

Sheet (2-b): Partial Differentiation (Chain Rule)

In the following problems, use the chain rule to find the indicated partial derivatives.

(1) $z = e^{uv^2}$; $u = x^3$, $v = x - y^2$; find: $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

(2) $z = u^2 \cos 4v$; $u = x^2 y^3$, $v = x^3 + y^3$; find: $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

(3) $z = 4x - 5y^2$; $x = u^4 - 8v^3$, $y = (2u - v)^2$; find: $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$

(4) $z = \frac{x-y}{x+y}$; $x = \frac{u}{v}$, $y = \frac{v^2}{u}$; find: $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$

(5) $w = (u^2 + v^2)^{3/2}$; $u = e^{-t} \sin \theta$, $v = e^{-t} \cos \theta$; find: $\frac{\partial w}{\partial t}$, $\frac{\partial w}{\partial \theta}$

(6) $w = \tan^{-1} \sqrt{uv}$; $u = r^2 - s^2$, $v = r^2 s^2$; find: $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$

(7) $R = rs^2 t^4$; $r = ue^{v^2}$, $s = ve^{-u^2}$, $t = e^{u^2 v^2}$; find: $\frac{\partial R}{\partial u}$, $\frac{\partial R}{\partial v}$

(8) $Q = \ln(pqr)$; $p = t^2 \sin^{-1} x$, $q = \frac{x}{t^2}$, $r = \tan^{-1} \frac{x}{t}$; find: $\frac{\partial Q}{\partial x}$, $\frac{\partial Q}{\partial t}$

(9) $w = \sqrt{x^2 + y^2}$; $x = \ln(rs + tu)$, $y = \frac{t}{u} \cos(rs)$; find: $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial t}$, $\frac{\partial w}{\partial u}$

Reference:

"Advanced Engineering Mathematics," by D. Zill and W. Wright, 4th edition, Jones & Bartlett, 2011.