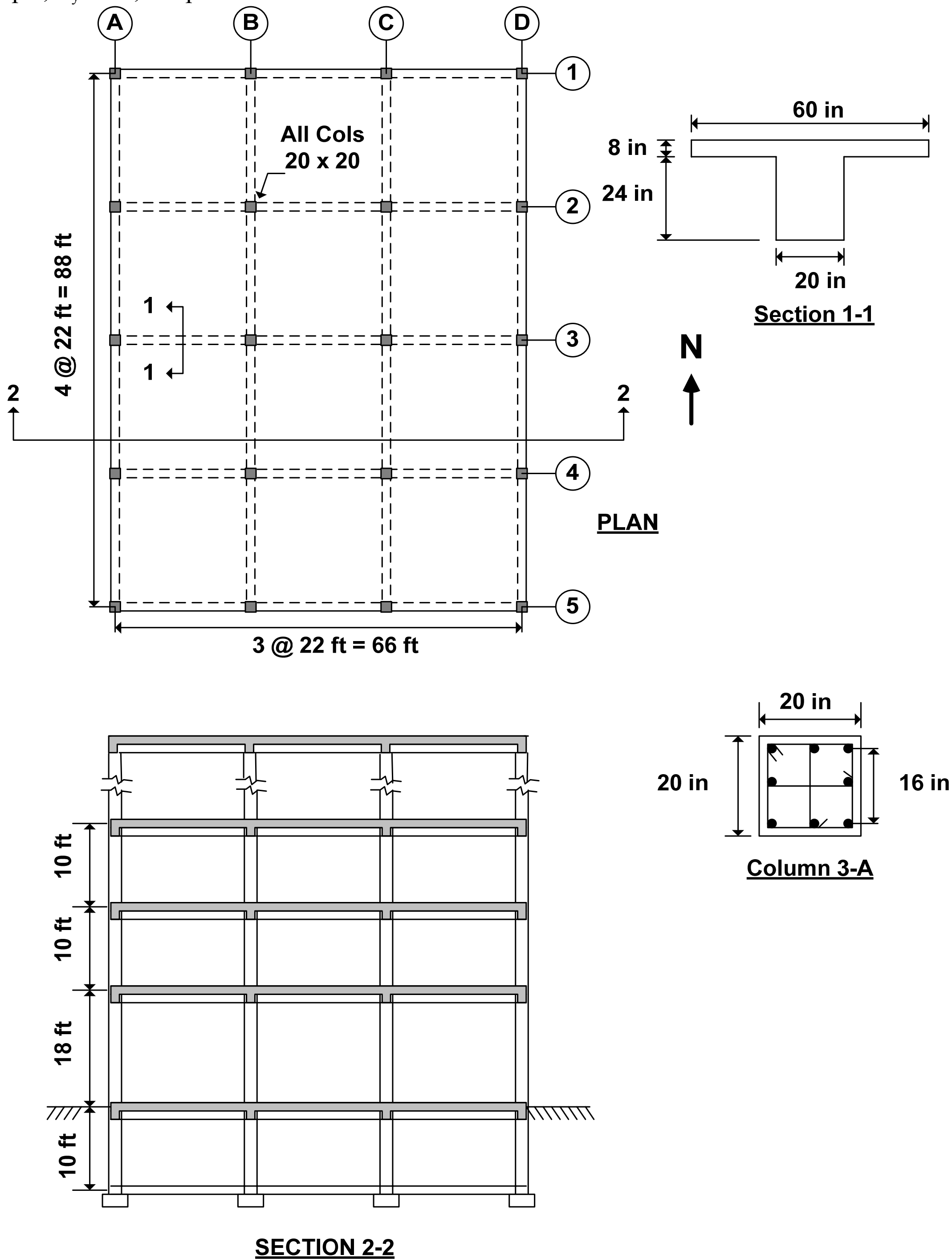
Compute the slenderness ratio	conservative value of k = 0.90) ℓ_u =28.9 – 1.67 = 26.3 ft $k\ell_u / r$ = 0.83 (26.3x12) / 6 = 45		
Check if slenderness can be neglected using Eq.(4-3): Apply the limit of Eq. (4-4)	$\frac{k\ell_u}{r} \le 34 - 12(M_1 / M_2)$ $(34 - 12M_1 / M_2) \le 40$ Note M_1/M_2 = - 1882/2214 = -0.85 (Bending in double curvature) or, for Load Combination I; M_1/M_2 = - 11871/1425 = -0.83 [34- 12 (-0.85)] = 44 > 40 use 40 $k\ell_u / r$ = 45 > 40 (limiting ratio for neglecting slenderness) Therefore, consider slenderness.	10.12.2 10.3.4 9.3.2	
Compute moment magnification factor (δ_{ns}) from Eq. (4-6): i) Compute critical load P_c from Eq (4-7) Use Eq. (4-8) to compute EI. Assume 2.5% column reinforcement, equally distributed along the perimeter of the square section with γ = 0.75 where γ is the ratio of the distance between the centres of the outermost bars to the column dimension perpendicular to the axis of bending.  Alternatively, compute EI from Eq,(4-9) Eq. (4-9) may further be simplified by assuming a value of β_d = 0.6. $EI = \frac{0.4E_cI_g}{1+\beta_d} = 0.25E_cI_g$	$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \ge 1.0$ $P_c = \frac{\pi^2 EI}{(k\ell_u)^2}$ E_c = 4415 ksi E_s = 29,000 ksi $(I_g)_{column}$ = 13,333 in^4 $I_{se} = 0.18\, \rho_t\, b\, h^3 \gamma^2$ (from **Slender Col. 4.5**) $I_{se} = 0.18(0.025)(20)(20)^3\,(0.75)^2$ = 405 in^4 β_d = 1.2D / (1.2D + 1.6L) = 1.2 x 520 / (1.2 x 520 + 1.6 x 410) = 624 / 1280 = 0.49 $EI = (0.2E_cI_g + E_sI_{se}) / (1 + B_d)$ EI = [(0.2 x 4415 x 13,333) + (29,000 x 405)] / (1 + 0.49) = 16 x 10^6 k-in^2 EI = (0.4 x 4415 x 13,333) / (1 + 0.49) EI = 16 x 10^6 k-in^2 EI = 0.25 E_cI_g = 0.25 x 4415 x 13,333 EI = 15 x 10^6 k-in^2	10.12.3 10.12.3 8.5.2 10.12.3 10.12.3 R10.12.3	 Slender Cols. 4.3 Slender Cols. 4.5

	$P_c = \pi^2 EI / (k \ell_u)^2$ $P_c = \pi^2 \times 16 \times 10^6 / (0.83 \times 26.3 \times 12)^2$ $P_c = 2301$ k	10.12.3	
ii) Compute C_m from Eq. (4-10):	$C_m = 0.6 + 0.4\ M_1/M_2 \geq 0.4$ $C_m = 0.6 + 0.4\ (-0.85) = 0.25 < 0.4$ use 0.4		
iii) Moment magnification factor	$\delta_{ns} = \dfrac{0.4}{1 - \dfrac{1280}{(0.75)(2301)}} = 1.55 \geq 1.0$	10.12.3	
Compute amplified moment M_c from Eq. (4-5)	$M_c = \delta_{ns} M_2$ = 1.55 (2214) = 3432 k-in	10.12.3	
Check against minimum design moment as per Eq. (4-11).	$M_{2,min} \geq P_u(0.6 + 0.03h)$ $M_{2,min}$ = 1280 (0.6 + 0.03 x 20) =1536 k-in M_c = 3432 k-in > $M_{2,min}$ = 1536 k-in Design for M_c = 3432 k-in	10.12.3.2	
Select reinforcement ratio and design the column section: Use Column Interaction Diagrams R6-60.7 and R6-60.9 for equal reinforcement on all sides and interpolate for $\gamma = 0.75$ (assumed above) A) Compute $K_n = \dfrac{P_n}{f_c' A_g}$ B) Compute $R_n = \dfrac{M_n}{f_c' A_g h}$ C) Read ρ_g for K_n and R_n values from the interaction diagrams D) Compute required A_{st} from $A_{st} = \rho_g A_g$ E) Find column reinforcement	Note: $\gamma = 0.75$ allows for more than 1.5 in clear cover required for interior columns, not exposed to weather. $K_n = \dfrac{P_u / \phi}{f_c' A_g} = \dfrac{1280 / 0.65}{(6)(20)^2} = 0.82$ $R_n = \dfrac{M_n / \phi}{f_c' A_g h} = \dfrac{3432 / 0.65}{(6)(20)^2(20)} = 0.11$ For $K_n = 0.82$ and $R_n = 0.11$ Read $\rho_g = 0.031$ for $\gamma = 0.7$ and $\rho_g = 0.029$ for $\gamma = 0.8$ Interpolating; $\rho_g = 0.030$ for $\gamma = 0.75$ (Note that the required steel ratio of 3% is slightly higher that the 2.5% assumed for computing EI. No revision is necessary). Required $A_{st} = 0.030 \times 400$ in.^{2} = 12.0 in.^{2} Try # 9 bars; 12.0 / 1.0 = 12.0 Use 12 # 9 Bars.	7.7.1	Flexure 9 Columns R6-60.7 and R6-60.8 Columns R6-60.7 and R6-60.8

SLENDER COLUMN EXAMPLE 2 - Design of an exterior column in a sway frame.

A typical floor plan and a section through a multi-story office building are shown below. Design column 3-A at the ground level for combined gravity and east-west wind loading. The results of first-order frame analysis under factored load combinations are given in the solution.

f'_c = 6,000 psi; f_y = 60,000 psi.



Slender Column Example 2

Procedure	Calculation	ACI 318-05 Section	Design Aid
Consider the applicable load combinations: The structure is *not* braced against sidesway. Therefore, the column will be designed considering the loads that cause sidesway. Note that sidesway in this structure is caused by wind loading. No significant sidesway is anticipated due to gravity loads since the structure is symmetric. However, the possibility of sidesway instability under gravity loads alone shall be investigated as per Sec. 10.13.6.	a)Load combinations that include wind; Comb. I: $U = 1.2D + 1.6L_r + 0.8W$ Comb. II: $U = 1.2D + 1.6W + 1.0L + 0.5L_r$ Comb.III: $U = 0.9D + 1.6W$ b) Load combinations for gravity loads; Comb. IV: $U = 1.4D$ Comb. V: $U = 1.2D + 1.6L + 0.5L_r$ Comb. VI: $U = 1.2D + 1.6\ L_r + 1.0L$	9.2.1 9.2.1	
Using the preliminary column section given in the figure, determine the effective length factor *k* for each column at the ground level. This requires the computation of beam and column stiffnesses. *Note:* All columns have the same section. Factor *k* reflects column end restraint conditions and depends on relative stiffnesses of columns to beams at top and bottom joints. Read *k* from **Slender Columns 4.2 and 4.1.**	$I_{beam} = 87{,}040\ in^4$ (for T-section) $I_{col} = (20)(20)^3/12 = 13{,}333\ in^4$ Find reduced EI values from **Slender Col. 4.3** for 6.0 ksi concrete; $(E_cI)_{beam} = 1545\ I_{beam} = (1545)(87{,}040)$ $= 134 \times 10^6$ k-in^2 $(E_cI)_{col} = 3091\ I_{col} = (3091)(13{,}333)$ $= 41 \times 10^6$ k-in^2 $(EI/\ell)_{beam} = 134 \times 10^6 / (22 \times 12)$ $= 507{,}576$ k-in $(EI/\ell_c)_{col,\ typical} = 41 \times 10^6 / (10 \times 12)$ $= 341{,}667$ k-in $(EI/\ell_c)_{col,\ atrium} = 41 \times 10^6 / (18 \times 12)$ $= 189{,}815$ k-in $\Psi = (3EI/\ell_c)_{col} / (3EI/\ell)_{beam}$ $\Psi = [(EI/\ell_c)_{col,\ above} + (EI/\ell_c)_{col,\ below}] / [(EI/\ell)_{beam,\ left} + (EI/\ell)_{beam,\ right}]$ i) For exterior columns (columns on lines A and D): $\Psi_A = (341{,}667 + 189{,}815) / 507{,}576 = 1.05$ $\Psi_B = \Psi_A = 1.05$; from **Slender Cols. 4.2**: k = 1.35 (for unbraced frames) k = 0.78 for a braced column, from **Slender Cols. 4.1.** This value is computed for further magnification of moments, if necessary for column 3-A as per Sec. 10.13.6.	10.11.1 10.13.6	Slender Cols. 4.3 Slender Cols. 4.3 Slender Cols. 4.2 Slender Cols. 4.1

<table>
<tr>
<td>Compute the slenderness ratio</td>
<td>ii) For interior columns (columns on lines B and C):
Ψ_A = (341,667 + 189,815)/(507,576 + 507,576) = 0.52
$\Psi_B = \Psi_A$ = 0.52; from Slender Cols. 4.2;
k = 1.15 (for unbraced frames)</td>
<td></td>
<td>Slender Cols. 4.2</td>
</tr>
<tr>
<td>Compute critical load P_c from Eq. 4.7 and EI from either Eq. 4.8 or 4.9. Note, if Eq. 4.9 is used for simplicity with β_d = 0 (since wind loading is a short term load)
$$EI = \frac{0.4E_c I_g}{1+\beta_d} = 0.4E_c I_g$$
For braced columns, Eq. 4.9 can be simplified by substituting β_d = 0.6. Then; $EI = 0.25\ E_c\ I_g$

P_c for braced columns may be needed if further magnification of moments is required as per Sec. 10.13.6..</td>
<td>i) For exterior columns (columns on lines A and D):
E_c = 4415 ksi for f'_c = 6 ksi

For sway columns;
EI = $0.4E_cI_g$ = 0.4(4415)(13,333)
= 23.5 x 10^6 k-in^2
ℓ_u = (18) (12) - 32 = 184 in
$P_c = \pi^2$ EI/(k ℓ_u)^2
= π^2(23.5x10^6)/(1.35x184)^2
= 3759 kips for a sway frame.

For braced columns;
EI = $0.25E_cI_g$ = 0.25(4415)(13,333)
= 14.7 x 10^6 k-in^2

$P_c = \pi^2$EI/(k ℓ_u)^2
= π^2(14.7x10^6)/(0.78x184)^2
= 7044 kips for braced columns.

ii) For interior columns (columns on lines B and C):

$P_c = \pi^2$ EI / (k ℓ_u)^2
= π^2 (23.5 x 10^6) / (1.15 x 184)^2
= 5180 kips for a sway frame.

ΣP_c = 10 (3759) + 10 (5180) = 89,390 kips</td>
<td>10.12.13</td>
<td>Slender Cols. 4.3</td>
</tr>
<tr>
<td>Compute magnified sway moment $\delta_s M_s$ Under Load Combination I.

Conduct first-order frame analysis using Load Combination I, and the stiffness values specified in Slender Cols. 4.3.

Note: Counterclockwise moment at column end is positive.</td>
<td>i) Load Comb. I: U = 1.2D + 1.6L_r + 0.8W
<table>
<tr><td>Load</td><td>1.2D +1.6L_r</td><td>0.8W</td></tr>
<tr><td>P_u (kips)
Corner Column</td><td>425</td><td>±12</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>682</td><td>±12</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1134</td><td>±4</td></tr>
<tr><td>$(M_u)_{top}$ (k-in)
Column 3-A</td><td>-1296</td><td>±765</td></tr>
<tr><td>$(M_u)_{bot}$ (k-in)
Column 3-A</td><td>-1296</td><td>±1111</td></tr>
</table></td>
<td>9.2.1

10.11.1</td>
<td></td>
</tr>
</table>

<table>
<tr>
<td>Compute sway magnification factor δ_s from Eq. 8.12. This requires the computation of ΣP_f in addition to ΣP_c Obtained in the previous step.</td>
<td>Sway magnification factor δ_s:

$\Sigma P_f = 4\ (425 + 12) + 10\ (682 + 12) + 6\ (1134 + 4) = 15{,}516$ kips

$\delta_s = 1 / [1 - \Sigma P_f / [(0.75)\Sigma P_c\ ]$
$= 1/[1 - 15{,}516 /(0.75 \times 89{,}390)] = 1.30$

$\delta_s\ M_{1s} = 1.30 \times 765 = 995$ k-in
$\delta_s\ M_{2s} = 1.30 \times 1111 = 1444$ k-in</td>
<td>10.13.4.3</td>
<td></td>
</tr>
<tr>
<td>Compute design moments M_1 and M_2</td>
<td>$M_1 = M_{1ns} + \delta_s M_{1s} = 1296 + 995 = 2291$ k-in
$M_2 = M_{2ns} + \delta_s M_{2s} = 1296 + 1444 = 2738$ k-in</td>
<td>10.13.3</td>
<td>Fig. 4.5</td>
</tr>
<tr>
<td>Check if further magnification of moments is required for Column 3-A due to the curvature of columns between the ends as per Sec. 10.13.5</td>
<td>$\dfrac{\ell_u}{r} > \dfrac{35}{\sqrt{P_u/(f'_c\ A_g)}}$

$P_u = 682 + 12 = 694$ k
$\ell_u/r = 184 / (0.3 \times 20) = 30.7$
$35 / \sqrt{694/(6x(20)^2)} = 65.1 > 30.7$
Therefore, no further magnification is required.</td>
<td>10.13.5</td>
<td></td>
</tr>
<tr>
<td>Compute magnified sway moment $\delta_s M_s$ Under Load Combination II.

Conduct first-order frame analysis using Load Combination I, and the stiffness values specified in Slender Cols. 4.4.

Note: Counterclockwise moment at column end is positive.

Compute sway magnification factor δ_s from Eq. 8.12. This requires the computation of ΣP_f in addition to ΣP_c Obtained in the previous step.</td>
<td>ii) Load Combination II:
$U=1.2D+1.6W+1.0L+0.5L_r$

<table>
<tr><td>Load</td><td>1.2D +1.0L + 0.5L_r</td><td>1.6W</td></tr>
<tr><td>P_u (kips)
Corner Column</td><td>493</td><td>±24</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>845</td><td>±24</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1459</td><td>±8</td></tr>
<tr><td>$(M_u)_{top}$ (k-in)
Column 3-A</td><td>-1756</td><td>±1530</td></tr>
<tr><td>$(M_u)_{bot}$ (k-in)
Column 3-A</td><td>-1756</td><td>±2222</td></tr>
</table>

Sway magnification factor δ_s:

$\Sigma P_f = 4\ (493 + 24) + 10\ (845 + 24) + 6\ (1459 + 8) = 19{,}560$ kips

$\delta_s = 1 / [1 - \Sigma P_f / (0.75\Sigma P_c\ )]$
$= 1/[1 - 19{,}560 /(0.75 \times 89{,}390)] = 1.41$

$\delta_s\ M_{1s} = 1.41 \times 1530 = 2157$ k-in
$\delta_s\ M_{2s} = 1.41 \times 2222 = 3111$ k-in</td>
<td>9.2.1

10.11.1

10.13.4.3</td>
<td></td>
</tr>
</table>

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Compute design moments M_1 and M_2</td><td>$M_1 = M_{1ns}+\delta_s M_{1s}$ =1756 +2157 = 3913 k-in
$M_2 = M_{2ns}+\delta_s M_{2s}$ =1756 +3111= 4867 k-in</td><td>10.13.3</td><td>Fig. 4.5</td></tr>
<tr><td>Check if further magnification of moments is required for Column 3-A due to the curvature of columns between the ends as per Sec. 10.13.5</td><td>$\frac{\ell_u}{r} > \frac{35}{\sqrt{P_u/(f'_c A_g)}}$

P_u = 845 + 24 = 869 k
ℓ_u/r = 184 / (0.3 x 20) = 30.7
35 / $\sqrt{869/(6x(20)^2)}$ = 58.2 > 30.7
Therefore, no further magnification is required.</td><td>10.13.5</td><td></td></tr>
<tr><td>Compute magnified sway moment $\delta_s M_s$ Under Load Combination II.

Conduct first-order frame analysis using Load Combination I, and the stiffness values specified in Slender Cols. 4.4.

Note: Counterclockwise moment at column end is positive.

Compute sway magnification factor δ_s from Eq. 8.12. This requires the computation of ΣP_f in addition to ΣP_c Obtained in the previous step.</td><td>iii) Load Comb. III: U = 0.9D + 1.6W
<table><tr><td>Load</td><td>0.9D</td><td>1.6W</td></tr><tr><td>P_u (kips)
Corner Column</td><td>258</td><td>±24</td></tr><tr><td>P_u (kips)
Edge Column</td><td>452</td><td>±24</td></tr><tr><td>P_u (kips)
Interior Column</td><td>790</td><td>±8</td></tr><tr><td>$(M_u)_{top}$ (k-in)
Column 3-A</td><td>-972</td><td>±1530</td></tr><tr><td>$(M_u)_{bot}$ (k-in)
Column 3-A</td><td>-972</td><td>±2222</td></tr></table>Sway magnification factor δ_s:
ΣP_f = 4 (258 + 24) + 10 (452 + 24) + 6 (790 + 8) = 10,676 kips

$\delta_s = 1 / [1 - \Sigma P_f / (0.75 \Sigma P_c)]$
= 1/[1– 10,676 /(0.75 x 89,390)] = 1.19

$\delta_s M_{1s}$ = 1.19 x 1530 = 1821 k-in
$\delta_s M_{2s}$ = 1.19 x 2222 = 2644 k-in</td><td>9.2.1

10.11.1

10.13.4.3</td><td></td></tr>
<tr><td>Compute design moments M_1 and M_2</td><td>$M_1 = M_{1ns}+\delta_s M_{1s}$ = 972 +1821 = 2793 k-in
$M_2 = M_{2ns}+\delta_s M_{2s}$ = 972 +2644 = 3616 k-in</td><td>10.13.3</td><td>Fig. 4.5</td></tr>
<tr><td>Check if further magnification of moments is required for Column 3-A due to the curvature of columns between the ends as per Sec. 10.13.5</td><td>$\frac{\ell_u}{r} > \frac{35}{\sqrt{P_u/(f'_c A_g)}}$

P_u = 452 + 24 = 476 k
ℓ_u/r = 184 / (0.3 x 20) = 30.7
35 / $\sqrt{476/(6x(20)^2)}$ = 78.6 > 30.7
Therefore, no further magnification is required.</td><td>10.13.5</td><td></td></tr>
</table>

<table>
<tr>
<td>Check the stability of column under gravity loads only (Load combinations IV, V and VI) as per Sec. 10.13.6.

Consider factored axial loads and bending moments obtained from a first-order frame analysis, conducted using the flexural rigidities given in Slender Cols. 4.3

Note: Counterclockwise moment at column end is positive.

Compute the sway magnification factor δ_s

Critical load, from earlier calculation.</td>
<td>iv) Load Comb. IV: U = 1.4D

<table>
<tr><th>Load</th><th>1.4D</th></tr>
<tr><td>P_u (kips)
Corner Column</td><td>402</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>703</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1229</td></tr>
<tr><td>$(M_u)_{top}$ kip-in
Column 3-A</td><td>-1512</td></tr>
<tr><td>$(M_u)_{bot}$ kip-in
Column 3-A</td><td>-1512</td></tr>
</table>

Sway magnification factor δ_s:

ΣP_f = 4 (402) + 10 (703) + 6 (1229) = 16,012 kips

ΣP_c = 10 (3759) + 10 (5180) = 89,390 kips

$\delta_s = 1 / [1 - \Sigma P_f / (0.75\Sigma P_c)]$
= 1 / [1 – 16,012 / (0.75 x 89,390)]
= 1.31

δ_s = 1.31 < 2.5 O.K.</td>
<td>10.13.4.3</td>
<td>Slender Cols. 4.3</td>
</tr>
<tr>
<td>Check the stability of column under Load combination V as per Sec. 10.13.6.

Consider factored axial loads and bending moments obtained from a first-order frame analysis, conducted using the flexural rigidities given in Slender Cols. 4.3

Note: Counterclockwise moment at column end is positive.

Compute the sway magnification factor δ_s

Critical load, from earlier calculation.</td>
<td>v) Load Comb. V:U =1.2D + 1.6L + 0.5L_r

<table>
<tr><th>Load</th><th>1.2D + 1.6L + 0.5L_r</th></tr>
<tr><td>P_u (kips)
Corner Column</td><td>568</td></tr>
<tr><td>P_u (kips)
Edge Column</td><td>976</td></tr>
<tr><td>P_u (kips)
Interior Column</td><td>1687</td></tr>
<tr><td>$(M_u)_{top}$ kip-in
Column 3-A</td><td>-2032</td></tr>
<tr><td>$(M_u)_{bot}$ kip-in
Column 3-A</td><td>-2032</td></tr>
</table>

Sway magnification factor δ_s:
ΣP_f = 4 (568) + 10 (976) + 6 (1687) = 22,154 kips
ΣP_c = 10 (3759) + 10 (5180) = 89,390 kips
$\delta_s = 1 / [1 - \Sigma P_f / (0.75\Sigma P_c)]$
= 1 / [1 – 22,154 / (0.75 x 89,390)]
= 1.49
δ_s = 1.49 < 2.5 O.K.</td>
<td>10.13.4.3</td>
<td>Slender Cols. 4.3</td>
</tr>
</table>

Check the stability of column under Load combination VI as per Sec. 10.13.6.

Consider factored axial loads and bending moments obtained from a first-order frame analysis, conducted using the flexural rigidities given in **Slender Cols. 4.3**

Note: Counterclockwise moment at column end is positive.

vi) Load Comb. VI: $U=1.2D+1.6\ L_r + 1.0L$

10.13.5

Load	$1.2D+ 1.6\ L_r + 1.0L$
P_u (kips) Corner Column	548
P_u (kips) Edge Column	900
P_u (kips) Interior Column	1514
$(M_u)_{top}$ kip-in Column 3-A	-1756
$(M_u)_{bot}$ kip-in Column 3-A	-1756

Slender Cols. 4.3

Compute the sway magnification factor δ_s

Sway magnification factor δ_s:

$\Sigma P_f = 4\ (548) + 10\ (900) + 6\ (1514)$
$= 20{,}276$ kips

$\Sigma P_c = 10\ (3759) + 10\ (5180) = 89{,}390$ kips

Critical load, from earlier calculation.

$\delta_s = 1\ /\ [1 - \Sigma P_f\ /\ (0.75 \Sigma P_c\ )]$
$= 1\ /\ [1 - 20{,}276\ /\ (0.75 \times 89{,}390)]$
$= 1.43$

$\delta_s = 1.43 < 2.5$ O.K.

Design the Column for the governing load combination.

Summary of Design Loads:

Load Combinations	P_u (kN)	(M_u) (kN.m)
I - $U = 1.2D + 1.6L_r + 0.8W$	682	-2738
II - $U = 1.2D+1.6W +1.0L+0.5L_r$	845	-4867
III - $U = 0.9D + 1.6W$	452	-3616
IV - $U = 1.4D$	703	-1512
V - $U = 1.2D + 1.6L + 0.5L_r$	976	-2032
VI - $U=1.2D+1.6\ L_r + 1.0L$	900	-1756

Note: Counterclockwise moment at column end is positive.

Select the interaction diagrams given in **Columns 3.4.3** from Chapter 3 for equal reinforcement on all sides for $\gamma = 0.80$ (assumed)

For Load Combination II;

Compute; $K_n = \dfrac{P_n}{f'_c A_g}$

$$K_n = \frac{P_u/\varphi}{f'_c A_g} = \frac{845/0.65}{(6)(20)^2} = 0.54$$

Compute; $R_n = \dfrac{M_n}{f'_c A_g h}$

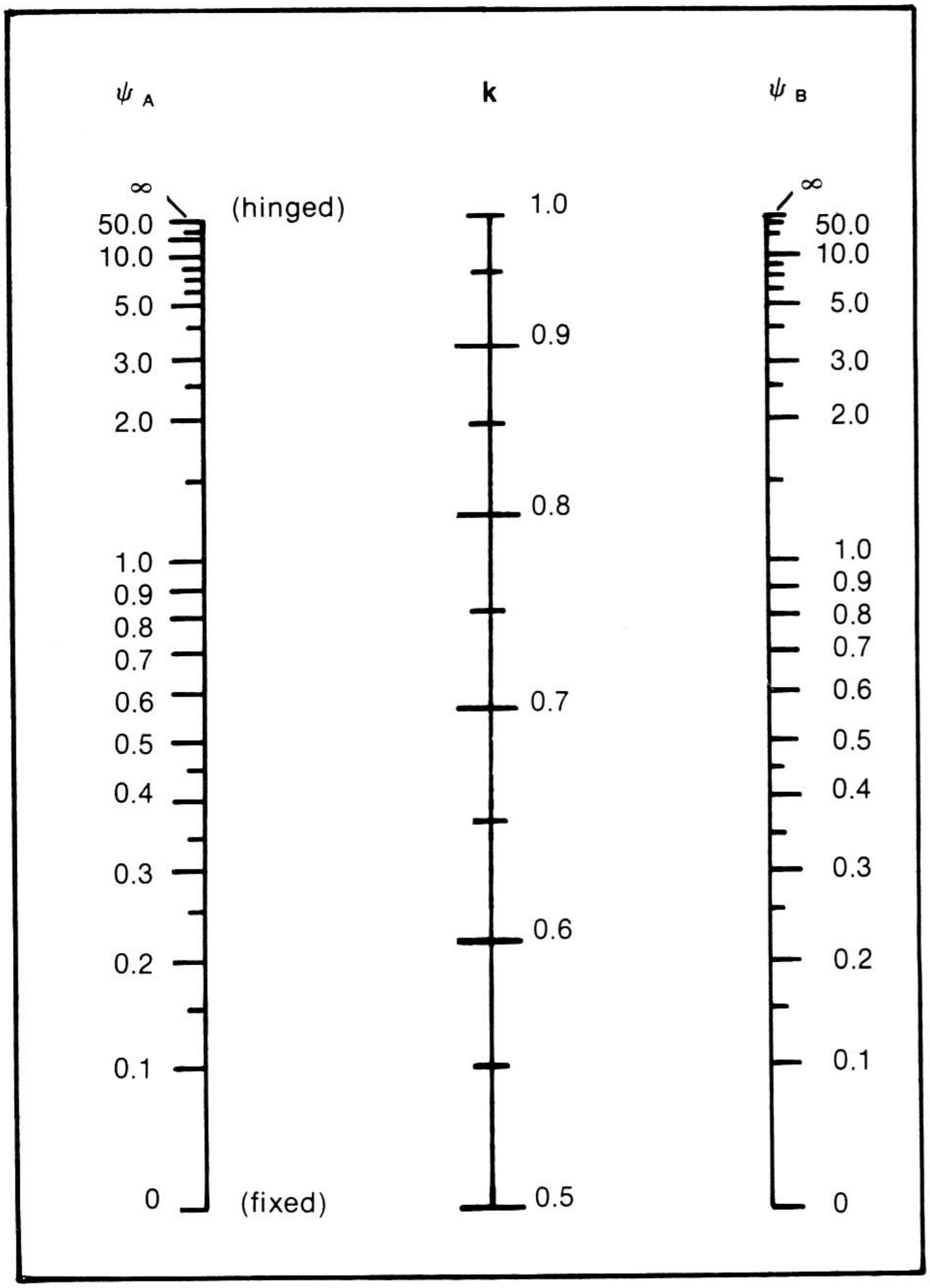
$$R_n = \frac{M_n/\varphi}{f'_c A_g h} = \frac{4867/0.65}{(6)(20)^2(20)} = 0.16$$

Columns 3.4.3

Read ρ_g for K_n and R_n values from the interaction diagrams	For K_n = 0.54 and R_n = 0.16 Read ρ_g = 0.030 Required A_{st} = 0.030 x 400 in.^{2} = 12.0 in.^{2} Try # 9 bars; 12.0 / 1.0 = 12.0 Try 12 # 9 Bars. Check for Load Combination V; $K_n = \dfrac{P_u/\varphi}{f_c' A_g} = \dfrac{976/0.65}{(6)(20)^2} = 0.63$ $R_n = \dfrac{M_n/\varphi}{f_c' A_g h} = \dfrac{2032/0.65}{(6)(20)^2 (20)} = 0.065$ For K_n = 0.63 and R_n = 0.065 ρ_g = 0.030 is sufficient Therefore, use 12 # 9 Bars. *Note:* For further details of cross-sectional design refer to Chapter 3.		Columns 3.4.3 Columns 3.4.3

4.6 Slender Column Design Aids

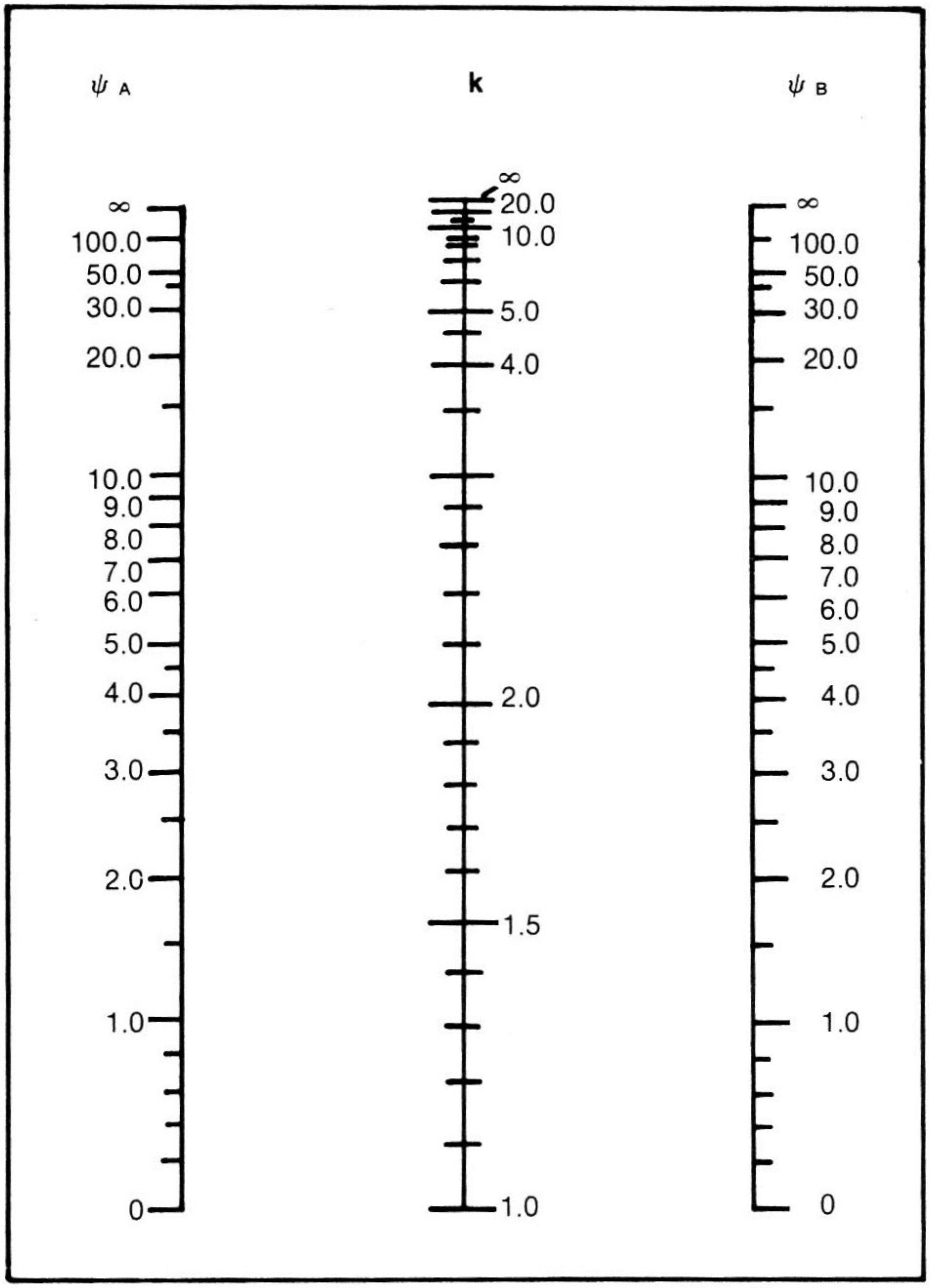
Slender Column - 4.1 Effective Length Factor – Jackson and Moreland Alignment Chart for Columns in Braced (Non-Sway) Frames[3]



$$\psi_i = \frac{(\Sigma EI/\ell_c)_{columns}}{(\Sigma EI/\ell)_{beams}} \quad \text{at end i of column}$$

[3] "Guide to Design Criteria for Metal Compression Members," 2nd Edition, Column Research Council, Fritz Engineering Laboratory, Lehigh University, Bethlehem, PA, 1966

Slender Columns - 4.2 Effective Length Factor – Jackson and Moreland Alignment Chart for Columns in Unbraced (Sway) Frames[4]



$$\psi_i = \frac{(\Sigma EI/\ell_c)_{columns}}{(\Sigma EI/\ell)_{beams}} \quad \text{at end i of column}$$

[4] "Guide to Design Criteria for Metal Compression Members," 2nd Edition, Column Research Council, Fritz Engineering Laboratory, Lehigh University, Bethlehem, PA, 1966

Slender Columns - 4.3 Recommended Flexural Rigidities (EI) for use in First-Order and Second Order Analyses of Frames for Design of Slender Columns

f'_c (ksi)	3	4	5	6	7	8	9	10	I/I_g
E_c (ksi)	3120	3605	4031	4415	4769	5098	5407	5700	
$E_c I / I_g$ (ksi)									
Beams	1092	1262	1411	1545	1669	1784	1892	1995	0.35
Columns	2184	2524	2822	3091	3338	3569	3785	3990	0.70
Walls (Uncracked)	2184	2524	2822	3091	3338	3569	3785	3990	0.70
Walls (Cracked)	1092	1262	1411	1545	1669	1784	1892	1995	0.35
Flat Plates Flat Slabs	780	901	1008	1104	1192	1275	1352	1425	0.25

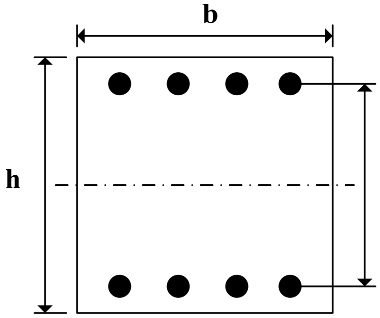
Notes:

1. The above values will be divided by $(1+\beta_d)$, when sustained lateral loads act or for stability checks made in accordance with Section 10.13.6 of ACI 318-05. For non-sway frames, β_d is ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination, $\beta_d = 1.2D / (1.2D + 1.6L)$.
2. For sway frames, except as specified in Section 10.13.6 of ACI 318-05, β_d is ratio of maximum factored sustained shear within a story to the maximum factored shear in that story.
3. The above values are applicable to normal-density concrete with w_c between 90 and 155 lb/ft^3.
4. The moment of inertia of a T-beam should be based on the effective flange width, shown in **Flexure 6.** It is generally sufficiently accurate to take I_g of a T-beam as two times the I_g for the web.
5. Area of a member will *not* be reduced for analysis.

Slender Column - 4.4 Effective Length Factor “k” for Columns in Braced Frames

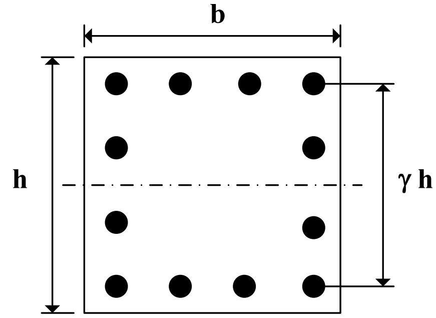
		k			
		BOTTOM: Stiff	BOTTOM: Elastic	BOTTOM: Elastic	BOTTOM: Hinged
TOP	Hinged	0.81	0.91	0.95	1.00
TOP	Elastic	0.77	0.86	0.90	0.95
TOP	Elastic	0.74	0.83	0.86	0.91
TOP	Stiff	0.67	0.74	0.77	0.81

Slender Columns - 4.5 Moment of Inertia of Reinforcement about Sectional Centroid[5]



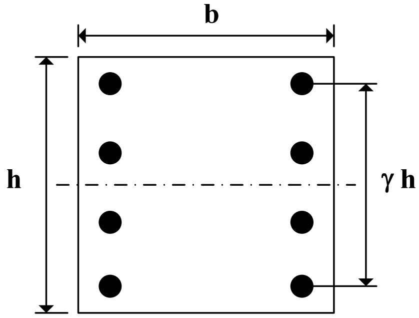
Bars in two end faces

$I_{se} = 0.25\,\rho_t\, b\, h^3 \gamma^2$



Equal reinforcement on four sides

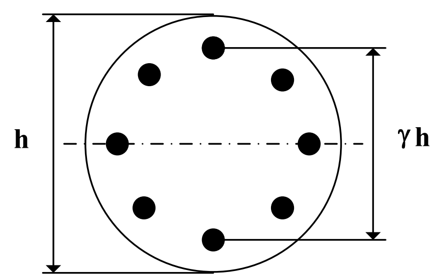
$I_{se} = 0.18\,\rho_t\, b\, h^3 \gamma^2$



Bars in two side faces

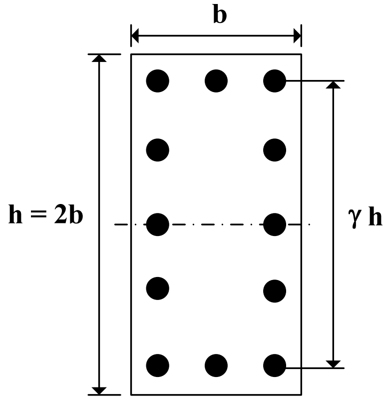
$I_{se} = 0.17\,\rho_t\, b\, h^3 \gamma^2$ **(3 bars per face)**

$I_{se} = 0.12\,\rho_t\, b\, h^3 \gamma^2$ **(6 bars per face)**



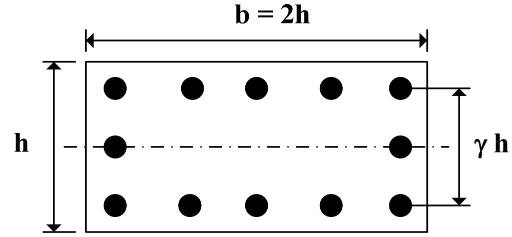
Uniformly distributed reinforcement

$I_{se} = 0.10\,\rho_t\, h^4 \gamma^2$



Bars uniformly spaced on all sides

$I_{se} = 0.13\,\rho_t\, b\, h^3 \gamma^2$



Bars uniformly spaced on all sides

$I_{se} = 0.22\,\rho_t\, b\, h^3 \gamma^2$

[5] This table is based on Table 12-1 of MacGregor, J.G., Third Edition, Prentice Hall, Englewood Cliffs, New Jersey, 1997.

Chapter 5

Footing Design

By S. Ali Mirza[1] and William Brant[2]

5.1 Introduction

Reinforced concrete foundations, or footings, transmit loads from a structure to the supporting soil. Footings are designed based on the nature of the loading, the properties of the footing and the properties of the soil.

Design of a footing typically consists of the following steps:

1. Determine the requirements for the footing, including the loading and the nature of the supported structure.

2. Select options for the footing and determine the necessary soils parameters. This step is often completed by consulting with a Geotechnical Engineer.

3. The geometry of the foundation is selected so that any minimum requirements based on soils parameters are met. Following are typical requirements:

    - The calculated bearing pressures need to be less than the allowable bearing pressures. Bearing pressures are the pressures that the footing exerts on the supporting soil. Bearing pressures are measured in units of force per unit area, such as pounds per square foot.
    - The calculated settlement of the footing, due to applied loads, needs to be less than the allowable settlement.
    - The footing needs to have sufficient capacity to resist sliding caused by any horizontal loads.
    - The footing needs to be sufficiently stable to resist overturning loads. Overturning loads are commonly caused by horizontal loads applied above the base of the footing.
    - Local conditions.
    - Building code requirements.

[1] Professor Emeritus of Civil Engineering, Lakehead University, Thunder Bay, ON, Canada.
[2] Structural Engineer, Black & Veatch, Kansas City, KS.

4. Structural design of the footing is completed, including selection and spacing of reinforcing steel in accordance with ACI 318 and any applicable building code. During this step, the previously selected geometry may need to be revised to accommodate the strength requirements of the reinforced concrete sections. Integral to the structural design are the requirements specific to foundations, as defined in ACI 318-05 Chapter 15.

5.2 Types of Foundations

Shallow footings bear directly on the supporting soil. This type of foundation is used when the shallow soils can safely support the foundation loads.

A *deep foundation* may be selected if the shallow soils cannot economically support the foundation loads. Deep foundations consist of a footing that bears on piers or piles. The footing above the piers or piles is typically referred to as a pile cap.

The *piers or piles* are supported by deeper competent soils, or are supported on bedrock. It is commonly assumed that the soil immediately below the pile caps provides no direct support to the pile cap.

5.3 Allowable Stress Design and Strength Design

Traditionally the geometry of a footing or a pile cap is selected using unfactored loads. The structural design of the foundation is then completed using strength design in accordance with ACI 318.

ACI Committee 336 is in the process of developing a methodology for completing the entire footing design using the strength design method.

5.4 Structural Design

The following steps are typically followed for completing the structural design of the footing or pile cap, based on ACI 318-05:

1. Determine footing plan dimensions by comparing the gross soil bearing pressure and the allowable soil bearing pressure.

2. Apply load factors in accordance with Chapter 9 of ACI 318-05.

3. Determine whether the footing or pile cap will be considered as spanning one-way or two-ways.

4. Confirm the thickness of the footing or pile cap by comparing the shear capacity of the concrete section to the factored shear load. ACI 318-05 Chapter 15 provides guidance on selecting the location for the critical cross-section for one-way shear. ACI 318-05 Chapter 11 provides guidance on selecting the location for the critical cross-section for two-way shear. Chapter 2 of this handbook on shear design also provides further design information and design aids.

5. Determine reinforcing bar requirements for the concrete section based on the flexural capacity along with the following requirements in ACI 318-05.

    - Requirements specific to footings
    - Temperature and shrinkage reinforcing requirements
    - Bar spacing requirements
    - Development and splicing requirements
    - Seismic Design provisions
    - Other standards of design and construction, as required

5.5 Footings Subject to Eccentric Loading

Footings are often subjected to lateral loads or overturning moments, in addition to vertical loads. These types of loads are typically seismic or wind loads.

Lateral loads or overturning moments result in a non-uniform soil bearing pressure under the footing, where the soil bearing pressure is larger on one side of the footing than the other. Non-uniform soil bearing can also be caused by a foundation pedestal not being located at the footing center of gravity.

If the lateral loads and overturning moments are small in proportion to the vertical loads, then the entire bottom of the footing is in compression and a $P/A \pm M/S$ type of analysis is appropriate for calculating the soil bearing pressures, where the various parameters are defined as follows:

P = The total vertical load, including any applied loads along with the weight of all of the components of the foundation, and also including the weight of the soil located directly above the footing.

A = The area of the bottom of the footing.

M = The total overturning moment measured at the bottom of the footing, including horizontal loads times the vertical distance from the load application location to the bottom of the footing plus any overturning moments.

S = The section modulus of the bottom of the footing.

If M/S exceeds P/A, then $P/A - M/S$ results in tension, which is generally not possible at the footing/soil interface. This interface is generally only able to transmit compression, not tension. A different method of analysis is required when M/S exceeds P/A.

Following are the typical steps for calculating bearing pressures for a footing, when non-uniform bearing pressures are present. These steps are based on a footing that is rectangular in shape when measured in plan, and assumes that the lateral loads or overturning moments are parallel to one of the principal footing axes. These steps should be completed for as many load combinations as required to confirm compliance with applicable design criteria. For instance, the load combination with the maximum downward vertical load often causes the maximum bearing pressure while the load combination with the minimum downward vertical load often causes the minimum stability.

1. Determine the total vertical load, *P*.

2. Determine the lateral and overturning loads.

3. Calculate the total overturning moment *M*, measured at the bottom of the footing.

4. Determine whether *P/A* exceeds *M/S*. This can be done by calculating and comparing *P/A* and *M/S* or is typically completed by calculating the eccentricity, which equals *M* divided by *P*. If e exceeds the footing length divided by 6, then *M/S* exceed *P/A*.

5. If *P/A* exceeds *M/S*, then the maximum bearing pressure equals *P/A* + *M/S* and the minimum bearing pressure equals *P/A*-*M/S*.

6. If P/A is less than M/S, then the soil bearing pressure is as shown in Fig. 5-1. Such a soil bearing pressure distribution would normally be considered undesirable because it makes the footing structurally ineffective. The maximum bearing pressure, shown in the figure, is calculated as follows:

Maximum Bearing pressure = 2 P / [(B) (X)]
Where X = 3(L/2 - e) and e = M / P

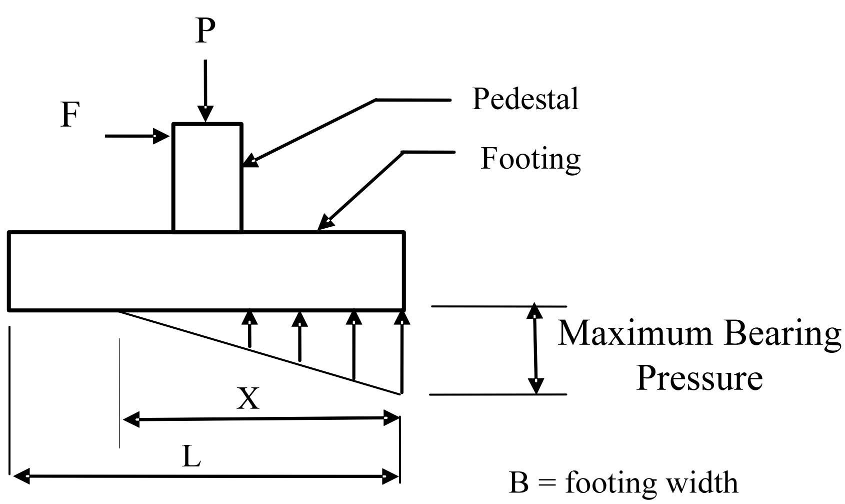


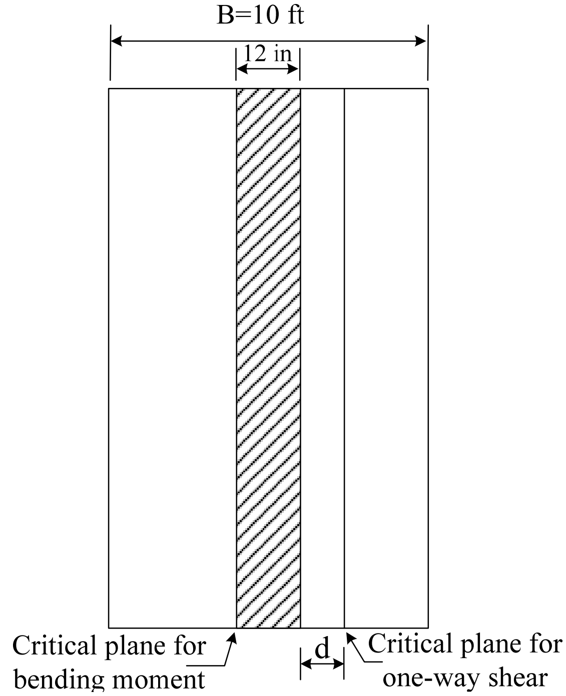
Fig. 5-1 Footing under eccentric loading

5.6 Footing Design Examples

The footing examples in this section illustrate the use of ACI 318-05 for some typical footing designs as well as demonstrate the use of some design aids included in other chapters. However, these examples do not necessarily provide a complete procedure for foundation design as they are not intended to substitute for engineering skills or experience.

FOOTINGS EXAMPLE 1 - Design of a continuous (wall) footing

Determine the size and reinforcement for the continuous footing under a 12 in. bearing wall of a 10 story building founded on soil.

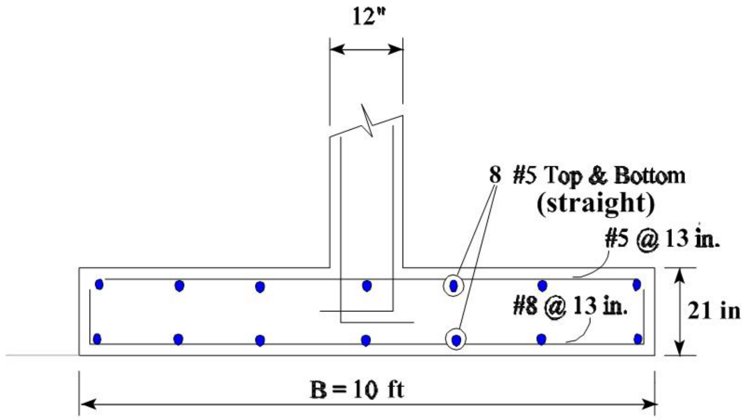


<u>Given:</u>
f'_c = 4 ksi
f_y = 60 ksi
Dead Load = D = 25 k/ft
Live Load = L = 12.5 k/ft
Wind O.T. = W = 4 k/ft
(axial load due to overturning under wind loading)
Seismic O.T. = E = 5 k/ft
(axial load due to overturning under earthquake loading)

Allowable soil bearing pressures:
D = 3 ksf = "a"
D + L = 4 ksf = "b"
D + L + (W or E) = 5 ksf = "c"

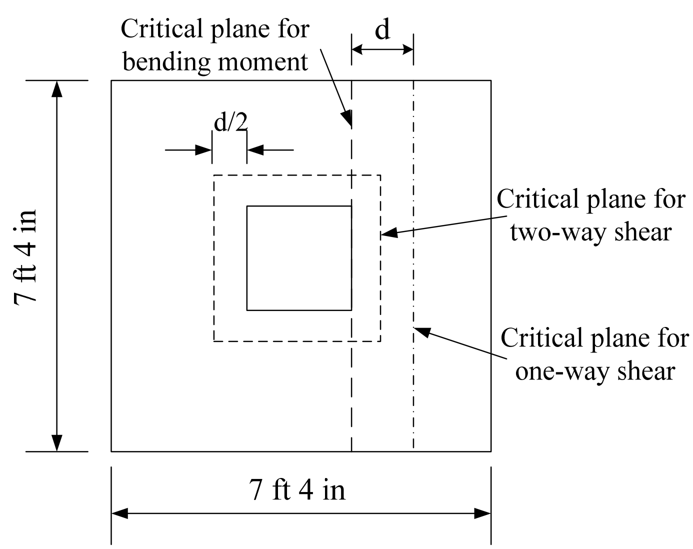
Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing.	Ignoring the footing self-weight; D/a = 25/3 = 8.3 ft (D + L)/b = 37.5/4 = 9.4 ft Z controls (D + L + W)/c = 41.5/5 = 8.3 ft (D + L + E)/c = 42.5/5 = 8.5 ft Use B = 10 ft		
Required strength.	U = 1.4D = 1.4(25) = 35 k/ft or 3.50 ksf U = 1.2D + 1.6L = 1.2(25) + 1.6(12.5) = 50 k/ft or 5.00 ksf (Controls) U = 1.2D + 1.6W + 1.0L = 1.2(25) + 1.6(4) +12.5 = 48.9 k/ft or 4.89 ksf U = 0.9D + 1.6W = 0.9(25) + 1.6(4) = 28.9 k/ft or 2.89 ksf U = 1.2D + 1.0E + 1.0L = 1.2(25) + (5) + 12.5	9.2	

	= 47.5 k/ft or 4.75 ksf U = 0.9D + 1.0E = 0.9(25) + (5) = 27.5 k/ft or 2.75 ksf		
Design for shear. Calculate V_u at d from the face of the wall	ϕ_{shear} = 0.75 Assume $V_s = 0$ (no shear reinforcement) $\phi V_n = \phi V_c$ $\phi V_c = \phi(2\sqrt{f'_c} b_w d)$ Try d = 17 in. and h = 21 in. $\phi V_c = 0.75(2\sqrt{4000})(12)(17)/1000$ =19.35 k/ft V_u = (10/2 - 6/12 - 17/12)(5.00) = 15.5 k/ft $\phi V_n = \phi V_c > V_u$ OK	9.3.2.3 11.1.1 11.3 11.1.3.1	
Calculate moment at the face of the wall Compute flexural tension reinforcement Shrinkage and temperature reinforcement	$M_u = (5)(4.5)^2/2$ = 50.6 ft-k/ft $\phi K_n = M_u (12{,}000)/(bd^2)$ $\phi K_n = 50.6 (12{,}000)/[(12)(17)^2]$ = 176 psi For ϕK_n = 176 psi, select ρ = 0.34% $A_s = \rho bd$ = 0.0034 (12) (17) = 0.70 in^2/ft Check for $A_{s,min}$= 0.0018 bh $A_{s,min}$=0.0018(12)(21)=0.46 in^2/ft <0.7in^2/ft OK Use bottom bars #8 @ 13 in c/c hooked at ends. If these bars are not hooked, provide calculations to justify the use of straight bars. **Note:** ε_t = 0.040 > 0.005 for tension controlled sections and ϕ = 0.9 Use top bars #5 @ 13 in c/c arbitrarily designed to take approximately 40% of bending moment due to possible reversal caused by earthquake loads. 8# 5 top and bottom longitudinal bars will satisfy the requirement for shrinkage and	15.4.2 7.12 10.5.4 10.3.4 9.3.2 7.12	 Flexure 1 Flexure 1

	temperature reinforcement in the other direction.		
Check shear for earthquake load effects. For structural members resisting earthquake loads, if the nominal shear strength is less than the shear corresponding to the development of nominal flexural resistance, then; $\phi_{shear} = 0.6$	M_n = 61.9 ft-k/ft and the corresponding V_{fn} = 18.6 k/ft $V_c = 2\sqrt{4000}(12)(17.5)/1000$ $= 26.5$ k/ft > V_{fn} = 18.6 k/ft Therefore, the use of $\phi_{shear} = 0.75$ above is correct.	9.3.4 (a)	
Final Design 			

FOOTINGS EXAMPLE 2 - Design of a square spread footing

Determine the size and reinforcing for a square spread footing that supports a 16 in. square column, founded on soil.



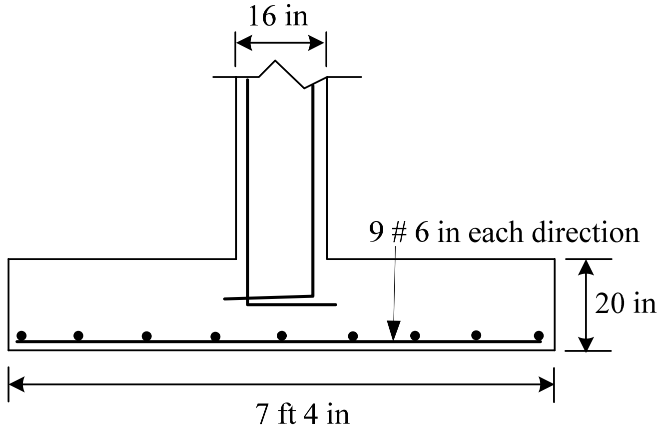
Given:

f'_c = 4 ksi
f_y = 60 ksi
Dead Load = D = 200 k
Live Load = L = 100 k
Allowable soil bearing pressures:
Due to D = 4 ksf = "a"
Due to D + L = 7 ksf = "b"

Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing.	Ignoring the footing self-weight; D/a = 200/4 = 50 sq. ft. (Controls) (D+L)/b = 300/7 = 42.9 sq. ft. **Use 7.33 ft x 7.33 ft** A = 53.7 > 50 sq. ft. OK		
Required strength.	U = 1.4D = 1.4(200) = 280 k or (280/53.7) = 5.3 ksf U = 1.2D + 1.6L = 1.2(200) + 1.6(100) = 400k or (400/53.7)= 7.5 ksf (Controls)	9.2	
Design for shear. Two-way action	ϕ_{shear} = 0.75 Assume $V_s = 0$ (no shear reinforcement) $\phi V_n = \phi V_c$ Try d = 16 in. and h = 20 in. $b_o = 4(16 + 16) = 128$ in. $V_c = (2 + \frac{4}{\beta})\sqrt{f'_c}b_o d$ $V_c = (2 + \frac{4}{16/16})\sqrt{f'_c}b_o d = 6\sqrt{f'_c}b_o d$ $V_c = (\frac{\alpha_s d}{b_o} + 2)\sqrt{f'_c}b_o d$ $V_c = (\frac{(40)(16)}{128} + 2)\sqrt{f'_c}b_o d$ $V_c = 7\sqrt{f'_c}b_o d$ $V_c = 4\sqrt{f'_c}b_o d$ (Controls) $\phi V_c = 0.75(4\sqrt{4000}(128)(16))/1000$ = 388.5 k $V_u = [(7.33)^2 - ((16+16)/12)^2](7.5) =$ 349.6 k	9.3.2.3 11.1.1 11.12.1.2 11.12.2.1 (a) 11.12.2.1 (b) 11.12.2.1 (c)	

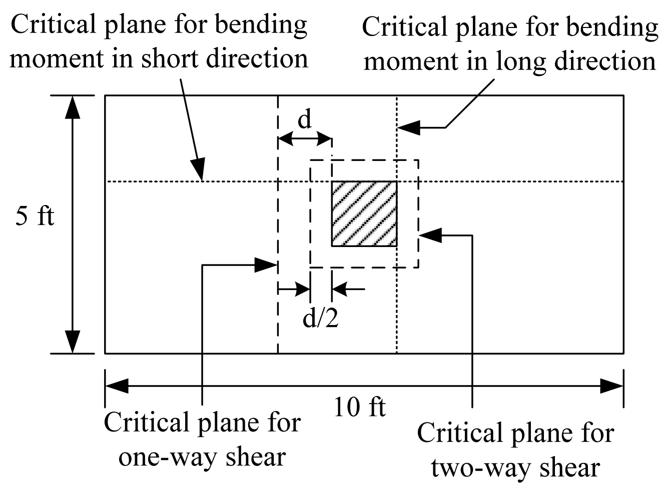
One-way action	$\phi V_n = \phi V_c > V_u$ OK b_w = 7.33 (12) = 88 in. and d = 15.5 in. $V_c = 2\sqrt{f'_c} b_w d$ $\phi V_c = 0.75(2\sqrt{4000})(88)(15.5)/1000$ = 129.4 k V_u =7.33 [(7.33/2) – (8+15.5)/12](7.5) = 94.0 k $\phi V_n = \phi V_c > V_u$ OK	11.12.1.1 11.3.1.1	
Bearing Bearing resistance of footing	$\phi_{bearing}$ = 0.65 $\sqrt{A_2 / A_1} = 2$ $B_r = \phi(0.85 f'_c A_1)\sqrt{A_2 / A_1}$ B_r = 0.65(0.85)(4)(16)^2 (2) B_r = 1131 k > 400 k OK	9.3.2.4 10.17.1	
Calculate moment at the column face Compute flexural tension reinforcement (bottom bars) using design aids in Chapter 1	M_u = (7.5)(3)^2 (7.33)/2 = 248 ft-k $\phi K_n = M_u$ (12,000)/(bd^2) ϕK_n = 248 (12,000)/[(7.33)(12)(15.5)^2] = 141 psi For ϕK_n = 141 psi, select ρ = 0.27% A_s = ρbd = 0.0027 (7.33)(12)(15.5) = 3.7 in^2 Check for $A_{s,min}$= 0.0018 bh $A_{s,min}$=0.0018(7.33)(12)(20)= 3.2 in^2 < 3.7in^2 OK Use 9 #6 straight bars in both directions **Note:** ε_t = 0.050 > 0.005 for tension controlled sections and ϕ = 0.9.	15.4.2 7.12 10.5.4 10.3.4 9.3.2	Flexure 1 Flexure 1
Development length: Critical sections for development length occur at the column face.	$\ell_d = \left(f_y \Psi_t \Psi_e \lambda / (25\sqrt{f'_c})\right) d_b$ $\ell_d = \left(\frac{(60{,}000)(1.0)(1.0)(1.0)}{25\sqrt{4{,}000}}\right) 0.75$ P_d =29 in. < P_d (provided) = (3)(12) – 3 = 33 in OK	15.6.3 15.4.2 12.2.2	

Final Design



FOOTINGS EXAMPLE 3 - Design of a rectangular spread footing.

Determine the size and reinforcing for a rectangular spread footing that supports a 16 in. square column, founded on soil.



Given:
f'_c = 4 ksi
f_y = 60 ksi
Dead Load = D = 180 k
Live Load = L = 100 k
Wind O.T. = W = 120 k
(axial load due to overturning under wind loading)
Allowable soil bearing pressures:
Due to D = 4 ksf = "a"
Due to D + L = 6 ksf = "b"
Due to D + L + W = 8.4 ksf = "c"

Design a rectangular footing with an aspect ratio ≤ 0.6

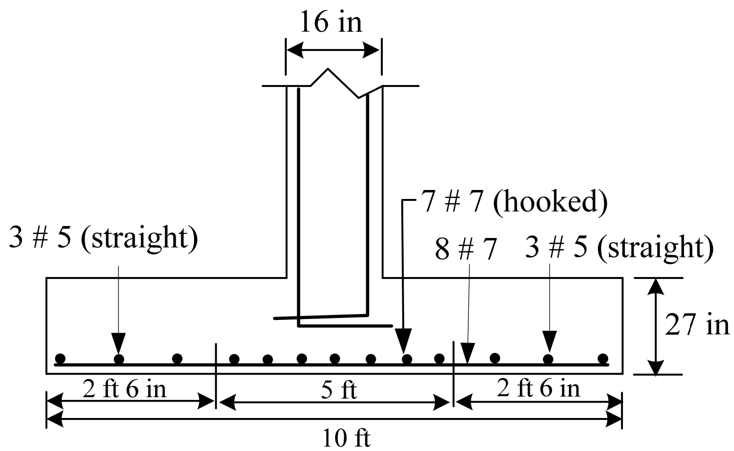
Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing.	Ignoring the self-weight of the footing; D/a = 180/4 = 45 sq.ft. (D+L)/b = 280/6 = 46.7 sq.ft. (D + L + W)/c = 400/8.4 = 47.6 sq.ft. Controls Use 5 ft x 10 ft A = 50 sq.ft. is OK		
Required Strength	U = 1.4D = 1.4(180) = 252 k or (252/50) = 5.1 ksf U = 1.2D + 1.6L	9.2	

	$= 1.2(180) + 1.6(100)$ $= 376$ k or $(376/50) = 7.6$ ksf $U = 1.2D + 1.6W + 1.0L$ $= 1.2(180) + 1.6(120) + 1.0(100)$ $= 508$ k or 10.2 ksf (Controls) $U = 0.9D + 1.6W$ $= 0.9(180) + 1.6(120)$ $= 354$ k or 7.1 ksf		
Design for shear. Two-way action One-way action (in short direction)	$\phi_{shear} = 0.75$ Assume $V_s = 0$ (no shear reinforcement) $\phi V_n = \phi V_c$ Try d = 23 in. and h = 27 in. $b_o = 4(16 + 23) = 156$ in. $V_c = (2 + \frac{4}{\beta})\sqrt{f'_c} b_o d$ $V_c = (2 + \frac{4}{16/16})\sqrt{f'_c} b_o d = 6\sqrt{f'_c} b_o d$ $V_c = (\frac{\alpha_s d}{b_o} + 2)\sqrt{f'_c} b_o d$ $V_c = (\frac{(40)(23)}{156} + 2)\sqrt{f'_c} b_o d$ $V_c = 7.9\sqrt{f'_c} b_o d$ $V_c = 4\sqrt{f'_c} b_o d$ (Controls) $\phi V_c = 0.75(4\sqrt{4000}(156)(23)) / 1000$ $= 680.7$ k $V_u = [(10)(5) - (16+23)/12)^2](10.2)$ $= 402.3$ k $\phi V_n = \phi V_c > V_u$ □ OK $b_w = 5(12) = 60$ in. and d = 23.5 in. $V_c = 2\sqrt{f'_c} b_w d$	9.3.2.3 11.1.1 11.12.1.2 11.12.2.1 (a) 11.12.2.1 (b) 11.12.2.1 (c) 11.12.1.1 11.3.1.1	

	$\phi V_c = 0.75(2\sqrt{4000})(60)(23.5)/1000$ $= 133.7$ k V_u=5[(10/2) – (8+23.5)/12](10.2)=121.2 k $\phi V_n = \phi V_c > V_u$ OK One-way action in the long direction is not a problem because the footing edge is located within the potential critical section for one-way shear.		
Bearing Bearing resistance of footing	$\phi_{bearing} = 0.65$ $\sqrt{A_2 / A_1} = 2$ $B_r = \phi(0.85 f'_c A_1)\sqrt{A_2 / A_1}$ $B_r = 0.65(0.85)(4)(16)^2 (2)$ $B_r = 1131$ k > 508 k OK	9.3.2.4 10.17.1	
Calculate moment in the long direction, at the column face. Compute flexural tension reinforcement (bottom bars) using design aids in Chapter 1.	$M_u = (10.2)(4.33)^2 (5)/2 = 479$ ft-k $\phi K_n = M_u (12,000)/(bd^2)$ $\phi K_n = 479 (12,000)/[(5)(12)(23.5)^2]$ = 173.5 psi For ϕK_n = 173.5 psi, select ρ = 0.335% $A_s = \rho bd = 0.00335 (5)(12)(23.5) = 4.72$ in^2 Check for $A_{s,min}$= 0.0018 bh $A_{s,min}$= 0.0018(5)(12)(27) = 2.92 in^2 < 4.72 in^2 OK Use 8 #7 bars distributed uniformly across the entire 5ft width of footing **Note:** ε_t = 0.041 > 0.005 for tension controlled sections and ϕ = 0.9.	15.4.2 7.12 10.5.4 15.4.4.1 10.3.4 9.3.2	 Flexure 1 Flexure 1
Calculate moment in the short direction, at the column face. Compute flexural tension reinforcement (bottom bars) using design aids in Chapter 1.	$M_u = (10.2)(1.83)^2 (10)/2 = 171.4$ ft-k $\phi K_n = M_u (12,000)/(bd^2)$ $\phi K_n = 171.4 (12,000)/[(10)(12)(22.5)^2]$ = 33.9 psi For ϕK_n = 33.9 psi, select ρ = 0.07% $A_s = \rho bd = 0.0007 (10)(12)(22.5)$ = 1.89 in^2	15.4.2	 Flexure 1

Check for minimum reinforcement	$A_{s,min}$= 0.0018 bh $A_{s,min}$= 0.0018(10)(12)(27) = 5.83 in^2 > 1.89 in^2 Use A_s = 5.83 in^2 (Reinf. In central 5-ft band) / (total reinf.) = 2/(β+1) β = 10/5 = 2; and 2/(β+1) = 2/3 Reinf. In central 5-ft band = 5.83(2/3) = 3.89 in^2 Use 7 #7 bars distributed uniformly across the entire 5ft band. Reinforcement outside the central band = 5.83 – 7(0.6) = 1.63 in^2 Use 6 #5 bars (3 each side) distributed uniformly outside the central band.	7.12 10.5.4 15.4.4.2	
Development length: Critical sections for development length occur at the column face.	$\ell_d = (3/40)(f_y/\sqrt{f'_c})$ $[(\Psi_t \Psi_e \Psi_s \lambda)/((c_b + K_{tr})/d_b)]d_b$ K_{tr} = 0; and $((c_b + K_{tr})/d_b)$ = 2.5 $\ell_d = (3/40)(60{,}000/\sqrt{4{,}000})$ $[(1.0)(1.0)(1.0)(1.0)/2.5]0.875$ P_d =25 in. for # 7 bars P_d =25 in < P_d (provided) = (4.33)(12) – 3 = 49 in in the long direction: use straight # 7 bars P_d =25 in > P_d (provided) = (1.83)(12) – 3 = 19 in in the short direction: use hooked # 7 bars $\ell_d = (3/40)(60{,}000/\sqrt{4{,}000})$ $[(1.0)(1.0)(0.8)(1.0)/2.5]0.625$ P_d =15 in. for # 5 bars P_d = 15 in < P_d (provided) = 19 in in the short direction: use straight # 5 bars	15.6.3 15.4.2 12.2.3 12.2.4	

Final Design



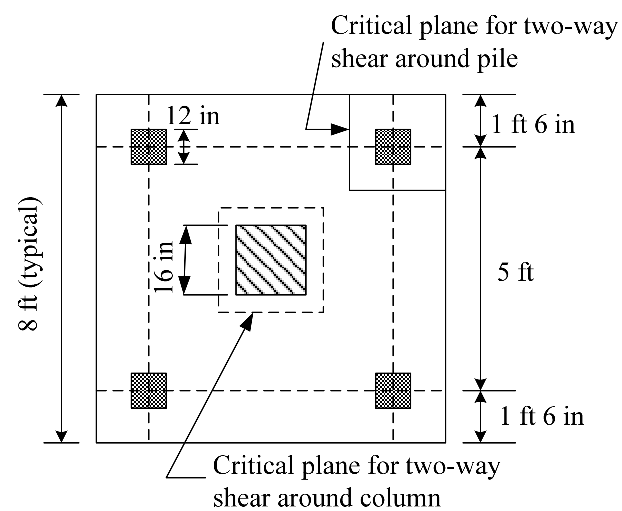
FOOTINGS EXAMPLE 4 - Design of a pile cap.

Determine the size and reinforcing for a square pile cap that supports a 16 in. square column and is placed on 4 piles.

Given:

f'_c = 5 ksi
f_y = 60 ksi
Dead Load = D = 250 k
Live Load = L = 150 k

16 x 16 in. reinforced concrete column
12 x 12 in. reinforced concrete piles
(4 piles each @ 5 ft on centers)

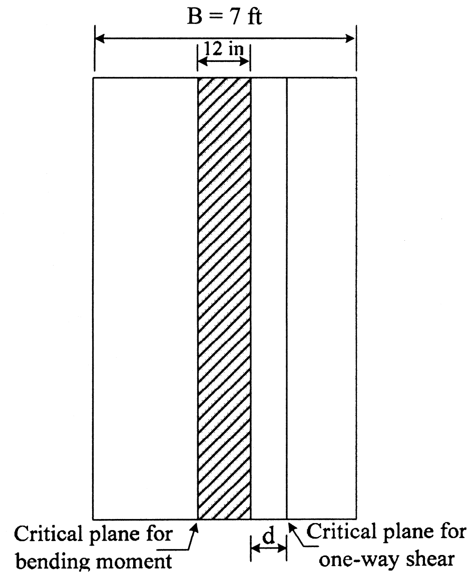


Procedure	Computation	ACI 318-05 Section	Design Aid
Factored Loads:	Column: U = 1.4D = 1.4(250) = 350 k U = 1.2D + 1.6L = 1.2(250) + 1.6(150) = 540 k = V_u (Controls) Piles: P_u = 540/4 = 135 k = V_u ignoring the self-weight of pile cap	9.2	

Design for shear. Two-way action	$\phi_{shear} = 0.75$ Assume $V_s = 0$, (no shear reinforcement) $\phi V_n = \phi V_c$ Try d = 26 in. and h = 33 in. <u>Around Column:</u> $b_o = 4(16 + 26) = 168$ in. $V_c = (2 + \frac{4}{\beta})\sqrt{f'_c}b_o d$ $V_c = (2 + \frac{4}{16/16})\sqrt{f'_c}b_o d = 6\sqrt{f'_c}b_o d$ $V_c = (\frac{\alpha_s d}{b_o} + 2)\sqrt{f'_c}b_o d$ $V_c = (\frac{(40)(26)}{168} + 2)\sqrt{f'_c}b_o d$ $V_c = 8.2\sqrt{f'_c}b_o d$ $V_c = 4\sqrt{f'_c}b_o d$ (Controls) $\phi V_c = 0.75(4\sqrt{5000}(168)(26))/1000$ = 926 k $V_u = 540$ k $\phi V_n = \phi V_c > V_u$ OK <u>Around Piles</u> $b_o = 2(18 + 6 + 13) = 74$ in. $V_c = (2 + \frac{4}{12/12})\sqrt{f'_c}b_o d = 6\sqrt{f'_c}b_o d$ $V_c = (\frac{(20)(26)}{74} + 2)\sqrt{f'_c}b_o d$ $V_c = 9\sqrt{f'_c}b_o d$ $V_c = 4\sqrt{f'_c}b_o d$ (Controls)	9.3.2.3 11.1.1 11.12.1.2 11.12.2.1 (a) 11.12.2.1 (b) 11.12.2.1 (c) 11.12.1.2 11.12.2.1 (a) 11.12.2.1 (b) 11.12.2.1 (c)	

	$\phi V_c = 0.75(4\sqrt{5000}(74)(26))/1000$ $= 408$ k $V_u = 135$ k $\phi V_n = \phi V_c > V_u$ OK Note: The effective depth is conservative for the two-way action but is O.K. considering the overlapping of the critical sections around the column and the piles		
One-way action	One-way action will not be a problem because the piles are located within potential critical sections for one-way shear.		
Design for flexure Find flexural tension reinforcement (bottom bars)	$M_u = 2(135)(2.5 - 0.67) = 495$ ft-k $\phi K_n = M_u (12{,}000)/(bd^2)$ $\phi K_n = 495 (12{,}000)/[(8)(12)(25.5)^2]$ $= 95.2$ psi For $\phi K_n = 95.2$ psi, select $\rho = 0.19\%$ $A_s = \rho bd = 0.0019 (8)(12)(25.5) = 4.7$ in^2 Check for $A_{s,min} = 0.0018$ bh $A_{s,min} = 0.0018(8)(12)(33) = 5.7$ in^2 > 4.7 in^2 A_s (required) = 5.7 in^2 Use 10 #7 each way (bottom reinforcement)	7.12 10.5.4	Flexure 1
Top reinforcement:	Not required.		

FOOTINGS EXAMPLE 5 - Design of a continuous footing with an overturning moment



Determine the size and reinforcing bars for a continuous footing under a 12-in. bearing wall, founded on soil, and subject to loading that includes an overturning moment.

Given:
f'_c = 4 ksi
f_y = 60 ksi

Depth from top of grade to bottom of footing = 3 ft
Density of soil above footing = 100 pcf
Density of footing concrete = 150 pcf

Vertical Dead Load = 15 k/ft (including wall weight)
Horizontal wind shear = V = 2.3 k/ft (applied at 1 ft above grade)

Allowable soil bearing pressure based on unfactored loads = 4 ksf

Procedure	Computation	ACI 318-05 Section	Design Aid
Sizing the footing	Try footing width = B = 7 ft Area = A = 1(7) =7 ft^2/ft Section Modulus = S =1(7)(7)/6=8.167 ft^3/ft Try a 14 inch thick footing Weight of footing + soil above footing = (14/12)(0.150) + (36-14)(0.100/12) = 0.175 + 0.183 = 0.358 ksf Total weight of footing + soil above footing + wall from top of grade to top of footing = (0.175)(7)+(.183)(7-1)+(36-14)(0.150/12) = 2.60 kips/ft Total Vertical Load = P = 15 + 2.6 =17.6k/ft (Dead Load) Vertical distance from bottom of footing to location of applied shear = H = 3 + 1 = 4 ft. Overturning moment measured at base of footing = M = (V)(H) = (2.3)(4) = 9.2 ft-kips/ft (Wind Load)		

	Eccentricity = e = M/P = 9.2/17.6 = 0.52 ft B/6 = 7/6 = 1.17 ft Since e < B/6, bearing pressure = P/A ± M/S Maximum bearing pressure =P/A + M/S = (15 + 2.6)/7 + 9.2/8.167 = 3.64 ksf Minimum bearing pressure =P/A – M/S = (15 + 2.6)/7 – 9.2/8.167 = 1.39 ksf Max bearing pressure < allowable: OK		
Required Strength	U = 1.4D = 1.4(17.6)/7 = 3.52 ksf U = 1.2D + 1.6W + 1.0L 1.2D = 1.2(17.6)/7 = 3.02 ksf 1.6W = 1.6(9.2)/8.167 = 1.80 ksf 1.0L = 0 e = 1.6(M)/(1.2(P)) = 1.6(9.2)/(1.2(17.6) = 0.70 ft Since e < B/6, bearing pressure = 1.2(P/A) ± 1.6(M/S) U = 4.82 ksf (maximum) U = 1.22 ksf (minimum) U = 0.9D + 1.6W 0.9D = 0.9(17.6)/7 = 2.27 ksf 1.6W =1.6(9.2)/8.167 = 1.80 ksf e = 1.6(M)/(0.9(P)) = 1.6(9.2)/(0.9(17.6) = 0.93 ft Since e < B/6, bearing pressure = 0.9(P/A) ± 1.6(M/S) U = 4.07 ksf (maximum) U = 0.47 ksf (minimum)	9.2	

Design for Shear	ϕ_{shear} = 0.75 Assume V_s = 0 (i.e. no shear reinforcement) $\phi V_n = \phi V_c$ $\phi V_c = \phi\left(2\sqrt{f_c^{'}}b_w d\right)$ Try d = 10 in. and h = 14 in. $\phi V_c = 0.75\left(2\sqrt{4000} \times 12 \times 10\right)/1000$ = 11.38 k/ft Calculate V_u for the different load combinations that may control. Calculate at the location d from the face of the wall. Delete the portion of bearing pressure caused by weight of footing and soil above footing. Distance d from face of wall = (7/2 – 6/12 – 10/12) = 2.17 ft measured from the edge of the footing U = 1.4D V_u = (3.52 – (1.4)(0.358))(2.17) = 6.55 k/ft U = 1.2D + 1.6W + 1.0L Bearing pressure measured at distance d from face of wall = 4.82 – (4.82 – 1.22)(2.17/7) = 3.70 ksf V_u = (3.70 – 1.2(0.358))(2.17) + (4.82 – 3.70)(2.17/2) = 8.31 k/ft (controls) $\phi V_n = \phi V_c > V_u$ OK	9.3.2.3 11.1.1 11.3 11.1.3.1	

Moment	Calculate the moment at the face of the wall = (7/2 – 6/12) = 3.0 ft measured from the edge of the footing U = 1.4D $M_u = (3.52 - 1.4(0.358))(3.0)^2 / 2$ $= 13.58$ ft-k/ft U = 1.2D + 1.6W + 1.0L Bearing pressure measured at face of wall $= 4.82 - (4.82 - 1.22)(3/7) = 3.28$ ksf $M_u = (3.28 - 1.2\times0.358)(3.0)^2 / 2$ $+ (4.82 - 3.28)(3.0)^2 / 3$ $= 17.45$ ft-k/ft (controls) Compute $\phi K_n = M_u(12{,}000)/(bd^2)$ $\phi K_n = 17.45(12{,}000)/(12\times10^2) = 175\,psi$ For $\phi K_n = 175\,psi$, select Δ = 0.34% Compute flexural tension reinforcement $A_s = \rho b\,d = 0.0034(12)(10) = 0.41 in^2/ft$ Check for A_{smin} = 0.0018 bh A_{smin} = 0.0018(12)(14) = 0.30 in^2/ft < 0.41 in^2/ft OK Use bottom bars #6 @12 in. Use 7#5 bottom longitudinal bars to satisfy the requirements for shrinkage and temperature reinforcement in the other direction.	15.4.2 7.12 10.5.4 7.12	Flexure 1

Chapter 6

Seismic Design

By Murat Saatcioglu[1]

6.1 Introduction

Seismic design of reinforced concrete buildings is performed by determining earthquake design forces for the anticipated seismic activity in the region, from the general building code adopted by the local authority. The structural elements are then proportioned and detailed following the requirements of Chapter 21 of ACI 318-05. Seismic design forces are determined on the basis of earthquake risk levels associated with different regions. Seismic risk levels have been traditionally characterized as low, moderate and high. These risk levels are considered in structural design to produce buildings with compatible seismic performance levels. ACI 318-05 has three design and performance levels, identified as *ordinary*, *intermediate* and *special*, corresponding to low, moderate and high seismic risk levels, respectively. Ordinary building design is attained for structures located in low seismic regions without the need to follow the special seismic design requirements of Chapter 21. These structures are expected to perform within the elastic range of deformations when subjected to seismic excitations. Buildings in moderate to high seismic risk regions are often designed for earthquake forces that are less than those corresponding to elastic response at anticipated earthquake intensities. Lateral force resisting systems for these buildings may have to dissipate earthquake induced energy through significant inelasticity in their critical regions. These regions require special design and detailing techniques to sustain cycles of inelastic deformation reversals without a significant loss in strength. The latter can be ensured by following the seismic provisions of ACI 318-05 outlined in Chapter 21.

The design and detailing requirements of ACI 318-05 are compatible with the level of energy dissipation assumed in selecting force modification factors and the resulting design force levels. While all earthquake resistant buildings must be designed following more stringent designed and detailing requirements, the level of detailing may be *intermediate* for buildings located in moderate seismic regions, and more stringent for buildings in regions of high seismicity, forming a *special* category. This ensures appropriate level of toughness in the building. It is permissible, however, to design buildings for higher toughness in the lower seismic zones to take advantage of reduced design forces.

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Structural members not designed as part of the lateral-force resisting system are designed as gravity load carrying members. These members, if present in special moment-resisting frames or special structural wall systems located in high seismic regions, must be protected during strong earthquakes as they continue carrying gravity loads tributary to them. They are often referred to as “gravity elements” and they “go for the ride” during the earthquake motion. Chapter 21 provides design and detailing requirements for such members in Sec. 21.11. Therefore, the structural engineer should first identify if the member under consideration for design is part of a lateral load resisting system, and if so, establish if the element is to be designed as intermediate or special seismic resisting element. **It is important to note that the requirements of Chapter 21 of ACI 318-05 are intended to be *additional* provisions, over and above those stated in other chapters of the Code for *ordinary* building design.**

6.2 Limitations on Materials

Certain limitations are imposed on materials used for seismic resistant construction to ensure deformability of members within the inelastic range of deformations. The following are the limits placed on concrete and reinforcing steel used in seismic resistant designs:

i) $f'_c \geq 3000$ psi

ii) $f'_c \leq 5000$ psi for light-weight concrete, unless suitability is demonstrated by tests.

iii) Reinforcement shall comply with ASTM A-706. ASTM A-615 Grades 40 and 60 are permitted if they satisfy items iv and v below.

iv) $(f_y)_{Actual} - (f_y)_{Specified} \leq 18{,}000$ psi

v) Actual tensile strength / actual yield strength ≥ 1.25

vi) $f_{yt} \leq 60{,}000$ psi for all transverse reinforcement, including spirals.

In addition to the above limitations, mechanical and welded splices of reinforcement and anchorage to concrete should meet the requirements of 21.2.6 through 21.2.8. Accordingly, a mechanical splice is expected to develop at least $1.25f_y$ of the bar in tension or compression, and is classified as either Type 1 or Type 2. Type 1 mechanical splice is not expected to develop the specified tensile strength of the spliced bar beyond $1.25f_y$ and is not permitted within potential plastic hinging regions (regions of yielding) where stresses may approach the tensile strength. Therefore, Type 1 mechanical splices are not permitted within a segment equal to twice the member depth from the critical section. Type 2 mechanical splice, on the hand, must be designed to develop the tensile strength of spliced bars and is permitted to be used within the plastic hinging region. Welded splices are treated the same as Type 1 mechanical splices with the same restrictions (21.2.7).

6.3 Flexural Members of Special Moment Frames

6.3.1 Flexural Design

Members designed to resist primarily flexure ($P_u \leq A_g f'_c/10$) are subject to additional design and detailing considerations for improved seismic performance. These requirements consist of geometric constraints, minimum positive and negative moment capacities along member length, confinement of critical regions of elements for improved deformability, promotion of ductile flexural response and the prevention of premature shear failure. Design aid **Seismic 1** illustrates the geometric constraints, as well as minimum top and bottom reinforcement requirements for minimum moment capacity in each section during lateral load reversals. The same design aid also shows the spacing requirements for concrete confinement at potential plastic hinge locations at member ends (within a distance equal to twice the member depth). The transverse confinement reinforcement consists of hoops, which may be made up of two pieces as illustrated in **Seismic 2**. Where hoops are not required outside the plastic hinge region, stirrups with seismic hooks should be used as also illustrated in **Seismic 2**.

6.3.2 Shear Design

Seismic induced energy in special moment resisting frames is expected to be dissipated through flexural yielding of members. During inelastic response, however, the members should be protected against premature brittle shear failure. This is ensured by providing sufficient shear capacity to resist seismic design shear forces. Seismic design shear V_e in plastic hinge regions is associated with maximum inelastic moments that can develop at the ends of members when the longitudinal tension reinforcement is in the strain hardening range (assumed to develop 1.25 f_y). This moment level is labeled as *probable flexural strength*, M_{pr}. Figure 6-1 illustrates the internal forces of a section that develop at probable moment resistance.

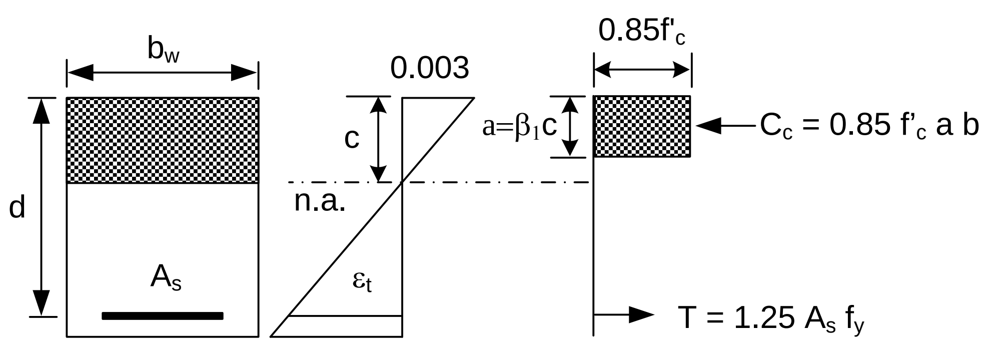


Fig. 6-1 Internal forces in a reinforced concrete section at probable flexural strength.

M_{pr} for a rectangular section with tension reinforcement can be obtained from **Seismic 3.** This design aid provides values for coefficient K_{pr}, which is used to solve the following equation:

$$M_{pr} = 1.25 A_s f_y (d - \frac{a}{2}) \qquad \textbf{(6-1)}$$

Where; $$a = \frac{1.25\, A_s\, f_y}{0.85\, f_c^{'}\, b} \tag{6-2}$$

$$A_s = \rho\, b_w d \tag{6-3}$$

Substituting Eqs. 6-2 and 6-3 into 6-1 gives;

$$M_{pr} = K_{pr} b_w d^2 \tag{6-4}$$

Where; $$K_{pr} = 1.25\rho\, f_y (1 - 0.735\rho \frac{f_y}{f_c^{'}}) \tag{6-5}$$

Once M_{pr} is obtained, the seismic design shear can be computed from the equilibrium of forces shown in **Seismic 4**.

The contribution of concrete to shear, V_c within the plastic hinge region (length equal to twice the member depth at each end) may be negligibly small upon the formation of hinge due to the deterioration of concrete. Therefore, when V_e within the hinging region is equal to one-half or more of the maximum required shear strength, and the factored axial compression including earthquake effects is less than $A_g\, f'_c / 20$, V_c should be ignored completely in design ($V_c = 0$).

6.4 Special Moment Frame Members Subjected to Bending and Axial Load

6.4.1 Flexural Design

Members designed to resist earthquake forces while subjected to factored axial compressive force of $P_u > A_g f'_c/10$ are designed following the requirements of Sec. 21.4 of ACI 318-05. Columns that fall in this category are designed using the interaction diagrams provided in Chapter 3, with minimum and maximum reinforcement ratios of 1% and 6%, respectively. The 2% reduction in the maximum limit of reinforcement ratio from the 8% limit specified for ordinary building columns is intended to reduce the congestion of reinforcement and development of high shear stresses in earthquake resistant columns. ACI 318-05 also provides limitations on column cross sectional dimensions as illustrated in **Seismic 5.**

6.4.2 Strong-Column Weak-Beam Concept

In multistory reinforced concrete buildings it is desirable to dissipate earthquake induced energy by yielding of the beams rater than the columns. The columns are responsible from overall strength and stability of the structure, with severe consequences of failure. Furthermore, columns are compression members and axial compression reduces the ductility of reinforced concrete columns, thus necessitating more stringent confinement reinforcement. Therefore, it is preferable to control inelasticity in columns, to the extent possible, while dissipating most of the energy through yielding of the beams. This is known as the “strong-column weak-beam concept.”

The strong-column weak-beam concept is enforced in the ACI Code through Sect. 21.4.2.2, which states that the flexural strength of columns should be 6/5 of that of the adjoining beams, as indicated below.

$$\sum M_{nc} \geq \frac{6}{5}\sum M_{nb} \quad \textbf{(6-6)}$$

Where;

$\sum M_{nc}$ is the sum of nominal flexural strengths of the columns framing into the joint, computed at the faces of the joint under factored axial forces such that they give the lowest flexural strength. Nominal flexural strengths of columns can be computed using the column interaction diagrams in Chapter 3.

$\sum M_{nb}$ is the sum of the nominal flexural strengths of the beams framing into the joint, computed at the faces of the joint. For negative moment capacity calculations, the slab reinforcement in the effective slab width, as defined in Sec. 8.10 of ACI 318-05 and illustrated in **Flexure 6** should also be included, provided that they have sufficient development length beyond the critical section. The nominal flexural strength of a beam can be computed using the appropriate design aids in Chapter 1. (**Flexure 1 to Flexure 8**)

If Eq. (6-6) is not satisfied, the confinement reinforcement required at column ends, as presented in the next section and as required by Sec. 21.4.4 of ACI 318-05 will continue through the full height of the column.

6.4.3 Confinement Reinforcement

The behavior of reinforced concrete compression members is dominated by concrete (as opposed to reinforcement), which tends to be brittle unless confined by properly designed transverse reinforcement. In seismic resistant columns, where inelastic response is expected, sufficient ductility must be ensured through the confinement of core concrete. This can be achieved by using spiral reinforcement or closely spaced hoops, overlapping hoops, and crossties. The increased inelastic deformability is assumed to be met if the column core is confined sufficiently to maintain column concentric load capacity beyond the spalling of cover concrete. This performance criterion results in the following minimum confinement reinforcement as stated in Sec. 21.4.4 of ACI318-05:

i) The volumetric ratio of spiral or circular hoop reinforcement, ρ_s shall not be less than;

$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}} \quad \textbf{(6-7)}$$

$$\rho_s = 0.12\frac{f_c'}{f_{yt}} \quad \textbf{(6-8)}$$

ii) The total cross sectional area of rectangular hoop reinforcement, A_{sh} shall not be less than;

$$A_{sh} = 0.3\, s\, b_c \frac{f_c^{'}}{f_{yt}} \left(\frac{A_g}{A_{ch}} - 1 \right) \tag{6-9}$$

$$A_{sh} = 0.09\, s\, b_c \frac{f_c^{'}}{f_{yt}} \tag{6-10}$$

Figure 6-2 illustrates the definition of some of the terms used for confined core concrete. The above reinforcement should be provided with due considerations given to their spacing, both along the column height and column cross-sectional plane, for increased effectiveness of concrete confinement. The spacing requirements of ACI 318-05 for transverse confinement reinforcement are indicated in **Seismic 5**. The volumetric ratio of spiral and circular hoop reinforcement for circular columns and the reinforcement ratio of rectilinear transverse reinforcement in square and rectangular columns can be obtained from **Seismic 6** and **Seismic 7**, respectively.

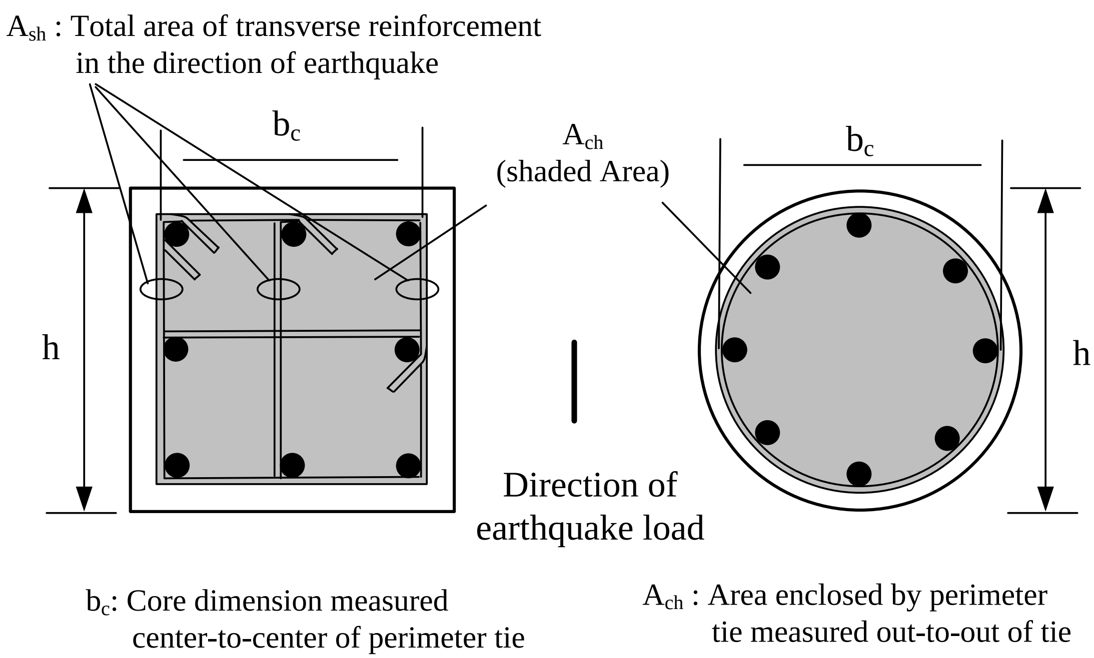


Fig. 6-2 Definitions of terms involving confined core concrete

6.4.4 Shear Design

Seismic design shear in columns is computed from **Seismic 4** as shear force associated with the development of probable moment strength (M_{pr}) at column ends when the associated factored axial force, P_u is acting on the column. These moments are computed with reinforcement strengths in tension equal to 1.25 f_y, reflecting the contribution of longitudinal column reinforcement in the strain hardening range. However, the column capacity is often governed by the crushing of compression concrete without excessive yielding of tension reinforcement. Therefore, the engineer should exercise

judgment in selecting the probable moment strength for columns, depending on the level of accompanying axial compression. A conservative approach for estimating column M_{pr} for shear calculations is to use nominal moment capacity at balanced section, since this would be the maximum moment capacity for the column. The seismic shear V_e obtained in this manner need not exceed the seismic shear force associated with the formation of plastic hinges at the ends of the framing beams, i.e., column shear balancing seismic shear at the ends of the beams when probable moment strength of beams, M_{pr} are developed at beam ends. However, at no case shall V_e computed above be less than the factored column shear force determined by analysis of the structure under seismic loading.

Once the seismic design shear force is computed, the plastic hinge regions at the ends of the column (ℓ_0 defined in **Seismic 5**) will be designed for V_e. In the design, however, the shear strength provided by concrete, V_c will be neglected ($V_c = 0$) if *both* of the following conditions are met:

i) $V_e \geq$ 50% of the maximum shear strength required within ℓ_0 due to the factored column shear force determined by structural analysis.

ii) P_u (including earthquake effects) < $A_g f'_c / 20$.

6.5 Joints of Special Moment Frames

The formation of plastic hinges at the ends of beams may result in significant shear force reversals in beam-column joints. Joint shear can be determined by computing the internal forces acting on the joint while assuming that the tension beam reinforcement anchored into the joint develops 1.25 f_y. **Seismic 8** and **Seismic 9** illustrate the shear force acting in interior and exterior joints.

6.5.1 Joint Shear Strength

The joint shear produces diagonal tension and compression reversals which may be critical for premature diagonal tension or compression failures, unless properly reinforced. The joint shear may especially be critical in edge and corner joints, which are not confined by the adjoining beams on all four faces. A member that frames into a joint face is considered to provide confinement to the joint if at least ¾ of the face of the joint is covered by the framing member. The shear capacity of beam-column joints in special moment resisting frames can be computed by the following expressions given in Sec. 21.5.3 of ACI 318-05.

i) For joints confined on all four faces: $\mathbf{V_n \leq 20\sqrt{f'_c}\, A_j}$ **(6-11)**

ii) For joints confined on three or two opposite faces: $\mathbf{V_n \leq 15\sqrt{f'_c}\, A_j}$ **(6-12)**

iii) For other joints: $\mathbf{V_n \leq 12\sqrt{f'_c}\, A_j}$ **(6-13)**

Where, A_j is the effective joint cross-sectional area defined in Fig. 6-3 as the column depth (column dimension in the direction of joint shear) times the effective width of the column, which is equal to the column width except where the beams frame into a wider column.

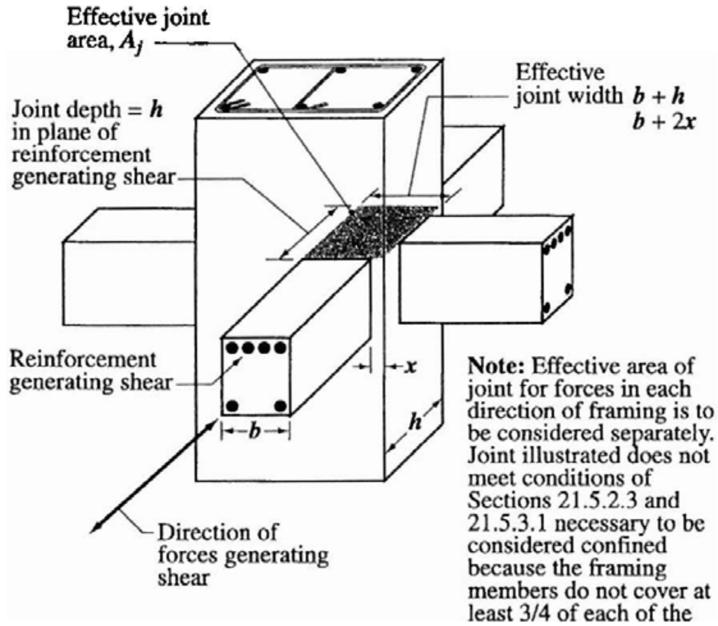


Fig. 6-3 Definition of effective joint area A_j

6.5.2 Joint Reinforcement

The column confinement reinforcement provided at the ends of columns should continue into the beam-column joint if the joint is *not* confined by the framing beams on all four faces, as described in the previous section. For interior joints, with attached beams externally confining the joint on all four faces, the spacing of joint reinforcement can be relaxed to 6 in.

6.6 Members of Intermediate Moment Frames

6.6.1 Flexural Design

Members of intermediate moment frames located in regions of moderate seismicity and are designed to resist primarily flexure ($P_u \leq A_g f'_c/10$), will meet the beam design requirements of **Seismic 10**. This design aid provides guidance for both the longitudinal flexural reinforcement and transverse confinement reinforcement. Members subjected to higher axial loads will be designed as columns following the requirements for columns outlined also in **Seismic 10,** unless the column is designed to have spiral reinforcement.

The transverse reinforcement in beam-column joints of intermediate moment frames will conform to Sec. 11.11.2 of ACI 318-05.

6.6.2 Shear Design

The shear strength ϕV_n of members of intermediate moment frames will be at least equal to the shear force associated with the development of nominal capacities of members at their ends while also subjected to the effects of factored gravity loads. Also, the shear strength should not be lower than the maximum shear obtained from the design load combinations where the earthquake loading is assumed to be twice the magnitude prescribed by the governing code. **Seismic 11** shows the design shear force V_u associated with the development of nominal member strengths at the ends.

6.7 Members not Designed as Part of the Lateral-Force-Resisting System

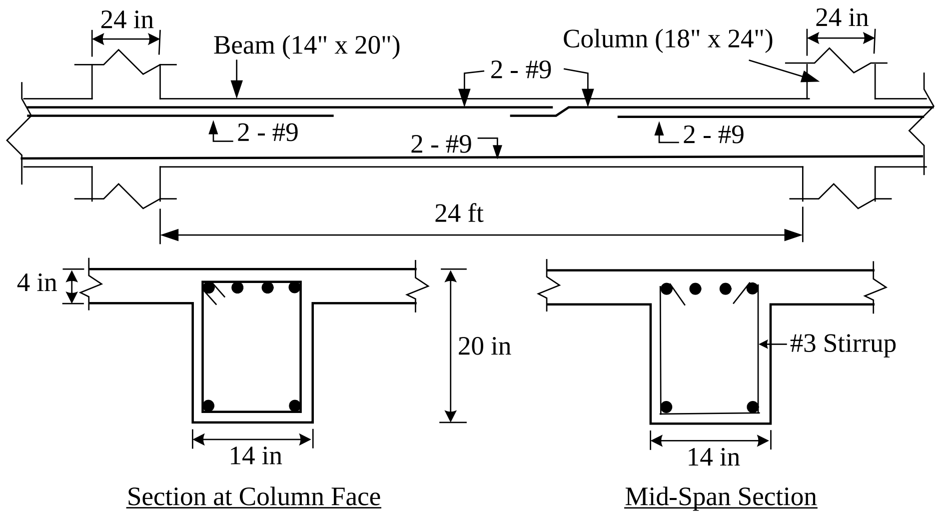
Members of structures located in regions of high seismic risk, but not forming part of the lateral force resisting system, must be investigated for sufficient deformability during seismic response. These members, although not designed to resist seismic forces will deform along with the seismic lateral force resisting system. Therefore, they should have adequate strength and deformability to allow the development of design displacement δ_u, as per the requirements of Sec. 21.11 of ACI 318-05.

6.8 Seismic Design Examples

SEISMIC DESIGN EXAMPLE 1 - Adequacy of beam flexural design for a special moment frame.

The interior-span beam shown is designed for flexure using factored design loads. Check if the beam meets the seismic design requirements for flexure if it is part of a special moment frame located in a seismically active region.

Given:
f'_c = 4,000 psi
f_y = 60,000 psi
Clear cover: 1.5 in

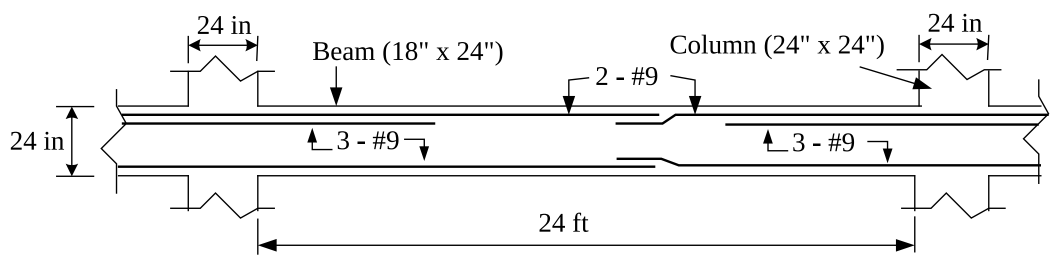


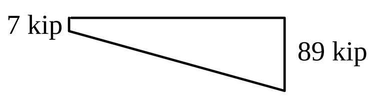
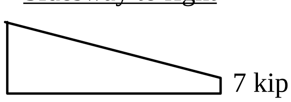
Procedure	Calculation	ACI 318-05 Section	Design Aid
Check geometric constraints for the beam.	d = 20 - 1.5 - 0.375 - 1.128/2 = 17.6 in i) Clear span l_n= 24 ft $\geq$ 4d = 5.8 ft O.K. ii) b_w/h = 14/20 = 0.7 > 0.3 O.K. iii) b_w > 10 in O.K. and $b_w < c_2$ O.K.	 21.3.1.2 21.3.1.3 21.3.1.3	Seismic 1
Check for minimum and maximum ratio of longitudinal reinforcement	i) $(\rho_{min})_{top} = (\rho_{min})_{bott.} = 3\sqrt{f'_c}/f_y$ = 0.32% ii) $(\rho_{min})_{top} = (\rho_{min})_{bott.}$ = 200/f_y = 0.33 % iii) 2 # 9 bars result in ρ = 0.81% O.K. 2 # 9 top and bottom continuous bars. iv) ρ_{max} = 2.5 % O.K.	21.3.2.1	Seismic 1
Check for minimum positive and negative moment capacity at each section.	i) $M_n^+ \geq 0.5M_n^-$ at column face; For 4- #9 bars; ρ^- = 4 (1.0) / [(14)(17.6)] = 1.62 % $M_n^- = K_n\, bd^2/12000$ = (751/09) (14)(17.6)^2 /12000 = 301 ft-kips For 2 - #9 bars; ρ^+ = 2 (1.0) / [(14)(17.6)] = 0.81 % $M_n^+ = K_n\, bd^2/12000$ = (407/0.9) (14)(17.6)^2 /12000 = 163 ft-kips 163 > 0.5 (301) = 151 ft-kips O.K. ii) $M_n^+ \geq 0.25\, (M_n^-)_{max}$ at any section; $(M_n^+)_{min}$=163 > 0.25 (301) = 75 ft-kip iii) $M_n^- \geq 0.25\, (M_n^-)_{max}$ at any section; $(M_n^-)_{min}$= 163 ft-k > 0.25(301)= 75 ft-k	21.3.2.2 21.3.2.2 21.3.2.2	Flexure 1 Flexure 1

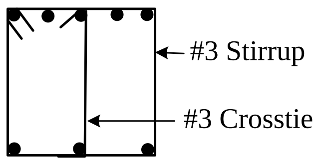
SEISMIC DESIGN EXAMPLE 2 - Design of the critical end regions of a beam in a special moment frame for shear and confinement.

The beam shown below is part of a special moment frame located in a high seismic risk area. Design transverse reinforcement at the potential hinging regions of the beam. Maximum shear strength required by analysis under factored loads is 82 kips.

<u>**Given:**</u>
f'_c = 4,000 psi
f_y = 60,000 psi
Clear cover: 1.5 in
Unfactored Live load: 1.20 k/ft
Unfactored Dead load: 2.45 k/ft
Interior beam
$(V_{max})_{req.}$ = 82 kips



Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine design shear force V_e associated with the formation of plastic hinges at beam ends. First compute probable moment strength (M_{pr}) for positive and negative bending.	Assuming #3 hoops, effective depth d: d = 24 - 1.5 - 0.375 - 1.128/2 = 21.6 in For 5 - #9 bars ; $\rho^- = 5\,(1.0) / [18(21.6)] = 0.0129$ K^-_{pr} = 830 psi; $M^-_{pr} = K^-_{pr}\, bd^2/12000$ $M^-_{pr} = 830(18)(21.6)^2/12000 = 581$ ft-k For 3 - #9 bars ; $\rho^+ = 3\,(1.0) / [18(21.6)] = 0.0077$ K^+_{pr} = 528 psi; $M^+_{pr} = K^+_{pr}\, bd^2/12000$ $M^+_{pr} = 528[(18)(21.6)^2]/12000 = 370$ ft-k	 21.3.4.1	 Seismic 3 Seismic 3
Compute design shear force V_e associated with the formation of M_{pr} at member ends while the member is loaded with factored gravity loads. Shear force diagrams;  <u>Sidesway to right</u>  <u>Sidesway to left</u>	w_u = 1.2 D + 1.0 L+0.2S Int. beam; S=0 w_u = 1.2(2.45) + (1.0)(1.20)] = 4.14 k/ft $V_e = \frac{M_{pr1} + M_{pr2}}{\ell_n} \pm \frac{w_u \ell_n}{2}$ $V_e = \frac{370 + 581}{20} \pm \frac{4.14(20)}{2}$ $V_e = 48 \pm 41 = 89$ kips	21.3.4.1	Seismic 4
Check the magnitude of seismic induced shear relative to the maximum design shear required under factored loads and determine the contribution of concrete to shear strength, V_c.	$(V_{max})_{req.}$ /2 = 82/2 = 42 kips $V_e > (V_{max})_{req.}$ /2 Also, $P_u < A_g\, f'_c$ / 20 (beam) Therefore, V_c = 0 (within the hinging region: 2h)	21.3.4.2	

Procedure	**Calculation**	**ACI 318-05 Section**	**Design Aid**
Determine vertical shear reinforcement at the critical section. 	Use #3 perimeter hoops and cross ties as shown in the figure. $\phi V_s = V_e$; $V_s = 89/0.75 = 119$ kips $s = (A_v\ f_y\ d)/V_s$ s = (3x0.11)(60)(21.6)/119 = 3.6 in	21.3.4.2 11.5.7	
Provide hoop steel in the potential hinge region at member ends for concrete confinement. Note, when applicable, confinement reinforcement will also be provided at potential plastic hinge regions away from the face of support. Where hoops are not required (outside the potential hinging regions) stirrups with seismic hooks at both ends should be provided at no more than d/2 distance throughout the length of the beam.	s < d/4 = 21.6/4 = 5.4 in < 8 $(d_b)_{long.}$ = 8(1.128) = 9 in < 24 $(d_b)_{hoop}$ = 24(0.375) = 9 in < 12 in spacing required for shear is 3.6 in. Therefore, use 14 sets of hoop and crosstie at s = 3.5 in within 2h = 2(24) = 48 in (4 ft) distance from the column face at each end, with the first hoop located not more than 2 in from the column face.	21.3.3 21.3.3.4	Seismic 1
Check hoop detailing.	Perimeter hoops and cross ties provide lateral support to at least every other longitudinal reinforcement on the perimeter by the corner of a hoop or the hook of a cross tie. No longitudinal bar is farther than 6 in from a laterally supported bar. Hooks should extend $6d_b$ or 3 in.	21.3.3.3 7.10.5.3	Seismic 2

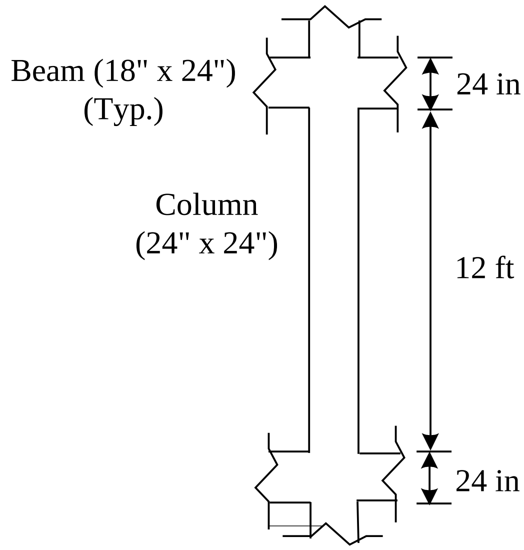
SEISMIC DESIGN EXAMPLE 3 - Design of a column of a special moment frame for longitudinal and confinement reinforcement.

The column shown has a 24 in square cross-section, and forms part of a special moment frame. Design the column for longitudinal and confinement reinforcement. Assume that the slenderness effects are negligible, and the framing beams are the same as those given in SEISMIC DESIGN EXAMPLE 2. Ignore the contribution of slab reinforcement to flexural strength of beams.

Given:
f'_c = 4,000 psi
f_y = 60,000 psi
Clear cover: 1.5 in
Slenderness is negligible
Column is bent in double curvature

i) Design forces for sidesway to right:
ϕP_n= 1079 kip
$(\phi M_n)_{top}$= 390 ft-kip
$(\phi M_n)_{bot.}$= 353 ft-kip

ii) Design forces for sidesway to left:
ϕP_n= 910 kip
$(\phi M_n)_{top}$= 367 ft-kip
$(\phi M_n)_{bot.}$= 236 ft-kip

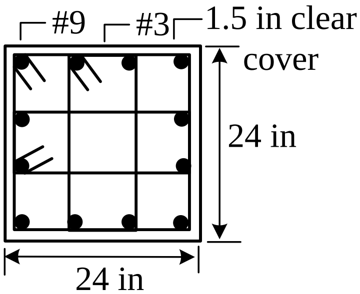


Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine column size	Given: h = b = 24 in		
Check if slenderness effects may be neglected.	Given : Slenderness effects are negligible.	10.12.2	
Check the level of axial compression	$A_g f'_c/10$ = (576)(4000)/[(10)(1000)] = 230 kips ϕP_n = 1079 kips > 230 kips. Therefore, the requirements of section 21.4 apply.	21.4.1	
Check geometric constraints. Note that the beam reinforcement is continuous over the support (#9 bars with b_d = 1.128 in).	h = b = 24 in > 12 in O.K. 24/24 = 1.0 > 0.4 O.K. h = 24 in > 20 d_b = 20(1.128) = 22.6 in O.K.	21.4.1.1 21.4.1.2 21.5.1.4	Seismic 5
Determine longitudinal reinforcement. First select the appropriate interaction diagram. Estimate γ for a column section of 24 in, cover of 1.5 in, and assumed bar sizes of #3 ties and #9 longitudinal bars.	γ = [24 - 2(1.5 + 0.375) - 1.128)] / 24 γ = 0.80 Square cross-section. If equal area of reinforcement is to be provided on four sides, select Columns 3.2.3 interaction diagrams.		Columns 3.2.3
Select the critical design loads and compute; $K_n = P_n/(f'_cA_g)$ and $R_n = M_n/(f'_cA_gh)$ Obtain reinforcement ratio ρ from	ϕP_n = 1079 kips; P_n =1079/0.65 =1660 kips ϕM_n = 390 ft-k; M_n = 390/0.65 = 600 ft-k A_g = (24)(24) = 576 in^2	10.2 10.3	Columns 3.2.3

interaction diagrams. Select longitudinal reinforcement	P_n / ($f'_c A_g$) = 1660 /[(4)(576)] = 0.72 M_n / ($f'_c A_g h$) = (600 x 12) / [(4)(576)(24)] = 0.13 for γ = 0.80; ρ = 0.019 $A_s = A_g \rho$ A_s = 576 x 0.019 A_s = 10.94 in^2 (req'd); use 12 #9 bars A_s = 12 x 1.00 = 12.0 in^2 (provided)		
Check the limits of reinforcement ratio ρ	$(\rho)_{prov.}$ = 12.0/576 = 0.021 0.01 < 0.021 < 0.006 O.K.	21.4.3.1	

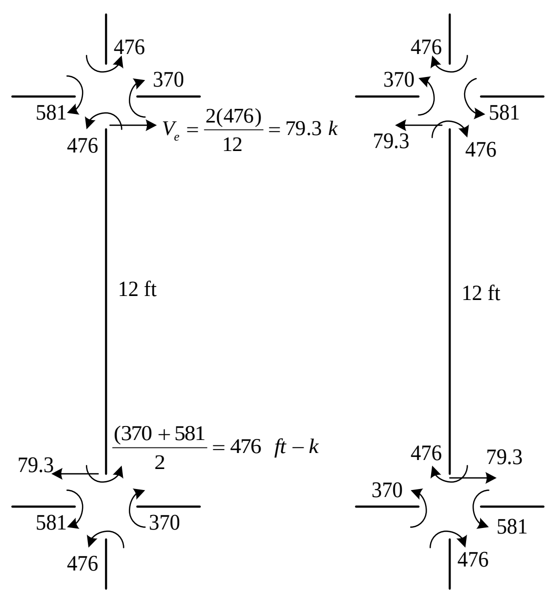
Check flexural strengths of columns and beams at each joint.
 	
Determine column strength M_{nc} from interaction diagram
 	
Determine strengths of the adjoining beams. Beams are the same as those given in SEISMIC DESIGN EXAMPLE 2, with 5-#9 top bars (ρ = 0.0129) and 3-#9 bottom bars (ρ =0.0077).
	$\sum M_{nc} \geq \frac{6}{5}\sum M_{nb}$
	<u>For sidesway to right:</u>
	ϕP_n = 1079 kips; P_n =1079/0.65=1660 kips
	for ρ = 0.021 and P_n / $f'_c A_g$ = 0.72;
	M_{nc} /($f'_c A_g h$) = 0.14 when γ = 0.80
	M_{nc}=(0.14)(4)(576)(24)/12= 645 ft-kips
	
	From Chapter 1- Design for flexure:
	For 5-#9 bars ρ = 0.0129
	0.9K_n = 617 psi and K_n = 617/0.9 = 686 psi
	M^-_{nb} = $K_n bd^2$ / 12000
	= (686)(18)$(21.6)^2$/12000 = 480 ft-kips
	For 3-#9 bars ρ =0.0077
	0.9K_n = 386 psi and K_n = 386/0.9 = 429 psi
	M^+_{nb} = $K_n bd^2$ / 12000
	= (429)(18)$(21.6)^2$/12000 = 300 ft-kips
	$\sum M_{nc} \geq \frac{6}{5}\sum M_{nb}$
	(2)(645)=1290>(6/5)(480+300)=936 O.K.
	
	<u>For sidesway to left:</u>
	ϕP_n = 910 kips; P_n = 910/0.65= 1400 kips
	for ρ = 0.021 and P_n / $f'_c A_g$ = 0.61;
	M_n /($f'_c A_g h$) = 0.16 when γ = 0.80
	M_n=(0.16)(4)(576)(24)/12= 737 ft-kips
	Beam strengths as determined above:
	M^-_{nb} = 480 ft-kips
	M^+_{nb} = 300 ft-kips
	(2)(737)=1474>(6/5)(480+300)=936 O.K.
		21.4.2.2
		
		21.4.2.2
			
			Columns 3.2.3
			
			Flexure 1
			
			Flexure 1
			
			Columns 3.2.3

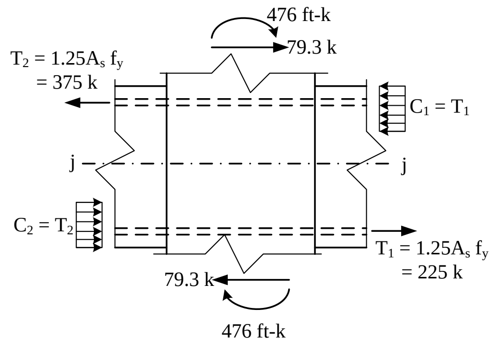
Design for confinement reinforcement	$A_g = (24)^2 = 576\ in^2$ $A_{ch} = [24 - 2(1.5)]^2 = 441\ in^2$ A_g/A_{ch} = 1.31 $\rho_s = 0.0062 = A_{sh}/(sb_c)$; Try #3 overlapping hoops as shown, A_{sh} = 4(0.11) = 0.44 in^2; b_c = 20.6 in s = 0.44/[(0.0062)(20.6)] = 3.45 in	21.4.4.1	Seismic 7

			
Check for maximum spacing of hoops.	$s < 24/4 = 6$ in O.K. $s < 6(d_d)_{long.} = 6(1.125) = 6.75$ in O.K. $s < s_0 = 4+(14-h_x)/3 = 4+(14-6.9)/3 = 6.36$ in O.K. spacing of hoop legs, $h_x < 14$ in O.K. use #3 overlapping hoops @ 3.5 in spacing. $\ell_0 \geq h = 24\,in$ $\ell_0 \geq \ell_c / 6 = 24\,in$ $\ell_0 \geq 18\,in$ Provide hoops over 24 in (2 ft) top and bottom, measured from the joint face. <u>Note:</u> Because the strong-column weak-beam requirement of 21.4.2.2 was met, the confinement reinforcement need not be provided throughout the entire column length. Also, the contribution of the column to lateral strength and stiffness of the structure can be considered.	21.4.4.2 21.4.4.3 21.4.4.4 21.4.2.3 21.4.2.1	Seismic 5 Seismic 5

SEISMIC DESIGN EXAMPLE 4 – Shear strength of a monolithic beam-column joint.

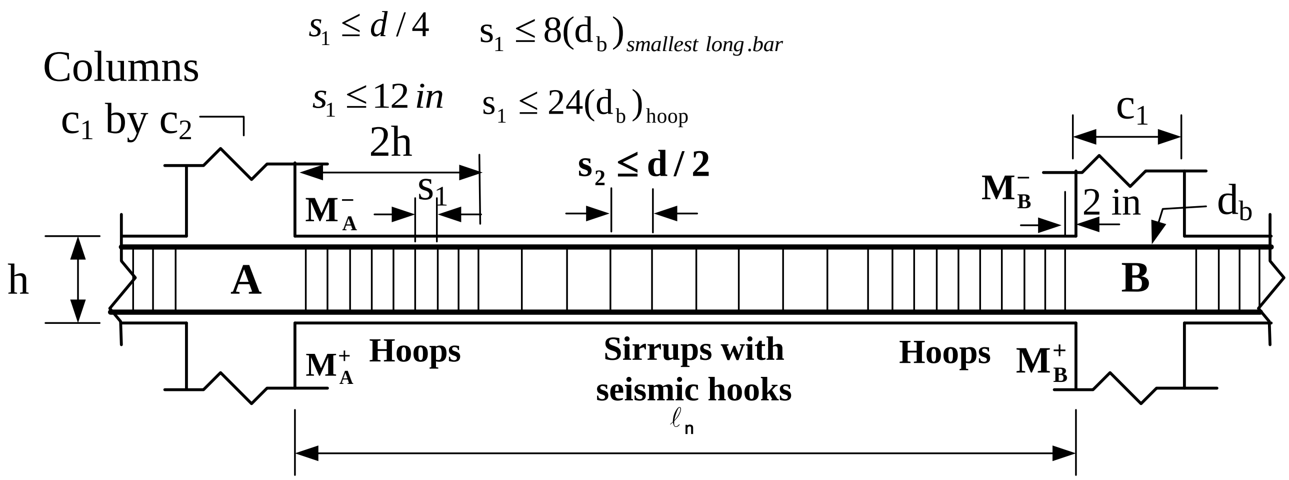
Consider a special moment frame and check the shear strength of an interior beam-column joint with four framing beams. The columns have a 24 in square cross-section, and a 12 ft clear height. The maximum probable moment strength of columns is $(M_{pr})_{col.}$= 520 ft-kips. The framing beams have the same geometry and reinforcement as those given in SEISMIC DESIGN EXAMPLE 2. f'_c = 4,000 psi; f_y = 60,000 psi.

Procedure	Calculation	ACI 318-05 Section	Design Aid
Compute column shear force V_e associated with the formation of plastic hinges at the ends of columns, i.e. when probable moment strengths, $(M_{pr})_{col.}$ are developed.	$V_e = 2(M_{pr})_{col.} / 12$ V_e = 2(520) / 12 = 86.7 kips	21.4.5.1	Seismic 4
Note that column shear need not exceed that associated with formation of plastic hinges at the ends of the framing beams. Compute V_e when probable moment strengths are developed at the ends of the beams.	From SEISMIC DESIGN EXAMPLE 2; M^-_{pr} = 581 ft-k and M^+_{pr} = 370 ft-k  Sidesway to right Sidesway to left V_e = 79.3 kips < 86.7 kips Therefore, use V_e= 79.3 kips.	21.4.5.1	Seismic 4

Compute joint shear, when stress in flexural tension reinforcement of the framing beams is $1.25f_y$. Note that, in this case, because the framing beams have symmetrical reinforcement arrangements, the joint shear associated with sidesway to left would be the same as that computed for sidesway to right.	 Sidesway to right $T_1 = 1.25(3)(1.0)(60) = 225$ k $C_1 = T_1 = 225$ k $T_2 = 1.25(5)(1.0)(60) = 375$ k $C_2 = T_2 = 375$ k $V_j = T_2 + C_1 - V_e$ $V_j = 375 + 225 - 79.3 = 520.7$ kips	21.5.1.1	Seismic 8
Compute shear strength of the joint. The joint is confined externally by four framing beams, each covering at least ¾ of the joint face.	$V_c = 20\sqrt{f_c'}A_j$ Joint depth = 24 in Effective joint width = smaller of {b + h} or {b + 2x} where x = (24 – 18)/2 = 3 in {b + h} = 18 + 24 = 42 in {b + 2x}=18 + (2)(3) = 24 in < 42 in use 24 in $A_j = (24)(24) = 576$ in^2 $\phi V_c = (0.75)20\sqrt{4000}(576)/1000$ $\phi V_c = 546$ kips > 520.7 kips O.K.	21.5.3.1	Fig. 6-3
Note that transverse reinforcement equal to at least one half the amount required for column confinement, has to be provided, with absolute maximum tie spacing relaxed to 6 in.		21.5.2.2	

6. Seismic Design Aids

Seismic 1 – Requirements for flexural members of special moment frames



Top, as well as bottom reinforcement, <u>each;</u>

- Shall not be less than:

- $A_{s,min} = \dfrac{3\sqrt{f'_c}}{f_y} b_w d$
- $200 b_w d / f_y$
- **Two continuous bars**

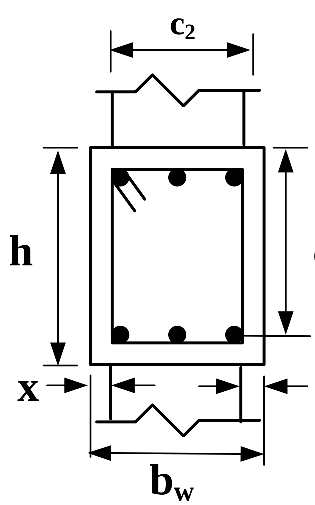
- Shall not exceed: $\rho = 0.025$

Minimum flexural capacities;

- $M_A^+ \ge 0.5\, M_A^-$
- $M_B^+ \ge 0.5\, M_B^-$
- **If the largest end moment is M_{max}; at any section along the beam:**

  $M^+ \ge 0.25\, M_{max}$

  $M^- \ge 0.25\, M_{max}$



$b_w \ge 0.3\,h$ $\qquad b_w \le c_2 + x + y$

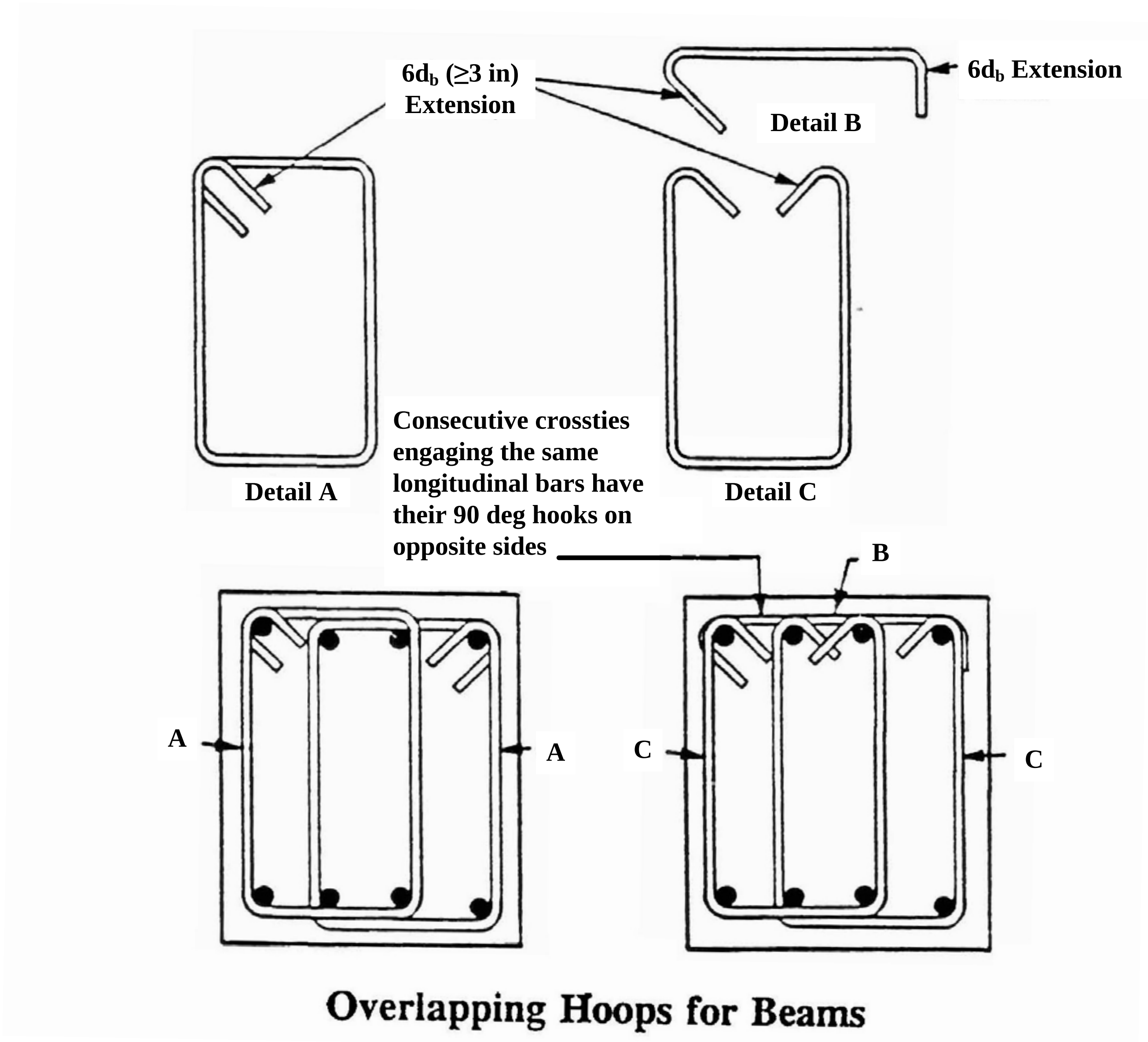
$b_w \ge 10\,in$ $\qquad x \le 3/4\,h$

$\ell_n \ge 4\,d$ $\qquad y \le 3/4\,h$

If beam reinforcement is continuous over the support:

$c_1 \ge 20 d_b$ **(NWC)** $\qquad c_1 \ge 26 d_b$ **(LWC)**

Seismic 2 – Details of transverse reinforcement for flexural members of special moment frames

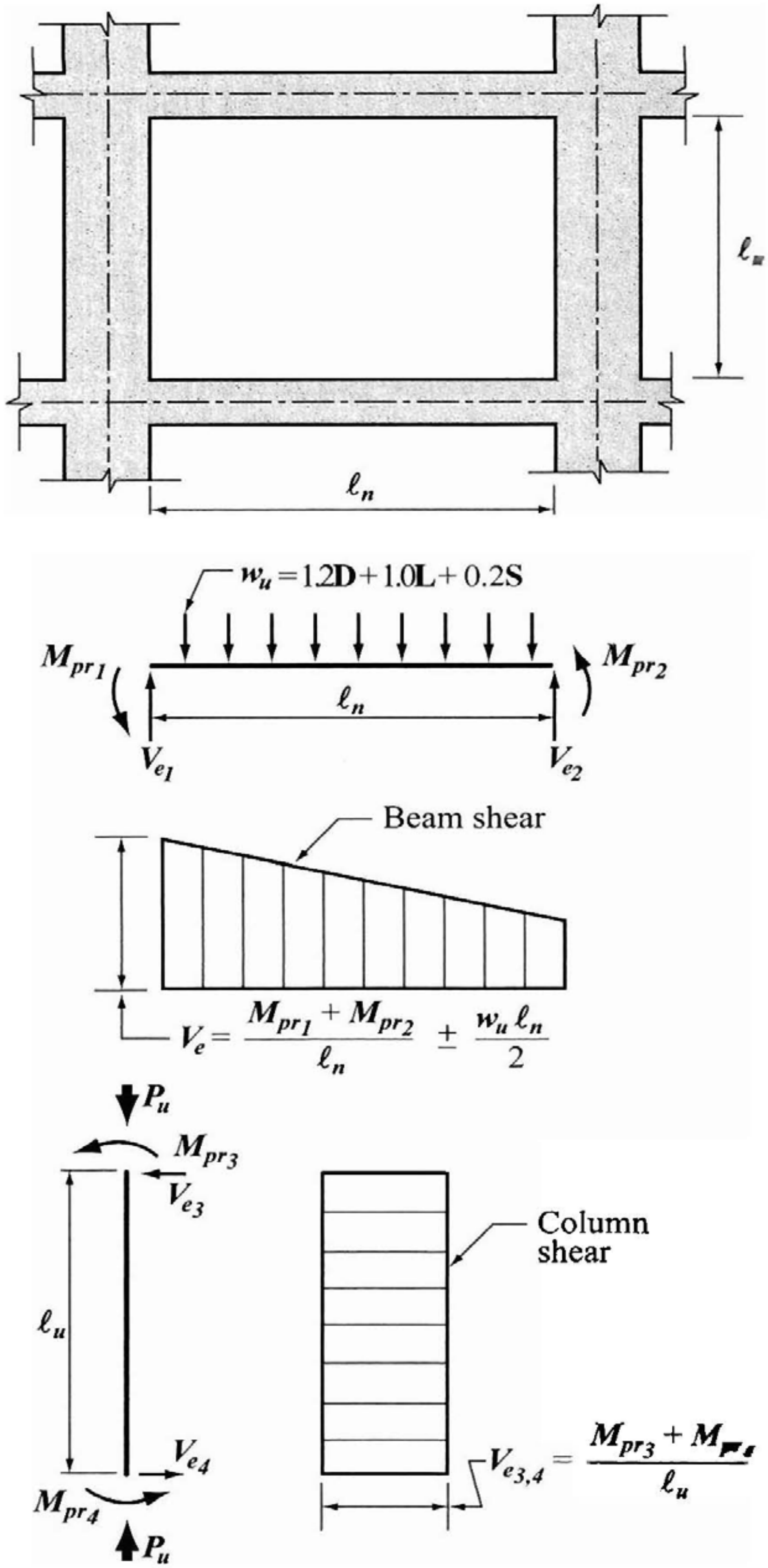


Seismic 3 - Probable moment strength for flexural members
f_y = 60,000;   1.25 f_y = 75,000 psi; ϕ = 1.0

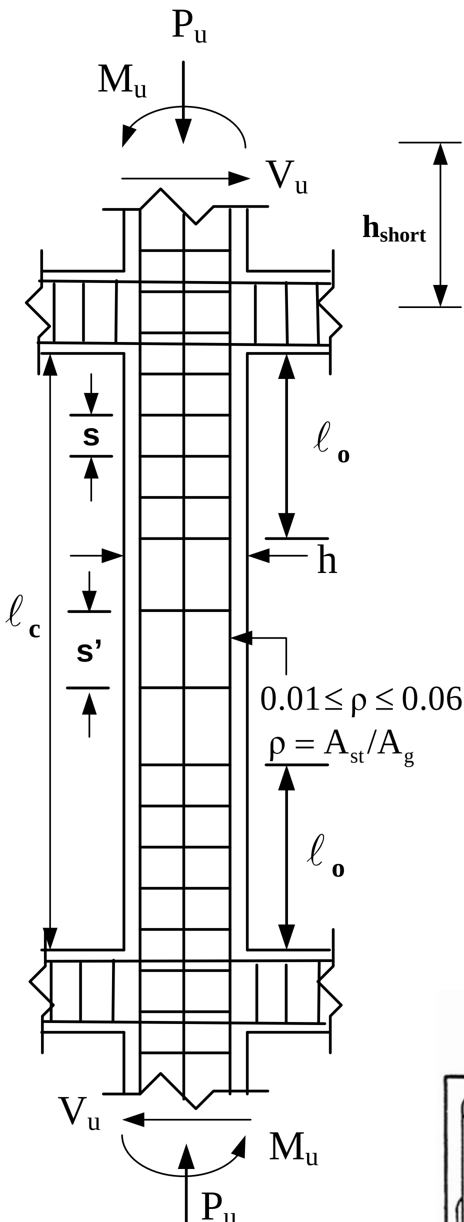
$M_{pr} = K_{pr}\, b_w d^2 /12{,}000$ ft-kips   **$\rho = A_s / bd$**

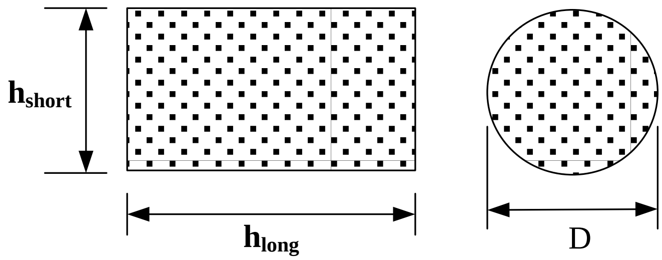
f'c (psi):	3000	4000	5000	6000	7000	8000	9000	10000
ρ	K_{pr} (psi)							
0.004	282	287	289	291	292	293	294	295
0.005	347	354	358	361	363	365	366	367
0.006	410	420	426	430	433	435	437	438
0.007	471	484	493	498	502	505	507	509
0.008	529	547	558	565	570	574	576	579
0.009	586	608	621	630	637	642	645	648
0.010	640	667	684	695	703	709	713	717
0.011	692	725	745	758	768	775	781	785
0.012	741	781	805	821	832	840	847	852
0.013	789	835	863	882	895	905	913	919
0.014	834	888	920	942	957	969	978	985
0.015	877	939	976	1001	1019	1032	1042	1051
0.016	918	988	1031	1059	1079	1094	1106	1115
0.017	956	1036	1084	1116	1138	1156	1169	1179
0.018	993	1082	1136	1171	1197	1216	1231	1243
0.019	1027	1126	1186	1226	1254	1276	1292	1306
0.020	1059	1169	1235	1280	1311	1335	1353	1368
0.021	1089	1210	1283	1332	1367	1393	1413	1429
0.022	1116	1250	1330	1383	1421	1450	1472	1490
0.023	1142	1288	1375	1433	1475	1506	1531	1550
0.024	1165	1324	1419	1482	1528	1562	1588	1609
0.025	1186	1358	1462	1530	1580	1617	1645	1668

Seismic 4 – Shear strength for flexural members and members subjected to bending and axial load of special moment frames



Seismic 5 – Requirements for members subjected to bending and axial load of special moment frames





$h_{short} \ge 12\,in$ $\quad D \ge 12\,in$

If beam reinforcement is continuous over the support:

$h \ge 20d_b$ **(NWC)**

$h \ge 26d_b$ **(LWC)**

$$\frac{h_{short}}{h_{long}} \ge 0.4$$

$s \le h/4$ $\qquad s' \ge 6\,in$

$s \le 6\,(d_b)_{long.}$ $\qquad s' \ge 6(d_b)_{long.}$

$s \le s_0$ **but need not be less than 4 in**

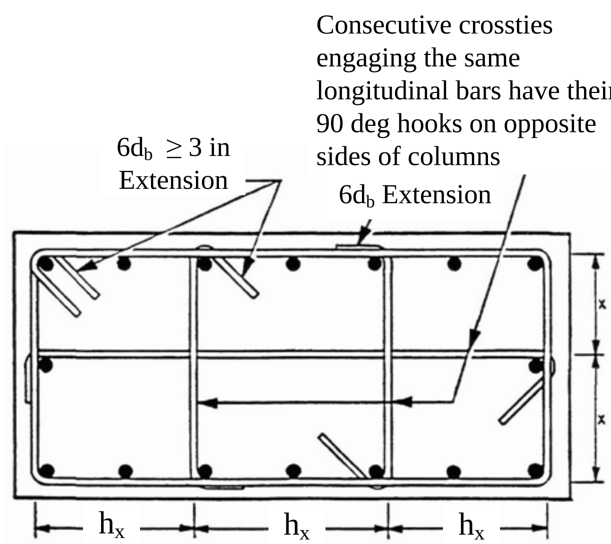
Where; $s_0 = 4 + \left(\frac{14 - h_x}{3}\right) \le 6\,in$

$\ell_o \ge h_{long}$

$\ell_o \ge \ell_c / 6$

$\ell_o \ge 18\,in$

Use transverse reinforcement at spacing “s” throughout the length of longitudinal reinforcement splice



h_x shall not exceed 14 inches

Transverse Reinforcement for Columns

Seismic 6 – Volumetric ratio of spiral reinforcement (ρ_s) for concrete confinement f_y = 60,000 psi

$$\rho_s \geq 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c^{'}}{f_{yt}} \quad \textbf{but} \quad \rho_s \geq 0.12\frac{f_c^{'}}{f_{yt}}$$

f'$_c$ (psi):	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9000**	**10000**
A_g/A_{ch}				**ρ_s**				
1.1	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020
1.2	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020
1.3	0.007	0.009	0.011	0.014	0.016	0.018	0.020	0.023
1.4	0.009	0.012	0.015	0.018	0.021	0.024	0.027	0.030
1.5	0.011	0.015	0.019	0.023	0.026	0.030	0.034	0.038
1.6	0.014	0.018	0.023	0.027	0.032	0.036	0.041	0.045
1.7	0.016	0.021	0.026	0.032	0.037	0.042	0.047	0.053
1.8	0.018	0.024	0.030	0.036	0.042	0.048	0.054	0.060
1.9	0.020	0.027	0.034	0.041	0.047	0.054	0.061	0.068
2.0	0.023	0.030	0.038	0.045	0.053	0.060	0.068	0.075
2.1	0.025	0.033	0.041	0.050	0.058	0.066	0.074	0.083
2.2	0.027	0.036	0.045	0.054	0.063	0.072	0.081	0.090
2.3	0.029	0.039	0.049	0.059	0.068	0.078	0.088	0.098
2.4	0.032	0.042	0.053	0.063	0.074	0.084	0.095	0.105
2.5	0.034	0.045	0.056	0.068	0.079	0.090	0.101	0.113

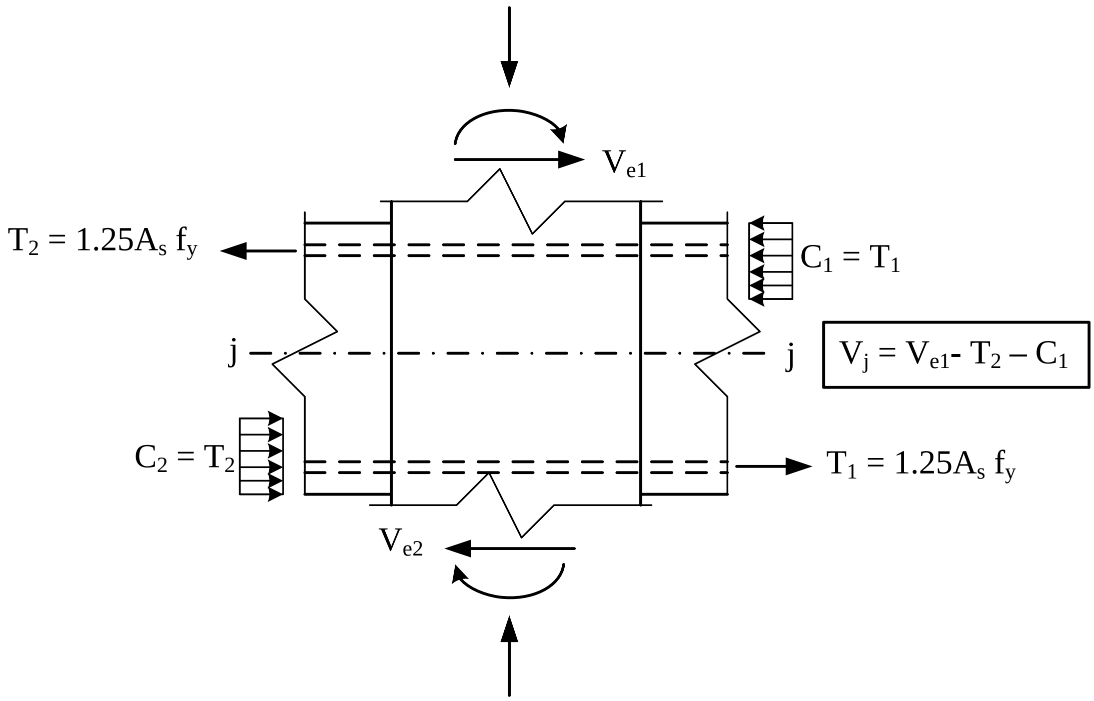
Note: Volumetric ratio, $\rho_s = \pi b A_s/(A_{ch} s)$ where; $A_{ch} = \pi(b_c/2)^2$ (See Fig. 6-2)

Seismic 7 – Area ratio of rectilinear confinement reinforcement (ρ_c) for concrete f_y = 60,000 psi

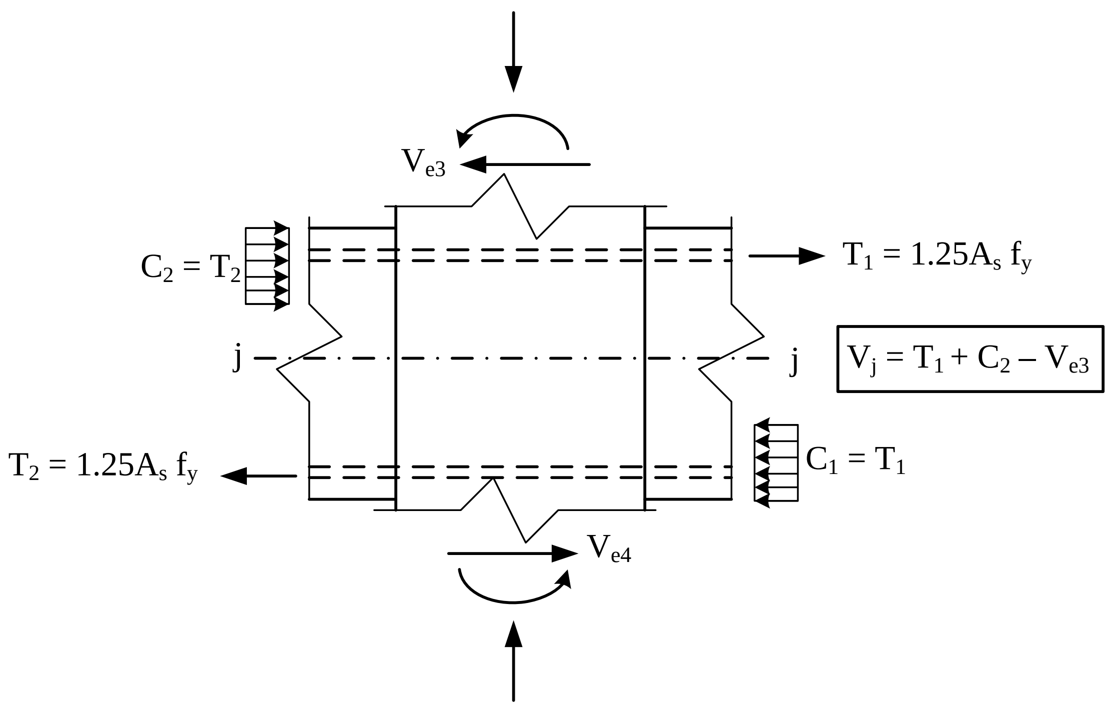
$$\rho_c = \frac{A_{sh}}{sb_c} \geq 0.3\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}} \qquad \text{but} \qquad \rho_c \geq 0.09\frac{f_c'}{f_{yt}}$$

f'_c (psi):	3000	4000	5000	6000	7000	8000	9000	10000
A_g/A_{ch}				ρ_c				
1.1	0.005	0.006	0.008	0.009	0.011	0.012	0.014	0.015
1.2	0.005	0.006	0.008	0.009	0.011	0.012	0.014	0.015
1.3	0.005	0.006	0.008	0.009	0.011	0.012	0.014	0.015
1.4	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020
1.5	0.008	0.010	0.013	0.015	0.018	0.020	0.023	0.025
1.6	0.009	0.012	0.015	0.018	0.021	0.024	0.027	0.030
1.7	0.011	0.014	0.018	0.021	0.025	0.028	0.032	0.035
1.8	0.012	0.016	0.020	0.024	0.028	0.032	0.036	0.040
1.9	0.014	0.018	0.023	0.027	0.032	0.036	0.041	0.045
2.0	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
2.1	0.017	0.022	0.028	0.033	0.039	0.044	0.050	0.055
2.2	0.018	0.024	0.030	0.036	0.042	0.048	0.054	0.060
2.3	0.020	0.026	0.033	0.039	0.046	0.052	0.059	0.065
2.4	0.021	0.028	0.035	0.042	0.049	0.056	0.063	0.070
2.5	0.023	0.030	0.038	0.045	0.053	0.060	0.068	0.075

Seismic 8 – Joint shear, V_j in an interior beam-column joint

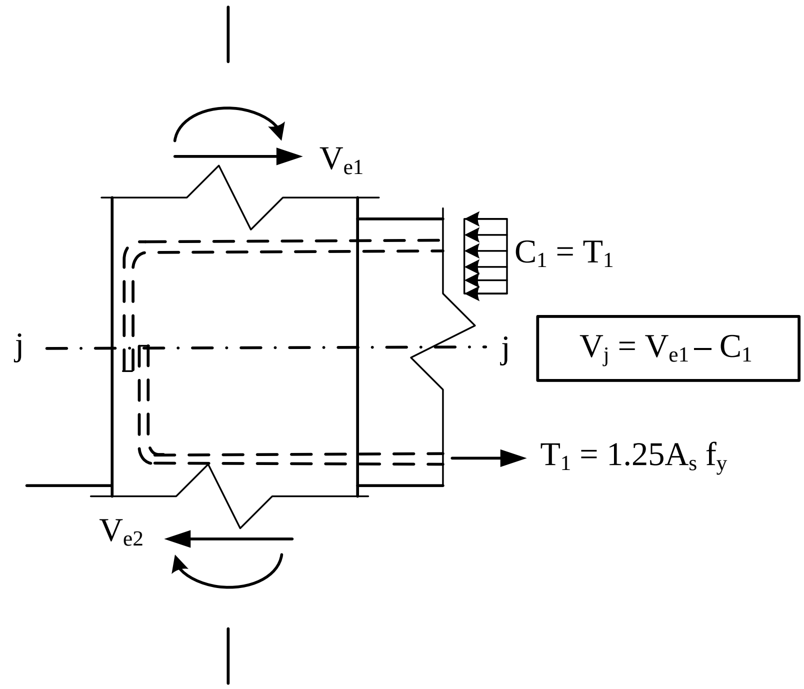


Sidesway to Right

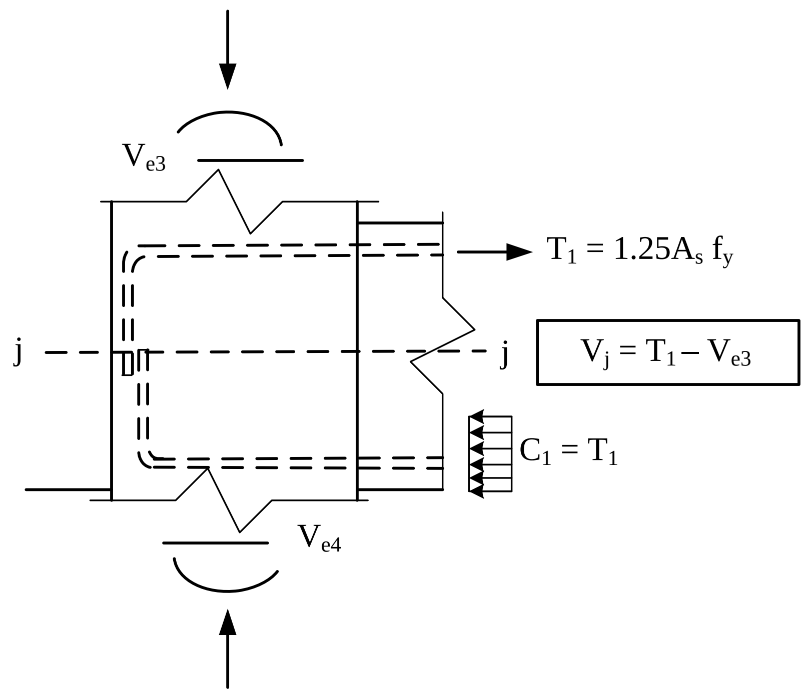


Sidesway to Left

Seismic 9 – Joint shear, V_j in an exterior beam-column joint

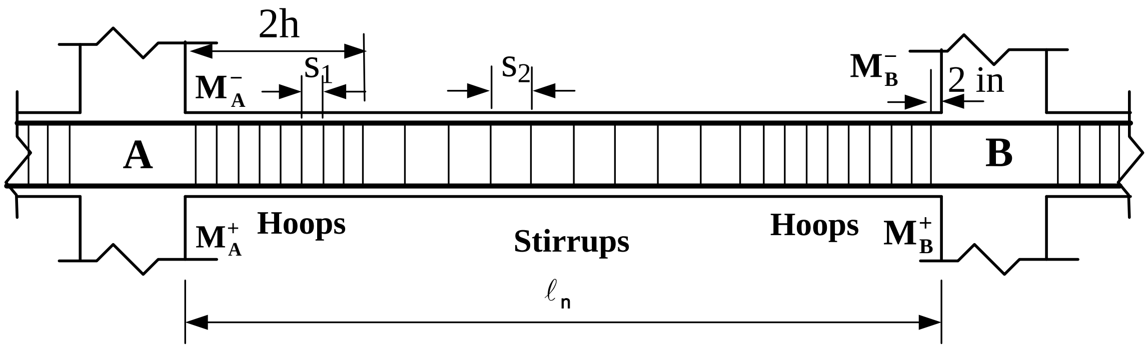


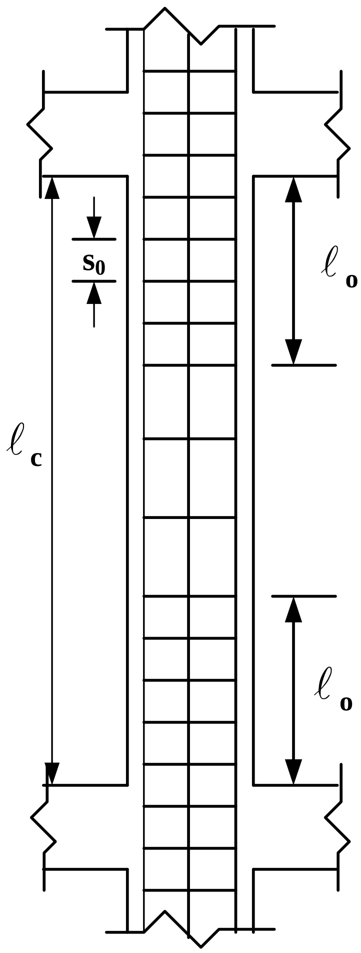
Sidesway to Right



Sidesway to Left

Seismic 10 – Requirements for flexural members and members subjected to bending and axial load of intermediate moment frames





The spacing of transverse reinforcement in columns, outside ℓ_o will be designed following the requirements of Sec. 7.10 and 11.5.5.1 of ACI 318-05, as for ordinary building columns.

Minimum flexural capacities;

- $\mathbf{M_A^+ \geq \frac{1}{3} M_A^-}$
- $\mathbf{M_B^+ \geq \frac{1}{3} M_B^-}$
- **If the largest end moment is $\mathbf{M_{max}}$; at any section along the beam:**

$$\mathbf{M^+ \geq \frac{1}{5} M_{max}}$$

$$\mathbf{M^- \geq \frac{1}{5} M_{max}}$$

Hoop/stirrup spacing;

$s_1 \leq d/4$ $\quad$ $s_1 \leq 8(d_b)_{smallest\ long.bar}$

$s_1 \leq 12\,in$ $\quad$ $s_1 \leq 24(d_b)_{hoop}$

$s_2 \leq d/2$ $\quad$ $s_0 \leq 24(d_b)_{hoop}$

$s_0 \leq 8(d_b)_{long.bar}$ $\quad$ $s_0 \leq 12\,in$

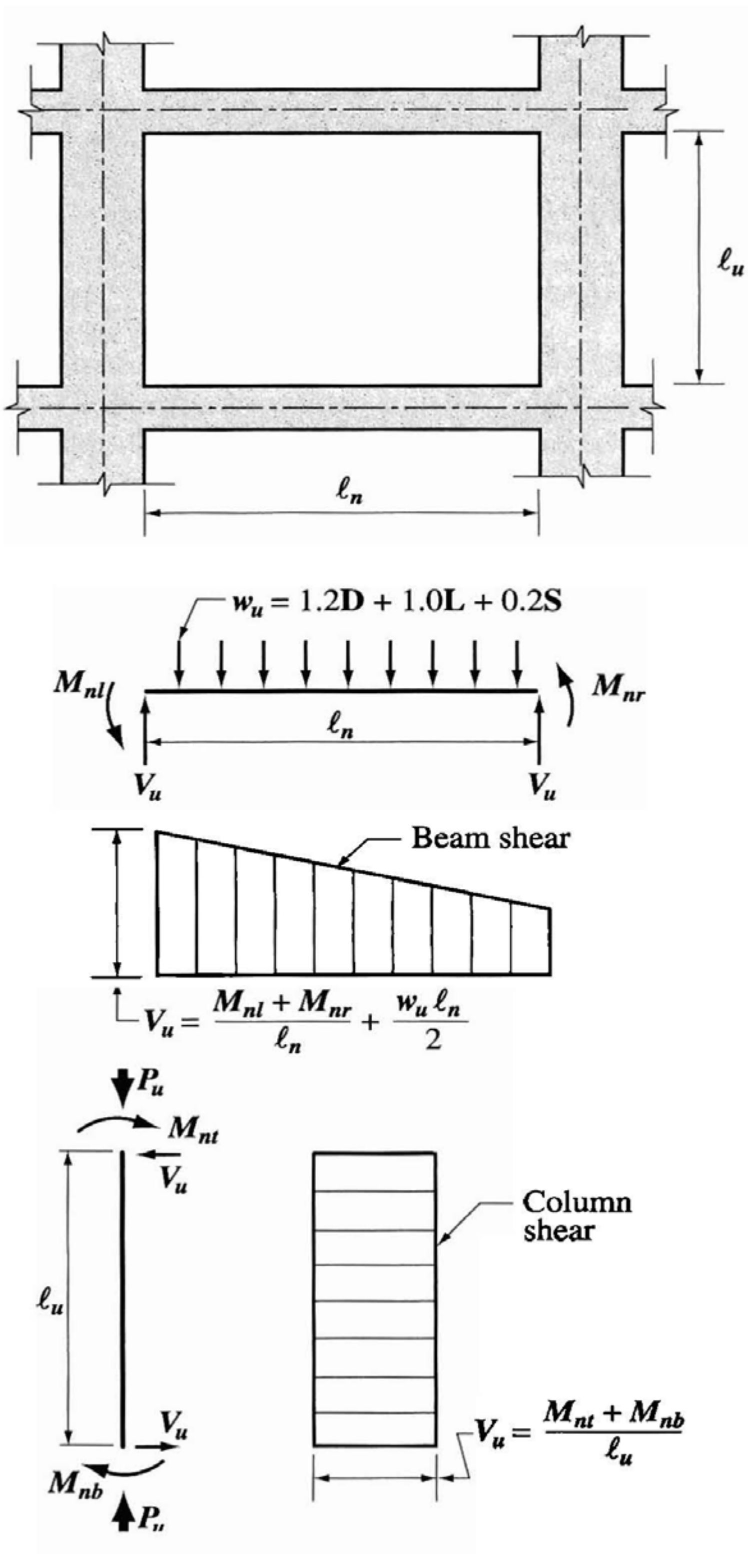
$\mathbf{s_0 \leq}$ ½ (smallest dimension of column section)

Potential plastic hinge length;

$\boldsymbol{\ell_o \geq \ell_c / 6}$ $\quad$ $\boldsymbol{\ell_o \geq 18\,in}$

$\boldsymbol{\ell_0 \geq}$ maximum dimension of column section

Seismic 11 – Shear strength for flexural members and members subjected to bending and axial load of intermediate frames



Appendix A – Properties of ASTM standard reinforcing bars

BAR SIZE DESIGNATION	NOMINAL CROSS SECTION AREA (sq. in.)	WEIGHT (lb/ft)	NOMINAL DIAMETER (in.)	NOMINAL PERIMETER (in.)
#3	0.11	0.376	0.375	1.18
#4	0.20	0.668	0.500	1.57
#5	0.31	1.043	0.625	1.96
#6	0.44	1.502	0.750	2.36
#7	0.60	2.044	0.875	2.75
#8	0.79	2.670	1.000	3.14
#9	1.00	3.400	1.128	3.54
#10	1.27	4.303	1.270	3.99
#11	1.56	5.313	1.410	4.43
#14	2.25	7.650	1.693	5.32
#18	4.00	13.600	2.257	7.09

Note: The nominal dimensions of a deformed bar are equivalent to those of a plain bar having the same mass per foot as the deformed bars.

Appendix – B Analysis Tables

Table B-1 Beam Diagrams

Simple Beam — uniformly distributed load

$R = V$ .... $= \dfrac{w\ell}{2}$

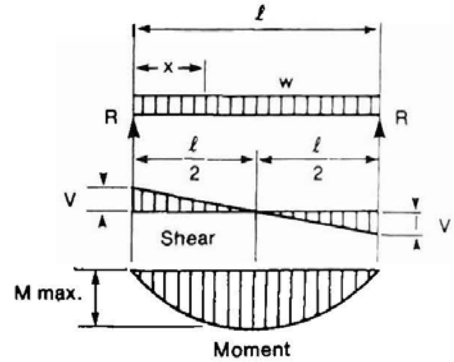
V_x .... $= w\left(\dfrac{\ell}{2} - x\right)$

M max. (at centre) .... $= \dfrac{w\ell^2}{8}$

M_x .... $= \dfrac{wx}{2}(\ell - x)$

Δmax. (at centre) .... $= \dfrac{5\,w\ell^4}{384\,EI}$

Δ_x .... $= \dfrac{wx}{24\,EI}(\ell^3 - 2\ell x^2 + x^3)$



Simple Beam — load increasing uniformly to one end

$R_1 = V_1$ .... $= \dfrac{w\ell}{6}$

$R_2 = V_2$ .... $= \dfrac{w\ell}{3}$

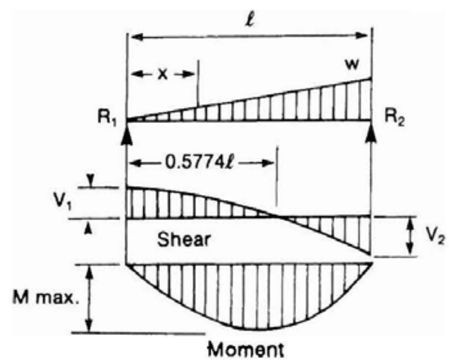
V_x .... $= \dfrac{w\ell}{3} - \dfrac{wx^2}{2\ell}$

M max. $\left(\text{at } x = \dfrac{\ell}{\sqrt{3}} = 0.5774\ell\right)$ .... $= \dfrac{w\ell^2}{9\sqrt{3}} = 0.06415w\ell^2$

M_x .... $= \dfrac{wx}{6\ell}(\ell^2 - x^2)$

Δmax. $\left(\text{at } x = \ell\sqrt{1 - \sqrt{\dfrac{8}{15}}} = 0.5193\ell\right)$ ...... $0.00652\,\dfrac{w\ell^4}{EI}$

Δ_x .... $= \dfrac{wx}{360\,EI\ell}(3x^4 - 10\ell^2x^2 + 7\ell^4)$



Simple Beam — load increasing uniformly to centre

$R = V$ .... $= \dfrac{w\ell}{4}$

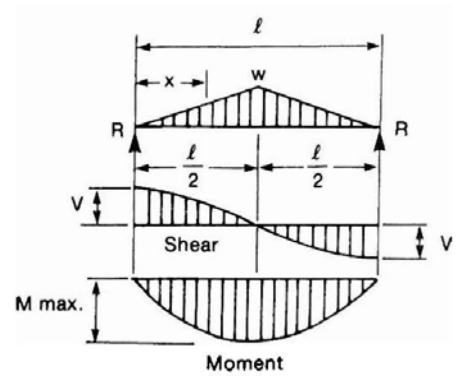
$V_x \left(\text{when } x < \dfrac{\ell}{2}\right)$ .... $= \dfrac{w}{4\ell}(\ell^2 - 4x^2)$

M max. (at centre) .... $= \dfrac{w\ell^2}{12}$

$M_x \left(\text{when } x < \dfrac{\ell}{2}\right)$ .... $= \dfrac{w\ell x}{2}\left(\dfrac{1}{2} - \dfrac{2x^2}{3\ell^2}\right)$

Δmax. (at centre) .... $= \dfrac{w\ell^4}{120\,EI}$

Δ_x .... $= \dfrac{wx}{960\,EI\,\ell}(5\ell^2 - 4x^2)^2$



Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.

P: Concentrated load.

Table B-1 Beam Diagrams (Cont'd)

Simple Beam — uniform load partially distributed

$R_1 = V_1$ (max. when $a < c$) .... $= \frac{wb}{2\ell}(2c + b)$

$R_2 = V_2$ (max. when $a > c$) .... $= \frac{wb}{2\ell}(2a + b)$

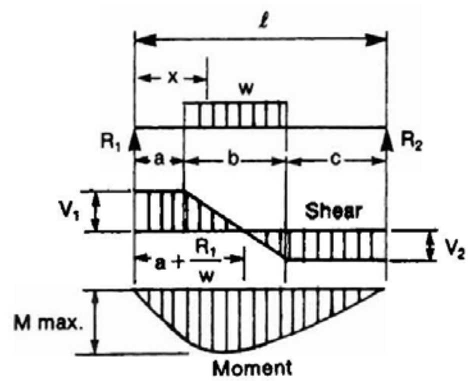
V_x (when $x > a$ and $< (a + b)$) .... $= R_1 - w(x - a)$

M max. $\left(\text{at } x = a + \frac{R_1}{w}\right)$ .... $= R_1\left(a + \frac{R_1}{2w}\right)$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $x > a$ and $< (a + b)$) .... $= R_1 x - \frac{w}{2}(x - a)^2$

M_x (when $x > (a + b)$) .... $= R_2(\ell - x)$



Beam fixed at both ends — symmetrical trapezoidal load

$R = V$ .... $= \frac{w\ell}{2}\left(1 - \frac{a}{\ell}\right)$

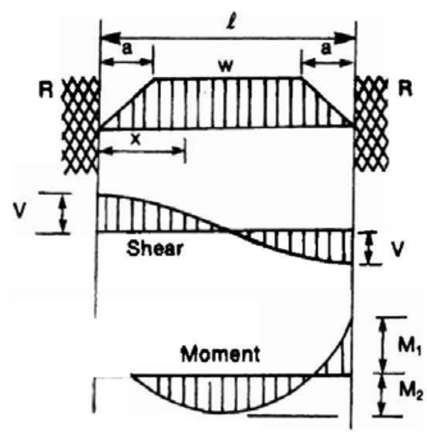
M_1 .... $= -\frac{w\ell^2}{12}\left(1 - 2\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3}\right)$

M_2 .... $= \frac{w\ell^2}{24}\left(1 - 2\frac{a^3}{\ell^3}\right)$

M_x (when $x < a$) .... $= M_1 + R_1 x - \frac{wx^3}{6a}$

M_x (when $a < x < \ell - a$) .... $= M_1 + R_1 x - \frac{wa}{2}\left(x - \frac{2}{3}a\right)$
$- \frac{w}{2}(x - a)^2$

Note: When $a = \ell/2$ loading is triangular



Simple Beam — concentrated load at any point

$R_1 = V_1$ (max. when $a < b$) .... $= \frac{Pb}{\ell}$

$R_2 = V_2$ (max. when $a > b$) .... $= \frac{Pa}{\ell}$

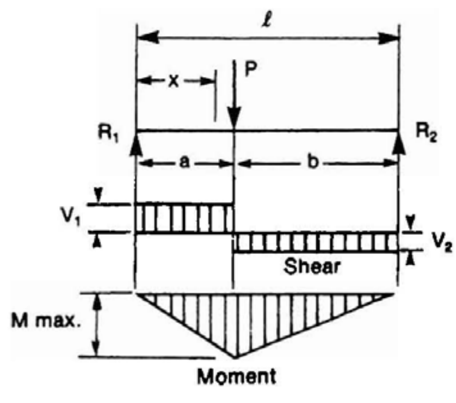
M max. (at point of load) .... $= \frac{Pab}{\ell}$

M_x (when $x < a$) .... $= \frac{Pbx}{\ell}$

Δmax. $\left(\text{at } x = \sqrt{\frac{a(a + 2b)}{3}} \text{ when } a > b\right)$ $= \frac{Pab(a + 2b)\sqrt{3a(a + 2b)}}{27\,EI\ell}$

Δa (at point of load) .... $= \frac{Pa^2b^2}{3\,EI\ell}$

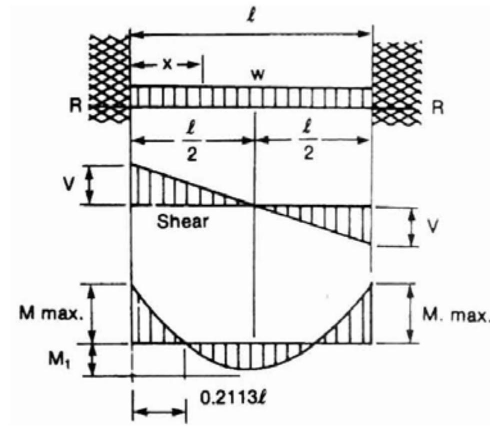
Δ_x (when $x < a$) .... $= \frac{Pbx}{6\,EI\ell}(\ell^2 - b^2 - x^2)$



Note:
w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.
P: Concentrated load.

Table B-1 Beam Diagrams (Cont'd)

Beam fixed at both ends — uniformly distributed loads



$R = V$ .... $= \frac{w\ell}{2}$

V_x .... $= w\left(\frac{\ell}{2} - x\right)$

M max. (at ends) .... $= -\frac{w\ell^2}{12}$

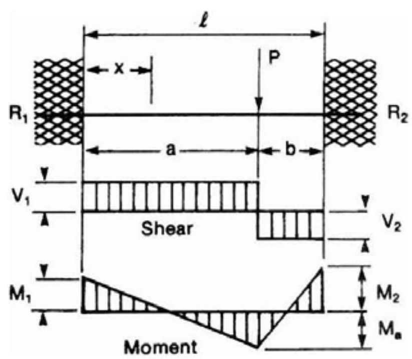
M_1 (at centre) .... $= \frac{w\ell^2}{24}$

M_x .... $= \frac{w}{12}(6\ell x - \ell^2 - 6x^2)$

Δmax. (at centre) .... $= \frac{w\ell^4}{384\,EI}$

Δ_x .... $= \frac{wx^2}{24\,EI}(\ell - x)^2$

Beam fixed at both ends — concentrated load at any point



$R_1 = V_1$ (max. when $a < b$) .... $= \frac{Pb^2}{\ell^3}(3a + b)$

$R_2 = V_2$ (max. when $a > b$) .... $= \frac{Pa^2}{\ell^3}(a + 3b)$

M_1 (max. when $a < b$) .... $= -\frac{Pab^2}{\ell^2}$

M_2 (max. when $a > b$) .... $= -\frac{Pa^2b}{\ell^2}$

M_a (at point of load) .... $= \frac{2Pa^2b^2}{\ell^3}$

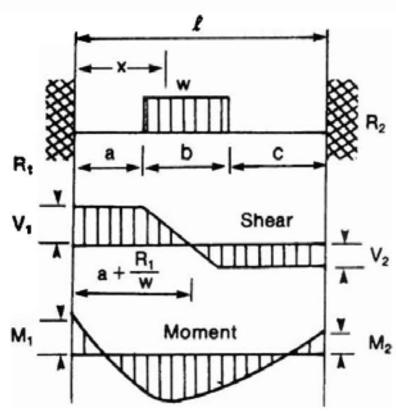
M_x (when $x < a$) .... $= R_1x - \frac{Pab^2}{\ell^2}$

Δmax. $\left(\text{when } a > b \text{ at } x = \frac{2a\ell}{3a + b}\right)$ .... $= \frac{2Pa^3b^2}{3\,EI\,(3a + b)^2}$

Δ_a (at point of load) .... $= \frac{Pa^3b^3}{3\,EI\ell^3}$

Δ_x (when $x < a$) .... $= \frac{Pb^2x^2}{6\,EI\ell^3}(3a\ell - 3ax - bx)$

Beam fixed at both ends — uniform load partially distributed



M_1 .... $= \frac{w}{12\ell^2}\left[(\ell - a)^3(\ell + 3a) - c^3(4\ell - 3c)\right]$

M_2 .... $= \frac{w}{12\ell^2}\left[(\ell - c)^3(\ell + 3c) - a^3(4\ell - 3a)\right]$

$R_1 = V_1$ .... $= \frac{1}{\ell}\left[M_1 - M_2 + wc\left(c + \frac{b}{2}\right)\right]$

$R_2 = V_2$ .... $= \frac{1}{\ell}\left[M_2 - M_1 + wc\left(a + \frac{b}{2}\right)\right]$

M_x (when $x < a$) .... $= R_1x - M_1$

M_x (when $a < x < (a + c)$) .... $= R_1x - M_1 - \frac{w}{2}(x - a)^2$

M_x (when $x > (a + c)$) .... $= R_2(\ell - x) - M_2$

Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.

P: Concentrated load.

Table B-1 Beam Diagrams (Cont'd)

Beam fixed at both ends — load increasing uniformly to one end

$R_1 = V_1$ .... $= \dfrac{3w\ell}{20}$

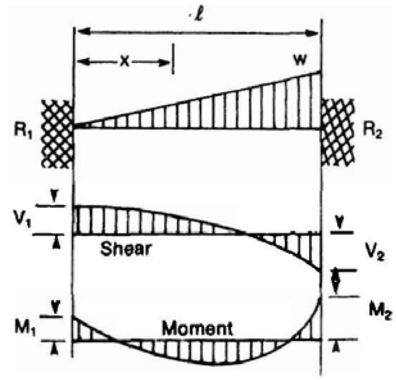
$R_2 = V_2$ .... $= \dfrac{7w\ell}{20}$

V_x .... $= \dfrac{3w\ell}{20} - \dfrac{wx^2}{2\ell}$

M_1 .... $= -\dfrac{w\ell^2}{30}$

M_2 .... $= -\dfrac{w\ell^2}{20}$

M_x .... $= \dfrac{3w\ell x}{20} - \dfrac{w\ell^2}{30} - \dfrac{wx^3}{6\ell}$



Beam fixed at one end, supported at other — uniformly distributed load

$R_1 = V_1$ .... $= \dfrac{3w\ell}{8}$

$R_2 = V_2$ max. .... $= \dfrac{5w\ell}{8}$

V_x .... $= R_1 - wx$

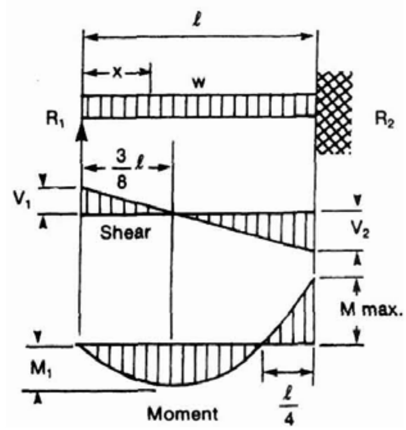
M max. .... $= -\dfrac{w\ell^2}{8}$

$M_1 \left(\text{at } x = \dfrac{3}{8}\ell\right)$ .... $= \dfrac{9}{128} w\ell^2$

M_x .... $= R_1 x - \dfrac{wx^2}{2}$

Δmax. $\left(\text{at } x = \dfrac{\ell}{16}(1 + \sqrt{33}) = 0.4215\ell\right)$. $= \dfrac{w\ell^4}{185\,EI}$

Δ_x .... $= \dfrac{wx}{48\,EI}(\ell^3 - 3\ell x^2 + 2x^3)$



Beam fixed at one end, supported at other — concentrated load at any point

$R_1 = V_1$ .... $= \dfrac{Pb^2}{2\ell^3}(a + 2\ell)$

$R_2 = V_2$ .... $= \dfrac{Pa}{2\ell^3}(3\ell^2 - a^2)$

M_1 (at point of load) .... $= R_1 a$

M_2 (at fixed end) .... $= -\dfrac{Pab}{2\ell^2}(a + \ell)$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $x > a$) .... $= R_1 x - P(x - a)$

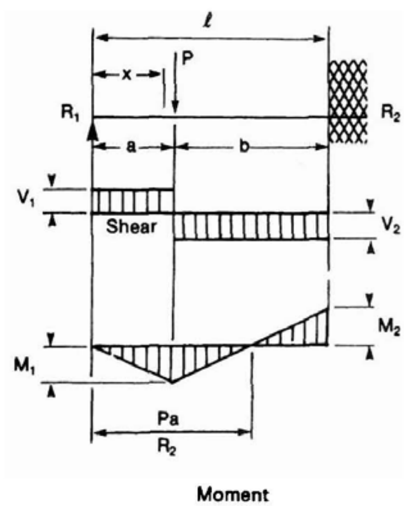
Δmax. $\left(\text{when } a < 0.414\ell \text{ at } x = \ell\,\dfrac{\ell^2 + a^2}{3\ell^2 - a^2}\right) = \dfrac{Pa}{3\,EI}\,\dfrac{(\ell^2 - a^2)^3}{(3\ell^2 - a^2)^2}$

Δmax. $\left(\text{when } a > 0.414\ell \text{ at } x = \ell\sqrt{\dfrac{a}{2\ell + a}}\right) = \dfrac{Pab^2}{6\,EI}\sqrt{\dfrac{a}{2\ell + a}}$

Δa (at point of load) .... $= \dfrac{Pa^2b^3}{12\,EI\ell^3}(3\ell + a)$

Δx (when $x < a$) .... $= \dfrac{Pb^2x}{12\,EI\ell^3}(3a\ell^2 - 2\ell x^2 - ax^2)$

Δx (when $x > a$) .... $= \dfrac{Pa}{12\,EI\ell^3}(\ell - x)^2(3\ell^2 x - a^2 x - 2a^2\ell)$



Note:
w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.
P: Concentrated load.
Moments are positive if they cause compression in the top of the beam.

Table B-1 Beam Diagrams (Cont'd)

Cantilever Beam — uniformly distributed load

$R = V$ .... $= w\ell$

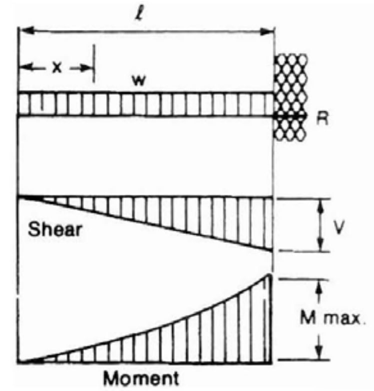
V_x .... $= wx$

M max. (at fixed end) .... $= -\dfrac{w\ell^2}{2}$

M_x .... $= \dfrac{wx^2}{2}$

Δmax. (at free end) .... $= \dfrac{w\ell^4}{8\,EI}$

Δ_x .... $= \dfrac{w}{24\,EI}(x^4 - 4\ell^3 x + 3\ell^4)$



Cantilever Beam — load increasing uniformly to fixed end

$R = V$ .... $= \dfrac{w\ell}{2}$

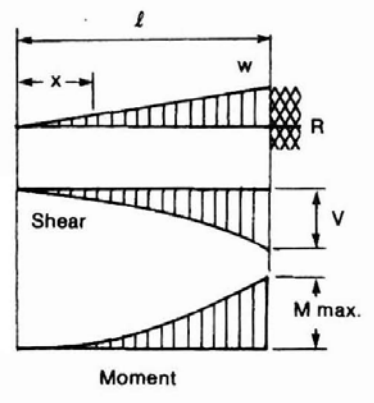
V_x .... $= \dfrac{wx^2}{2\ell}$

M max. (at fixed end) .... $= -\dfrac{w\ell^2}{6}$

M_x .... $= \dfrac{wx^3}{6\ell}$

Δmax. (at free end) .... $= \dfrac{w\ell^4}{30\,EI}$

Δ_x .... $= \dfrac{w}{120\,EI\ell}(x^5 - 5\ell^4 x + 4\ell^5)$



Cantilever Beam — concentrated load at any point

$R = V$ .... $= P$

M max. (at fixed end) .... $= -Pb$

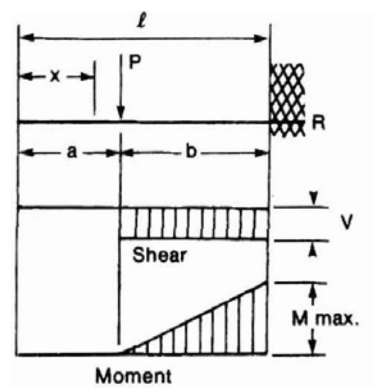
M_x (when $x > a$) .... $= P(x - a)$

Δmax. (at free end) .... $= \dfrac{Pb^2}{6\,EI}(3\ell - b)$

Δa (at point of load) .... $= \dfrac{Pb^3}{3\,EI}$

Δ_x (when $x < a$) .... $= \dfrac{Pb^2}{6\,EI}(3\ell - 3x - b)$

Δ_x (when $x > a$) .... $= \dfrac{P(\ell - x)^2}{6\,EI}(3b - \ell + x)$



Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.

P: Concentrated load.

Moments are positive if they cause compression in the top of the beam.

Table B-1 Beam Diagrams (Cont'd)

Beam overhanging one support — uniformly distributed load

$R_1 = V_1 \dots = \frac{w}{2\ell}(\ell^2 - a^2)$

$R_2 = V_2 + V_3 \dots = \frac{w}{2\ell}(\ell + a)^2$

$V_2 \dots = wa$

$V_3 \dots = \frac{w}{2\ell}(\ell^2 + a^2)$

V_x (between supports) $\dots = R_1 - wx$

V_{x1} (for overhang) $\dots = w(a - x_1)$

$M_1 \left(\text{at } x = \frac{\ell}{2}\left[1 - \frac{a^2}{\ell^2}\right]\right) \dots = \frac{w}{8\ell^2}(\ell + a)^2(\ell - a)^2$

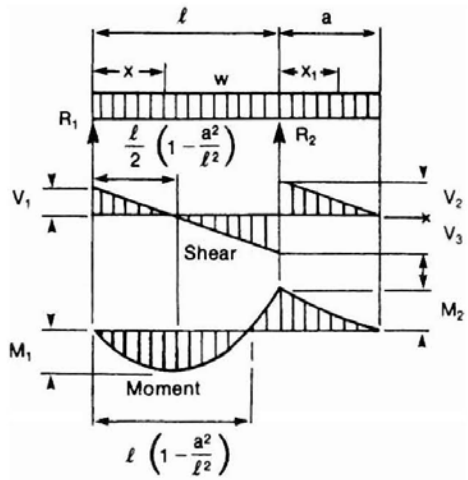
M_2 (at R_2) $\dots = \frac{wa^2}{2}$

M_x (between supports) $\dots = \frac{wx}{2\ell}(\ell^2 - a^2 - x\ell)$

M_{x1} (for overhang) $\dots = \frac{w}{2}(a - x_1)^2$

Δ_x (between supports) $\dots = \frac{wx}{24 EI\ell}(\ell^4 - 2\ell^2x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2x^2)$

Δ_{x1} (for overhang) $\dots = \frac{wx_1}{24 EI}(4a^2\ell - \ell^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$



Beam overhanging one support — uniformly distributed load on overhang

$R_1 = V_1 \dots = \frac{wa^2}{2\ell}$

$R_2 = V_1 + V_2 \dots = \frac{wa}{2\ell}(2\ell + a)$

$V_2 \dots = wa$

V_{x1} (for overhang) $\dots = w(a - x_1)$

M max. (at R_2) $\dots = \frac{wa^2}{2}$

M_x (between supports) $\dots = \frac{wa^2x}{2\ell}$

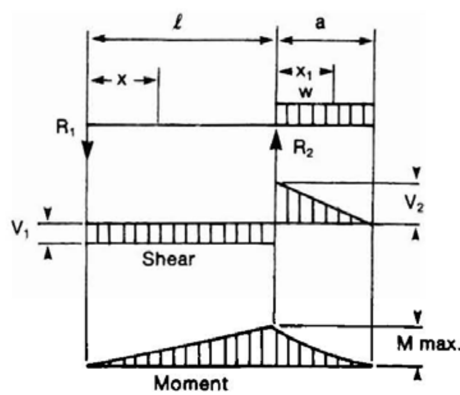
M_{x1} (for overhang) $\dots = \frac{w}{2}(a - x_1)^2$

Δmax. $\left(\text{between supports at } x = \frac{\ell}{\sqrt{3}}\right) \dots = \frac{wa^2\ell^2}{18\sqrt{3}\,EI} = 0.03208 \frac{wa^2\ell^2}{EI}$

Δmax. (for overhang at $x_1 = a$) $\dots = \frac{wa^3}{24 EI}(4\ell + 3a)$

Δ_x (between supports) $\dots = \frac{wa^2x}{12 EI\ell}(\ell^2 - x^2)$

Δ_{x1} (for overhang) $\dots = \frac{wx_1}{24 EI}(4a^2\ell + 6a^2x_1 - 4ax_1^2 + x_1^3)$



Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length

P: Concentrated load.

Moments are positive if they cause compression in the top of the beam.

Table B-1 Beam Diagrams (Cont'd)

Beam overhanging one support — concentrated load at end of overhang

$R_1 = V_1$ .... $= \frac{Pa}{\ell}$

$R_2 = V_1 + V_2$ .... $= \frac{P}{\ell}(\ell + a)$

V_2 .... $= P$

M max. (at R_2) .... $= Pa$

M_x (between supports) .... $= \frac{Pax}{\ell}$

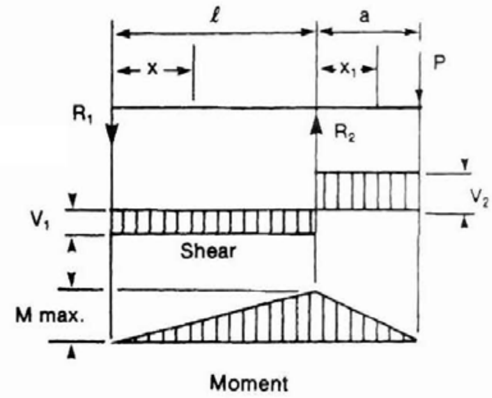
M_{x_1} (for overhang) .... $= P(a - x_1)$

Δmax. $\left(\text{between supports at } x = \frac{\ell}{\sqrt{3}}\right)$ ... $= \frac{Pa\ell^2}{9\sqrt{3}\,EI} = 0.06415\frac{Pa\ell^2}{EI}$

Δmax. (for overhang at $x_1 = a$) .... $= \frac{Pa^2}{3\,EI}(\ell + a)$

Δ_x (between supports) .... $= \frac{Pax}{6\,EI\ell}(\ell^2 - x^2)$

Δ_{x_1} (for overhang) .... $= \frac{Px_1}{6\,EI}(2a\ell + 3ax_1 - x_1^2)$



Note:

w: Distributed load per unit length. In the case of triangular distribution, w represents the maximum intensity of load per unit length.

P: Concentrated load.

Moments are positive if they cause compression in the top of the beam.

Table B-2 Moments and Reactions in Continuous Beams under Uniformly Distributed Loads

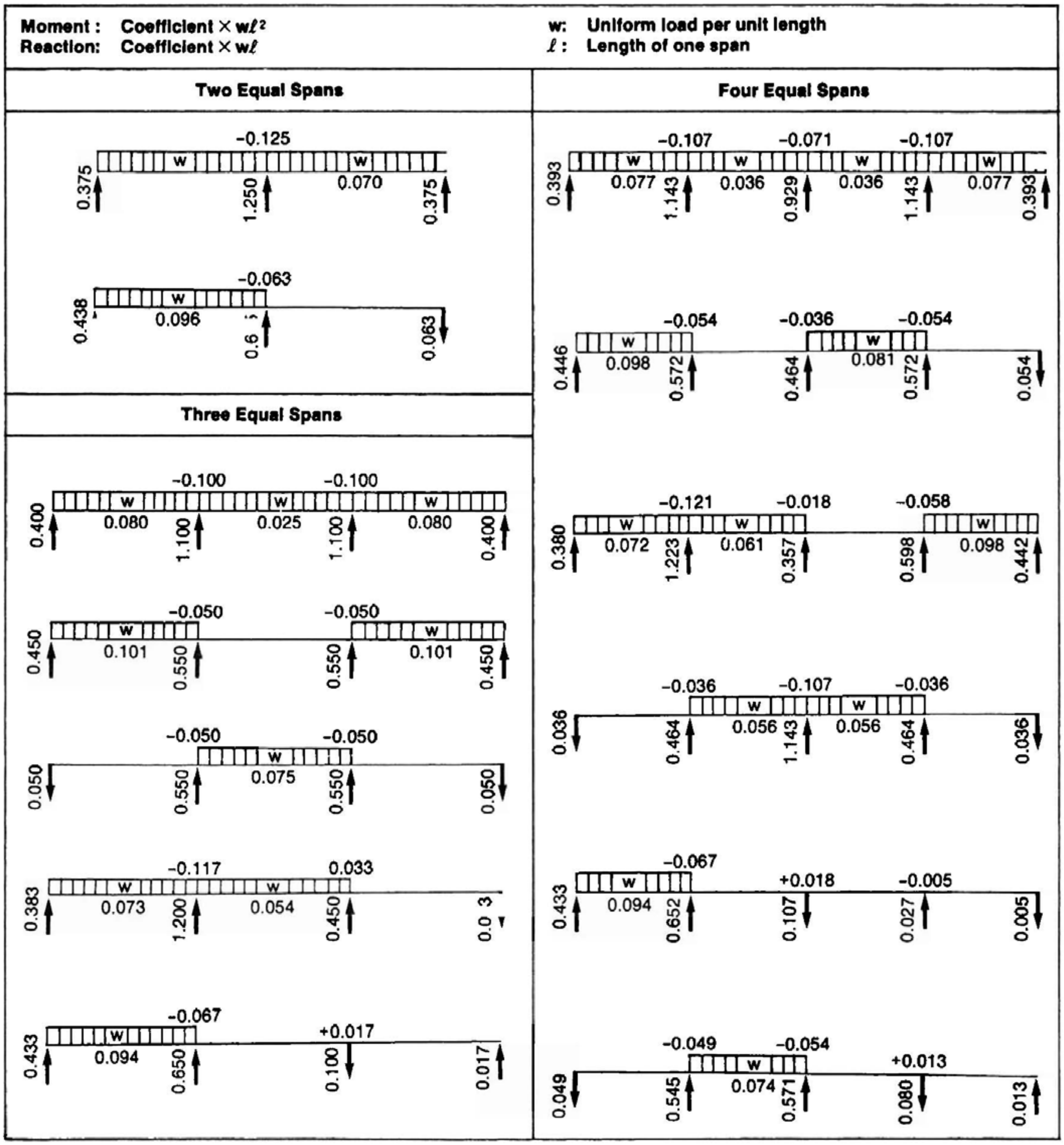


Table B-3 Moments and Reactions in Continuous Beams under Central Point Loads

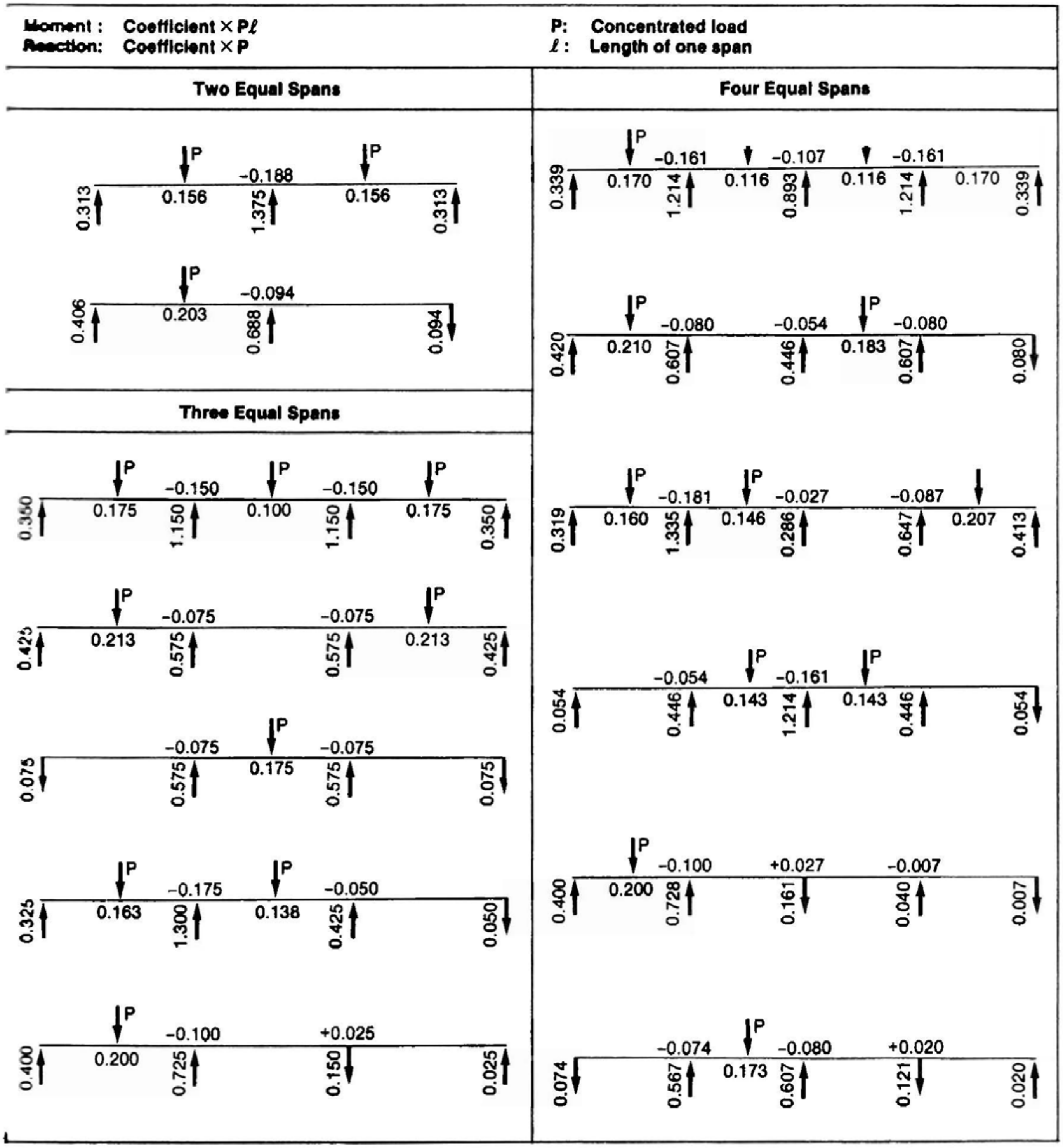


Table B-4 Moments and Reactions in Continuous Beams, Point Loads at Third Points of Span

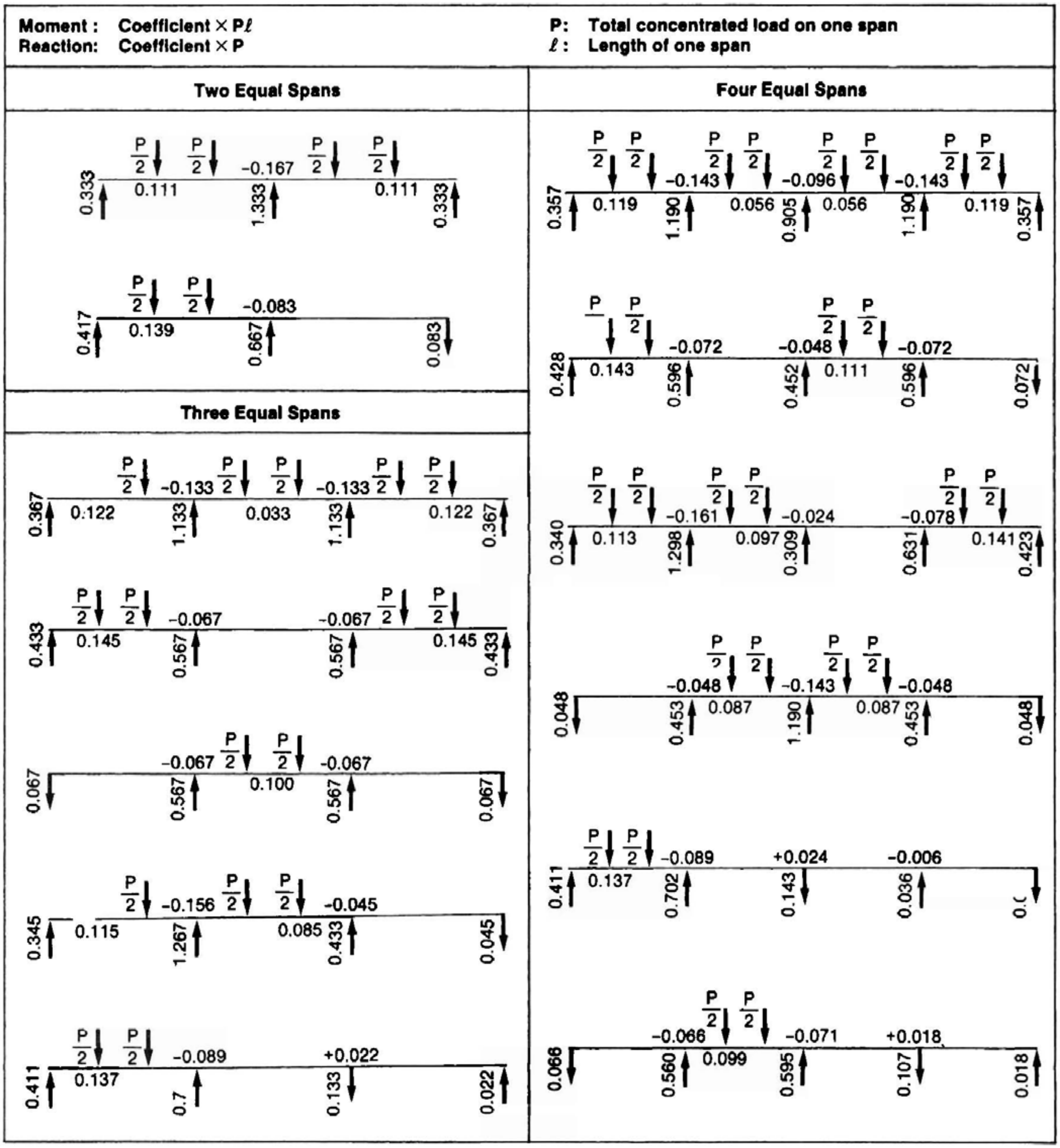
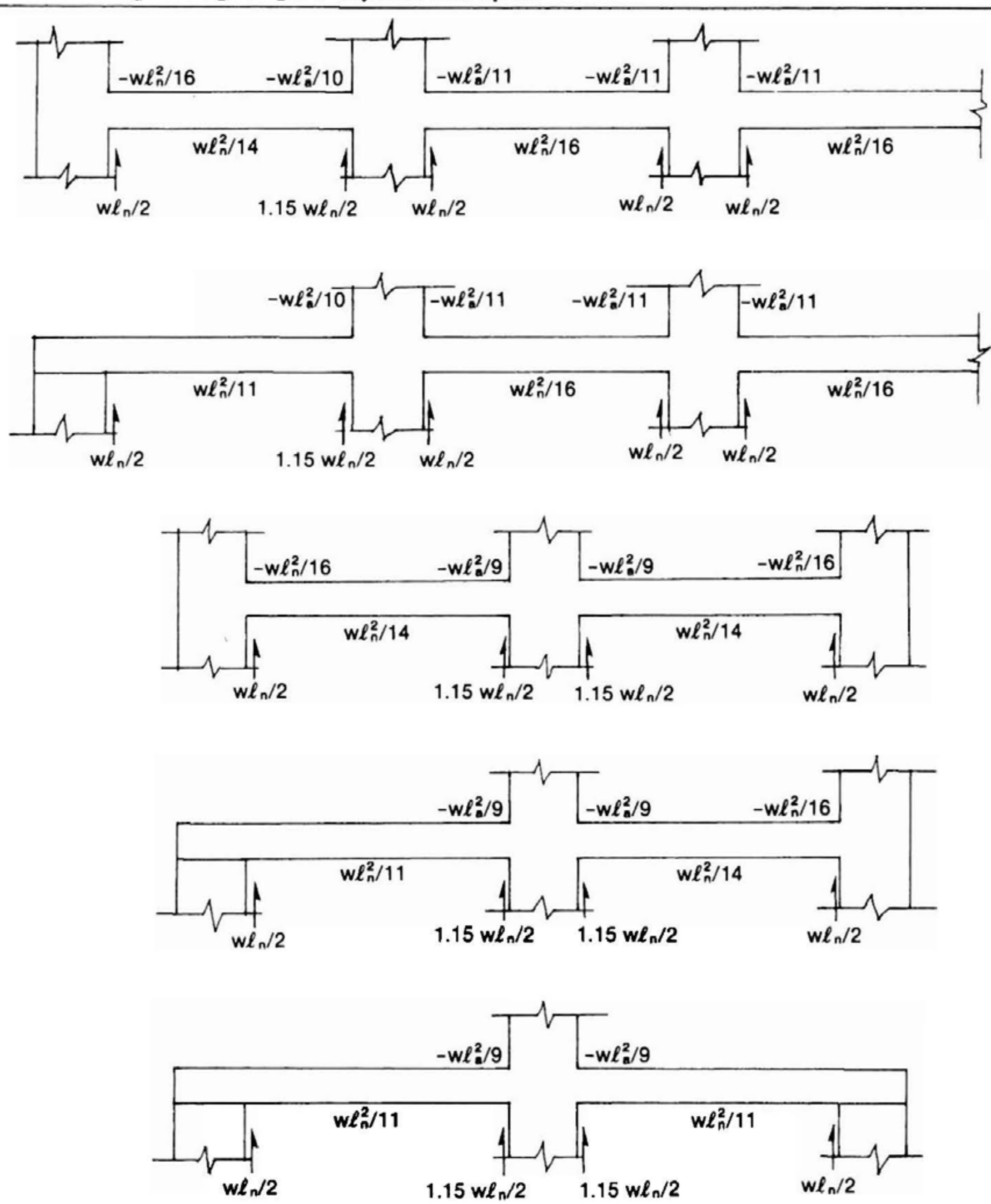


Table B-5 Approximate Moments and Shears for Continuous Beams and One-way Slabs

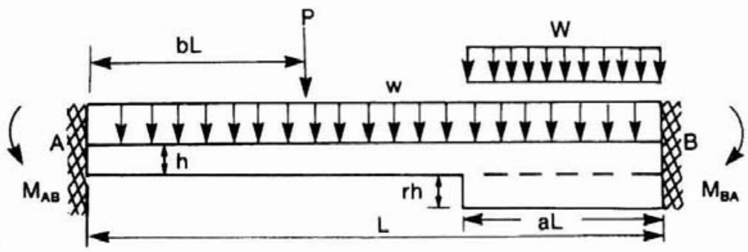
w: Factored load per unit length
ℓ_n: Clear span length
ℓ_a: Average length of adjacent clear spans



Notes:

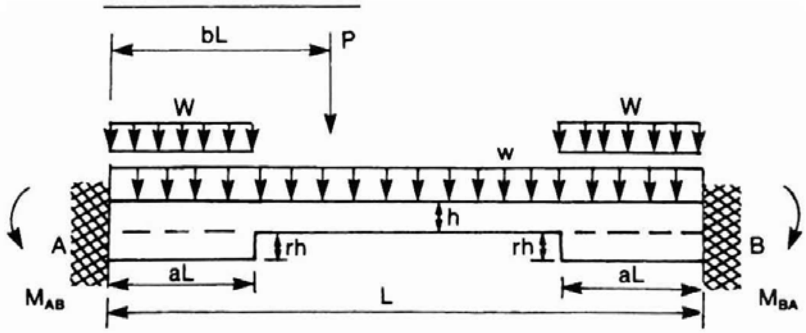
1. This figure is applicable to prismatic members loaded with uniformly distributed load where the ratio of factored live load to factored dead load is not greater than 3.0, and the span lengths approximately equal, with the longer of the two adjacent spans not greater than the shorter by more than 20 per cent.
2. If the exterior support is a spandrel or girder, the negative moment at the interior face of exterior support is $w\ell_n^2/24$.
3. For slabs with spans not exceeding 3 m or beams with the ratio of the sum of the column stiffnesses to the beam stiffness exceeds eight at each end, the negative moment at all supports can be taken equal to $w\ell_n^2/12$.

Table B-6 Beams with Prismatic Haunch at One End



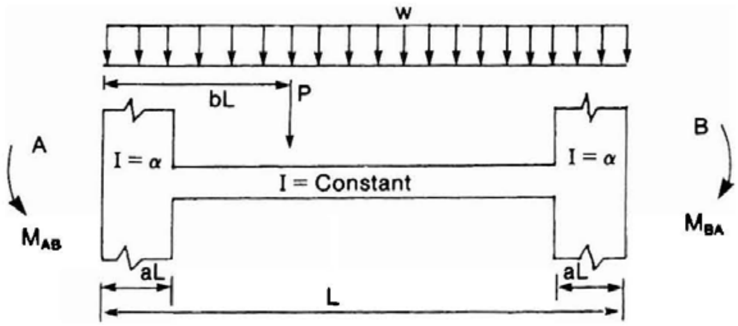
Right haunch		Carry-over factors		Stiffness factors		Unif. load F.E.M. coef. × wL^2		Concentrated load F.E.M. — coef. × PL										Haunch load F.E.M. coef. × WL^2	
								b											
								0.1		0.3		0.5		0.7		0.9			
a	r	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
0.1	0.4	0.593	0.491	4.24	5.12	0.0749	0.1016	0.0799	0.0113	0.1397	0.0788	0.1110	0.1553	0.0478	0.1798	0.0042	0.0911	0.0001	0.0047
	0.6	0.615	0.490	4.30	5.40	0.0727	0.1062	0.0797	0.0119	0.1378	0.0828	0.1074	0.1630	0.0439	0.1881	0.0029	0.0937	0.0001	0.0048
	1.0	0.639	0.488	4.37	5.72	0.0703	0.1114	0.0794	0.0125	0.1358	0.0873	0.1035	0.1716	0.0396	0.1974	0.0016	0.0966	0.0001	0.0049
	1.5	0.652	0.487	4.40	5.89	0.0690	0.1143	0.0792	0.0129	0.1346	0.0898	0.1012	0.1764	0.0373	0.2026	0.0008	0.0982	0.0000	0.0049
	2.0	0.658	0.487	4.42	5.97	0.0684	0.1156	0.0791	0.0131	0.1341	0.0910	0.1002	0.1786	0.0361	0.2050	0.0005	0.0990	0.0000	0.0050
0.2	0.4	0.677	0.469	4.42	6.37	0.0706	0.1126	0.0791	0.0134	0.1345	0.0925	0.1020	0.1788	0.0409	0.1975	0.0050	0.0890	0.0013	0.0171
	0.6	0.730	0.463	4.56	7.18	0.0664	0.1225	0.0785	0.0149	0.1302	0.1025	0.0942	0.1972	0.0335	0.2148	0.0037	0.0917	0.0010	0.0178
	1.0	0.793	0.458	4.74	8.22	0.0610	0.1353	0.0777	0.0168	0.1248	0.1154	0.0843	0.2207	0.0242	0.2368	0.0022	0.0951	0.0006	0.0187
	1.5	0.831	0.455	4.86	8.88	0.0576	0.1434	0.0772	0.0180	0.1214	0.1235	0.0781	0.2355	0.0182	0.2507	0.0012	0.0973	0.0003	0.0193
	2.0	0.849	0.453	4.91	9.20	0.0559	0.1473	0.0769	0.0186	0.1197	0.1276	0.0750	0.2429	0.0153	0.2576	0.0007	0.0984	0.0002	0.0196
0.3	0.4	0.741	0.439	4.52	7.63	0.0698	0.1155	0.0787	0.0149	0.1319	0.1013	0.0987	0.1899	0.0420	0.1929	0.0056	0.0868	0.0045	0.0338
	0.6	0.831	0.427	4.75	9.24	0.0642	0.1296	0.0777	0.0175	0.1255	0.1182	0.0877	0.2185	0.0338	0.2130	0.0045	0.0893	0.0036	0.0359
	1.0	0.954	0.415	5.09	11.69	0.0559	0.1511	0.0762	0.0215	0.1158	0.1440	0.0711	0.2621	0.0217	0.2436	0.0028	0.0930	0.0023	0.0391
	1.5	1.036	0.409	5.34	13.53	0.0497	0.1673	0.0751	0.0245	0.1085	0.1633	0.0587	0.2948	0.0128	0.2665	0.0017	0.0959	0.0014	0.0415
	2.0	1.078	0.407	5.48	14.54	0.0464	0.1762	0.0745	0.0262	0.1045	0.1740	0.0520	0.3129	0.0080	0.2792	0.0010	0.0974	0.0008	0.0448
0.4	0.4	0.774	0.405	4.55	8.70	0.0703	0.1117	0.0786	0.0156	0.1315	0.1035	0.0992	0.1855	0.0445	0.1773	0.0059	0.0849	0.0106	0.0509
	0.6	0.901	0.86	4.83	11.28	0.0646	0.1269	0.0774	0.0192	0.1240	0.1254	0.0875	0.2182	0.0377	0.1932	0.0049	0.0869	0.0089	0.0547
	1.0	1.102	0.367	5.33	16.03	0.0549	0.1548	0.0752	0.0257	0.1105	0.1658	0.0671	0.2780	0.0267	0.2222	0.0034	0.0904	0.0063	0.0616
	1.5	1.260	0.357	5.79	20.46	0.0462	0.1807	0.0732	0.0319	0.0982	0.2035	0.0485	0.3339	0.0173	0.2491	0.0022	0.0938	0.0037	0.0679
	2.0	1.349	0.352	6.09	23.32	0.0407	0.1975	0.0719	0.0358	0.0903	0.2278	0.0367	0.3699	0.0113	0.2664	0.0014	0.0959	0.0027	0.0720
0.5	0.4	0.768	0.371	4.56	9.45	0.0700	0.1048	0.0786	0.0154	0.1312	0.0993	0.0983	0.1679	0.0442	0.1663	0.0059	0.0836	0.0189	0.0656
	0.6	0.919	0.343	4.84	12.94	0.0651	0.1176	0.0774	0.0193	0.1240	0.1218	0.0884	0.1935	0.0386	0.1769	0.0051	0.0849	0.0167	0.0702
	1.0	1.200	0.316	5.42	20.61	0.0561	0.1451	0.0749	0.0280	0.1096	0.1709	0.0706	0.2486	0.0299	0.1993	0.0038	0.0877	0.0131	0.0802
	1.5	1.470	0.301	6.10	29.74	0.0466	0.1777	0.0720	0.0384	0.0934	0.2290	0.0516	0.3137	0.0215	0.2255	0.0027	0.0909	0.0094	0.0918
	2.0	1.647	0.295	6.63	37.04	0.0393	0.2036	0.0698	0.0466	0.0807	0.2755	0.0370	0.3655	0.0153	0.2463	0.0019	0.0934	0.0067	0.1011
0.6	0.4	0.726	0.341	4.62	9.84	0.0675	0.0986	0.0782	0.0146	0.1280	0.0916	0.0923	0.1519	0.0419	0.1603	0.0056	0.0829	0.0283	0.0769
	0.6	0.872	0.305	4.88	13.97	0.0630	0.1072	0.0771	0.0183	0.1214	0.1096	0.0835	0.1664	0.0368	0.1666	0.0048	0.0837	0.0254	0.0813
	1.0	1.196	0.267	5.43	24.35	0.0560	0.1277	0.0748	0.0274	0.1092	0.1537	0.0705	0.1999	0.0299	0.1804	0.0038	0.0854	0.0212	0.0913
	1.5	1.588	0.247	6.18	39.79	0.0482	0.1572	0.0718	0.0408	0.0939	0.2183	0.0572	0.2478	0.0237	0.1997	0.0030	0.0878	0.0171	0.1055
	2.0	1.905	0.237	6.92	55.51	0.0412	0.1870	0.0688	0.0544	0.0792	0.2839	0.0455	0.2960	0.0186	0.2189	0.0023	0.0901	0.0136	0.1197
0.7	0.4	0.657	0.321	4.86	9.96	0.0631	0.0954	0.0770	0.0138	0.1175	0.0846	0.0844	0.1461	0.0392	0.1582	0.0053	0.0827	0.0372	0.0854
	0.6	0.770	0.275	5.14	14.39	0.0580	0.1006	0.0758	0.0167	0.1097	0.0955	0.0745	0.1543	0.0335	0.1621	0.0045	0.0832	0.0330	0.0890
	1.0	1.056	0.224	5.62	26.45	0.0516	0.1122	0.0738	0.0243	0.0992	0.1213	0.0626	0.1710	0.0269	0.1694	0.0035	0.0841	0.0280	0.0965
	1.5	1.491	0.196	6.24	47.48	0.0463	0.1304	0.0714	0.0371	0.0890	0.1633	0.0537	0.1959	0.0223	0.1796	0.0028	0.0854	0.0241	0.1076
	2.0	1.944	0.183	6.95	73.85	0.0417	0.1523	0.0687	0.0530	0.0793	0.2149	0.0468	0.2255	0.0191	0.1915	0.0024	0.0869	0.0210	0.1210
0.8	0.4	0.583	0.319	5.46	9.97	0.0585	0.0951	0.0741	0.0137	0.1040	0.0837	0.0793	0.1456	0.0380	0.1580	0.0053	0.0826	0.0452	0.0917
	0.6	0.645	0.263	5.89	14.44	0.0516	0.0990	0.0721	0.0160	0.0921	0.0907	0.0667	0.1520	0.0311	0.1614	0.0043	0.0831	0.0388	0.0951
	1.0	0.818	0.196	6.47	27.06	0.0435	0.1053	0.0696	0.0211	0.0781	0.1025	0.0521	0.1615	0.0232	0.1660	0.0031	0.0838	0.0314	0.1004
	1.5	1.128	0.155	6.98	50.85	0.0385	0.1130	0.0676	0.0296	0.0692	0.1175	0.0432	0.1715	0.0184	0.1705	0.0024	0.0844	0.0268	0.1064
	2.0	1.533	0.135	7.47	84.60	0.0355	0.1222	0.0658	0.0412	0.0638	0.1357	0.0384	0.1824	0.0159	0.1750	0.0020	0.0849	0.0242	0.1133
0.9	0.4	0.524	0.356	6.87	10.10	0.0604	0.0948	0.0674	0.0157	0.1031	0.0835	0.0844	0.1439	0.0418	0.1568	0.0059	0.0824	0.0550	0.0942
	0.6	0.542	0.295	7.95	14.58	0.0497	0.0991	0.0623	0.0184	0.0866	0.0913	0.0691	0.1510	0.0339	0.1605	0.0048	0.0830	0.0460	0.0985
	1.0	0.594	0.206	9.44	27.16	0.0372	0.1052	0.0553	0.0226	0.0642	0.1023	0.0484	0.1609	0.0231	0.1656	0.0032	0.0837	0.0337	0.1044
	1.5	0.695	0.142	10.48	51.25	0.0289	0.1098	0.0506	0.0266	0.0492	0.1105	0.0346	0.1680	0.0159	0.1692	0.0021	0.0842	0.0255	0.1089
	2.0	0.842	0.107	11.07	86.80	0.0245	0.1147	0.0481	0.0306	0.0414	0.1159	0.0274	0.1723	0.0121	0.1714	0.0016	0.0845	0.0213	0.1117

Table B-7 Beams with Prismatic Haunch at Both Ends



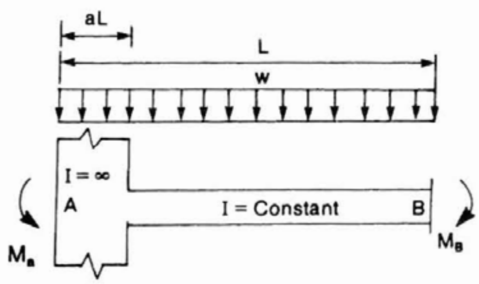
		Carry-over factors	Stiffness factors	Unif. load F.E.M. coef. × WL^2	Concentrated load F.E.M. — coef. × PL										Haunch load both haunches F.E.M. coef. × wL^2
					b = 0.1		0.3		0.5		0.7		0.9		
a	r	$C_{AB} = C_{BA}$	$k_{AB} = k_{BA}$	$M_{AB} = M_{BA}$	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	$M_{AB} = M_{BA}$
0.1	0.4	0.583	5.49	0.0921	0.0905	0.0053	0.1727	0.0606	0.1396	0.1396	0.0606	0.1727	0.0053	0.0905	0.0049
	0.6	0.603	5.93	0.0940	0.0932	0.0040	0.1796	0.0589	0.1428	0.1428	0.0589	0.1796	0.0040	0.0932	0.0049
	1.0	0.624	6.45	0.0961	0.0962	0.0023	0.1873	0.0566	0.1462	0.1462	0.0566	0.1873	0.0023	0.0962	0.0050
	1.5	0.636	6.75	0.0972	0.0980	0.0013	0.1918	0.0551	0.1480	0.1480	0.0551	0.1918	0.0013	0.0980	0.0050
	2.0	0.641	6.90	0.0976	0.0988	0.0008	0.1939	0.0543	0.1489	0.1489	0.0543	0.1939	0.0008	0.0988	0.0050
0.2	0.4	0.634	7.32	0.0970	0.0874	0.0079	0.1852	0.0623	0.1506	0.1506	0.0623	0.1852	0.0079	0.0874	0.0187
	0.6	0.674	8.80	0.1007	0.0899	0.0066	0.1993	0.0584	0.1575	0.1575	0.0584	0.1993	0.0066	0.0899	0.0191
	1.0	0.723	11.09	0.1049	0.0935	0.0046	0.2193	0.0499	0.1654	0.1654	0.0499	0.2193	0.0046	0.0935	0.0195
	1.5	0.752	12.87	0.1073	0.0961	0.0029	0.2338	0.0420	0.1699	0.1699	0.0420	0.2338	0.0029	0.0961	0.0197
	2.0	0.765	13.87	0.1084	0.0976	0.0018	0.2410	0.0372	0.1720	0.1720	0.0372	0.2410	0.0018	0.0976	0.0198
0.3	0.4	0.642	9.02	0.0977	0.0845	0.0097	0.1763	0.0707	0.1558	0.1558	0.0707	0.1763	0.0097	0.0845	0.0397
	0.6	0.697	12.09	0.1027	0.0861	0.0095	0.1898	0.0700	0.1665	0.1665	0.0700	0.1898	0.0095	0.0861	0.0410
	1.0	0.775	18.68	0.1091	0.0890	0.0084	0.2136	0.0627	0.1803	0.1803	0.0627	0.2136	0.0084	0.0890	0.0426
	1.5	0.828	26.49	0.1132	0.0920	0.0065	0.2376	0.0492	0.1891	0.1891	0.0492	0.2376	0.0065	0.0920	0.0437
	2.0	0.855	32.77	0.1153	0.0943	0.0048	0.2555	0.0366	0.1934	0.1934	0.0366	0.2555	0.0048	0.0943	0.0442
0.4	0.4	0.599	10.15	0.0937	0.0825	0.0101	0.1601	0.0732	0.1509	0.1509	0.0732	0.1601	0.0101	0.0825	0.0642
	0.6	0.652	14.52	0.0966	0.0833	0.0106	0.1668	0.0776	0.1632	0.1632	0.0776	0.1668	0.0106	0.0833	0.0668
	1.0	0.744	26.06	0.1067	0.0847	0.0112	0.1790	0.0835	0.1833	0.1833	0.0835	0.1790	0.0112	0.0847	0.0711
	1.5	0.827	45.95	0.1131	0.0862	0.0113	0.1919	0.0852	0.1995	0.1995	0.0852	0.1919	0.0113	0.0862	0.0746
	2.0	0.878	71.41	0.1169	0.0876	0.0108	0.2033	0.0822	0.2089	0.2089	0.0822	0.2033	0.0108	0.0876	0.0766
0.5	0.0	0.500	4.00	0.0833	0.0810	0.0090	0.1470	0.0630	0.1250	0.1250	0.0630	0.1470	0.0090	0.0810	0.0833

Table B-8 Prismatic Member with Equal Infinitely Stiff End Regions



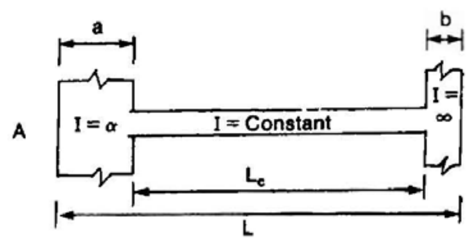
a	Carry-over Factors	Stiffness Factors	Unif. Load F.E.M. Coef. × wL^2	Concentrated Load F.E.M. — Coef. × PL									
				b									
				0.1		0.2		0.3		0.4		0.5	
	C	k	M	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
0.05	0.575	5.23	0.0913	0.0940	0.0030	0.1505	0.0245	0.1711	0.0595	0.1640	0.0999	0.1375	0.1375
0.10	0.648	7.11	0.0983	0.1000	0.0000	0.1722	0.0152	0.1968	0.0532	0.1856	0.1019	0.1500	0.1500
0.15	0.719	10.17	0.1046	0.1000	0.0000	0.1909	0.0056	0.2247	0.0431	0.2095	0.1013	0.1625	0.1625
0.20	0.786	15.56	0.1100	0.1000	0.0000	0.2000	0.0000	0.2546	0.0286	0.2369	0.0964	0.1750	0.1750
0.25	0.846	26.00	0.1146	0.1000	0.0000	0.2000	0.0000	0.2830	0.0118	0.2699	0.0851	0.1875	0.1875

Table B-9 Prismatic Member with Infinitely Stiff Region at One End



a	Carry-over Factors		Stiffness Factors		Unif. Load F.E.M. Coef. × WL^2	
	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}
0.05	0.496	0.579	4.91	4.21	0.1002	0.0752
0.10	0.486	0.657	6.00	4.44	0.1175	0.0675
0.15	0.471	0.765	7.64	4.71	0.1352	0.0602
0.20	0.452	0.875	9.60	5.00	0.1533	0.0533
0.25	0.429	1.000	12.44	5.33	0.1719	0.0469

Table B-10 Prismatic Member with Unequal Infinitely Stiff End Regions



$$k' = \left[1 - \left[\frac{3\,(L_c + 2b)\,(L_c + 2a)}{L_c^2}\right]\right] \frac{L}{L_c}$$

$$k_{AB} = \left[1 + 3\left[\frac{L_c + 2a}{L_c}\right]^2\right] \frac{L}{L_c}$$

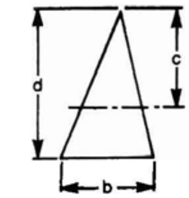
$$k_{BA} = \left[1 + 3\left[\frac{L_c + 2b}{L_c}\right]^2\right] \frac{L}{L_c}$$

$$C_{AB} = k'/k_{AB}$$

$$C_{BA} = k'/k_{BA}$$

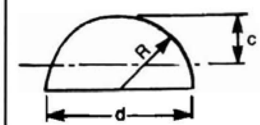
Appendix – C Sectional Properties

Triangle
Axis of moments through centre of gravity



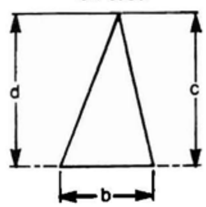
$$A = \frac{bd}{2}$$
$$c = \frac{2d}{3}$$
$$I = \frac{bd^3}{36}$$
$$S = \frac{bd^2}{24}$$
$$r = \frac{d}{\sqrt{18}}$$

Half Circle
Axis of moments through centre of gravity



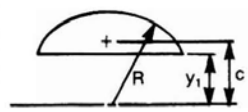
$$A = \frac{\pi R^2}{2}$$
$$c = R\left(1 - \frac{4}{3\pi}\right)$$
$$I = R^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$$
$$S = \frac{R^3}{24}\,\frac{(9\pi^2 - 64)}{(3\pi - 4)}$$
$$r = R\frac{\sqrt{9\pi^2 - 64}}{6\pi}$$

Triangle
Axis of moments on base



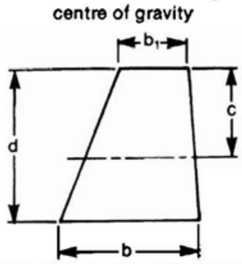
$$A = \frac{bd}{2}$$
$$c = d$$
$$I = \frac{bd^3}{12}$$
$$S = \frac{bd^2}{12}$$
$$r = \frac{d}{\sqrt{6}}$$

Segment of a Circle
Axis of moments through circle centre



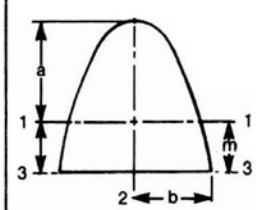
$$I = \frac{\pi R^4}{8} + \frac{y_1}{2}\sqrt{(R^2 - y_1^2)^3}$$
$$- \frac{R^2}{4}\left[y_1\sqrt{R^2 - y_1^2 + R^2 \sin^{-1}}\frac{y_1}{R}\right]$$
$$A = \frac{\pi R^2}{2} - y_1\sqrt{R^2 - y_1^2}$$
$$- R^2 \sin^{-1}\left(\frac{y_1}{R}\right)$$
$$c = \frac{2\,(R^2 - y_1^2)^{3/2}}{3A}$$

Trapezoid
Axis of moments through centre of gravity



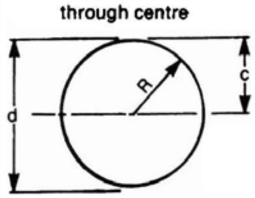
$$A = \frac{d\,(b + b_1)}{2}$$
$$c = \frac{d\,(2b + b_1)}{3\,(b + b_1)}$$
$$I = \frac{d^3\,(b^2 + 4bb_1 + b_1^2)}{36\,(b + b_1)}$$
$$S = \frac{d^2\,(b^2 + 4bb_1 + b_1^2)}{12\,(2b + b_1)}$$
$$r = \frac{-}{6\,(b + b_1)}\sqrt{2\,(b^2 + 4bb_1 + b_1^2)}$$

Parabola



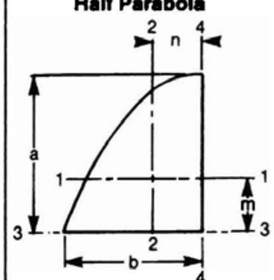
$$A = \frac{4}{3}\,ab$$
$$m = \frac{2}{5}\,a$$
$$I_1 = \frac{16}{175}\,a^3b$$
$$I_2 = \frac{4}{15}\,ab^3$$
$$I_3 = \frac{32}{105}\,a^3b$$

Circle
Axis of moments through centre



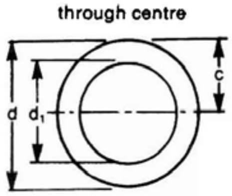
$$A = \frac{\pi d^2}{4} = \pi R^2$$
$$c = \frac{d}{2} = R$$
$$I = \frac{\pi d^4}{64} = \frac{\pi R^4}{4}$$
$$s = \frac{\pi d^3}{32} = \frac{\pi R^3}{4}$$
$$r = \frac{d}{4} = \frac{R}{2}$$

Half Parabola



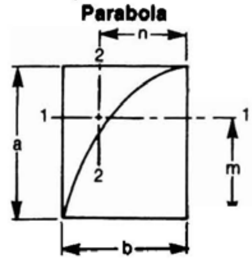
$$A = \frac{2}{3}\,ab$$
$$m = \frac{2}{5}\,a$$
$$n = \frac{3}{8}\,b$$
$$I_1 = \frac{8}{175}\,a^3b$$
$$I_2 = \frac{19}{480}\,ab^3$$
$$I_3 = \frac{16}{105}\,a^3b$$
$$I_4 = \frac{2}{15}\,ab^3$$

Hollow Circle
Axis of moments through centre



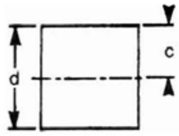
$$A = \frac{\pi(d^2 - d_1^2)}{4}$$
$$c = \frac{d}{2}$$
$$I = \frac{\pi(d^4 - d_1^4)}{64}$$
$$S = \frac{\pi(d^4 - d_1^4)}{32d}$$
$$r = \frac{\sqrt{d^2 + d_1^2}}{4}$$

Complement of Half Parabola



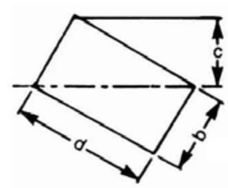
$$A = \frac{1}{3}\,ab$$
$$m = \frac{7}{10}\,a$$
$$n = \frac{3}{4}\,b$$
$$I_1 = \frac{37}{2100}\,a^3b$$
$$- \frac{1}{80}$$

Square
Axis of moments through centre



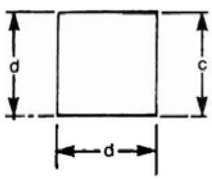
$$A = d^2 \qquad c = \frac{d}{2} \qquad I = \frac{d^4}{12} \qquad S = \frac{d^3}{6} \qquad r = \frac{d}{\sqrt{12}} = 0.288675\,d$$

Rectangle
Axis of moments on diagonal



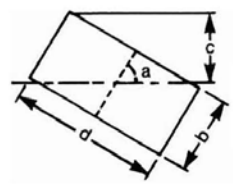
$$A = bd \qquad c = \frac{bd}{\sqrt{b^2 + d^2}} \qquad I = \frac{b^3 d^3}{6\,(b^2 + d^2)} \qquad S = \frac{b^2 d^2}{6\sqrt{b^2 + d^2}} \qquad r = \frac{bd}{\sqrt{6\,(b^2 + d^2)}}$$

Square
Axis of moments on base



$$A = d^2 \qquad c = d \qquad I = \frac{d^4}{3} \qquad S = \frac{d^3}{3} \qquad r = \frac{d}{\sqrt{3}} = 0.577350\,d$$

Rectangle
Axis of moments any line through centre of gravity

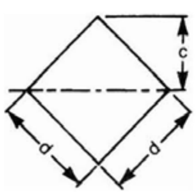


$$A = bd \qquad c = \frac{b \sin a + d \cos a}{2}$$

$$I = \frac{bd\,(b^2 \sin^2 a + d^2 \cos^2 a)}{12} \qquad S = \frac{bd\,(b^2 \sin^2 a + d^2 \cos^2 a)}{6\,(b \sin a + d \cos a)}$$

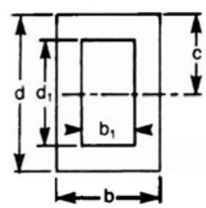
$$r = \sqrt{\frac{b^2 \sin^2 a + d^2 \cos^2 a}{12}}$$

Square
Axis of moments on diagonal



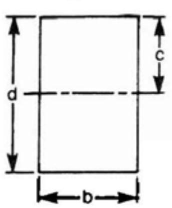
$$A = d^2 \qquad c = \frac{d}{\sqrt{2}} = 0.707107\,d \qquad I = \frac{d^4}{12} \qquad S = \frac{d^3}{6\sqrt{2}} = 0.117851\,d^3 \qquad r = \frac{d}{\sqrt{12}} = 0.288675\,d$$

Hollow Rectangle
Axis of moments through centre



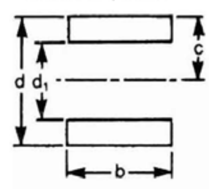
$$A = bd - b_1 d_1 \qquad c = \frac{d}{2} \qquad I = \frac{bd^3 - b_1 d_1^3}{12} \qquad S = \frac{bd^3 - b_1 d_1^3}{6d} \qquad r = \sqrt{\frac{bd^3 - b_1 d_1^3}{12\,A}}$$

Rectangle
Axis of moments through centre



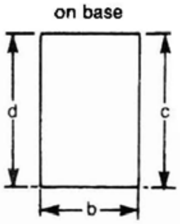
$$A = bd \qquad c = \frac{d}{2} \qquad I = \frac{bd^3}{12} \qquad S = \frac{bd^2}{6} \qquad r = \frac{d}{\sqrt{12}} = 0.288675\,d$$

Equal Rectangles
Axis of moments through centre of gravity



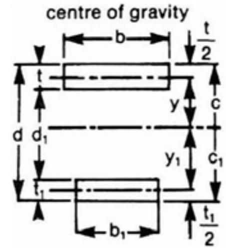
$$A = b\,(d - d_1) \qquad c = \frac{d}{2} \qquad I = \frac{b\,(d^3 - d_1^3)}{12} \qquad S = \frac{b\,(d^3 - d_1^3)}{6d} \qquad r = \sqrt{\frac{d^3 - d_1^3}{12\,(d - d_1)}}$$

Rectangle
Axis of moments on base



$$A = bd \qquad c = d \qquad I = \frac{bd^3}{3} \qquad S = \frac{bd^2}{3} \qquad r = \frac{d}{\sqrt{3}} = 0.577350\,d$$

Unequal Rectangles
Axis of moments through centre of gravity

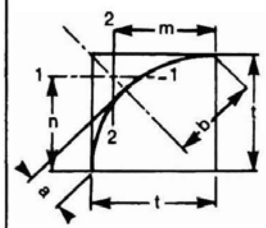


$$A = bt + b_1 t_1 \qquad c = \frac{\tfrac{1}{2}\,bt^2 + b_1 t_1 \,(d - \tfrac{1}{2}\,t_1)}{A}$$

$$I = \frac{bt^3}{12} + bty^2 + \frac{b_1 t_1^3}{12} + b_1 t_1 y_1^2$$

$$S = \frac{I}{c} \qquad S_1 = \frac{I}{c_1} \qquad r = \sqrt{\frac{I}{A}}$$

Parabolic Fillet in Right Angle



$$a = \frac{t}{2\sqrt{2}}$$

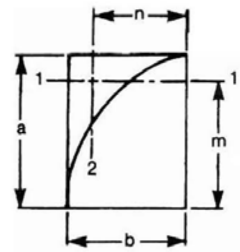
$$b = \frac{t}{\sqrt{2}}$$

$$A = \frac{1}{6}t^2$$

$$m = n = \frac{4}{5}t$$

$$I_1 = I_2 = \frac{11}{2100}t^4$$

***Elliptic Complement**



$$A = ab\left(1 - \frac{\pi}{4}\right)$$

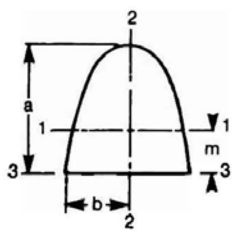
$$m = \frac{a}{6\left(1 - \frac{\pi}{4}\right)}$$

$$n = \frac{b}{6\left(1 - \frac{\pi}{4}\right)}$$

$$I_1 = a^3b\left[\frac{1}{3} - \frac{\pi}{16} - \frac{1}{36\left(1 - \frac{\pi}{4}\right)}\right]$$

$$I_2 = ab^3\left[\frac{1}{3} - \frac{\pi}{16} - \frac{1}{36\left(1 - \frac{\pi}{4}\right)}\right]$$

***Half Ellipse**



$$A = \frac{1}{2}\pi ab$$

$$m = \frac{4a}{3\pi}$$

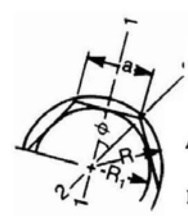
$$I_1 = a^3b\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$$

$$I_2 = \frac{1}{8}\pi ab^3$$

$$I_3 = \frac{1}{8}\pi a^3b$$

Regular Polygon

Axis of moments through centre



n = Number of sides

$$\phi = \frac{180°}{n}$$

$$a = 2\sqrt{R^2 - R_1^2}$$

$$R = \frac{a}{2\sin\phi}$$

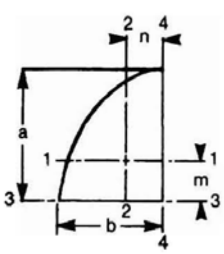
$$R_1 = \frac{a}{2\tan\phi}$$

$$A = \frac{1}{4}na^2\cot\phi = \frac{1}{2}nR^2\sin 2\phi = nR_1^2\tan\phi$$

$$I_1 = I_2 = \frac{A(6R^2 - a^2)}{24} = \frac{A(12R_1^2 + a^2)}{48}$$

$$r_1 = r_2 = \sqrt{\frac{6R^2 - a^2}{24}} = \sqrt{\frac{12R_1 + a^2}{48}}$$

***Quarter Ellipse**



$$A = \frac{1}{4}\pi ab$$

$$m = \frac{4a}{3\pi}$$

$$n = \frac{4b}{3\pi}$$

$$I_1 = a^3b\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$$

$$I_2 = ab^3\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$$

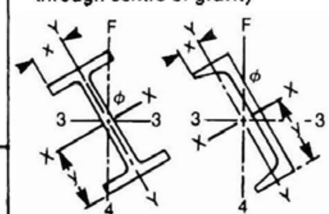
$$I_3 = \frac{1}{16}\pi a^3b$$

$$I_4 = \frac{1}{16}\pi ab^3$$

*To obtain properties of half circles, quarter circle and circular complement, substitute a = b = R.

Beams and Channels

Transverse force oblique through centre of gravity



$$I_3 = I_x\sin^2\phi + I_y\cos^2\phi$$

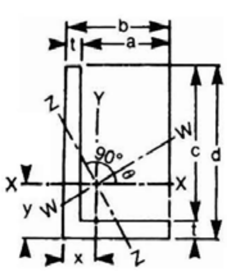
$$I_4 = I_x\cos^2\phi + I_y\sin^2\phi$$

$$f_b = M\left[\frac{y}{I_x}\sin\phi + \frac{x}{I_y}\cos\phi\right]$$

where M is bending moment due to force F.

Angle

Axis of moments through centre of gravity



Z-Z is axis of minimum I

$$\tan 2\theta = \frac{2K}{I_y - I_x}$$

$$A = t(b + c) \quad x = \frac{b^2 + ct}{2(b + c)} \quad y = \frac{d^2 + at}{2(b + c)}$$

K = Product of Inertia about X-X & Y-Y

$$= \pm\frac{abcdt}{4(b + c)}$$

$$I_x = \frac{1}{3}\left[t(d - y)^3 + by^3 - a(y - t)^3\right]$$

$$I_y = \frac{1}{3}\left[t(b - x)^3 + dx^3 - c(x - t)^3\right]$$

$$I_z = I_x\sin^2\theta + I_y\cos^2\theta + K\sin 2\theta$$

$$I_w = I_x\cos^2\theta + I_y\sin^2\theta - K\sin 2\theta$$

K is negative when heel of angle, with respect to c.g., is in 1st or 3rd quadrant, positive when in 2nd or 4th quadrant.

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ACI SP-17(09)
ACI Design Handbook
1st printing

Page 34, Step 8, Calculation revised as:

$$\cancel{A_{v,min} = 0.75\sqrt{f'_c}\frac{b_{ws}}{f_y}}$$

$$\cancel{\text{but not less than } \frac{50b_{ws}}{f_y}}$$

$$A_{v,min} = 0.75\sqrt{f'_c}\frac{b_w s}{f_y}$$

$$\text{but not less than } \frac{50b_w s}{f_y}$$

Page 35, Step 1, Calculation revised as:

– 4.47 kip/ft~~(23.5 in.²)~~**(23.5 in.)²**/24 = 983. in.-kip

Page 38, Step 5, Calculation revised as:

$$\cancel{\phi V_u = \frac{24.7\text{ kip} + 0.75(0.22\text{ in.}^2)60\text{ kip/in.}^2(20\text{ in.})}{10}}$$

$$\phi V_u = 24.7\text{ kip} + \frac{0.75(0.22\text{ in.}^2)60\text{ kip/in.}^2(20\text{ in.})}{10}$$

Page 38, Step 7, code references revised as:

~~11.5.4.1~~ 11.5.6.1	Step 7 – Determine distance ℓ_{v1}, distance beyond x_1 at which no stirrups are required. Find $\ell_{v1} = (V_u - V_c/2)/w_u$ Compute $x_1 + \ell_{v1}$ Conclude: use s = 7 in. until ϕV_u < 44.5 kip and use s = 10 in. until $\phi V_u < 0.5\phi V_c$	ℓ_{v1} = (22.3 kip – 24.7 kip/2)/4.6 kip/ft = 2.16 ft $x_1 + \ell_{v1}$ = 4.50 ft + 2.16 ft = 6.66 ft = 80 in. From face of support use 3 in. space then 5 spaces @ 7 in. (35 in.) and 5 spaces @ 10 in. (50 in.) 88 in. > 80 in.

Page 40, Step 1, Calculation revised as:

~~$\phi V_c = 4(0.75)(\sqrt{5000}\text{ psi})(24\text{ in.} + d + 16\text{ in.} + d)d\text{ in.}$~~

$\phi V_c = 4(0.75)(\sqrt{5000}\text{ psi})2(24\text{ in.} + d + 16\text{ in.} + d)d\text{ in.}$

Page 41, ALTERNATE METHOD, Step 3, Calculation revised as:

$K2 = 0.778 + (0.763 - 0.778)(3.67 -$ ~~0.60~~ **3.6**$)/(3.8 - 3.6)$

Page 42, Step 2, Calculation revised as:

= 252.6 kip – (1.188 ~~in./kip~~ **kip/in.**)d – (0.0372 ksi)d^2

Page 42, Step 3, Calculation revised as:

~~$\phi V_c = 0.75[4(\sqrt{3000}\text{ lb/in.}^2)4(16 + d)\text{ in.}[(d)\text{ in.}]/$~~

$\phi V_c = 0.75[4(\sqrt{3000}\text{ lb/in.}^2)4(16 + d)\text{ in.}(d)\text{ in.}]/$

Page 43, Shear Example 9 revised as:

Given:

~~$V_n = 4(\alpha_s d/b_o + 2)(\sqrt{f'_c})b_o d \le 4(\sqrt{f'_c})b_o d$~~

$V_n = (\alpha_s d/b_o + 2)(\sqrt{f'_c})b_o d \le 4(\sqrt{f'_c})b_o d$

Page 44, Shear Example 10 revised as:

Given:

~~$V_n = 4(\alpha_s d/b_o + 2)(\sqrt{f'_c})b_o d \le 4(\sqrt{f'_c})b_o d$~~

$V_n = (\alpha_s d/b_o + 2)(\sqrt{f'_c})b_o d \le 4(\sqrt{f'_c})b_o d$

Page 45, Shear Example 11 revised as:

f'_c = ~~4000~~ **3000** psi

Page 46, Step 2, Calculation revised as:

~~$T_{cr} = 4(0.75)(\sqrt{5000}\text{ psi})(384\text{ in.})^2/80 = 391{,}000\text{ in.}\cdot\text{lb}$~~

$T_{cr} = 4(0.75)(\sqrt{5000}\text{ psi})(384\text{ in.}^2)^2/80 = 391{,}000\text{ in.}\cdot\text{lb}$

Page 46, Step 3, Calculation revised as:

~~$f_{vt} = 53\text{ ft}\cdot\text{kip }(12\text{ in./ft})66\text{ in./}[1.7(256\text{ in.}^2)] = 0.377\text{ ksi}$~~

$f_{vt} = 53\text{ ft}\cdot\text{kip }(12\text{ in./ft})66\text{ in./}[1.7(256\text{ in.}^2)^2] = 0.377\text{ ksi}$

Page 46, Step 4, Procedure revised as:

~~$A_v/s = [V_u - 2\phi f'_c(b_w d)]/(\phi f_y d)$~~

$A_v/s = \left[V_u - 2\phi\sqrt{f'_c}(b_w d)\right]/(\phi f_y d)$

Page 47, Step 5, Calculation revised as:

~~$A_{\ell,min} = 5(\sqrt{5000}\text{ psi})(384\text{ in.}^2/60{,}000\text{ psi} - (0.0324)(60\text{ ksi}/60\text{ ksi}) = 1.90\text{ in.}^2$~~

$A_{\ell,min} = 5(\sqrt{5000}\text{ psi})(384\text{ in.}^2/60{,}000\text{ psi} - (0.0324)(66\text{ in.})(60\text{ ksi}/60\text{ ksi}) = 0.12\text{ in.}^2$

Page 47, ALTERNATE METHOD, Step 1, Calculation revised as:
K_{vs} = 1290 ~~ksi~~ **kip/in.**

Page 47, ALTERNATE METHOD, Step 1, code references revised as:

ALTERNATE METHOD using design aid			
~~11.2.1.1~~ ~~11.5.6.2~~ **11.3.1.1** **11.5.7.2** 11.6.2.2a 11.6.3.1 11.6.3.6	Step 1 – Look up parameters for f_c' = 5000 psi, Grade 60 reinforcement, b_w = 16 in., h = 24 in	$K_{fc}K_{vc}$ = (1.118)43.5k = 48.6 kip K_{vs} = 1290 ksi $K_{fc}K_t$ = (1.118)89.1 ft·kip = 99.6 ft·kip $K_{fc}K_{tcr}$ = (1.118)38.9 ft·kip = 43.5 ft·kip K_{ts}= 1089 ft·kip/in.	Shear 2 Table 2a Table 2b Shear 6.1a Shear 6.1b Shear 6.2b

Page 47, ALTERNATE METHOD, Step 2, Calculation revised as:
53 ft·kip > 0.25ϕ(43.5 ft·kip) = ~~10.9~~ **8.2** ft·kip

Page 47, ALTERNATE METHOD, Step 3, Calculation revised as:

$$\cancel{= \sqrt{\{61\,\text{kip} / 5[(0.75)48.6\,\text{kip}]\}^2 + \{53\,\text{ft·kip} / [(0.75(1089\,\text{ft·kip})]\}^2}}$$

$$\underline{= \sqrt{\{61\,\text{kip} / 5[(0.75)48.6\,\text{kip}]\}^2 + \{53\,\text{ft·kip} / [(0.75(99.6\,\text{ft·kip})]\}^2}}$$

Page 47, ALTERNATE METHOD, Step 4, Calculation revised as:
(61 kip/0.75 – 48.6 kip)/ 1290 ~~ksi~~**kip/in.** + 53.0 ft·kip**/**
[(0.75)1089 ft·kip**/in.**] = 0.0254 + 0.0649 = 0.0903 in.^{2}/in.

Page 48, Step 6, Calculation revised as:

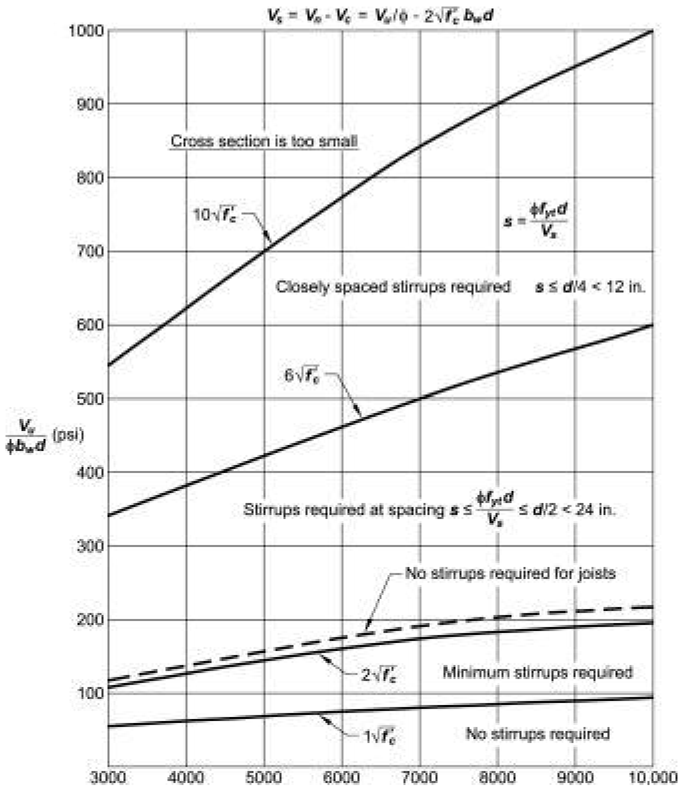
A_ℓ = (0.020 in.)66 in.(1.00) = ~~0.13~~**1.32** in.^{2}

Page 48, Step 1, code references revised as:

~~11.2.1.1~~ **11.3.1.1** ~~11.5.6.2~~ **11.5.7.2** 11.6.2.2a 11.6.3.1 11.6.3.6	Step 1 – Look up parameters for f_c' = 5000 psi, Grade 60 reinforcement, b_w = 16 in, h = 24 in.	$K_{fc}K_{vc}$ = (1.118)43.5 = 48.6 kip K_{vs} = 1290 ksi $K_{fc}K_t$ = (1.118)89.1 = 99.6 ft·kip $K_{fc}K_{tcr}$ = (1.118)38.9 = 43.5 ft·kip K_{ts} = 1089 ft·kip/in.	Shear 2 Table 2a & 2c Shear 2a & 2b Shear 6.1a Shear 6.1b Shear 6.2b

Page 49, Shear 1, Reference Sections revised as:
~~11.11.1~~ **11.1.1**, 11.3.1.1, 11.5.4, ~~11.5.6.2~~ **11.5.7.2**, ~~11.2.6.8~~ **11.5.7.9**, and 8.11.8.

Page 49, revised as:



$\cancel{\sqrt{f'_c}}$ $\underline{\boldsymbol{f'_c}}$ (psi)

Page 50, Shear 2, Reference Sections revised as:
~~11.2.1.1~~ **<u>11.3.1.1</u>** and ~~11.5.6.2~~ **<u>11.5.7.2</u>**

Page 50, Table 2b, heading revised as:

Table 2b

Values K_{vs} (~~psi~~ **<u>lb/in.</u>**)

Page 50, Shear 2 revised as:
K_{vs} = $f_y d$ (~~kip~~ **<u>kip/in.</u>**) (Table 2(b))

Page 55, Table 5.1b heading revised as;

Values ***K*2**~~, ksi~~

Page 68, top of page, Calculation revised as:

$$\cancel{\gamma \approx \frac{15-5}{18} = 0.67}$$

$$\underline{\gamma \approx \frac{15-5}{15} = 0.67}$$

Page 69, "B" Calculation revised as;

$$\frac{686}{(5)(201)(16)} \quad \frac{686}{(5)(201)(16)}$$

$\cancel{= 0.43}$ **= 0.043**

Page 69, "C" Calculation revised as:

~~0.64~~ **0.58**	0.69	~~0.72~~ **0.75**

Page 69, "D" Calculation, second sentence revised as:
Use interaction diagrams C5-60.6, C5-60.7, ~~C5-60.7,~~ and C5-60.8.

Page 69, "F" Procedure revised as:

F) Compute $\cancel{A_g = \frac{P_n}{f'_c k_n}, \text{in.}^2}$

$$\underline{A_g = \frac{P_n}{f'_c K_n}, \text{in.}^2}$$

Page 179, 10.13.5,

Calculation revised as:

$\cancel{\ell_u / r > 35 / \sqrt{P_u / (f'_c / A_g)}}$

$\ell_u / r > 35 / \sqrt{P_u / (f'_c A_g)}$

Page 180, 10.13.5, Calculation revised as:

$\cancel{\ell_u / r > 35 / \sqrt{P_u / (f'_c / A_g)}}$

$\ell_u / r > 35 / \sqrt{P_u / (f'_c A_g)}$

Page 183, Calculation revised as:
Summary of design loads:

Load combinations	P_u, **(kip)**	**(M_u) (in.-kip)**
I—$U = 1.2D + 1.6L_r + 0.8W$	694 or 670	–2738
II—$U = 1.2D + 1.6W + 1.0L + 0.5L_r$	869 or 821	~~–4967~~ **–4867**
III—$U = 0.9D + 1.6W$	476 or 428	–3616
IV—$U = 1.4D$	703	–1512
V—$U = 1.2D + 1.6L + 0.5L_r$	976	–2032
VI—$U = 1.2D + 1.6L_r + 1.0L$	900	–1756

Page 231, Table A2 heading revised as:

Cross section area of bar ~~As~~ $\underline{A_s}$ (or ~~As'~~ $\underline{A_s'}$), in.^{2}

Page 234, revised as:

~~Centroid of group, W =~~ **For the bundled bars configuration shown here, the centroidal distance is calculated by the following equation:**

$$x = \frac{\frac{5}{2} A_{si} d_{b1} + A_{s2}(d_{b1} + d_{b2}/2)}{\Sigma A_{si}}$$