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ENGINEERING DESIGN HANDBOOK
ARMY WEAPON SYSTEMS ANALYSIS, PART ONE

TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

Paragraph *Page*

CHAPTER 11
FACTORS AFFECTING TARGET SELECTION

TABLE OF CONTENTS (cont'd)

Paragraph *Page*

CHAPTER 12
THE SCENARIO

CHAPTER 13
WEAPON DELIVERY ERROR CHARACTERISTICS AND DISTRIBUTION

CHAPTER 14
PROBABILITY OF HITTING FOR SINGLE ROUNDS (SINGLE SHOT HIT PROBABILITIES)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

CHAPTER 20
MULTIPLE ROUND HIT PROBABILITIES, TARGET COVERAGE, AND TARGET DAMAGE

TABLE OF CONTENTS (cont'd)

CHAPTER 21
RELIABILITY, LIFE TESTING, RELIABILITY GROWTH, AVAILABILITY, AND MAINTAINABILITY

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

Paragraph *Page*

CHAPTER 24
THE WSEIAC EVALUATION MODEL

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (cont'd)

LIST OF ILLUSTRATIONS (cont'd)

LIST OF TABLES

LIST OF TABLES (cont'd)

PREFACE

The US Army now has approximately thirty years of experience in the evaluation or analysis of weapon systems. Military operations research studies, or weapon systems evaluations, were initiated in a very natural way in the mid-1940's at the US Army Ballistic Research Laboratories (BRL) Aberdeen Proving Ground, MD—for here resided the talent and personnel necessary to pursue the rigorous scientific approaches demanded by this important discipline. A glance at the References and Bibliographies reveals that many of the techniques and methodologies applicable for the comparison of weapons and weapon systems were developed by the BRL. The size of the task, the continuing requirement to revise and improve analytic techniques, and the increased sophistication of the systems being studied led to the establishment of a Weapon Systems Laboratory within the BRL. Today this laboratory has grown into the US Army Materiel Systems Analysis Activity (AMSAA).

The objective of the *Army Weapon Systems Analysis* Handbook is to give the appropriate background for young analysts entering the field of military operations research, and to record some of the more useful or recommended methodology for evaluation of Army weapon systems and materiel. This handbook may also be used as a text for teaching weapon systems analysis; other purposes are given in Chapter 1. The handbook has been written hopefully in an introductory manner; derivations have been kept to a minimum by proper references to the appropriate literature. In fact, the References and Bibliographies cite valuable source material for those who may wish to acquire an extensive knowledge of any of the subjects introduced herein, or to do further research on the methodology.

In view of the extensiveness of the subject and literally hundreds of references which cover the field of Army weapon systems analyses, it seemed convenient to publish the handbook in two parts. The contents of *Part One* are listed in the Table of Contents. For *Part Two*, additional topics in Army weapon systems analysis—including measure of effectiveness, target detection phenomena and chances of target detection, combat theory for homogeneous and heterogeneous forces, weapon equivalence studies, analysis of costs and other resource measures, survivability, an introduction to war gaming and simulation, example applications, and other pertinent systems analysis techniques—will be covered.

Although some of the earlier chapter material of the handbook was drafted in the early 1970's by personnel of ARINC, the contents and subject matter presented here are predominantly the contributions of Dr. Frank E. Grubbs—formerly Chief Operations Research Analyst of the US Army Ballistic Research Laboratories—prepared for the Engineering Design Handbook Office, Research Triangle Institute, NC, prime contractor to the US Army Materiel Development and Readiness Command.

In connection with the preparation of this handbook, we are indebted to Dr. Robert J. Eichelberger, Director of the US Army Ballistic Research Laboratories for his support, which contributed in a major way to the accomplishments. The expert technical typing services of Mrs. Thelma B. Springer is also acknowledged.

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CHAPTER 1

BACKGROUND AND PURPOSE OF THE ARMY WEAPON SYSTEMS ANALYSIS HANDBOOK

A brief sketch is given of the historical development and value of military operations research and systems analysis in the US Army. The purposes of the handbook are also outlined.

1-1 HISTORICAL BACKGROUND OF ARMY WEAPON SYSTEMS ANALYSIS

With the advent of World War II, and the consequent large-scale production of weapons and ammunition, much effort was naturally placed on research, development, and analytical investigations to find new principles and exploit them in the production of new and better weapons. Indeed, many scientific, engineering, and professional people of our country were drawn into defense work to aid the war effort. With such talent available, a natural consequence was the development of many new ideas and new fields of interest. As a matter of fact, the field of military operations research (or OR) grew out of such effort as a dividend of expanded science in the newly organized defense establishment. Now it can be argued that operations research and later the field of systems analysis really sprung up much earlier, and that staff type studies had always been made anyway. So what was new? The point is that it took World War II to bring the interdisciplinary approach to bear on military operational problems and to attempt, also, to find optimum solutions to the various problems of weaponry. It is, therefore, of interest to trace briefly some of the military operations research work in our account here.

At the outbreak of the war in Europe in 1939, the British Government quickly mobilized the scientists from the Bawdsey Research Station to assist military personnel in analyzing the problems of integrating the newly developing radar technology into the country's early warning system. These studies were extended to include all facets of early air warning, radar detection and night operations. Investigations were initiated to determine the best way to coordinate the early warning radar network with the weapons of the British Army's Antiaircraft Command and to resolve the problems of detecting submarines and surface ships by the use of airborne radar equipment. These studies marked the origin of operational research (the UK term for OR) in all three British military services. Two examples will illustrate the usefulness of the analytical or operations analysis approach.

During the Battle of Britain, extensive bomb damage and casualty data were collected. Analysis of this information enabled the operational planners of the Allied Forces to select profitable targets and to estimate accurately in advance the effects of bombing during subsequent raids over Germany. Data analysis of the Battle of Britain also showed that the percentage of loss of attacking aircraft decreased as the formation size increased, a finding directly responsible for the first thousand-plane raid over Germany by the R.A.F. in 1942.

The relative ineffectiveness of the antisubmarine tactics employed by British aircraft during 1941-42 also warranted investigation. Variation of explosive size and aircraft attack pattern did little to improve results. Investigation revealed that enemy submarines were usually on the surface or in the initial stages of diving when attacked and that the charges being dropped were set to detonate at depths of 100 ft or more. The cushioning effect of the sea at that depth rendered the charge little more than a nuisance to the submarine. The investigators therefore recommended setting the charges to explode at a depth of about 25 ft, but the firing device had a minimum setting restriction of 35 ft. It was found that setting the charges to detonate at 35 ft increased antisubmarine effectiveness between 400 and 700%. The analysis also pointed out the need for developing a shallow-depth firing device or fuze.

Within months after the entry of the United States into World War II, both the US Navy and the US Army Air Force had organized operations research groups. Early problems studied by these American groups were strikingly similar to those occupying their British counterparts. By the end of the war, a wide range of problems had been studied — including protection of convoys, antisubmarine search and destroy tactics, detection of surface ships, denial of sea lanes, selection of bombs and fuzes, bombing accuracy, jungle warfare operations, aircraft pursuit tactics, and amphibious operations. A solid foundation for weapon systems analysis, therefore, had been constructed.

In the US Army, techniques of military operations research and weapon systems analysis began to evolve during the mid- to late 1940's at the Ballistic Research Laboratories (BRL) in a rather natural way also. At that time, the standard infantry rifle was the M1 which fired a caliber .30 bullet weighing 150 grains at a muzzle velocity of 2956 ft/s. Dr. Robert H. Kent, then Associate Technical Director of the BRL, had earlier advocated a higher muzzle velocity (MV) (3500 ft/s) and a caliber .22 bullet of about 50 grains weight, which would have a much flatter trajectory and thereby reduce the effect of errors in estimating range to the target. Also, the caliber .22 bullet could be designed so that it would be as lethal as the caliber .30 bullet in human targets. A committee was formed of representatives of the BRL to study the overall problem of effectiveness for such a new infantry rifle and the result was a recommendation very similar to the current relatively light weight M16 Rifle (3200 ft/s MV).

Another military operations research study or weapon systems analysis of the BRL in the late 1940's had to do with an optimum caliber type of effectiveness study for the family of field artillery. In fact, BRL, as a result of an overall effectiveness investigation, advanced the concept of a longer, more lethal projectile that had thinner walls and more explosive than formerly to produce smaller high velocity fragments and hence improved antipersonnel effectiveness. The results of this new family of field artillery study are published in BRL Report No. 771 (Ref. 1).

In the late 1940's much of the weapon systems analysis work of the BRL was carried out by the Weapons Systems Branch of the Terminal Ballistics Laboratory under the direction of Mr. Herbert K. Weiss with the aid of personnel from the other BRL laboratories as required. Weiss subsequently headed an Ordnance Engineering Laboratory which in 1952 evolved into the Weapons Systems Laboratory of the BRL. The Weapon Systems Laboratory was separated from the Ballistic Research Laboratories in the organization of the Aberdeen Research and Development Center in 1968 and was designated the US Army Materiel Systems Analysis Agency (AMSAA). Since 1971, AMSAA has reported directly to the Commanding General, US Army Materiel Command, Washington, DC — now US Army Materiel Development and Readiness Command (DARCOM). In 1974, AMSAA was renamed the Army Materiel Systems Analysis Activity.

DARCOM-R 11-1 (Ref. 2) encourages installations and activities of DARCOM to use systems analysis techniques rather widely.

The Army's Operations Research Office (ORO) was fully activated in 1949 at Fort Leslie J. McNair, Washington, DC. Since that time, military operations research and systems analysis have flourished as major activities of the US Army R & D establishment.

The first Army Ordnance Conference of Operations Research was held at Frankford Arsenal in 1954, and the first of the annual Army Operations Research Symposia was held in 1962 at the Army Research Office (then the Office of Ordnance Research) Durham, NC.

The 1970's find operations research and systems analysis (OR/SA) as a prime tool to aid Army management in its major activities and its laboratories as well.

Many authors and investigators have pointed out that operations research did not really start as a result of World War II. In fact, Frederick William Lancaster (1916) was apparently the first to model combat in mathematical form (Ref. 3). Also, Whitmore (Ref. 4) points out that Thomas A. Edison, consulting for the Secretary of the Navy, Josephus Daniels, during World War I, recommended, as a

result of a study on shipping, that submarine attacks on our ships could be reduced to a very low level if our merchant ships were to sail into and out of the danger zones at night.

For interested readers, Trefethen (Ref. 5), for example, gives an account of operations research for management type endeavors.

1-2 DESCRIPTION OF WEAPON REQUIREMENTS AND WEAPON SYSTEMS ANALYSIS

The materiel acquisition process (Refs. 6 and 7) requires continuous communication between research personnel, the system developer, and the combat developer. The combat developer generally defines the military requirements necessary to achieve the Army combat readiness posture. A military requirement is defined as "An established need justifying the timely allocation of resources to achieve a capability to accomplish approved miltary objectives, missions, or tasks" (Ref. 8). Determination of these requirements relies heavily on the analysis process called requirement analysis. The major orientation of requirement analysis is directed toward the Army's mission and the capabilities necessary to accomplish efficiently that mission in future conflicts.

Military capabilities and weapon requirements generally are stated in terms of military characteristics, which are, "Those characteristics of equipment upon which depend its ability to perform desired military functions. Military characteristics include physical and operational characteristics but not technical characteristics" (Ref. 8). The establishment of military characteristics, through the process of requirement analysis, is primarily the responsibility of the combat developer, with aid from the weapon development personnel.

At this point, the systems developer begins to define technical approaches that will feasibly satisfy the weapon requirements. The major effort then is toward establishing technical characteristics, which are "Those characteristics of equipment which pertain primarily to the engineering principles involved in producing equipments possessing desired military characteristics. For example, for electronic equipment, technical characteristics include items as circuitry, and types and arrangement of components" (Ref. 8). As feasible alternatives are identified, the weapon systems analysis process is used. The purpose of the systems analysis process is to assist decision makers by providing a logical systematic, and objective framework that defines and evaluates all pertinent factors influencing each alternative course of action in the decision making process (Ref. 2).

Systems analysis is defined here momentarily as, "An orderly study of a management system or an operating system using the techniques of management analysis, operations research, industrial engineering, or other methods to evaluate the effectiveness with which missions are accomplished and to recommend improvements" (Ref. 8). In its most general application, systems analysis is concerned with a system or group of systems. A system is defined as, "An integrated relationship of components aligned to establish proper functional continuity towards the successful performance of a defined task or tasks" (Ref. 8).

1-3 PURPOSE OF THE HANDBOOK AND GENERAL GUIDELINES

The objective of this Handbook is focused on the status and application of weapon systems analysis methodology and philosophy to evaluate, compare, select, or cost a weapon system. Weapon systems analysis is devoted to the comparison of various means of satisfying the military requirements and is totally apart from, but also dependent on, the detailed engineering effort that is required to design and produce new or improved weapons.

Useful weapon systems analysis information of current applications and methodology is presented in this Handbook. Accordingly, the Handbook should:

1. Provide orientation and guidance for new weapon systems analysts;

2. Communicate to the analysts, in capsule form, the allied technical fields with which they must be concerned;

3. Conserve time, materials, and funds by outlining approaches to problems which have proven to be helpful over the years;

4. Provide a summary of current weapon systems analysis methodology.

This Handbook is intended primarily for use by analysts and engineers in the following categories:

1. The recently graduated physical scientists, mathematicians, and engineers who have limited knowledge of the principles of Army weapon systems analysis

2. Those specialists in a particular field of Army systems analysis who have only superficial knowledge of allied fields

3. Those systems analysts and engineers who are employed by Army contractors

4. Those analysts who occasionally are in need of an authoritative reference or standard technique of systems analysis.

Although it is written for the primary audiences indicated, the Handbook also will be a valuable tool in the training of inexperienced analysts and engineers, and provide information to augment the knowledge of personnel concerned with procurement, production, inspection, drawings, specifications, testing, maintenance, and administration.

To accomplish the stated purpose of this Handbook, concepts and procedures will be presented which some readers may consider more aligned with economic analysis, operations research, effectiveness analysis, cost-effectiveness analysis, or cost-benefit analysis. The intent, therefore, is to assemble information necessary or useful in performing overall weapon systems analyses.

1-4 OBJECTIVES AND PHASING OF WEAPON SYSTEMS ANALYSIS

Weapon systems analysis is concerned with and applied at the major milestones and decision points throughout the life cycle of weapon systems and is particularly applicable in early phases of research and development. In the initial phase, for example, studies may be conducted to investigate the possible uses, strategy, and tactics of a weapon system based on entirely new technology; the potential advantages which a proposed new Army weapon system would possess when compared with existing systems; or the range of parameters which would be most useful for a potentially new system. Similarly, during the operation and disposal portions of the life cycles, studies may be initiated to investigate alternative applications for a weapon system after it has served its original purpose, or to determine whether an existing weapon system should be retained in use, modified, or phased out in favor of a new replacement weapon system.

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CHAPTER 2

WHAT IS OPERATIONS RESEARCH/SYSTEMS ANALYSIS?

Definitions are given for the relatively new fields of operations research and systems analysis (OR/SA), and some current OR/SA terminology is discussed.

2-1 DEFINITIONS

Since the term "systems analysis" is more recent and really grew out of "operations research" type of activities initiated back in the early 1940's, we should first consider definitions of the latter term. Of course, we might attempt to dispense with definitions of both terms easily by saying that (1) operations research is research on, or analytical investigations of, operations within the purview of management's technical mission; and that (2) systems analysis is actually an appropriate, accurate analysis of systems of various kinds. However, many of the key people involved in the early establishment of OR/SA type activities did not come to any such simple agreements, and they spent much time trying to generate proper, agreed upon, universal and descriptive definitions. The term "operational research" was coined by A. P. Rowe in England during 1937-39 at the Air Ministry Research Station at Bawdsey in order to distinguish the new evaluation type of activity from the normal or usual research and development activities then in progress. In America, the term "operational research" never took hold, and instead the term "operations research" became widespread to indicate the new type of approach to what we will call scientific management procedures. Thus, we might say that the era had arrived for management to employ tools of science toward analyzing their problems, at least in many areas, and consequently make better decisions in their day to day operations.

Perhaps a reason for some of the difficulty in defining OR precisely was that the problems of management varied so widely and that for a given activity some of the branches of science were needed, or were applicable, while others were not. Then, again, some other problems might be of such general scope and cut across so many fields of the physical sciences that a large variety of techniques was needed in any useful overall evaluation or analysis. It is somewhat descriptive to say that in the early stages of applications of the new OR/SA techniques a number of sciences such as physics, engineering, mathematics, probability and statistics, and economics were brought to bear on the overall effort. For the more recent applications, the social sciences are playing increasingly important roles. A brief account of some of the difficulties and history of defining operations research is of interest here. (Ref. 1).

In their book *Methods of Operations Research,* (1st edition revised, 1950) Morse and Kimball (Ref. 2) define operations research as a new "scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control". They point out that operations research studies and activities should be separate from those of executive responsibilities since the operations researcher is to carry out the study, while the matter of making an operational decision (which is sometimes in conflict with scientific work) should be performed by another person, i.e., the executive.

To indicate the difficulty of defining OR, Professor Morse himself apparently was not satisfied with the previous definition. In May 1953 at a meeting of the Operations Research Society of American (which was formed during 1952) Professor Morse said, "Operations Research is an activity carried on between members of the Operations Research Society of America; its methods are those reported in

their Journal. Very soon this definition will be more instructive and convincing than any number of special exploratory articles and talks."

In May 1954 at the first Ordnance Conference on Operations Research held at the Frankford Arsenal, Professor Morse said, "Operations research is what operations research workers do." (!)

Perhaps the difficulty with defining the term was that there were too many definitions initially. Some of these were:

1. "OR is the science of decision."
2. "OR is the application of methods of the physical science to providing quantitative answers to executives with regard to operations under their control." (A variation of Morse's definition)
3. "OR is quantitative common sense."
4. "OR is scientific common sense."
5. "OR is at present undefined, but in time will become defined by the subject matter appearing in (its journal) JORSA."

O. W. Hamilton of the US Time Corporation defined OR as "the application of the methods of analysis of physical science to the solution of administrative problems involved in the control of industrial, governmental or military operations."

LeRoy A. Brothers, Department of the Air Force said, "Operations Research, or operations analysis, cannot be defined in a sentence or two nor independent of the environment within its operations."

Two Indian investigators, A. Rhaman and S. Husian Zaheer, in the Operational Research Quarterly (Vol 3, 1952) indicated that, "there are three levels of research: fundamental research, applied research, and operations research. Indeed, operations research is a task of integrating or cutting across a number of disciplines to arrive at a complete or whole solution."

We see that the definitions thus indicate some rather different opinions of various investigators probably due to many diverse areas of application of a new area of science being developed at the time to attack the technical problems of management.

We say momentarily that the last definition is perhaps a good one, but that the particular point of specifying three "levels" of research to define OR seems very restrictive and also inappropriate since there is continuing fundamental and applied research in the field of OR as one observes in papers of the journals of OR.

With this in mind, we can begin to piece together the ingredients of the term and field of operations research. In fact, OR does not involve the usual type of activity known as staff studies. Rather, it uses all of the principles of science that may apply, and scientifically trained people conduct the overall study. Operations research is an interdisciplinary activity generally using team research. Also, it is quantitative and often uses models of probabilistic character. Trade-offs should be considered to arrive at the best overall solution. The primary purpose of OR is to improve the operations under examination, with the final results pointing to very definite courses of action to be taken. It is, moreover, characteristic of OR that an unbiased procedure always is taken for the study since management wants the facts concerning the problem under investigation in order that a proper course of action will follow.

We will not attempt to develop any difference between operations research on one hand and its companion activity of systems analysis on the other. Many learned people have concluded that they are much the same in approach and overall procedures. A point that might be argued is that systems analysis is already a very descriptive activity, indicating that it involves the analysis of systems of all kinds, and hence that it helps to explain the term OR! Indeed, systems analysis studies should involve technical feasibility investigations of the system, its design, or alternative approaches to be used, the

development of the methodology or measures of effectiveness criteria for judgment, the acquisition of input data, the application of the methodology in trade-off studies, the optimization of parameters and, if possible, presentation of the single overall solution, especially on an economical basis.

Ref. 3, 1 June 1972, defines (military) operations research as, "the analytical study of military problems, undertaken to provide responsible commanders and staff agencies with a scientific basis for decision on action to improve military operations. Also known as operational research, or operations analysis." We might even conclude by saying that the original OR people who coined the term were very fortunate indeed since they chose a somewhat self-explanatory title! That is, OR is indeed just what it says; namely the application of research principles to a variety of operations or problems, whatever they might be!

A widely recognized field of interest is that of management science. Investigators in management science problems or activities approach their studies very much like operations research analysts do. Hence, an operations researcher and the management scientist are essentially the same. A recent field of activity is that of "risk analysis", a term coined by a former Deputy Secretary of Defense, David Packard. A thorough look into the activities of risk analysis reveals that it essentially just covers the same approaches as those of operations research, although the term nevertheless is an apt one, since the various risks in weaponry, for example, should be evaluated properly — preferably under field or combat conditions.

A simple but useful description of systems analysis is that it involves a systematic search for preferred alternatives in an attempt to optimize specified system objectives, i.e., minimize cost or maximize effectiveness. The process of systems analysis determines and quantifies comparative performances, effectiveness, and cost considerations for each alternative, using the scientific methods of exhaustive, reproducible investigation. Systems analysis also may be directed toward an improvement in the development, selection, modification, or use of existing systems.

In the application of systems analysis, two considerations should be kept in mind. First, the function of systems analysis is to provide appropriate information to assist the decision maker in the selection of alternatives for the attainment of desired objectives: Should the results be inconclusive, the analysis should provide means for formulating new alternatives. Secondly, the investigation of a particular system may be facilitated if one accepts the premise that the system being studied may possibly be considered a subsystem of a larger system. However, such a study of a single system alone could in fact lead to suboptimization which might result in a more costly, or less effective combination of systems (Ref. 9).

2-2 THE EVOLUTION OF SYSTEMS ANALYSIS

We have seen that systems analysis, operations analysis, operations research, cost effectiveness, and management science all have a common foundation — the scientific approach to problem solving. The major impetus toward the formulation of such disciplines has been attributed to the studies conducted by the British during World War II, and perhaps to some extent, to the limited analytical studies carried out in World War I. However, the scientific origins of the body of knowledge possibly may be traced back to primitive mathematical models advanced by Quesnay in 1759 and Walrus in 1874. More sophisticated economic models were proposed by von Neumann in 1937 and Kantorovich (1939). The mathematical foundations of linear models were advanced by Jordan (1873), by Minkowski in 1896, and Farkas in about 1903. The seminal work on dynamic models was achieved by Markov (1856-1922) (Ref. 5). An important antecedent to the operations research approach was the work of Frederick W. Taylor (1856-1915), the primogenitor of scientific management (Ref. 6). Of significance is the use of multidisciplined teams by Taylor in his approach.

When compared to the complexity of many military analytical problems being undertaken today, problems posed during World War II were perhaps rather simple ones. Objectives then were immediate and rather well defined as the basis for selection of alternative systems, and applications then usually involved small numbers of independent factors and few variables. In contrast, consider the objective of maximum deterrence in optimizing US strategic weapons force posture. How much is deterrent value related to the effectiveness of weapons (quantity, reliability, accuracy)? How much is related to the vulnerability of weapon systems (hardness, sensitivity to electromagnetic pulse or "EMP", and also mobility)? How much is related to the political doctrine or policy of use implied in the planning strategy? Answers to these and other inquiries may be very necessary to establish the criteria in terms of proper objectives before any attempt can be made to search for preferred alternatives.

The exponential rate of advancement in technology in the past few decades has introduced a seemingly unlimited number of variables and parameters which, in turn, have introduced a different set or higher order amount of uncertainties. It is axiomatic that the nuclear age and the attendant cold war have now transformed both the orientation and structure of the military engagement, and resources and objectives associated with the conflict — Vietnam is an example.

Competition between military programs for the defense dollar is the subject of continuing debate at all levels up to and including the Congress of the USA. The operations research development programs conducted during World War II were characterized by somewhat critical needs, fairly obvious criteria for measuring benefits of particular systems, and minor cost considerations. Today, however, the systems analyst must not only demonstrate the measure of effectiveness versus cost for preferred system alternatives, but often may be required also to present elaborate evidence of benefits to be derived beyond the boundaries of the system. Thus, selection criteria not only must be considered in terms of the attainment of the specified objective but also must be considered in terms of the overall military system objectives and constraints concurrently. The pressures of limited budgets, advancing technology, and proliferation of alternatives can be expected to continue in the future, placing a premium on system analyses which can demonstrate cost benefits at successively higher system levels.

In 1966 an Executive Order (Ref. 7) to all departments and agencies of the Government described in detail the requirements for a formalized planning, programming, and budgeting procedure in each department and further demanded that decisions in the management of department programs be based on modern analysis techniques. One quote from the President's memorandum makes very clear the role that systems analysis is to play in Government decision making: "I intend on a Government-wide basis to question objectives, evaluate programs, seek alternatives, and make hard choices based on careful analysis. And I want you to do the same in your agencies."

2-3 BASIC TERMINOLOGY OF OPERATIONS RESEARCH, SYSTEMS ANALYSIS, AND COST EFFECTIVENESS

We first recall that a system is an integrated relationship of components aligned to establish proper functioning continuity towards the successful performance of a defined task or tasks. The disciplines we are interested in immediately have a nomenclature of their own, and we give here the definitions of some of the terms and phrases rather frequently encountered in Army systems analysis studies. Additional definitions of terms may be found in Refs. 3, 8, 9, 10, and 11.

1. *Materiel Systems Analysis* involves an analytical investigation, and quantitative appraisal and comparison of materiel programs or courses of action in terms of the effectiveness or benefits expected versus the costs required or anticipated. We note generally for systems analysis on materiel items or programs that benefit and costs of concern are considered on a life cycle basis. In addition, one also

may be faced with more current constraints such as the total R & D budget to be allocated among competing programs. Systems analysis may be applied at any point in the life cycle. In general, systems analysis takes the form of studies, projects, and investigations applying modern analytical and costing procedures. Such overall activities may take the form and title of cost-effectiveness studies (or analyses), parametric design/cost effectiveness type studies, cost-benefit studies, cost and performance studies, trade-off studies or optimum mix studies, quantitative inventory mix studies or analyses, product-improvement determinations, risk analyses, or qualitative assessments of approaches in functional activities and programs. In fact, the techniques of OR/SA are equally applicable to all of these. As a function, systems analysis seeks to contribute to the decision making process during any part of the life cycle process.

2. The *objective* is the purpose to be achieved by a systems analysis. Objectives vary with the level of suboptimization of the study, of course. It is clearly of great importance to determine the most appropriate objective for an analysis.

3. *Alternatives* are the means by which objectives can be attained. They need not be obvious substitutes for one another or perform exactly the same function. As an example, to protect civilians against air attacks, shelters, "shooting" defenses, or retaliatory striking power are all alternatives.

4. The *criterion* is the test of preferredness needed to tell how to choose one alternative in preference to the other. For each alternative, the criterion used evaluates the extent to which the objectives are attained in view of costs and resources used.

5. *Suboptimization* refers to maximizing or minimizing the effectiveness of a characteristic of one or more system elements rather than optimizing the system as a whole. It should be noted that suboptimization of all the system components or elements does not necessarily result in an optimum system.

6. *Effectiveness* is a quantitative measure of the degree to which a system can achieve a set of specific mission requirements. Various criteria — e.g., fractional target damage, number of targets destroyed, or costs — can be used as some of the measures of effectiveness (MOE).

7. *Efficiency* is a quantitative measure of system output attainable from a given amount of resources.

8. A *model* is a mathematical or computer coding representation of an operation or a physical relationship. The roles of a model are to provide a standard methodology of comparison of alternatives and to judge the extent to which the overall effectiveness of the system is attained.

9. The adjective *iterative* describes a procedure or process which is executed repeatedly until some condition is attained or satisfied.

10. *Risk,* in cost-effectiveness analysis and operations research, is a probabilistic description of possible outcomes.

11. *Uncertainty* implies a falling short of certainty, or something so far removed from it that one can only guess or surmise. Probabilities often are used to describe the degrees of uncertainty — although perhaps uncertainty should be regarded as a qualitative term, whereas probability or chance as the quantitative term. As an example, the probability that a foreign nation will continue to furnish the U.S. with base rights is an uncertainty.

12. *Simulation* is an "abstract" representation of a physical system, situation, or phenomena by a computer, model, or perhaps other device. We say the terms simulation and model are rather closely allied terms, since they often may be used more or less interchangeably, with simulation being the tool to use in those cases where the situation is so complex that mathematical models cannot adequately describe them. In a simulation, the model or computerized representation is manipulated to imitate the significant features, parameters, aspects, etc., of the situation being studied.

13. *Cost-effectiveness analysis* (study) is the process of comparing alternative solutions to mission requirements in terms of the value received (effectiveness) for the resources expended (costs).

14. *Design to unit production cost* is that cost established prior to the development of an item to guide design and to control program costs. It is the cost to the Government to acquire a production item based on a stated level of production. It is established early in development to insure from the start that engineers design and develop an item that will not cost more than the Army can afford to pay for the item.

15. *Fixed costs* are those amounts of cost that do not vary with the volume produced.

16. *Incremental costs* are the added charges resulting from a change in the level or nature of activity. Indeed, they can result from any kind of change — adding a new product, adding new machinery, changing distribution channels, etc. Although the incremental costs sometimes are interpreted to be the same as marginal costs, the term "marginal costs" has a much more limited meaning, referring only to costs of an added unit of output.

17. The *total system cost* is the total R & D costs, plus the investment and operating costs (for a specified number of years of operation) that are required to develop, procure, and operate the particular weapon system.

18. *Life cycle cost analysis* is an analysis of total resources required for R & D, investment, and operation of an item of equipment throughout its life cycle including a consideration of the salvage value or cost of disposal at the end of the life cycle. Life cycle costing is used to estimate the total resources required for a weapon system, and hence as a basis for a comparison of alternative systems. Rather frequently, life cycle estimates are based on a fixed time frame, e.g., ten years of full operation, instead of the actual system life time. A life cycle cost analysis may be performed at any time during the life of the system; however, past expenditures are considered as sunk, and only future costs are considered relevant to any decision. The reader might consult Ref. 9.

19. A *feasibility study* is an evaluation of the applicability or desirability of a system from the standpoint of advantages versus disadvantages in any given case; a study to determine the best time to install such a system; or a study to determine whether a plan is capable of being accomplished successfully.

20. A *parametric design/cost effectiveness study* is the process of formulating and evaluating a complete range of alternative intrasystem trade-offs of component designs to provide the optimum capability for fulfilling a given system mission considering both technical feasibility and cost effectiveness.

21. *Reliability* is the probability that an item or system will perform its intended function satisfactorily for a specified interval under stated conditions.

22. *Maintainability* is a measure of the ease and rapidity with which a system or equipment can be restored to operational status following a failure. It is a characteristic of equipment design and installation, personnel availability in the required skill levels, adequacy of maintenance procedures and test equipment, and the physical environment under which maintenance is performed. Maintainability is expressed as the probability that an item will be retained in or restored to a specified condition within a given period of time, when the maintenance is performed in accordance with prescribed procedures and resources.

23. *Survivability* is a measure of the degree, often expressed as a probability, to which an item or system will withstand a hostile (manmade) environment and not suffer abortive impairment of its ability to accomplish its designated mission.

24. *System effectiveness* may be described as a quantitative measure of the degree to which a system may be expected to achieve a set of specific mission requirements, and which may be expressed as a function of (a) availability, (b) dependability, and (c) capability:

(a) *Availability* is a measure of the degree to which an item (or is the chance that a system) is in

the operable or commitable state at the start of a mission when the mission is called for at an unknown (random) point in time.

(b) *Dependability* is a measure of the system operating condition during the performance of the mission, given the condition of the system at the start of the mission (availability). Reliability, survivability, and maintainability are included in the dependability concept.

(c) *Capability* is a measure of or the chance that a system will achieve mission objectives given the conditions during the mission (dependability). (The analysis of system effectiveness based on the concepts of availability, dependability, and capability was first advanced in the so-called WSEIAC study, Ref. 11).

25. A *Cost and Operational Effectiveness Analysis* (COEA) is a documented investigation of: (a) comparative effectiveness of alternative means of meeting a requirement for eliminating or reducing a force or mission deficiency; (b) the validity of a requirement in a scenario which has the approval of Headquarters US Army Training and Doctrine Command and Headquarters Department of the Army; and (c) the cost of developing, producing, distributing, and sustaining the alternatives in a military environment for a time preceding the combat application. (This is a definition of the US Army Training and Doctrine Command, and it emphasizes "operational" effectiveness" of a system, which is the degree to which the ability of the force to perform its mission is improved or degraded by the introduction of the system, operation, or tactic into the force.)

26. *Human Factors Engineering* is the application of scientific principles concerning human physical and psychological characteristics to the design of equipment so as to increase speed and precision of operations, provide maximum maintenance efficiency, reduce fatigue, and simplify operations.

27. A *sensitivity analysis* involves repetitions of the evaluation with different quantitative values of the parameters and/or changed assumptions in order to ascertain whether the conclusions are sensitive to departures from nominal values, i.e., to determine the "robustness" of the systems analysis carried out. Such an investigation may lead to new or improved models or deficiencies in the study.

2-4 ESSENTIAL ELEMENTS OF SYSTEMS ANALYSIS

The essential elements of systems analysis are the objective, alternatives, costs, criteria or measures of effectiveness, models, results, and recommendations. Those familiar with military studies will note the similarity of these items to the elements of a staff study — statement of the problem, assumptions, facts bearing on the problem, discussion, conclusions, and recommendations.

2-4.1 OBJECTIVE

Unless the objective is understood and the proper questions are asked to solve the specific problem, the whole analytical process may be invalid beyond this stage. Problem definition is often the weakest point in the analysis. Experience is indispensible here. Either the analyst has the needed experience, or he otherwise must depend upon the experience of others. The sponsor, who initiated the problem in the first place, must be involved in the exact definition of the objective.

2-4.2 ALTERNATIVES

The next step in formulating the analysis is to identify all conceivable logical and economical alternatives. If alternatives do not exist, there may be no basis for any analysis. However, doing nothing or continuing the status quo is usually one alternative. An alternative should remain under consideration until it is shown to be infeasible or is dominated by another alternative.

2-4.3 COSTS

There are a number of approaches to costing that can be used, depending upon the particular problem. Costs of resources in cost-effectiveness analyses are in monetary units, but the dollar evaluation of these inputs may have to be made in comparative terms with other systems that must be deferred or abandoned in favor of the system selected and their alternative uses in the economy. It is axiomatic, however, that all alternatives in an individual analysis must be costed by the same or equivalent method, even though individual systems have widely different cost documentation and data tabulation formats. In order to make meaningful cost-effectiveness comparisons of such systems, it is necessary that cost-building blocks be identified and aggregated on a common basis. Cost estimation procedures are covered later in this Handbook.

2-4.4 CRITERIA

Measures of effectiveness are the criteria that are common to the evaluation of all competing alternative systems and are used to evaluate each system in terms of the objective; therefore, they must be relevant and measurable. The proper choice of measures of effectiveness is often the most subtle and elusive element of the analysis technique.

2-4.5 MODELS

A model for judging the performance or effectiveness of a system, as already indicated, is an "abstract" representation of the system in its real world environment. A full discussion of models is given in later chapters. Naturally, the formulation of the model is related closely to the identification of the problem, and all constraints and assumptions used in the model should be delineated, with a proper presentation of the rationale for their consideration.

2-4.6 RESULTS

All too frequently, only members of the system study team and others who have been closely associated with the conduct of the analysis are cognizant of the objective, alternatives, facts, assumptions, and methodologies involved. The salient information needed to make and support a decision is not necessarily provided to the decision maker or to other reviewers. This is indeed paradoxical because systems analysts strive to use scientific techniques in their tasks which means, in turn, that all pertinent factors should be evident or repeatable. An inadequate statement of findings by such a team is usually the result of inexperience, inadequate documentation, and failure at the outset to consider the form the results should take and the use to which they will be put. Often, however, the cause may be attributed to inadequate support, which allocates insufficient time and money to reporting the results. The results must not be expressed in scientific jargon, but rather in terms of the practical objectives of management.

2-4.7 RECOMMENDATIONS

The analyst should make scientific recommendations as to a course of action if his recommendations are supported by adequate analyses. He should keep in mind, however, that his role is primarily that of providing information and clarification of issues. He should illuminate, and not exhort. He should display analytic conclusions in a parametric form.

2-5 ADVANTAGES AND LIMITATIONS

The major advantage in using systems analysis is that it provides a more rational basis for decision making than heretofore. Its use for weapon system decisions will promote better understanding of

broad defense objectives and functional responsibilities, as well as better understanding of management problems that have to be solved within the local organization. The clarification provided by systems analysis should reduce the burden of the decision maker or, perhaps more accurately, extend the capabilities and activities of the decision maker. Areas of uncertainty are highlighted and addressed in the most appropriate terms available.

Finally, sensitivity analysis is an important factor in the value of systems analysis. Optimized solutions can be only as valid as the available input data, but the sensitivity established for different variables provides the decision maker with an understanding of the risks involved in his overall decision processes.

It should not be expected that the discipline of systems analysis is a panacea that will solve all military problems. Many remain beyond precise mathematical formulation and, in such cases, experience and judgment are at a premium. For a counterview of operations research/systems analysis in the civil sector, see "Systems Analysis: No Panacea for Nation's Domestic Problems", Science **158**, pp. 1028-30 (24 November 1967).

2-6 SUMMARY

Weapon systems analysis provides a useful quantitative technique to help management analyze its operational and technical process, and represents quite an advance in the decision making process. It employs the tools of science to analyze alternative courses of action and to model the process studied in sufficient detail so that uncertainties are identified, calculated risks are quantitatively measured or evaluated, and the relative merit and/or probability of success of each alternative are addressed, with system costs being taken into account. Systems analysis represents one of the most promising techniques in recent years to aid management in many important and timely decisions.

Methods involved in systems analysis need not be complex. The results should not be difficult to understand nor should an unnecessarily high level of mathematics be employed. The use of appropriate models is indispensable, however. A frequent requirement is an extended point of view in the physical sense (the system instead of a component) and in the temporal sense (a period of perhaps 5 to 20 yr).

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CHAPTER 3

HANDBOOK CONTENT AND USE

An overview of the content and use of this Handbook is presented.

3-1 CONTENT OF PART ONE OF THE HANDBOOK

The *Army Weapon Systems Analysis* Handbook presents a comprehensive coverage of some of the more useful methods currently being applied for the purpose of analyzing weapon systems performance. Topics of interest are presented in a logical sequence, starting with an introduction to the subject, followed by presentation of methodology in a concise form for the analyst. Such methodology is presented in a fairly general way insofar as possible, not peculiar to any particular weapon system, and the techniques of analysis are illustrated by numerous examples. It is thought that this method of approach would lead to a more useful handbook and at the same time help to train the young analyst quickly, thus allowing him to undertake weapon systems evaluation studies on his own in an expeditious manner. Also, and wherever possible, every attempt has been made to organize the material so that the chapters are self-contained.

The field of military operations research in general—and even that of Army weapon systems analyses—has grown rapidly in the past twenty years. So many important methodological techniques of evaluation are encompassed that it is not possible to cover even the introductory material in a single volume. Therefore, the present *Part One* attempts to give the young weapon systems analyst some of the more basic knowledge, methodology, and topics which will continue in effect for some years of use as a handbook for the practicing analyst. A brief discussion of the organization and character of *Part One* of the *Army Weapon Systems Analysis* Handbook is contained in the paragraphs that follow.

Naturally, it was necessary to give the background and purpose of the handbook in Chapter 1; and then in Chapter 2 to define the terms operations research and systems analysis, and indicate the scope of application to military problems.

For background material, the weapon systems analyst must have an appreciation of the objectives and applications of weapon systems analyses (Chapter 4), and he must be well aware of necessary documentation and management of weapon system resources in the Army research and development programs (Chapter 5).

There is a very important and sometimes tenuous area concerning the role of analyst (Chapter 6) and that of the decision maker (Chapter 7). The systems analyst must always be an impartial, unbiased technical evaluator of weapon system performance and he must also make recommendations to the manager in a useful form, militarily speaking. For the process of systems analysis to work properly, the analyst on one hand and the decision maker on the other should have the proper appreciation for and respect each other's role since they represent counterparts of the team in any weapon acquisition process. The manager's broad military experience (Chapter 7) enables him to improve upon analytical judgments, and hence pinpoint more proper directions of future effort toward improved weapons.

Chapter 8 aims to cover the sphere of conflict — presenting a discussion of the types of war; the intensities of conflict and levels of commitment; and Army combat functions, objectives, operations, and trends. Current thinking on Army combat organizations to accomplish missions also are discussed.

The physical environment has a very pronounced and significant effect on weapon system performance in the field (Chapter 9). The skills of commanders are brought to test in the process of striking the most appropriate balance for tactics, weapon movement, and employment under conditions of adverse weather and terrain, and the added problems of an otherwise hostile environment. The keen analyst would do well to make proper judgments of the probable consequences of the physical environment in order that the type and level of analysis he applies are sufficiently valid.

In additiori to having proper appreciation for the physical environment, the weapon systems analyst must be thoroughly conversant with the fundamentals of offensive actions, defensive actions, and maneuver as they affect weapon performance (Chapter 10). Also, the capability to conduct target damage assessment during battle is important and may lead to the optimization of the "shoot-look-shoot" strategy.

The scope of weapon systems analyses is enlarged considerably by the problems of detecting, acquiring, locating, and engaging enemy targets (Chapter 11). The importance of timely detection and prompt engagement of targets cannot be stressed too much since efficient employment of friendly weapons depends critically on such advantages. Moreover, since target detection is a stochastic process, target detection and acquisition chances must be taken into consideration and quantified.

The Scenario (Chapter 12) is a narrative description of conflict, including the more important elements that affect the evaluation of weapons, and must be properly outlined in terms of probable engagements in two-sided conflicts. Variations in the scenario through manipulation of factors affecting weapon employment and engagement may lead to optimum tactics and weapons.

It is seen that the first twelve chapters undertake to acquaint the weapon systems analyst with a very complex background of knowledge that the military planner and field commander must be trained in for our national defense effort. It is only then that weapon delivery error characteristics and distributions (Chapter 13) start the analytical process of characterizing weapon effectiveness. The concepts of round-to-round "internal" or ballistic dispersion on one hand, and the weapon aiming errors or "external" dispersion on the other, must be properly modeled by the analyst. Estimation of parameters for delivery error distributions round out this first phase of the evaluation processes.

With weapon delivery error distributions in hand, the analyst then can undertake the analytical problem of estimating single shot hit probabilities for weapons fired at targets of various shapes (Chapter 14). Some of the mathematical details are simplified considerably by the use of approximate techniques, which nevertheless lead to sufficient accuracy. Comparisons with exact probability of hit calculations give the analyst confidence that certain approximations are entirely adequate.

The topics of target vulnerability and the lethality of weapons are introduced next in Chapter 15. Determination of target vulnerability and lethality depends on experimental or engineering processes, and the analyst must summarize such target characteristics in terms of "vulnerable areas" and "lethal areas".

Needless to say, the rate of fire of weapons (Chapter 16) is one of the more important characteristics which has to be taken into consideration and entered into weapon effectiveness equations. Increased rates of fire usually mean increased kill rates — an important overall measure of effectiveness of weapons.

The three weapon characteristics — hit probabilities, conditional chances that hits are kills (vulnerability and lethality), and rates of fire — represent the basic input parameters for the discussion of stochastic and other types of duels (Chapter 17). These quantities are combined to determine kill rates (or attrition rates) which, in turn, determine the chance of winning a duel. Moreover, the chances of winning a duel can be studied for various firing strategies so that the operational performance of weapons can possibly be optimized.

Response time of a weapon system is defined as the delay experienced between the time a firing demand is placed upon the system and the time at which the weapon is actually fired at a target (Chapter 18). Although response time for a weapon is sometimes overlooked, it may be important in many applications; it also represents a variable which requires stochastic quantification.

The weapon systems analyst must possess expertise in the area of fuzing problems (Chapter 19), for the fuze is a key component of a projectile or missile and has decided effects on delivery accuracy and target vulnerability or warhead lethality. The reliability and safety of fuzing systems represent major considerations to be reckoned with by the analyst, and the distributions of terminal trajectory geometry have to be established for overall system effectiveness.

It should be clear that a major area of interest for the practicing analyst relates to multiple round hit probabilities, target coverage, and target damage. Proper analytical treatment of volleys or salvos of rounds fired from weapons is an involved subject for any exact treatment, although one which can be handled satisfactorily and often simplified by approximate theories. The aim of Chapter 20 is that of presenting the analyst with a thorough foundation of available methods for calculating kill probabilities when ballistic and aim errors enter the problem in various modes or applications of firing multiple rounds in combat.

Chapter 21 brings us to the increasingly important topics of reliabilty, life testing, availability and maintainability of systems, and reliability growth. There is hardly any weapon system today which can or should escape analyses in terms of these fields of interest, and the analyst must be highly competent in evaluations associated with life-time or failure distributions such as the exponential, the Weibull, the lognormal, and the binominal probability distributions. Statistical testing for high reliability and safety of systems is introduced in Chapter 21, as well as a brief account of reliability growth. A major topic, and current effort, concerning systems today is that of being able to place confidence bounds on the true, unknown reliability of complex systems; accordingly, coverage of the more recent and accurate techniques is given for the practicing analyst. Finally, reliability now is often one of the major, or sole, characteristics of some weapon systems, and hence may represent a prime activity for the systems analyst in many applications of his knowledge.

An introduction is given to the subject of mobility, maneuverability, and agility of weapon systems and vehicles in combat areas. Some of the problems of quantifying such parameters are discussed in Chapter 22. The terms — mobility, maneuverability, and agility — have for many years almost defied definition, quantification, and adequate modeling; yet the analyst finds that these subjects often are major problems to be dealt with in his evaluations. For example, the optimum relation, or trade-off, among firepower, mobility, and armor protection for combat vehicles is still of great interest after all the years of effort on the subject; accordingly, suitable model developments for such problems must continue. Hopefully, Chapter 22 will give the analyst a good introduction to, and some confidence in dealing with, battlefield mobility.

Although the principles of military operations research and statistics — and more recently reliability — have been applied now for many years to the evaluations of weapon systems, it was not until recently, so to speak, that these techniques of stochastic analyses have made adequate inroads into many problems of logistic planning and support (Chapter 23). Nevertheless, it has become increasingly clear that the design, development, production, and deployment of weapon systems need to consider the highly dependent problems of logistic planning and support since these factors deserve much effort toward the accomplishment of new and improved evaluations. The supply and support of combat units in the field naturally involve life-time or reliability probability distributions, and hence so

does inventory management, the medical workloads in the theatres, or ammunition supply, et al. Indeed, there are so many problems demanding solution in logistic planning and support that these are clearly very fertile areas of application for the military operations research analyst.

After discussions of many basic topics in current military operations research endeavors for Army systems evaluations, *Part One* ends with the WSEIAC effectiveness analysis model for overall treatment and unification of analytical techniques concerning system performance (Chapter 24). WSEIAC, the Weapon Systems Effectiveness Industry Advisory Committee, methodology was a cleverly conceived approach for Air Force evalution problems which should be kept in mind for some possible Army applications also. The WSEIAC model unifies the concepts of system availability (readiness), dependability (reliability), and capability (performance); moreover, it can easily be applied in some generality to give a single, overall figure of merit, or measure of effectiveness, for the system evaluated. Hence, the attractiveness of such a thought for the weapon systems analyst.

Thus the 24 chapters of *Part One* should provide analysts with a suitable backgrond of numerous techniques and much information for the broad field of Army weapon systems analysis. With this type of training the sytems analyst should be sufficiently well equipped to proceed to many of the ensuing procedures in *Part Two* for comparing or expressing the performance of weapon systems.

3-2 USE OF PART ONE OF THE HANDBOOK

It is not easy to advise someone just how to carry out a valid, appropriate, and comprehensive analysis for each military operations research type of problem. The involved nature of many weapon systems applications is such that a "cookbook" will not guarantee proper selection of models, alternatives, or optimum use of resources available. Nevertheless, the present Handbook, and its coverage of topics and models, should provide an adequate base from which an analyst can accomplish many worthwhile goals in the process of evaluating weapons or military systems.

As we have really already indicated, the aim of *Part One* of the Handbook is to give the new systems analyst a good understanding of the more important aspects of the very broad problems in military technology and weapon acquisition processes, along with typical analytical techniques contributing to the evaluation of weapons in combat usage. Also, the practicing systems analyst must reacquaint himself from time to time with various aspects of the problem of fielding reliable and effective weapons. He also will have a continuing need for a handbook which describes suitable methodology for weapon evaluation processes as well as indications of the directions of future effort toward any improved theories or models of analysis. Indeed, many relevant references are listed, and a fairly extensive bibliography is given for many chapters and subjects of interest.

Part One has been prepared so that the chapters are rather self-contained and much of the textual material can be used easily for the teaching of weapon systems analysis techniques in the classroom.

Finally, there has long existed the need for some systematic account of the methods of Army weapon systems analyses, and this volume hopefully should approach the problem of filling the gap or satisfying some of the need for such usage or application.

CHAPTER 4

OBJECTIVES AND APPLICATIONS OF WEAPON SYSTEMS ANALYSIS

This chapter describes the objectives and benefits sought by the Army through the performance of weapon systems analysis. The chapter also addresses the progression of a weapon system from concept through development and deployment to disposal.

4-1 GENERAL

Weapon systems analysis is a tool applied to insure that any expenditure of dollars buys the most appropriate security for the U.S. Since World War II, the major powers of the world have developed and deployed, in short periods of time, generations of successively more complex weapon systems. Each generation has been dependent upon a more sophisticated techology than the preceding one, and each involves an unprecedented commitment of time and resources. Consequently, decisions to develop a weapon system tend to become irrevocable in the short run (three to five years) and the penalties for "choice error" become rather severe, e.g., unfavorable strategic balance in the long run (eight to ten years). Systems analysis must not only maximize weapon system effectiveness for resources expended, but also should minimize the risk of choice error in weapon system selection.

4-1.1 THE WEAPON SYSTEMS ANALYSIS ENVIRONMENT

From a fundamental perspective, the value of a weapon system is constrained by:

1. Finite national resources which can be committed to military programs
2. The temporal effectiveness of a weapon system
3. The lack of military experience in the use of new weapon systems.

It is impractical to try to meet all possible contingencies which could threaten national security by procurement of more and more weapon systems at the expense of the myriad of competing nonmilitary demands for finite national resources. Further, each high technology weapon system requires an increasing level of sophistication in both operating and maintenance support systems, thereby consuming a greater proportion of available resources, which otherwise could have been directed toward the development of additional systems (opportunity costs).

Weapon-effectiveness-lifetime has become one of the most important concepts in a weapon systems analysis. Weapon system development currently requires about six to eight years, and many selection decisions must be made with only limited knowledge of the technological achievements being made by other world powers. As a result, the security value of a new weapon system may become marginal well before the deployment stage due to the development of countermeasures or other improvements in the capabilities of potential adversaries.

US national security is becoming more dependent upon weapon systems that have never been tried under combat conditions. Most of the weapon systems currently supporting our strategic forces and some supporting our tactical forces fall into this category. However, a great deal of systems analysis effort has been directed toward establishing doctrines and concepts of use for the employment of these weapon systems. Within these constraints, the purpose of systems analysis is to provide assistance to military judgment by identifying those weapon systems which would contribute most to military effectiveness per unit of expenditure.

In general, weapon systems analyses help defense managers determine a course of action from the choices available to them. These studies address the full problem of the decision maker by searching out the objectives a weapon system must meet, identifying alternative concepts which might meet these objectives, and comparing the alternatives in light of their potential value.

4-1.2 FUNDAMENTAL OBJECTIVES OF WEAPON SYSTEMS ANALYSIS

Weapon systems analyses generally address one or more of the following objectives:

1. Determination of the objectives of a weapon system development program. What is the intended mission of the completed weapon system? What is the policy governing system availability and survivability in combat? When is the system to be operable? In how many and in what types of military units is the system to see service?
2. Establishment of the performance requirements to be satisfied by the operating weapon system. In quantitative terms, what are both operational and support characteristics which the operating weapon system should possess to perform its mission?
3. Evaluation of alternatives. What are the costs, performance, and effectiveness characteristics of competing weapon systems concepts, or those already operating, which are possible candidates for further development and deployment? Studies of this type also may develop additional alternatives for evaluation.
4. Identification of critical influences. What are the key issues and assumptions underlying the performance, effectiveness, cost, and scheduling estimates made for the alternatives? What are the trade-offs available in developing the system? In general, such studies attempt to present the alternatives in such a manner that decision makers can understand their similarities and differences while allowing them to use their own criteria to make a choice.

A weapon systems analysis successfully meeting any one of these four objectives has made a significant contribution toward the fundamental objective of more efficient use of resources.

4-2 WEAPON SYSTEMS ACQUISITION

In order to conduct effective analyses in support of weapon systems acquisitions, it is essential that the analyst become familiar with the policies, objectives, and other facets of the weapon systems acquisition process. These guidelines and objectives provide the basic goals of analysis and the fundamental approach to the conduct of weapon systems analysis within the US Army. The basic policies for weapon systems acquisition are given in Army Regulation (AR) 1000-1, *Basic Policies for Systems Acquisition* (Ref. 1). AR 1000-1 states that the materiel needs of the Army generally are satisfied by one of four methods:

1. Product improvement of current standard equipment
2. Buying equipment already developed—commercial (domestic or foreign) or military (other Services or Allies)
3. Modification of commercially available items
4. Initiation of a new materiel development program.

The objective of the weapon systems acquisition process is to minimize costs in acquiring weapon systems whose performance is adequate to meet operational requirements or select the best of competing systems for a fixed level of effort. Materiel system design will emphasize simplicity, austerity, supportability, interoperability with systems of Allies, and planned future growth potential to accommodate anticipated future needs when additional cost for such growth can be justified.

The Headquarters of the Department of the Army (HQDA) has developed a model describing the steps in weapon systems acquisition from inception through development into production. As might be

expected, not all systems require all the steps presented in the model, e.g., the acquisition of already developed or commercial equipment requires fewer steps and simpler procedures than improving or developing a system. The four phases of weapon system acquisition are identified in AR 70-1, *Army Research, Development, and Acquisition* (Ref. 2) and Department of Army Pamphlet (DA Pam) 11-25, *Life Cycle System Management Model for Army Systems* (Ref. 3) as:

1. Conceptual Phase
2. Validation Phase
3. Full-Scale Development Phase
4. Production and Deployment Phase.

AR 15-14, *System Acquisition Review Council Procedures* (Ref. 4) and AR 71-9, *Materiel Objectives and Requirements** (Ref. 5) provide guidance for the preparation of the various documents and analyses (to be discussed in the paragraphs that follow) required during the acquisition process.

For the purpose of presentation to and approval by the Congress, the Army budget is broken into 12 major groupings, called appropriations (Ref. 6). Funds for weapon systems acquisition are drawn primarily from three of the appropriations—research, development, test, and evaluation (RDTE); procurement; and operation and maintenance, Army (OMA). The procurement appropriation (PA) provides the money for the production of weapon systems, and the OMA appropriation provides the funds to operate and maintain the systems after production.

The title "research, development, test, and evaluation" very well describes the use of this appropriation. Much of the work of a systems analyst with regard to weapon systems acquisition is done prior to the production and deployment phase, and is supported with RDTE funds. Therefore, it is well that the analyst become more familiar with the RDTE appropriation than the others that support weapon system acquisition. The RDTE program—also referred to as program six—is organized with the following categories:

1. Category 6.1, Research. This program includes scientific study and experimentation directed toward increasing knowledge and understanding in those scientific fields that are related to national security needs. It provides fundamental knowledge for the solution of identified military problems. It also provides part of the base for subsequent exploratory and advanced developments in the defense-related technologies of new or improved military functional capabilities.

2. Category 6.2, Exploratory Development. Included in this program are efforts directed toward solving specific military problems from fairly fundamental applied research to sophisticated prototype hardware, study, programming, and planning efforts. The dominant characteristic of this effort is that it is pointed toward specific military problem areas with a view toward developing and evaluating the feasibility and practicability of proposed solutions and determining their parameters.

3. Category 6.3, Advanced Development. Advanced development includes all projects which have moved into developing hardware and nonmateriel technological prototypes or techniques for experimental or operational test. Advanced development is divided into two subcategories—nonsystem advanced development (6.3A) and system advanced development (6.3B). Nonsystem advanced development (6.3A) is advanced development efforts addressing technological options or uncertainties. These efforts are categorized by the development of component, subsystem, technology demonstrators; or nonmateriel technological demonstrators which have a potential application to a variety of similar generic end products rather than for application to one specific, well-defined system. System advanced development (6.3B) is the design of items being directed toward hardware for test or experimentation.

*Title is correct August 1977 but is subject to change when the AR is revised.

4. Category 6.4, Engineering Development. This program includes those development projects which are being engineered for military service use but have not been approved for procurement or operations.

5. Category 6.5, Maintenance and Support. The maintenance and support program includes the effort directed toward supporting installations or operations required for general research and development use. It includes general and administrative expenses for personnel costs, travel, and other expenses for performance of RDTE functions at command headquarters level (except HQDA) and general and administrative activities; support of test ranges; support of scientific, technical, and management information functions and activities; investment for minor construction; investment cost for special purpose equipment peculiar to research and development (R&D) functions not identifiable with specific RDTE projects; and, when in support of two or more research and development projects, maintenance support of laboratories and operation and maintenance of test aircraft and ships.

6. Category 6.7, Operational System Development. RDTE funds included in other programs (see Chapter 5 for a discussion of the programs which constitute the Five Year Defense Program (FYDP)) are identified as operational system development funds. This category includes research and development effort directed toward development, engineering, and test of systems, support programs, vehicles, and weapons that have been approved for production and service employment.

In the progression through the acquisition cycle, approval to proceed from one phase to the next is contingent upon a formal review process to assure that previous phase objectives are met. The DA decision body conducting the reviews is designated the Army Systems Acquisition Review Council (ASARC). For major systems requiring Department of Defense (DOD) approval, a formal review will be conducted by the Defense Systems Acquisition Review Council (DSARC). Nonmajor materiel acquisitions require only in-process reviews (IPR)—face-to-face meetings between systems developers, users, trainers, and logisticians. Fig. 4-1 shows the acquisition phases and decision milestones. Fig. 4-2 identifies the decision level of the Defense Department, decision body, and monetary thresholds for approval of major and nonmajor materiel acquisitions. As shown in Fig. 4-2, the Decision Coordinating Paper (DCP), a DOD acquisition management summary document, is the official Office of the Secretary of Defense (OSD)/Army decision-recording document. It defines the latitude of the Army in managing a program within the approved thresholds of cost, performance, and schedule. Also recorded in it are the decisions of the Secretary of Defense (SECDEF). At the DA level, the decision approval document corresponding to the DCP is the Army Program Memorandum (APM) in which is recorded the final decisions of the Army authority. Both documents are similar in content and define approved thresholds of cost, performance, and schedules acceptable to OSD and the Army.

4-2.1 CONCEPTUAL PHASE

This is the first phase of a weapon system acquisition. Threat projections, technological forecast, and Joint and Army plans are examined to determine future Army needs. These needs are compared with programmed capabilities to ascertain capability gaps. In most cases, these capability gaps can be translated into science and technology objectives (STO's) designed to overcome specific problems. The STO's are one of the requirement documents supporting the Army research and development program. During the conceptual phase, research (category 6.1), exploratory development (category 6.2), and nonsystem advanced development (category 6.3A) are conducted in response to the STO's.

In addition to the STO, a Mission Element Need Statement (MENS) is required for major systems under DOD management. The MENS, discussed in Refs. 7 and 8, is prepared by the appropriate Service (Army, Navy, or Air Force) stating the need for a program initiation and submitted to the SECDEF for approval.

PHASE	CONCEPTUAL	VALIDATION	FULL-SCALE DEVELOPMENT	PRODUCTION AND DEPLOYMENT	
RDTE PROGRAM CATEGORY	6.1 → 6.2 → 6.3A →	6.3B →	6.4 →		
HARDWARE CONFIGURATION	Experimental Prototype Breadboard	Advanced Development Prototype/Brassboard	Engineering Development Prototype	Initial Production Items	Full Production Items
SUPPORTING DOCUMENTS	STO MENS*	LOA & OAP	ROC & AP		
TESTING		DT I OT I	DT II OT II	DT III OT III	Post Production Testing
	ASARC I DSARC I	IPR ASARC II DSARC II	IPR ASARC III DSARC III	IPR ASARC IIIa DSARC III	

STO = Science and Technology Objective
* MENS = Mission Element Need Statement
(Only for systems under DOD management)
LOA = Letter of Agreement
OAP = Outline Acquisition Plan
ROC = Required Operational Capability

AP = Acquisition Plan
DT = Development Test
OT = Operational Test
IPR = In-Process Review
ARSAC = Army Systems Acquisition Review Council
DSARC = Defense Systems Acquisition Review Council

Figure 4-1. Weapon Systems Acquisition Process

Type Acquisition	Level of Approval for Major Decisions	Decision Body	Decision Recording Document	General Monetary Thresholds (Millions)	Remarks
Major Programs	Secretary of Defense	DSARC & ASARC	Decision Coordinating Paper	75+ RDTE / PA* 300+	Special reviews may be held to resolve issues that are delegated by the ASARC/DSARC review process.
	Vice Chief of Staff US Army	ASARC	Army Program Memorandum	As Directed	Special reviews may be held to resolve issues that are delegated by the ASARC review process.
Nonmajor Programs	HQDA DCSRDA	IPR	Section I, Acquisition Plan	As Directed	
	Command (Materiel Developer)	IPR	Section I, Acquisition Plan	0-2 RDTE / PA* 0-3/FY or 0-15 for 5 yr Program.	IPR recommendations will be approved by the materiel developer. Formal HQDA participation limited to annual budget review, unless otherwise directed.

* PA = Procurement Appropriation

Figure 4-2. Materiel Acquisition Management

4-2.1.1 Letter of Agreement

When the materiel developer and the combat developer agree that a materiel concept has sufficient interest, importance, and operational and technical potential to warrant the commitment of resources to obtain additional information, they will prepare a Letter of Agreement (LOA). The purpose of the LOA is to insure agreement between the combat developer and the materiel developer on the nature and characteristics of the proposed system and the investigations needed to develop and validate the system concept; to define the associated operational, technical, and logistical support concepts; and to promote synchronous interaction between the combat developer and the materiel developer during the conduct of these investigations. The LOA is the document of record to support effort in the system advanced development (6.3B) subcategory of the RDTE program. Also a LOA may support the nonsystem advanced development (6.3A) subcategory of the RDTE program if the conceptual application to improved or new materiel systems can be defined adequately. The format for the LOA is given in Ref. 5.

4-2.1.2 Concept Formulation Package

Based upon agreements specified in the LOA and after advanced development has progressed to the point where operational and technical feasibilities have been demonstrated (and in some areas possibly even confirmed by performance test data), the combat developer and the materiel developer will prepare a Concept Formulation Package (CFP). Much of the weapon systems analysis work done during weapon systems acquisition is in support of the CFP. Included in the CFP are the systems analysis studies which follow:

1. Trade-Off Determination. The Trade-Off Determination (TOD) is a document prepared by the materiel developer to convey the apparent technical feasibility of a potential system, including technical risks associated with each approach, estimated RDTE, and procurement costs and schedules.

2. Trade-Off Analysis. The Trade-Off Analysis (TOA) is prepared jointly by the combat developer and materiel developer to determine which technical approach(es) offered in the TOD is (are) best.

3. Best Technical Approach. The Best Technical Approach (BTA)—prepared jointly by the materiel developer and combat developer—identifies the best technical approach(es) based on the results of the TOD and TOA, and an analysis of trade-offs among integrated logistical support concepts, technical concepts, life cycle costs, and schedules.

4. Cost and Operational Effectiveness Analysis. The Cost and Operational Effectiveness Analysis (COEA) is a documented investigation of:

a. Comparative effectiveness of alternative means of meeting a requirement for eliminating or reducing a force or mission deficiency

b. The validity of the requirement in a scenario which is approved by Headquarters, US Army Training and Doctrine Command (HQ TRADOC), and HQDA.

c. The cost of developing, producing, distributing, and sustaining each alternative in a military environment for a time preceding the combat application.

4-2.1.3 Outline Acquisition Plan

The Outline Acquisition Plan (OAP) is the last document which must be prepared during the Conceptual Phase. It contains the materiel system concept agreed upon by the materiel developer and the combat developer. Program decisions and appropriate analysis of technical options and plans for development of the materiel system concept during the Validation Phase are recorded in it. Also, it supports the LOA by providing a definitive plan for management of the advanced development effort to

achieve the materiel objective addressed by the LOA. The OAP will consist of the same sections as the Acquisition Plan (AP) (par. 4-2.2.2.2); however, the specific content, scope, and level of detail will be tailored to the needs and stage of development for the particular program. The sections of the OAP are (Ref. 9):

1. Section I, System Concept Summary. The System Concept Summary will contain the LOA and any implementing instructions which may be issued by HQDA or the developers (materiel and/or combat). Also, all significant decisions (IPR/ASARC/DSARC) will be recorded in this section.

2. Section II, System Concept Requirements and Analyses. This section contains the system concept as agreed upon in the LOA and any additional information which will assist in definition of the concept. Also, it contains the emerging CFP and the organization and operational concept. The requirement stated in this section is the requirement of record which supports the advanced development effort. All refinements and modifications to the materiel system concept(s) are documented in this section.

3. Section III, Plans for System Development. This section consists of appropriate tasking and supporting plans required for the acquisition of the weapon system. The plans establish the interface required of all participants and include a milestone schedule, event oriented and time phased, indicating how the total materiel acquisition program will be managed as a continuous process. Section III includes the following plans:
   a. Technical Development Plan
   b. Management Plan
   c. Financial Plan
   d. Facilities and Resources Plan
   e. Producibility Plan
   f. Advance Procurement Plan
   g. Threat Support Plan.

4. Section IV, Coordinated Test Plan. This section provides a coordinated test plan (CTP) of all testing to be accomplished during system acquisition. Critical issues and criteria against which tests will be designed and data evaluated are included. Specific guidance for the CTP is contained in Ref. 10.

5. Section V, Plan for Personnel and Training Requirements. This plan includes a description of the training devices, methods, and media to include television, training extension courses, skill qualification tests, and field manuals and other requirements necessary to provide for both individual and unit-crew training.

6. Section VI, Plan for Logistic Support. This section contains a broad general plan for logistic support including milestones for verification. Also, it includes identification of alternative support concepts; anticipated critical supportability issues (to include those for testing), recommended reliability, availability, and maintainability (RAM) objectives; life cycle support cost goals; and the anticipated logistic environment in which the system is expected to operate.

4-2.2 VALIDATION PHASE

A decision by an appropriate level decision review body (DSARC or ASARC) signals the start of the Validation Phase. This phase consists of those steps required to verify preliminary design and engineering, accomplish necessary planning, analyze trade-off proposals, resolve or minimize logistic problems identified during the conceptual phase, prepare a formal requirements document, and validate a concept for full-scale development. Advanced development prototypes (brassboard) should

be used and tested during this phase to provide data to estimate the military utility, cost, environmental impact, safety, operational effectiveness and suitability; and to refine the configuration prior to entering full-scale development. These prototypes should represent a complete system, subsystem, or component to permit a thorough evaluation. The quantity and level of prototype/hardware and software validation depend on the nature of the program, and the risks and trade-offs involved.

4-2.2.1 Testing

4-2.2.1.1. General

The formal testing to be conducted during acquisition is introduced during the Validation Phase. Testing is conducted to demonstrate how well the materiel system meets its technical and operational requirements; to provide data to assess developmental and operational risk for decision-making; to verify that the technical, operational, and support problems identified in previous testing have been corrected; and insures that all critical issues to be resolved by testing have been adequately considered. For the purpose of this policy, testing is considered to be grouped into two basic test categories:

1. Development Testing. Development Testing (DT) is that test and evaluation conducted to demonstrate that the engineering design and development process is complete; demonstrate that the design risks have been minimized; demonstrate that the system will meet specifications; and estimate the military utility of the system when introduced. DT is planned, conducted, and monitored by the materiel developer and the results are reported directly by the developer to the appropriate decision review body (DSARC/ASARC/IPR).
2. Operational Testing. Operational Testing (OT) is that test and evaluation conducted to estimate the military utility, operational effectiveness, operational suitability, and need for any modification of the prospective system. In addition, OT provides information on organization, personnel requirements, doctrine, and tactics. All OT is the responsibility of and is managed by the Operational Testing and Evaluation Agency (OTEA) who presents test results directly to the appropriate decision review body. The OT is conducted by operational and support personnel of the type and qualifications of those expected to use and maintain the system.

The DT and OT are divided into three phases—titled DT1, OT1, etc.—which are discussed in pars. 4-2.2.1.2, 4-2.2.1.3, 4-2.3, and 4-2.4. Although there are three phases of DT and OT, it may not be necessary for a given weapon system to undergo all three phases of both tests.

4-2.2.1.2 Development Test I

Development Test I (DT I) is conducted during the Validation Phase to demonstrate conclusively that technical risks have been identified and that solutions are in hand. Components, subsystems, brassboard configuration, or advanced development prototypes are examined to evaluate the potential application of technology and related design approaches prior to entering Full-scale Development.

4-2.2.1.3 Operational Test I

Operational Test I (OT I) is conducted on brassboard configurations, experimental prototypes, or advanced prototypes to provide data leading to the decision to enter Full-scale Development. OT I estimates:

1. The potential of the new item/system in relation to existing capabilities
2. The relative merits of available competing prototypes/systems from the aspect of military utility
3. The adequacy of the concepts for employment, supportability, organization; doctrinal, tactical, and training requirements; and related critical issues.

4-2.2.2 Documentation

4-2.2.2.1 Required Operational Capability and Letter Requirement

The basic document to support entry into full-scale development is the Required Operational Capability (ROC). It is prepared by the combat developer, coordinated with the materiel developer, and submitted to HQDA for decision. An executive summary of the supporting COEA is submitted with the ROC. The ROC is also the document proposing procurement of materiel already developed. The length of the document should be kept to a minimum; four pages seen to be a reasonable goal for most systems. The ROC contains:

1. Statement of need
2. Time frame
3. Threat/operational deficiency
4. Operational/organizational concept
5. Essential characteristics
6. Technical assessment
7. Logistic assessment
8. Life cycle cost assessment including design-to-cost goals.

The Letter Requirement (LR) is an abbreviated procedure for acquisition of low value items and may be used in lieu of the ROC where applicable. The LR is not appropriate for system components.

4-2.2.2.2 Acquisition Plan

The contents of the OAP are incorporated into the Acquisition Plan (AP). The AP is the document which records program decisions; contains the approved materiel requirement; and provides appropriate analysis of technical options and life cycle plans for development, testing, production, training support, and logistic support of the weapon system. It is a dynamic document and is refined and updated during the acquisition process and ensuing life cycle when product improvement or other changes to the materiel system occur. The AP has the same six sections as found in the OAP.

4-2.3 FULL-SCALE DEVELOPMENT PHASE

At the conclusion of the validation phase and after approval of the ROC, the decision to enter full-scale development is made by the appropriate decision authority (DSARC/ASARC/IPR). During this phase, the system—including all items necessary for the support of it—is fully developed and engineered, fabricated, tested, and a decision is made whether the item is acceptable to enter the inventory. Concurrently, nonmateriel aspects required to deploy an integrated system are developed, refined, and finalized. The intended output of this phase is a prototype system which closely approximates the final product, the documentation necessary to enter the production and deployment phase, and the test results to support the decision to enter production.

Producibility engineering and planning (PEP) is conducted early in this phase to assure producibility of the system prior to quantity procurement. PEP measures include, but are not limited to, developing technical data packages, designing special purpose production equipment and tooling, and computer modeling or simulation of the production process to better assess producibility.

During this phase, Development Test II (DT II) is conducted. This test provides the final technical data for determining the readiness of the system to enter production. DT II is characterized by using engineering and scientific approaches under controlled conditions to provide quantitative and qualitative data for use in an independent DT evaluation. This test demonstrates whether engineering is reasonably complete and that solutions to all significant design problems are in hand.

Operational Test II (OT II), also conducted during this phase, has as a goal the estimation of the military utility, operational effectiveness, and operational suitability of a system in as realistic an operational environment as possible. The test, conducted by a troop unit with normally assigned soldiers, uses controlled field exercises to examine the organization and doctrine, logistic support, threat, communication and control, and tactics associated with the planned operational employment of the system, stressing continuous tactical operations.

4-2.4 PRODUCTION AND DEPLOYMENT PHASE

The primary purpose of this phase is to produce efficiently and deliver to the operating unit an effective, supportable system in a timely manner and at minimum costs. The results of DT II and OT II are evaluated by the appropriate decision review body (DSARC/ASARC/IPR) to determine if the system is ready to enter low-rate initial production portion of this phase. The production is authorized to obtain a quantity of representative production items for use in Development Test III (DT III) and Operational Test III (OT III). The main purposes of these tests are to determine that deficiencies found in previous tests have been corrected and that the system is ready for full production and issue to units.

The results of DT III/OT III are reviewed by the appropriate level decision body (DSARC/ASARC/IPR) and, if acceptable, the system enters full-scale production. Operational units are trained and provided with the system.

4-3 OWNERSHIP PHASE

Although major emphasis is on systems analysis support to materiel acquisition decisions, systems analysis support will continue to be required during the ownership (operations and maintenance) lifetime of the materiel system. Systems analyses during this phase are concerned with the management of operations and are generally performed either to improve the logistic support for the materiel system or to increase its efficiency in performing either a primary or secondary mission. Where a new secondary mission is proposed, the analysis may be used to investigate and recommend modification in a component, subsystem, or in the overall system.

The last application of systems analysis in the life of a materiel system is that in which the system is shown to be less cost effective in comparison with alternatives. The cost-effective alternative is then developed and introduced into the force structure as the replacement of the old system. This study is also the first in the life of a new system.

REFERENCES

1. AR 1000-1, *Basic Policies for Systems Acquisition.*
2. AR 70-1, *Army Research, Development, and Acquisition.*
3. DA PAM 11-25, *Life Cycle System Management Model for Army Systems.*
4. AR 15-14, *System Acquisition Review Council Procedures.*
5. AR 71-9, *Materiel Objectives and Requirements.**
6. AR 37-100-76, *The Army Management Structure—Appropriations and Funds Available for Obligation Expense and Expenditures.*
7. DOD Directive 5000.1, *Major System Acquisitions.*
8. DOD Directive 5000.2, *Major System Acquisition Process.*
9. AR 70-27, *Outline Development Plan/Development Plan/Army Program Memorandum/Defense Program Memorandum/Decision Coordinating Paper.**
10. AR 70-10, *Test and Evaluation During Development and Acquisition of Materiel.*

*Titles are correct August 1977 but are subject to change when the AR's are revised.

CHAPTER 5

DOCUMENTATION AND MANAGEMENT OF WEAPON SYSTEM RESOURCES

An account is given of some of the goals, documentation, and management of weapon systems resources in the Department of Defense (DOD) and the Army.

5-1 MANAGEMENT GOALS OF THE SECRETARY OF DEFENSE

The overall goals of the Secretary of Defense (SECDEF) are succinctly stated in two general instructions of President Kennedy to Robert S. McNamara (then SECDEF) in the early 1960's (Ref. 1):

1. To determine what military forces are necessary to support our foreign policy
2. To procure and support these forces as economically as possible.

Within this broad charter, certain more precise management goals have emerged as criteria for maintaining continuity, balance, and flexibility among the requirements, programs, and resources of the several claimants for Department of Defense (DOD) budget support. These goals are:

1. Planning and budgeting by each Service (Army, Navy, Air Force, and Marine Corps) should be predicated upon accomplishment of major missions by the particular Service involved. In implementation, this means devoting priority attention to major force-oriented issues as reflected by estimates of future requirements and by the sequence of programs.

2. The system must provide an ability to relate resource inputs to military outputs; i.e., clearly defined relationships must be established between budget apportionments (resource inputs), which group all resources of a similar nature in accordance with Congressional precedent, and program aggregation built around the force-oriented missions.

3. The relationship between the budget cycle and the defense planning cycle should provide for continuity and mutual reinforcement of long-term objectives by shorter-term requirements and by application of resources to requirements. While, on the one hand, budget actions are on an annual cyclic basis, the defense planning cycle must project potential requirements out to some 20 yr in the future.

4. The management system should provide for continuous review and appraisal of the approved programs and to facilitate control should provide for periodic reporting of program progress and status.

5. Provisions must be made to furnish both physical and financial data in forms suitable for making cost and operational effectiveness studies of alternative force structures proposed to satisfy recognized requirements.

5-2 THE PLANNING, PROGRAMMING, AND BUDGETING SYSTEM

The management system devised to satisfy the goals listed in the preceding paragraph is called the Planning, Programming, and Budgeting System (PPBS) (Refs. 2 and 3). It was developed to bridge the gap between the planning and budgeting activities, thereby tying all facets of the defense effort together—relating national security objectives to strategy, strategy to military forces, forces to resources, and resources to costs, all within the same conceptual framework and all projected several years into the future. This relationship is depicted in Fig. 5-1. Only by integrating its activities can so large and complex an organization as the DOD manage its operations efficiently.

DOD Directive 7045.7 (Ref. 2) defines the Planning, Programming, and Budgeting Systems as, "An integrated system for the establishment, maintenance, and revision of the FYDP [Five Year Defense

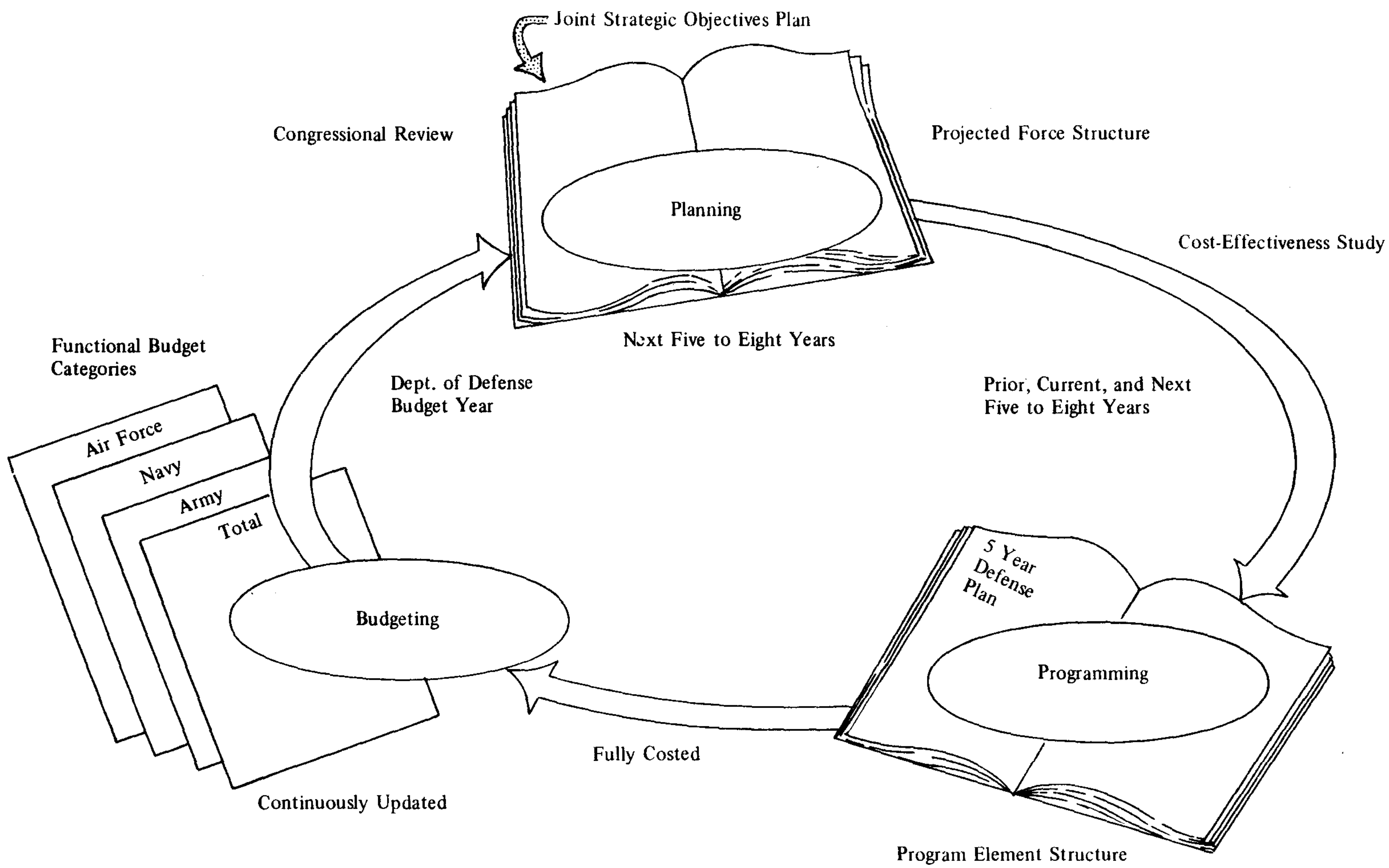

Figure 5-1. DOD Planning, Programming, and Budgeting System (PPBS)

Plan] and the DOD budget." A total of 34 major planning, programming and budgeting events occurs within a 30-month PPBS cycle resulting in the budget for a particular fiscal year. Since at any particular point in time three fiscal years are being addressed, each of the 34 events occurs once each 12 months. Roughly the cycle may be compared to a three-part musical round in which the second and third singers start as the preceding singer reaches the next phase. Fig. 5-2 shows the sequence and relationship of the events by DOD organizational level and month of scheduled occurrence within the 30-month cycle. Pars. 5-2.1 through 5-2.3 discuss separately the planning, programming, and budgeting phases.

As stated in the DOD Directive 7045.7 (Ref. 2) definition of the PPBS, the Five Year Defense Plan (FYDP) is one of the major documents of the system. The FYDP, which is the official summary of programs approved by the SECDEF, includes force, manpower, and cost data covering the prior, current, and succeeding fiscal years. The force structure includes data for the prior fiscal years, current fiscal year, budget year, and seven succeeding fiscal years. Cost and manpower data are included for the prior fiscal years, current fiscal year, budget year, and the four succeeding fiscal years. The FYDP documents resources under one of the ten defense programs listed in Table 5-1. Each program comprises a major management-emphasis area of the DOD. Programs one through six and ten have a force-mission or combat-mission orientation while programs seven, eight, and nine have a support orientation.

5-2.1 PLANNING

Planning forms the basis for assessing defense resource needs. The planning phase of the PPBS defines military objectives and devises the strategy to implement Presidential guidance and at the same time creates the requirement for forces. As such, planning underlies the SECDEF guidance and provides the basis for force development and program formulation. The Joint Strategic Planning System (JSPS) defines the DOD planning activities and AR 1-1 (Ref. 3) discusses Army planning. The JSPS and Army planning are discussed in pars. 5-2.1.1 and 5-2.1.2, respectively.

5-2.1.1 The Joint Strategic Planning System (JSPS)

The JSPS—a responsibility of the Joint Chiefs of Staff (JCS)—addresses all planning aspects of intelligence, strategy, requirements, and capabilities and takes into consideration Allied as well as US forces. The JSPS promulgates the planning product in a series of seven documents as follows:

1. Joint Intelligence Estimate for Planning (JIEP)
2. Joint Long Range Estimative Intelligence Document (JLREID)
3. Joint Long Range Strategic Study (JLRSS)

TABLE 5-1. THE TEN PROGRAMS OF THE FIVE YEAR DEFENSE PROGRAM

PROGRAM NUMBER	TITLE
1	Strategic Forces
2	General Purpose Forces
3	Intelligence and Communications
4	Airlift/Sealift
5	Guard and Reserve Forces
6	Research and Development
7	Central Supply and Maintenance
8	Training, Medical, and Other General Personnel Activities
9	Administrative and Associated Activities
10	Support of Other Nations

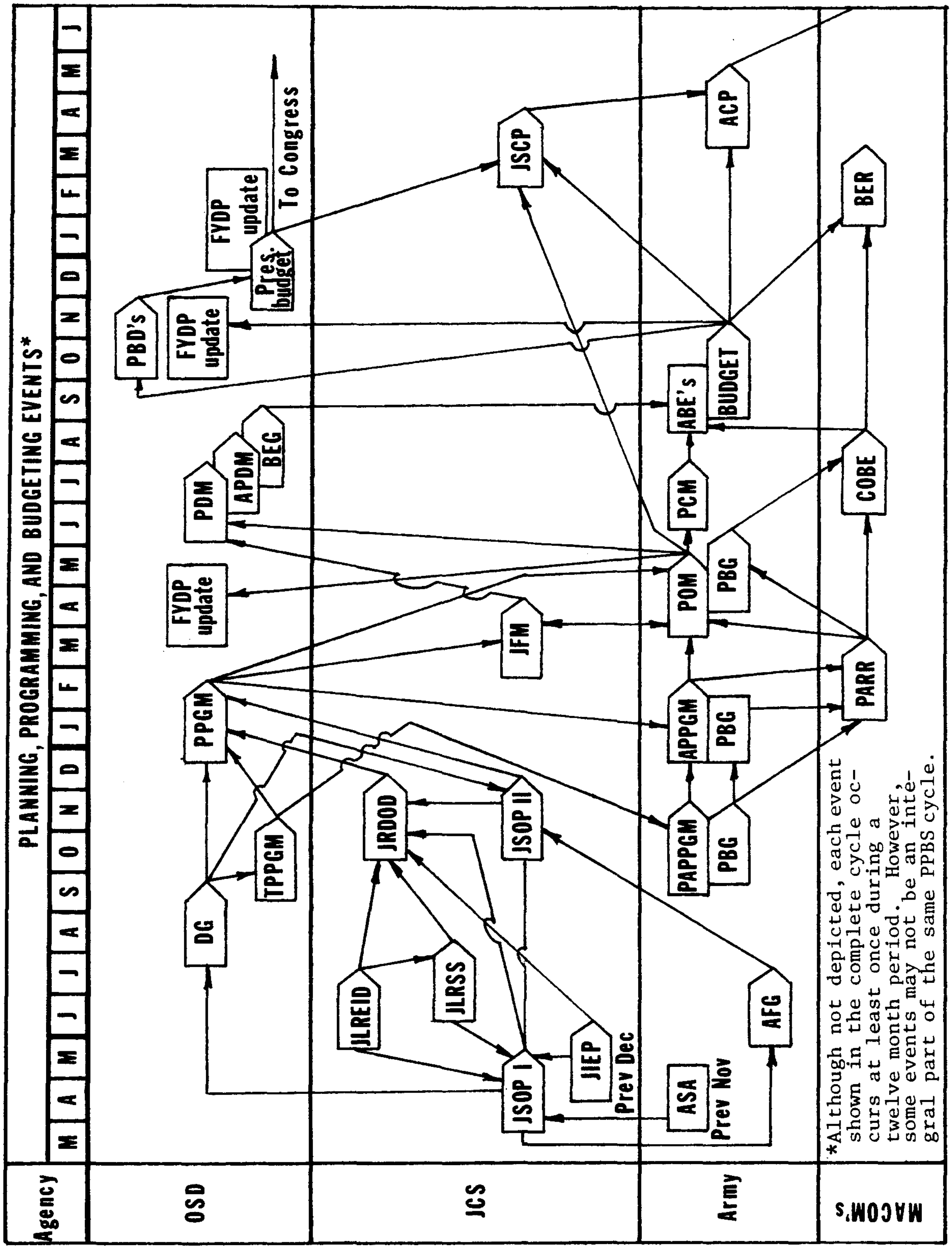

Figure 5-2. Planning, Programming, and Budgeting System Cycle

4. Joint Strategic Objectives Plan (JSOP)
5. Joint Research and Development Objectives Document (JRDOD)
6. Joint Force Memorandum (JFM)
7. Joint Strategic Capabilities Plan (JSCP).

The JLREID summarizes factors and trends affecting world power relationships in the long-range period (10-20 yr), including an intelligence estimate of the likelihood and capabilities of important foreign nations to undertake courses of action which would materially affect the national interests of the United States. This document—published annually in July—provides the principal basis for the JLRSS and the long-range period of the JRDOD.

The JLRSS is a long-range source document that addresses the strategic implications of worldwide and national economic, political, social, technical, and military trends. It deals with national objectives, policies, and military constraints and relates these to world and regional trends. As a source document, it is intended to stimulate more sharply focused strategic studies and to be useful in developing military policies, plans, and programs having long-range implications. The JLRSS is reviewed annually in August and updated at least every four years by the JCS.

The JRDOD advises the SECDEF on the composition and priorities of the DOD Research and Development (R&D) program. Proposed R&D objectives are derived from an analysis of the threats as outlined in the JLREID and the JIEP as well as the strategy, capabilities, and force recommendations in the JSOP. The JRDOD addresses both the long-range and the mid-range (2-10 yr) period. It is reviewed annually and revised at least every four years.

The JIEP contains global and regional appraisals including estimates of the external and internal threats to countries of significance to the United States and estimates of the Warsaw Pact and Asian Communist Military forces. It focuses on the mid-range and short-range (0-2 yr) periods and provides the principal intelligence basis for the JSOP, JFM, and the mid-range period of the JRDOD.

The JSOP provides the President, the National Security Council (NSC), and the SECDEF with advice of the JCS on the military strategy and force structure required in the mid-range period to attain the national security objectives of the United States. The JSOP consists of two volumes as follows:

1. Volume 1, Strategy and Force Planning Guidance, is published in May. It states national security objectives and the basic military objectives derived in consideration of US National interest and commitments and the estimate of the threat. Strategy advice is provided in the form of military appraisals and strategic concepts on both a worldwide and regional basis. Also, this volume contains (1) mid-range force planning guidance to the Chiefs of the Services and commanders of the unified and specified commands, and (2) concise evaluations of the military risk associated with the military strategy and force planning guidance. This document is considered by the SECDEF in the preparation of the Defense Guidance (DG) (the DG is discussed in par. 5-2.1.3).

2. Volume II, Analysis and Force Tabulations, is published each December. It develops the major US force requirements to execute the strategy. In addition, this volume recommends (1) major US, Allied, and friendly objective force levels to execute the strategy, and (2) appraises the capabilities of major programmed forces to meet the threat and execute the strategy successfully at a prudent level of risk considering reasonable attainability and fiscal responsibility. Also, it presents conclusions and recommendations of the JCS concerning mobilization requirements.

The JFM is published in the latter part of April of the second year and, although it is part of the JSPS, it overlaps into the programming phase. It receives significant input from the Planning and Programming Guidance Memorandum (PPGM) (the PPGM is discussed in par. 5-2.2) issued in February, and it and the Program Objective Memorandum (POM) (the POM is discussed in par. 5-2.2) provide each other reciprocal input. The JFM provides JCS recommendations on the fiscally-

constrained force levels and support programs that will require trade-off decisions by program managers during the current year.

The JSCP which is prepared late in the PPBS cycle—April of the third year—provides JCS guidance to the commanders of the unified and specified commands and the Chiefs of the Services for the accomplishment of military tasks based on projected military capabilities, estimates of the threat during the short-range period, and current guidance of the SECDEF. The JSCP specifically tasks the commanders of unified and specified commands with the preparation of contingency plans. This document, reviewed annually and republished at least biennially, consists of two volumes:

1. Volume 1, Concepts, Tasks, and Planning Guidance, presents the basic plan and provides a statement of the national security objective and derived military objectives, presents global and regional concepts, sets forth capabilities, assigns tasks, and provides planning guidance to the commanders of unified and specified commands.
2. Volume II, Forces, identifies the forces available for the development of operational plans.

5-2.1.2 Army Planning

Army long-range planning consists of long-range studies to meet specific requirements and the Army contributions to three JSPS documents: the JLREID, the JLRSS, and the JRDOD. The specific titled Army plans address the mid-range and short-range periods. These plans provide guidance and assistance to the Secretary of the Army (SA) and serve the planning needs of the Chief of Staff of the Army (CSA), major commands (MACOM's), and Army component commanders of the unified commands. The titles of these plans are:

1. Army Strategic Appraisal (ASA)
2. Army Force Guidance (AFG)
3. Army Capabilities Plan (ACP).

The ASA addresses strategic issues facing the military in the mid-range period and provides a basis for Army positions and initiatives related to those strategic issues. It supports Army participation in the development of the JSOP, Volume I, and the Defense Guidance.

The AFG provides guidance to the MACOM's and the Army Staff agencies for objective force planning in the JSOP, Volume II, and summarized key aspects of the program force.

The ACP, like the JSCP, is prepared late in the PPBS cycle—May of the third year. It provides mobilization and operational planning guidance to the Army staff agencies, MACOM's, and Army component commands of unified commands for the probable employment of Army forces in the short-range period. This document reflects specific tasks and capabilities attainable within existing programs and budget limitations. The ACP implements the JSCP and uses the same planning assumptions.

5-2.1.3 Defense Guidance

The Defense Guidance (DG) is the principal policy, strategy, and objectives planning document issued by the SECDEF. It develops from the foreign policy guidance of the President and the unconstrained military strategy and force planning guidance in the JSOP, Volume I. The DG covers all phases of management and, in broad terms, addresses force development to achieve the capability required to carry out national policy and strategy. It indicates parameters for force composition and capabilities, and it presents scenarios and planning constraints. It provides input into the JSOP, Volume II, and controls Army programming primarily through the influence of the DG on the Tentative Planning and Programming Guidance Memorandum (TPPGM) and the subsequent Planning

and Programming Guidance Memorandum (PPGM). The TPPGM and the PPGM are discussed in par. 5-2.2.

5-2.1.4 Interrelationship of Events

Fig. 5-3 shows the interrelationship of the planning documents by agency and month in which each is normally issued or reviewed. As shown on Fig. 5-3, the planning documents and actions culminate in February of the second year with the PPGM—a key guidance document that begins the programming phase of the PPBS.

The planning input to the PPGM begins with the publication of the ASA in November of the first year—15 months earlier. It and the JIEP, published a month later, provide direct input to the JSOP, Volume I. The JSOP, Volume I, in turn provides much of the basis for the DG and the preparation of JSOP, Volume II. Just as the ASA provides Army input in the JSOP, Volume I, the AFG summarizes key aspects of objective force planning for the JSOP, Volume II. Three JSPS documents—the JLREID, JLRSS, and JIEP—influence the content of the PPGM indirectly through the JRDOD. In summary, the DG, JSOP, Volume II, and the JRDOD provide direct input into the PPGM, which, as mentioned, is a key guidance document that initiates the programming phase.

Three documents, each a part of the JSPS or the Army planning documents, actually fall in other phases of the PPBS. The JFM, although a product of the JSPS, actually is a programming document. The JSCP and the ACP, both planning documents prepared during the budgeting phase, describe current force capabilities as a basis for contingency military planning.

5-2.2 PROGRAMMING

Program development is the translation of the Office of the Secretary of Defense (OSD) planning and programming guidance into a comprehensive and detailed allocation of forces, manpower, and funds for a five-year period. Program development culminates in the annual publication of the POM which presents to OSD the Army's proposal for a balanced allocation of its resources within specified constraints. Program approval is the process whereby the Army POM is reviewed by OSD and modified by SECDEF decisions to establish the Army's five-year program. The first year of the approved program serves as the basis for development of the Army Budget Estimate (ABE). (The ABE is discussed in par. 5-2.3.) The programming events are shown in Fig. 5-4.

5-2.2.1 Transition from Planning to Programming

As stated in par. 5-2.1.4, the programming phase of the PPBS formally begins in February of the second year with the publication of the PPGM. Several documents are prepared prior to February of the second year to assist in the transition from planning to programming. In September, the OSD publishes the Defense Guidance document which provides information regarding structuring the forces to support approved military strategy. (Defense Guidance is discussed in par. 5-2.1.3) Other activities during the period from September to February prepare the way for programming.

By the end of October, a computer-assisted technique called Total Army Analysis (TAA) is used to translate gross aggregations of force-structure requirements into more detailed elements of force content. The result bears the name of the Program Objective Memorandum (POM) force. It lists the major combat, combat support, and combat service support forces; the ammunition and resupply requirements; and the associated manpower spaces needed to satisfy given scenarios.

In November, the OSD issues the Tentative Planning and Programming Guidance Memorandum (TPPGM). This memorandum reflects previously coordinated service inputs, and consolidates and

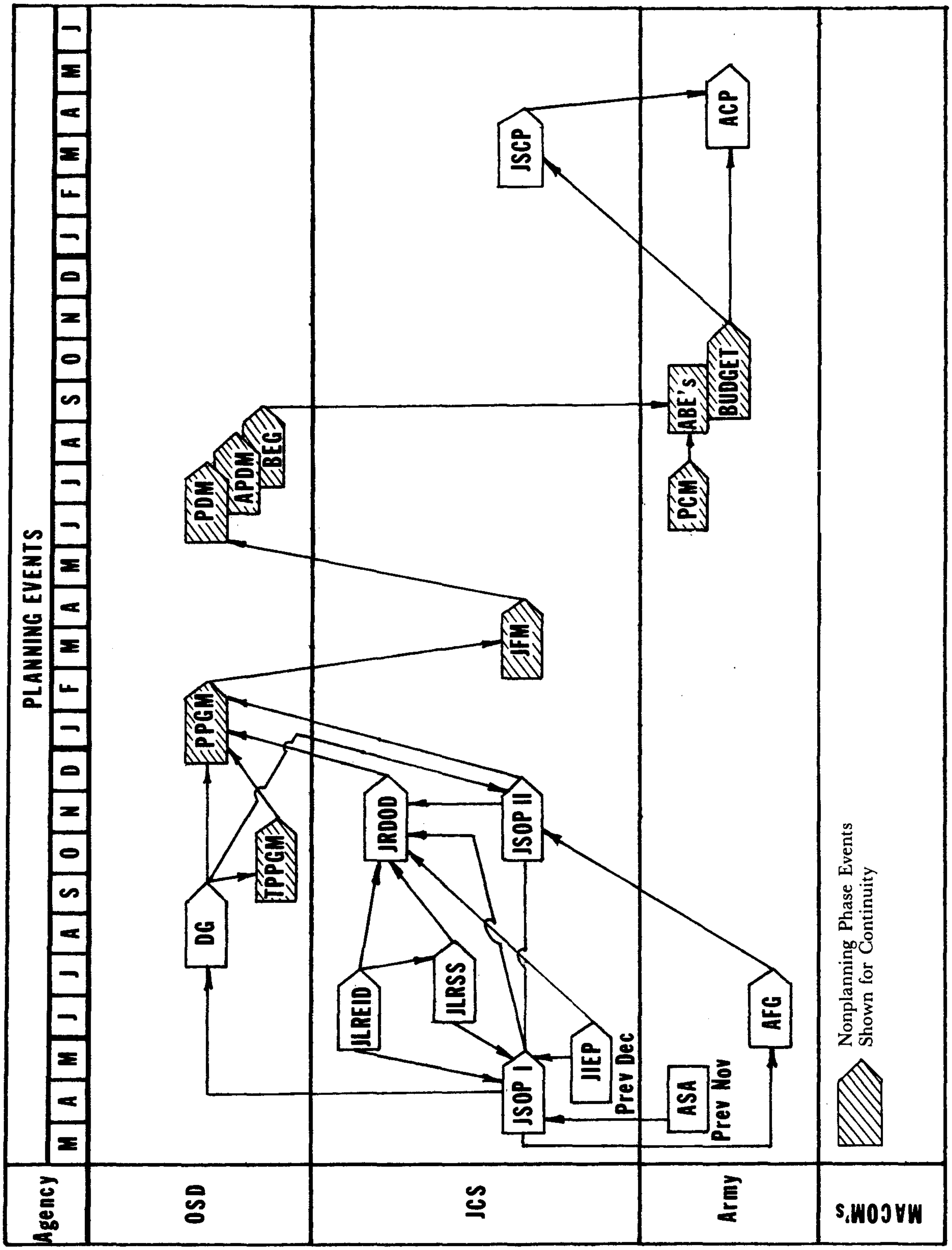

Figure 5-3. Planning Phase Events

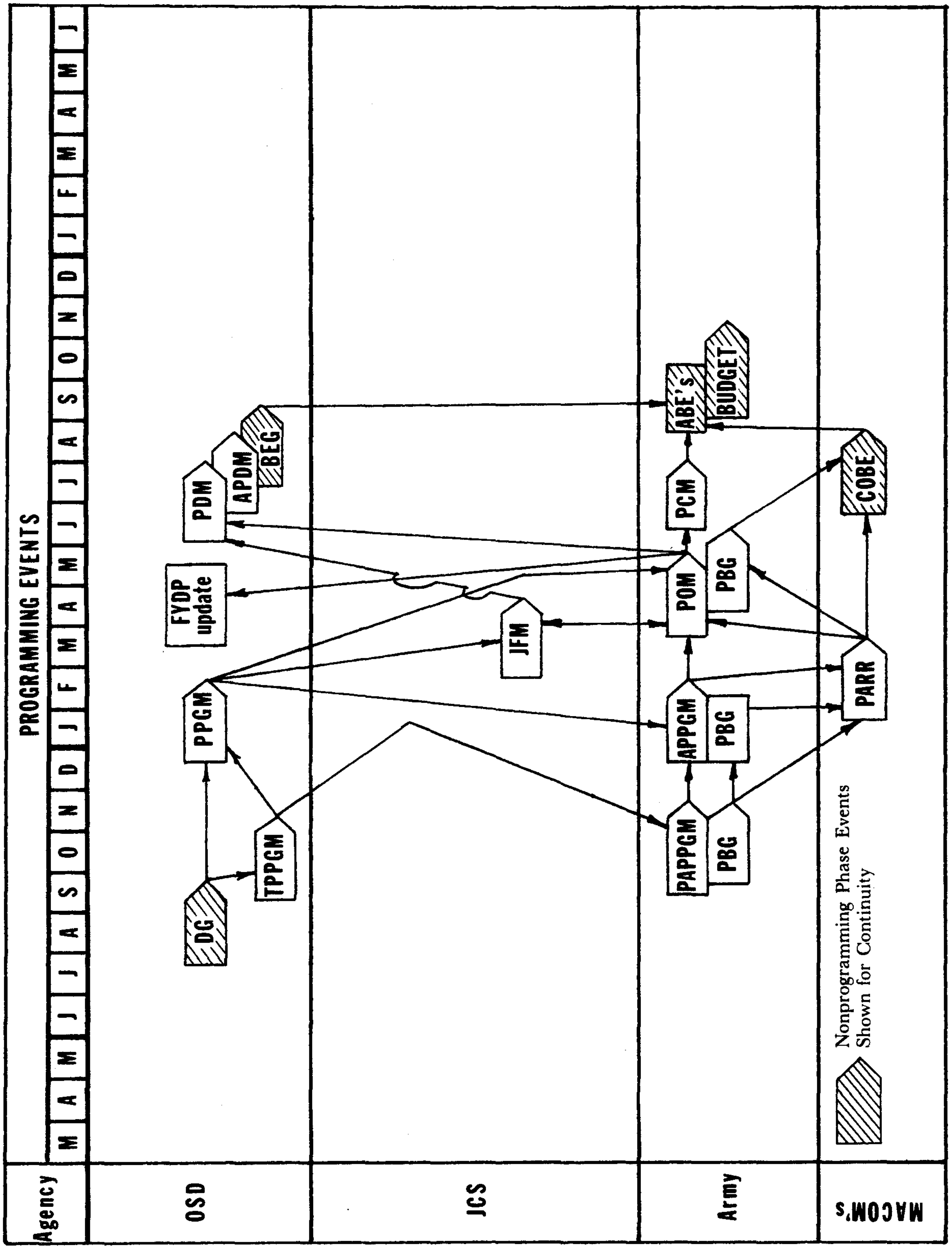

Figure 5-4. Programming Phase Events

elaborates earlier OSD guidance in both qualitative and quantitative terms. It defines tentative fiscal guidelines for each of the five program years and provides fiscal constraints affecting force levels, manpower, and supporting missions. The TPPGM is an advance document. Rather than present final guidance, it identifies potential issues likely to arise during OSD program reviews. Additionally, the TPPGM affords early opportunity to assess quantitative impacts of changes in past policy and proposed policy, and to assess assumptions on which subsequent resource allocations could be based.

The Preliminary Army Planning and Programming Guidance Memorandum (PAPPGM)—issued in mid-November shortly after receipt of the TPPGM—complements the TPPGM. This document interprets OSD guidance in consonance with that of the Chief of Staff and the Secretary of the Army. Also, it expresses Army objectives and resource constraints to Army staff agencies and major commands. Concurrent development of the TPPGM and PAPPGM brings all participants into the program formulation process early and enables them to influence both the form and substance of the final program guidance.

The first of three separate distributions of Program and Budget Guidance (PBG) occurs immediately after publication of the PAPPGM in October. The PBG—issued to selected MACOM's—provides the participating MACOM's with an update of resource data covering the first of the five-year programs to be addressed in the POM. The MACOM's that are involved in the programming process use the TPPGM, the PAPPGM, and the PBG update to develop the command programs for the first year of the POM.

The second distribution of the PBM occurs in January of the second year. This is a complete PBG revision that addresses the current year and budget year for all commands and agencies. It also contains program-year data for the MACOM's. The budget-year data provide guidance for preparing the Command Operating Budget Estimate (COBE) (see par. 5-2.3 for a discussion of the COBE); the program-year data give final guidance for preparing the Program Analysis and Resource Review (PARR) documents.

5-2.2.2 Program Development

Publishing the Planning and Programming Guidance Memorandum (PPGM) during February of the second year formally initiates the programming phase of the PPBS. This document provides final guidance for preparation of Service programs. In developing this guidance, the SECDEF considers the DG; JSOP, Volume II; Service and JCS comments on the TPPGM; and events of the previous several months. Final specific fiscal guidance for POM programs is provided in this document.

Shortly after the PPGM is distributed, the program development guidance of the Chief of Staff of the Army is published as the Army Planning and Programming Guidance Memorandum (APPGM). This memorandum complements the Defense PPGM and provides the Army Staff specific guidance for the POM preparation. Also, the fiscal guidance received from the OSD is reflected in the projected appropriation levels announced in this document.

Program Analysis and Resource Reviews (PARR's) provide the selected MACOM's a vehicle for making a formal contribution to the Army Program. The PARR's—submitted annually to Headquarters, Department of the Army (HQDA)—are based on the October PBG, the PAPPGM, and specific guidance in the program year portion of the January PBG. The PARR's address resource requirements and deficiencies in terms of impact on mission acomplishment. A major benefit of the PARR's for both the Commands and the Army Staff derives from highlighting in the document necessary adjustments in resource levels for a three fiscal-year period—current, budget, and program year—with emphasis on the program year.

The Joint Force Memorandum (JFM) which basically comprises a joint Program Objective Memorandum (POM) is submitted by the Joint Chiefs of Staff to the OSD in April.* While a programming event, the JFM is nevertheless a product of the JSPS as discussed in par. 5-2.1. This memorandum reflects the preliminary structuring of program forces developed by the Services as influenced by the guidance in the PPGM. Moreover, the resource data in it came largely from the Services. Therefore, as previously stated, the JFM tends to have the characteristics of a compilation of Service POM's. Nevertheless, the document does address issues raised in the POM's of the Services—a feature of significant benefit to the SECDEF during program review and approval.

Next in the programming sequence is the publication of the Program Objective Memorandum. In effect, the POM is what the previously described activities were developing. It is prepared concurrently with the program review and approval process. This document culminates the annual program development process of the Army and transmits to OSD the Army's proposal for resource allocation in consonance with OSD program guidance. The POM, published in May, describes all aspects of Army programs highlighting the forces, manpower, materiel acquisition, equipment distribution, and logistic support required to meet the strategy and objectives specified by the SECDEF. As a related action, the Army updates the Five Year Defense Plan to coincide with the POM resource allocations.

In late May the third distribution of PBG is made. This update reflects revised guidance based on the Army Staff's review of the PARR's and the content of the POM submitted to OSD earlier in the month. This PBG addresses the budget and program year.

During development and publication of the POM many issues and actions are identified that must be resolved or taken prior to submission of the Army budget in October or publication of the next POM the following May. These required follow-on actions are compiled in the Program Continuity Memorandum (PCM) and issued in mid-June to the Army Staff for action or referral to a MACOM.

5-2.2.3 Program Review and Approval

The first major action in the review and approval portion of the programming phase is the receipt, beginning in June, by the Army of issue papers from the OSD. These papers evaluate the POM proposals in terms of their relation to the policy and planning guidance; the balance between force structures, modernization, and readiness; and efficiency trade-offs. Also, they define issues, list alternatives, and evaluate the capabilities and costs of those alternatives in terms of their ability to accomplish a DOD mission. The primary Army Staff agency responsibility for review and comment is determined by the nature of the issue paper. The responsible agency prepares the draft reply, and processes it through the CSA to the Secretary of the Army for signature and forwarding. The OSD issue papers with the comments from the services are submitted to the SECDEF for decision. The decisions made as a result of reviewing the issue papers are released in July as a Program Decision Memorandum (PDM).

The PDM is analyzed by the Army Staff to identify those issues for which reclamas should be prepared and those major issues which the SA should discuss with the SECDEF. Also, the Army participates in the development of the JCS reclama of the Service PDM's. After receiving the written reclama from the Army, the SECDEF holds a major issues meeting with the SA and CSA. After

*The JFM is projected for replacement by the Joint Program Assessment Memorandum (JPAM) in FY78. The JPAM will probably be issued in June following publication of the POM's by the Services and contain detailed risk assessments of the programs developed.

reviewing the reclama and the major issues discussion, the SECDEF publishes in August the Amended Program Decision Memorandum (APDM) containing his final program decisions. This memorandum approves the POM as amended by the specific decisions of the PDM and the major issues meeting and is the final action of the programming phase of the PPBS.

5-2.3 BUDGETING

The Department of the Army budget is the detailed expression of approved plans and programs in terms of resource requirements. The budgeting phase of the PPBS is divided into two major parts—budget formulation and budget execution. The budgeting events are shown in Fig. 5-5.

Budget formulation is the process of developing detailed fund estimates to support plans and programs. The primary purpose of budget formulation is to obtain the funds required for program execution. This process terminates with Congressional enactment of the Authorization and Appropriation Bills.

Budget execution is the development and maintenance of operating and investment budgets necessary for execution of approved programs. It includes apportionment request and allocation, obligation, expenditure, and reporting of funds.

5-2.3.1 Budget Formulation

The MACOM's receive a copy of the POM when it is published in May which they use along with additional information furnished by the May PBG as bases for formulating command requirements for the budget year. In mid-July, the commands submit these requirements to HQDA as the first of a two-part Command Operating Budget Estimate (COBE)*. The input provides the Army Staff with the detail essential to the development and evaluation of budget estimates. The submission not only supports the formulation and justification of the Army Budget Estimates but also furnishes the commands an opportunity to inform the Army Staff of any foreseeable changes in previously projected program requirements.

Following approval of the POM, the OSD prepares the Budget Estimates Guidance (BEG) which provides specific guidance for development and submission of budget and authorization estimates to the OSD and the Office of Management and Budget (OMB). It identifies any supplemental request to be submitted for the current budget year together with items to be considered in the request. The BEG establishes the POM, as modified by the APDM, as the basis for budget authorization estimates. It provides guidance concerning outlay rules and the submission and use of FYDP data. Also it provides guidance concerning the use of inflation rates and the use of DOD level contingency funds. In addition, the document specifies the level of detail to be provided Congress for those submissions requiring authorizing legislation.

Based upon information from the POM, COBE, PDM and APDM, and BEG, the Appropriations Directors, who are members of the Army Staff, prepare their proposed budgets. During August and September, the Appropriation Directors present and defend their proposed budgets before the Budget Review Committee, the Comptroller of the Army, and the Under Secretary of the Army to insure adequate implementation of approved plans and programs.

Following the presentation of their budgets by the Appropriation Directors, the Director of the Army Budget (DAB), a member of the Office of the Comptroller of the Army, presents the overall Army Budget Estimates (ABE) to the Vice Chief of Staff of the Army (VCSA), CSA, and SA for final Army Approval. When the ABE's are approved by the SA, they become the Army budget.

The Army budget is provided to the OSD and the OMB at the end of September. Early in October the FYDP is updated to reflect the budget estimates and the PDM/APPM. Starting in October, the

*The second part submitted in mid-August addresses operating requirements. (see par. 5-2.3.2.)

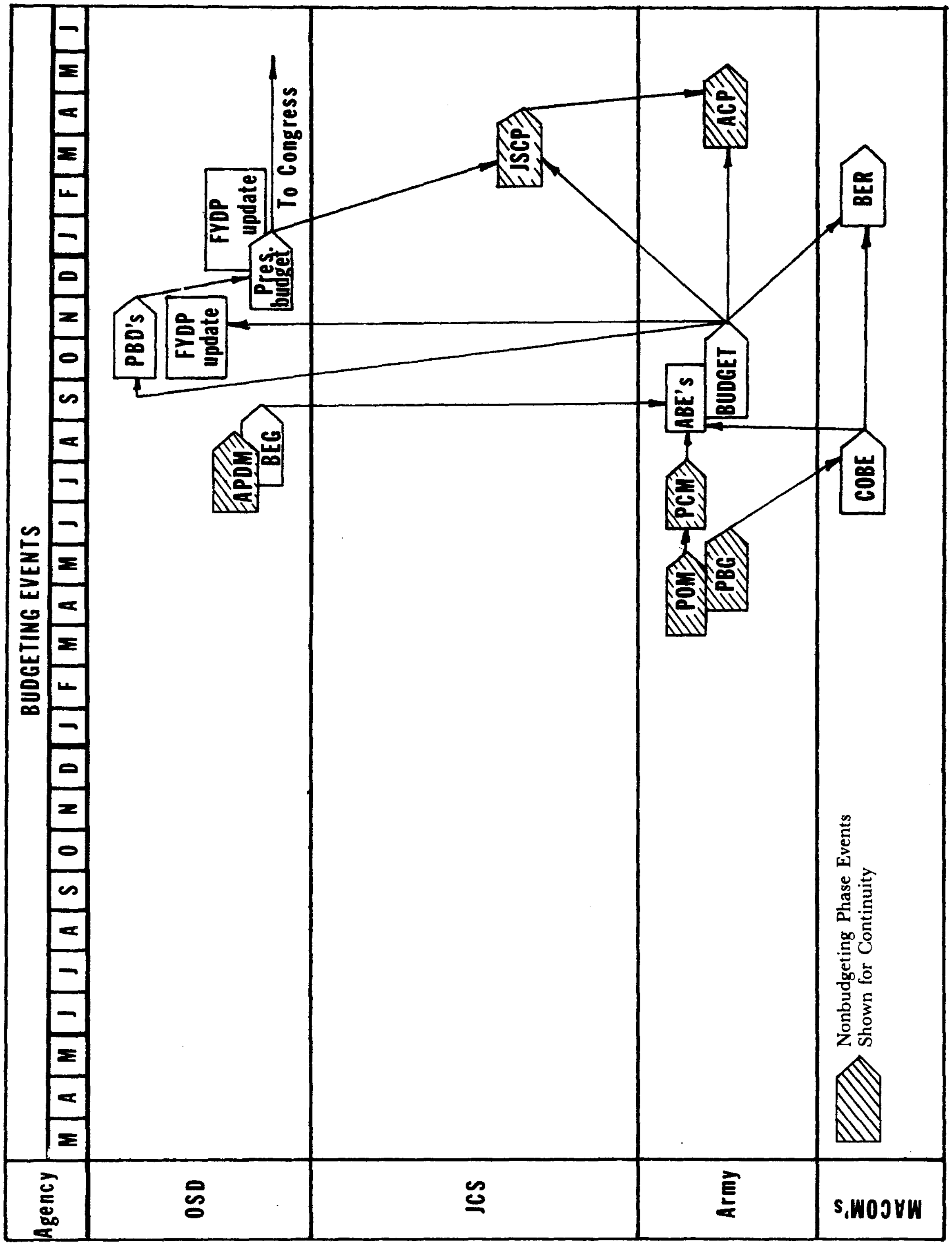

Figure 5-5. Budgeting Phase Events

OSD/OMB hold hearings with the Appropriation Directors to review the specific appropriation budget submissions. During these reviews the OSD/OMB analysts examine the budgets to insure that the approved program is implemented and that only necessary resources are requested. From late October through early December, Program Budget Decisions (PDM) are issued by the OSD either approving or revising specific programs based on the review of the budget submission. The PDM's may be appealed, and a revised PBD issued when needed. In December, the SA and the CSA meet with the SECDEF on major budget issues if required. Early in January the OMB publishes and transmits to Congress the approved DOD budget as part of the President's budget. The OSD updates the FYDP to reflect this program. The Congressional action period extends from presentation and justification of the budget request before Congressional committees to the passage of Authorization and Appropriation Bills.

5-2.3.2 Budget Execution

The budget execution process is initiated during the Congressional budget review period. This overlap with budget formulation is necessary to insure that command/agency budgets have been prepared and approved when actual execution is initiated.

The second part of the two-part COBE, which is received in HQDA in mid-August, addresses command operating requirements for the forthcoming year. The requirements are based on the May PBG and supplemental guidance furnished the MACOM's. The second part of the COBE provides Appropriation Directors with the detail necessary to prepare apportionment requests which are forwarded to OSD/OMB to obtain funds required by the Army for budget execution.

Each command/agency is notified of the approved operating levels through the October PBG and by issuance of the initial Funding Authorization Document (FAD) for the new fiscal year. This PBG provides detailed guidance regarding the DA actions on the command requests while the FAD contains the authority to obligate funds to execute the approved programs.

The MACOM's submit a Budget Execution Review (BER) at mid-year (1 April) to reflect four or more months of actual performance against the approved program and specific performance expected for the remainder of the fiscal year, and to request revisions to the approved operating budgets. Following DA review and approval of the BER's, the decisions are provided to the commands by issuing an updated FAD and by the revised guidance in the May issue of the PBG.

The final event in the budget execution is the preparation of the Prior Year Report (PYR) that provides actual data after the end of the fiscal year. From this report the Army staff updates the performance estimates to the actual experience. This information becomes the basis for future year budget projections.

5-3 SYSTEM ACQUISITION MANAGEMENT DOCUMENTS

The two major documents used in the management of system acquisition programs are the Decision Coordinating Paper (DCP) and the Army Program Memorandum (APM). These documents are mutually exclusive as only one of them applies to a particular materiel acquisition program.

5-3.1 DECISION COORDINATING PAPER

The DCP, discussed in detail in Ref. 4, is an OSD acquisition decision recording document which presents the rationale for starting, continuing, reorienting, or stopping a selected program at each critical milestone in the acquisition process. It identifies the objectives, conditions, and issues pertinent to each decision and assesses all important factors which influence the decision. The DCP is the principal discussion document at a review by an Army Systems Acquisition Review Council

(ASARC)/Defense Systems Acquisition Council (DSARC) and is the official document in which the SECDEF records his decisions regarding the program. The thresholds of cost (research and development, procurement, and life cycle ownership), performance, and schedule acceptable to HQDA and OSD are established in this document. Normally the materiel developer is the proponent for a DCP, and the Deputy Chief of Staff for Research, Development, and Acquisition (DCSRDA) has Army Staff responsibility for all DCP's. After SECDEF approval, the DCP constitutes a contract between the OSD and the Army and defines the latitude of the Army in managing the program.

5-3.2 ARMY PROGRAM MEMORANDUM

The APM, discussed in Ref. 4, is an Army acquisition recording document that presents the rationale for starting, continuing, reorienting, or stopping a selected program at each critical milestone in the materiel acquisition process. It is the principal discussion document at an ASARC meeting. The APM identifies the objectives, conditions, and issues pertinent to each decision and assesses important factors that influence the decision. The VCSA, under the authority delegated to him by the SA, approves the APM following recommendation(s) by the ASARC. This document establishes approved thresholds of cost (research and development, procurement, and life cycle ownership), performance, and schedule acceptable to HQDA and defines the materiel developer's latitude in managing the program subject to review by the ASARC. Normally the materiel developer is the proponent for the APM; the DCSRDA has Army Staff responsbility for all APM's. After approval, the APM is a contract between the HQDA and the materiel developer who is authorized to proceed consistent with cost and funding thresholds and the program defined in the APM.

5-4 REQUIREMENTS DEVELOPMENT

The US Army Training and Doctrine Command (TRADOC) is the focal point for the development of weapon systems or materiel requirements. According to AR 71-9 (Ref. 5), TRADOC has the responsibility for "Conducting conceptual and analytical studies to support the development of doctrine, materiel requirements, organization, training, and designated functional systems in accordance with DA force development planning guidance." As noted from this quote, TRADOC is not the only activity within the Army that is conducting materiel related studies.

The Army War College (AWC) is responsible for conducting strategic studies as directed by the Deputy Chief of Staff for Operations and Plans (DCSOPNS) HQDA. If these studies affect weapon system or materiel requirements, they will be coordinated with TRADOC.

The Concepts Analysis Agency (CAA) conducts mid- and long-range concept studies, as directed by DCSOPNS, to establish the framework and guidance for TRADOC development of weapon system and materiel requirements for Army forces. TRADOC provides input to CAA studies as tasked by HQDA and conducts mid- or short-range studies supporting the CAA studies separate from the CAA.

Deficiencies identified by this study effort form the basis for the preparation of many of the Science and Technology Objectives (STO's) or other requirement documents mentioned in Chapter 4. Deficiencies identified during tests and evaluation, and experience provide the basis for additional STO's or other requirement documents.

5-5 ARMY DOCUMENTS STATING RESEARCH AND DEVELOPMENT REQUIREMENTS

The principal Army documents listing requirements for research and development are the Science and Technology Objectives Guide (STOG) and the Catalog of Approved Requirements Documents (CARDS).

5-5.1 THE SCIENCE AND TECHNOLOGY OBJECTIVES GUIDE (STOG)

The STOG, a Department of the Army document compiling all the individual STO's, is the principal Army research, development, test and evaluation (RDTE) guidance document for the Science and Technology (S&T) Base, to include the categories of research (6.1), exploratory development (6.2), and nonsystem advanced development (6.3A). The four basic goals of the STOG are to provide:

1. From the user proponents to the developer community (Army in-house laboratories, civilian industry, and universities) priorities of the science and technology objectives
2. A synopsis of the concepts and background information from which the STO's are formulated so that developers may use their initiative to meet or surpass the objectives
3. A point of departure for further review, discussion, and clarification of STO's between users and developers which will insure that all capability gaps are identified and addressed
4. A baseline against which the productivity and accomplishments of the S&T base program can be measured and the relevance of it to the Army's needs can be evaluated.

5-5.2 THE CATALOG OF APPROVED REQUIREMENTS DOCUMENTS (CARDS)

The CARDS, a Department of the Army prepared document, provides a compilation of approved requirement documents to include:

1. Letter of Agreement (LOA). Discussed in par. 4-2.1.1.
2. Required Operational Capability (ROC). Discussed in par. 4-2.2.2.1
3. Letter Requirement (LR). Discussed in par. 4-2.2.2.1
4. Joint Service Operational Requirement (JSOR). A JSOR is a statement of need for the same end item of materiel for operational employment by the Army and at least one additional US military Service. Army sponsored JSOR's are prepared and processed following ROC procedures to the maximum extent possible.
5. Training Device Requirement (TDR). A TDR is a document prepared in a prescribed format which gives operational, technical, and cost information needed to obtain HQDA approval to acquire a training device. When approved by HQDA, the TDR is the document of record of the Army's requirement and will contain the guiding factors against which the developers and contractors meet the user's needs.
6. Training Device Letter Requirement (TDLR). The TDLR is identical to the LR in format, content, and purpose. The training device (TD) identifier is used to distinguish training device items from other items of materiel.

5-6 EPILOGUE

Through the years, the process by which the Army receives the funds needed to accomplish the missions has changed from time to time. The most comprehensive of these changes was the introduction of the PPBS because this system provided a bridge (programming) between Army planning and Army budgeting. Since the introduction of the PPBS, there have been refinements from time to time but no major changes in the system. It is only reasonable to expect that additional refinements will be made in the future, but the basic principles and concepts of the system should remain unchanged.

REFERENCES

1. R. S. McNamara, "Managing the Department of Defense", Civil Service Journal **4** (1964).
2. DOD Instruction 7045.7, *The Planning, Programming, and Budgeting System.*
3. AR 1-1, *Planning, Programing, and Budgeting Within the Department of the Army.*
4. AR 15-14, *Systems Acquisition Review Council Procedures.*
5. AR 71-9, *Materiel Objectives and Requirements.**

*Title is correct August 1977 but is subject to change when the AR is revised.

CHAPTER 6

ROLE OF THE SYSTEMS ANALYST

The role of the weapon systems analyst is discussed in sufficient detail to indicate the character, scope, and boundaries of his general activities.

6-1 GENERAL

The role of the systems analyst is that of scientific staff advisor; the analyst is neither a policy maker nor a decision maker. To the extent that a study has an impact on matters of policy, it is within the analyst's responsibility to present the significance of his results on policy in a complete, clear, concise, and coherent manner, and to provide sound justification for their consideration — he should explain rather than exhort. The proximity of the analyst's activity to the decision level presents a great temptation to force policy to conform to orderly scientific methodology. However, human psychology, political considerations, and other factors often defy probabilistic quantification. Many subtle influences of this sort cannot be included in a quantitative or strictly logical study and properly belong within the scope of judgment by the decision maker or the sponsor of the study.

The first task of the systems analyst is to assist the sponsor in defining his problem. When the problem is adequately defined, the analyst then engages in research* to identify all options available to solve the problem and the consequences of each option. The results should be reported in the terminology of the sponsor. Recommendations may be made regarding the most feasible option(s), but all options should be presented with unbiased arguments for and against each.

Research efforts can only be as thorough as available time and resources permit. Should these limitations threaten to impair the validity of the results, the analyst should notify the sponsor as soon as possible and appraise him of alternative courses of action he may take to redirect subsequent effort.

Ref. 1 is an interesting, comprehensive, and rather indispensible guide for systems analysts and is therefore highly recommended reading.

6-2 IDENTIFYING THE PROBLEM

The most critical, essential, and difficult task in systems analysis is that of accurately defining the problem to be studied. As succinctly stated by E. S. Quade (Ref. 2), "The difficulty lies more in deciding what ought to be done than in deciding how to do it." The majority of studies that have proven to be a disappointment can be associated with incorrect or misguided efforts as a result of inadequate problem definition. Though it is incumbent upon the analyst to ration the time devoted to the entire project, effort devoted to precise problem definition is time well spent even to the extent that the scope of the study must be somewhat sacrificed. A limited study which is pertinent to the needs of the decision maker is preferred to one which is more extensive and detailed in coverage, but provides a solution to *another* problem.

In general, those who identify the need, establish the objectives, and assign the study seek the aid of systems analysis because they recognize the need for technical expertise and lack the time necessary to investigate all the factors, considerations, and relationships affecting the objectives they desire and the most effective means to attain them. These limitations are often manifest in the statement of study

*Research is used here in a rather broad sense to include all means of arriving at new information and insight.

requirements as expressions of abstract concepts, imprecise technical language, and vague interrelationships subject to varied interpretation. Often, the real issues to be resolved are clouded by preconceived notions, standard procedures, traditional precedents, etc., or more commonly, a narrow perspective of the problem due to inward orientation.

The systems analyst, therefore, must engage in extensive and searching dialogue with the study sponsor to understand fully the decision to be made; the purpose of the study; how the result of the study will be used; the relationship of the system or systems to be studied to parent, lateral, and subordinate systems; and other considerations he perceives as bearing on the problem. He must consult appropriate technical and military professionals wherever necessary. Problem definition may be viewed as an iterative process of mutual enlightment wherein, by successive dialogues between the analyst and the study sponsor, each broadens the other's perspective as to the true nature of the problem and develops insights into the necessary goals to be attained and the most effective means of attaining them. It is not uncommon that the study objectives emanating from the problem definition activity are changed materially from those originally stated. Further, problem definition is not limited to the initial study stages; it also may continue throughout the study as research uncovers critical issues, policy conflicts, alternatives, etc.

It is unrealistic to outline a standard approach that will guarantee success in problem definitions. Each study should be viewed as a unique entity, for — regardless of the apparent similarity between problems, even within the same family of weapon systems — the requirements, policies, skills, technology, time, and other factors often present changing relationships. However, the following features are common to many excellent studies:

1. Examine at least superficially how the particular problem fits into the broad picture of Army capabilities, resources, missions, etc. In this way, the important interfaces, relationships, and trade-off areas can be reasonably identified. The system of interest is composed of smaller subsystems and is often a subsystem of a larger system. Knowing where the system is to be studied fits into a larger system, e.g., a new tank into an armored division structure, helps separate true from apparent relationships.

2. From system relationships and interfaces, the analyst should formulate assumptions and constraints that must be applied to limit, yet focus, the study toward useful and attainable results. Valid constraints and assumptions enable the analyst to focus on pertinent data and criteria in reducing the problem to manageable proportions. Caution must be exercised, however, for unwise limitation may assume away the key elements of the problem.

3. Bounding the problem is an essential and critical step wherein important interfaces, critical parameters, pertinent relationships, and trade-off areas are synthesized within the constraints and assumptions. This step forms the basis for criteria development and must be taken in collaboration with those who know the real world implications of each assumption and constraint to assure that the problem to be solved is capable of solution.

4. Once the boundaries of the problem have been tentatively defined, the next step is to establish appropriate criteria for evaluating problem solutions. This is a crucial step in problem definition and involves isolation of essential considerations which, if satisfied, would result in effective attainment of objectives. There is little in the way of constructive guidance that can be given in support of this task. Often, the approach used is simply one of trial and error. However, the criteria chosen will set the framework for defining the problem, and inadequate attention to this step may seriously impair the utility of the entire study.

5. The statement of the problem also should outline the work to be done. This includes the tasks to be performed, interrelationships and sequence of subtasks, and a general indication of the effort re-

quired to accomplish each task. Areas for suboptimization are identified to permit the integration of lower level analyses into studies of higher order systems.

These five features need not necessarily be considered in order. They must be examined in a coordinated effort between the analyst and the study sponsor through an iterative process. Further, the reader will note two important aspects in the course of problem definition. First, at least tentative approaches to problem solution will have been identified. Only in this way could the problem be assessed as capable of solution. Secondly, in defining the problem, the analyst will have an appropriate outline and accurate definition of the problem; these constitute a very significant part of the analysis. In fact, some authorities in the field contend that defining the problem implies the result of a systems analysis, while development of the study is merely an organized, mechanical application of resources.

6-3 GUIDELINES FOR THE ANALYST

6-3.1 RESPONSIBILITY TO THE DECISION MAKER

The analyst should review with the decision maker or sponsor any new turn of the study which may affect the definition of the problem, the scope of the analysis, or the validity of the results. The decision maker's responsibility is to provide program guidance and maintain overall control over the scope and tempo of the effort to insure effective and timely results. To carry out this responsibility, the sponsor must be apprised of significant deviations from the planned program, unexpected difficulties, schedule slippages, etc., in order that he may lend his expert judgment on how they might be resolved. The analyst must assist by making recommendations as to alternative courses of action necessary to pursue problem objectives in precise and meaningful terms which the *decision maker can understand and use.*

The credibility and validity of a study will be judged by the decision maker who will apply critically his military and engineering experience in judging study results. He tolerates, and even respects, analytical detail provided that the analyst is able to relate such detail to supportable, logical conclusions. Therefore, throughout the study, reports should be oriented continuously to meet the needs of the decision maker in a frame of reference that he can understand. The report must give him an intelligible description of the overall design, scope, and general approach used. The specific models used and the methods used to perform validity and sensitivity analyses on assumptions, constraints, and input data should be made clear. The decision maker also should be given an indication of the confidence that can be placed in the findings and how far they reasonably can be extrapolated. Gaps and unresolved conflicts should be highlighted. The validity of the results will depend on the completeness and integrity of the analysis, rather than the complexity of the calculation methods.

Common criticisms of system analysis studies (Ref. 3) are:

1. Studies often are seriously out of balance, applying great amounts of detailed treatment to some parts of the problem, while other important parts are handled in relatively crude ways or not at all.
2. Despite increased sophistication and detail, studies often seem illogical.
3. Studies are weak in the logical design of analysis and in the definition of simple, understandable comparisons of measurable capabilities.
4. Studies using complex models and simulations occasionally may be very difficult for anyone except possibly their authors to understand.

Preliminary analyses should establish the logical framework of the study and the level of detail which is necessary. In general, the level of analysis should be uniform throughout the study unless it can be shown that certain areas warrant more emphasis.

Studies tend to become confusing and comparison of capabilities obscure when presented in a morass of detail. To avoid this tendency, a useful technique is to present succinctly the major findings,

conclusions, and critical interfaces in the body of the report, relegating supporting detail to appendices. This not only improves understanding of important relationships, but also assists in making sensitivity and validity checks on the technical material.

The use, or development and use, of models and their validity depends on how adequately the model reflects the important relationships and behavior that exist in the application under study. As detail is added to a model, however, it may tend to obscure the more important relationships and to increase the time required to develop the model. It may, therefore, become very difficult to manipulate the model and to examine its sensitivity to input data. It is well to remember that the purpose of the model is to establish and measure the important relationships and not to mirror completely the real environment. The real trick to developing a model is to include as few variables as possible, but still capture the more important elements and relationships of the problem.

The study must be completed in time to make its results useful. It should be designed from its onset to provide the best analysis possible within the time available. The analyst and the decision maker must discuss candidly the time available so that they can make the trade-offs in time and resource requirements, and accuracy and scope that are required to provide the best basis for an informed decision at the time it must be made.

6-3.2 SENSITIVITY TO THE CHANGING NEEDS OF THE ARMY

The systems analyst must ensure that he continually improves his knowledge of the Army's mission and its operational environment. The interplay between an Army weapon system and the variety of combat conditions it must face increases the difficulties that are encountered in performing a useful systems analysis. While a system study must reduce these complexities to those most significant to the system under study, this cannot be done adequately unless it is based on a sound knowledge of the conditions in which the system must function. Materiel acquisition policy and guidance should be studied carefully. Such documentary material should be augmented by other available operational and technical guides (field manuals, technical manuals, etc.) that deal with the system under study or similar and related systems. The help of experienced military and engineering personnel should be sought. Such background preparation will be invaluable in understanding the complexities that must be faced in designing the study, and it will pay off in more constructive working relations with associated engineers, military analysts, and users of the system.

6-3.3 RESPONSIBILITY OF THE ANALYST TO IMPROVE HIS TECHNICAL PROFICIENCY

It is expected, of course, that the systems analyst will strive constantly to increase his technical competence in military operations research and in the techniques of applying his disciplines to real world problems. He must recognize that true professionalism does not rely on techniques alone to produce practical results. A sound knowledge of techniques is essential, but techniques are to be considered adjuncts to the basic logic of the study. The ability to cooperate effectively with workers in other disciplines in order to develop a coherent approach and study plan is a true mark of professionalism. Ref. 1 is especially recommended reading.

6-4 PROBLEMS OF BIAS

No matter how diligent the effort toward objectivity, it must be recognized that some degree of bias always will be present in an analysis. It is an inherent characteristic of human nature and is a composite of the background, experience, environment, and attitude of the group as well as the individual.

Some bias may be unconscious or deliberate and, if not recognized and controlled, will act to inhibit perceptivity and creativity. Bias may be obvious as in the case of dogmatic adherence to a personal philosophy, or subtle as in the case of implied organizational policy. In any of its forms, it is important to *recognize* bias in order to avoid it inhibiting imaginative thinking and broadening one's own perspective. This chapter will address the two basic categories of bias: personal and environmental, some examples of each, and their effects. The intent is to promote an awareness of the importance of recognizing and controlling bias.

6-4.1 PERSONAL BIAS

Personal bias is more easily recognized (especially in someone else) and is often the most difficult to control. Subclasses within this category include:

1. *Authoritative bias* results from blind acceptance of authoritative positions on important issues rather than examining and evaluating such principles as advice or guidance in light of study objectives. Here the analyst is persuaded more by the pundit's position than by his argument. An example of authoritative bias is uncritically allowing someone regarded as an "expert" in the field to specify the criteria by which objectives will be evaluated based on the principle "He ought to know best". The analyst should realize that misplaced emphasis which may result from such blind acceptance will ultimately be his own responsibility.

2. *The standard approach bias* involves the adaptation of study objectives to standard methods and techniques as a substitute for innovative and creative thinking. Here the analyst assumes some universal conformity in problems and, therefore, instead of seeking an appropriate approach to problem solution, attempts to force study objectives into pre-established alternatives using simple iteration. This type of modus operandi runs the risk of defining a neat and tidy solution for an inappropriate problem.

3. *The confidence bias* most often is encountered where the analyst withdraws to the safety of his own particular specialty or experience pattern. As in the "standard approach" bias, he attempts to force the problem to conform to his frame of reference, usually considering only a narrow range of variables or alternatives with which he has the most familiarity and competence. Similarly, he runs the same risk of having a well defined solution to the wrong problem.

6-4.2 ENVIRONMENTAL BIAS

Environmental bias is usually more subtle than personal bias and often appropriately may be called collective personal bias. Subclasses within this category include:

1. *Institutional bias* results from traditional organizational doctrines or policies that dictate or imply an established framework for the conduct of analyses. This may be manifest as implied restrictions on interpersonal relationships (chain of command authority), formal procedures which inhibit flexibility (interdepartmental autonomy), or simply adherence to the "party line". These may inhibit communication and expression, and are extremely difficult to overcome.

2. *Disciplinary bias* refers to an obvious form of collective bias where the structure of the study group reflects one predominant background (e.g., mathematics, statistics, or psychology). Study results will tend to reflect such bias and may not satisfy the needs of the decision maker. This bias often may be resolved only through the use of multidisciplined teams, however.

3. *Classification bias* is an environmental bias which can take many forms. Often relationships are described in relative terms for ease of reference and use; they are qualitative rather than quantitative. Strategic weapon systems normally are classified with the general war level of conflict; tactical weapon

systems, with limited war conflict levels. Assumptions based on conflict levels (e.g., general war) may overlook the adaptability, with minor modifications, of a strategic weapon to tactical delivery systems supporting certain limited war situations. The categorization or classification of relationships is often artificial and has meaning for limited purposes. Adaptation of their use for other functions may be inappropriate, therefore.

4. *System bias* refers to any bias based on supporting studies which has an effect on the present analysis. Most often the system under study must be integrated into a larger system and may require interface with several coordinate systems. The studies supporting these systems materially influence the evaluation of critical relationships. The analyst must insure that any bias in these supporting studies is identified and its significance for his analysis evaluated. There are many instances where bias has been propagated in successive studies, grossly inflating its effect.

6-5 FURTHER CONSIDERATIONS

In the course of developing a systems analysis study, the analyst will encounter a series of temptations that may warp or reduce the usefulness of his results. Instead of designing the analysis so that it can answer important policy questions, many analysts are diverted to study portions of the problem that are of particular interest to them. On the other hand, it is easy to fall into the pitfall of trying to do too big a job. To overcome this tendency, questions should be developed to determine what can be answered in the time available to complete the study. It is essential to limit the study to questions that can be investigated and to which sound and defensible answers can be given. While it is easy for large studies to get bogged down in irrelevant or unimportant details, it is equally easy for small studies to go astray because the analyst attempts to answer questions that are beyond his capabilities. The dangers of overelaborating a computational model are manifold. So much time is wasted on minor details and on complex computer programming that possibly invalid shortcuts may be taken in the final hectic days of the study.

The analyst is encouraged to read *Models, Data, and War: A Critique of the Study of Conventional Forces* by Stockfisch (Ref. 4), which apparently was motivated by some concern about the quality of "back of the envelope" type analyses and input information actually used in various weapon selection processes in the recent past. This reference also covers the use of firepower scores and indexes to estimate force ratios and the relative effectiveness of divisions. Our approach in this Handbook is not along such lines, as will be seen.

It is essential that systems analyses reflect the best professional judgment and methodology available. To achieve effective use of systems analysis as a management tool, there is constant need for teamwork and imagination. To achieve these, each participant must always be aware of the skills and needs of the others.

Finally, we should mention the concept of Red and Blue teams in connection with the function of weapon systems analysis. In recent years, there has been a considerable amount of discussion on this type of subject, and the weapon systems analyst has often been viewed or considered as the "devil's advocate", so to speak. In any event, it is believed that the systems analyst should not be on either side of a problem or study — especially in line with his major role of performing competent, impartial studies so that the best decisions will be made ultimately. In Chapter 8 we discuss the idea of using Red Teams in the materiel acquisition process in connection with the overall type of staff work that is now performed for the decision maker. Otherwise, the role of impartial studies on the part of the systems analyst is always the proper one.

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CHAPTER 7

ROLE OF THE DECISION MAKER

The key role of the decision maker in the review and implementation of the weapon systems analysis studies is characterized and highlighted.

7-1 DECISION ECHELONS IN THE ARMY

Military plans establish requirements for military capabilities. The programming and budgeting system generates time-phased programs using resources approved by the Congress to provide the most effective forces and weapon systems to satisfy these requirements. While organizational elements within the Army management structure are subject to change, the command relationships described in the paragraphs that follow are representative of the decision echelons expected to remain in effect for the near future.

The planning, programming, and budgeting cycle is directed by the Secretary of the Army and executed by the Chief of Staff, US Army, assisted by an appropriate staff. The key staff elments that affect and control the systems analysis programs within the Army are the Director of the Army staff and the Comptroller, US Army. The Office of the Director of the Army Staff — through the Program Analysis and Evaluation Directorate, the Management Directorate, and the Management Information Systems Directorate — monitors all efforts under way to improve the operational effectiveness of the US Army. The Comptroller monitors the use of resources made available to execute approved programs and establishes, through the Director of Cost Analysis, the ground rules for conducting cost analyses of the various weapon systems (Ref. 1). The various Deputy Chiefs of Staff (Logistics; Military Operations and Plans; Personnel; and Research, Development, and Acquistion) and other staff offices monitor and direct activities within functional areas. Directly beneath the Secretary of the Army in the command line are two major commands of primary interest to the systems analyst — the US Army Training and Doctrine Command and the US Army Materiel Development and Readiness Command.

At each decision level of command or operation, the commander is responsible to the next higher echelon for the effective accomplishment of his assigned mission and the proper use of his resources. Within the authority assigned to him, each commander must contribute to the planning process by making specific recommendations as to how his assigned missions should be programmed. He must continuously supervise and direct the execution of approved programs within the budget (resource) constraints that are established and report continuously on the progress of his programs against Army and DOD objectives.

Each level of command (and its supporting staff) is part of a tightly interlocked decision process that attempts to improve the combat effectiveness of the Army. At each level, conflicting demands for resources must be evaluated and decisions made or recommendations made to higher levels. Major weapon systems acquisitions involve a continuous dialogue between all command and staff levels engaged in the decision process. At the upper levels, hard decisions as to trade-offs between desirable systems must be made; at the lower levels, hard decisions must be made as to the most effective approach, among alternative approaches, to meet desired performance and cost objectives. In the complex area of defense readiness, a systems analyst may become a key contributor in a decision process that involves several command levels and that may reach to the upper echelons of the Office of the

Secretary of Defense and other Government agencies. As a result, and as time and means allow, each systems analysis should be designed to contribute information that is pertinent and useful in the decision making process. Consideration of the decision interests of associated and higher levels of command will assist in identifying the significant facets which a sound system study must encompass.

7-2 FUNCTIONS OF THE DECISION MAKER

Since systems analyses will influence decisions that will determine the equipment, organization, and doctrine of the future Army, commanders at all decision levels must assure that such analyses reflect their own best professional judgment and that of their staff scientists and engineers. It is not enough to sit in judgment at the final in-process review, for by then most of the assumptions, intuitive judgments, and estimates of future values of important parameters may be hidden from view. The professional judgments of the commander and his supporting technical staff must be incorporated into the study from the very beginning and must be thoroughly coordinated with the systems analyst who is making the study. The analyst should record all the professional judgments, evaluate their impact on the scope and direction of the study, and provide feedback regarding their implications on study results, including recommendations as to proper courses of action to be taken. The results of the study must be structured so that they are related only to technical performance, cost, and schedule, but also to the decision problems faced by the sponsor and higher echelons as well.

The commander, in coordination with the systems analyst, must establish objectives, develop avenues of approach, establish control boundaries, determine control and reporting procedures, and, most importantly, know the capabilities of those assigned to carry out the analysis. In this context, he must assure that resources available are sufficient to accomplish the task in the time available and that expected results will assist in answering criticial decision questions. Resources must be focused toward specific objectives which, if achieved, will help solve his problem, or at least will pave the way for continued progress toward the desired objectives. Unless he plans well and retains control over the project, resources may be expended with little to show in the way of more complete understanding of the capabilities and limitations of the system studied.

7-3 INITIATING THE ANALYSIS

The initiation of a systems analysis involves three basic actions: establishment of a clear objective (i.e., statement of the problem), defining appropriate criteria for evaluating the analysis results, and development of a logical set of assumptions that limit and bound the study. The initial guidance should specify the kind of results anticipated and the purpose to be served by such results. If, for example, an improved aerial reconnaissance and surveillance (R&S) capability is required, the initial system studies may be directed to a determination of the R&S imagery acquistion capability that should be sought in view of the probable constraints of on-board display, processing, data links, and ground-processing capabilities. If earlier studies have not established the intelligence tasks to be performed by aerial systems (Army and/or other services) in support of the total intelligence needs of the commander, these studies may have to be performed before the initial aerial R&S system study is started.

The commander who initiates a new study should be aware of any related studies that have been performed, or are under way, or planned, and what the results of these studies are, or are likely to be. The initial study guidance must establish not only the specific objectives of the study, but also must provide a clear understanding to the analysts how this study relates to others that will have influence on the interpretation of results by decision makers at higher echelons.

When the study has been appropriately defined in relation to supporting studies, a valid and consistent set of assumptions must be established to guide and limit the detailed analysis. The art of establishing such assumptions involves simplifying the problem by narrowing considerations to a limited set of variables that are most significant to system performance or cost. The difficulty, of course, is that simplifying assumptions may oversimplify the problem and remove it too far from the real world environment the study must address. The commander must use his judgment and that of his technical staff to ensure that the assumptions are reasonable and acceptable at all command levels.

The commander must review assumptions to determine their validity (real world relationy), potency (change in results due to change in assumptions), scope (the bounds and direction given to the study), timeliness (relation to future environment), and intelligibility. The initial assumptions are not sacrosanct. They should be reviewed continously to assure that they are valid and complete, and they should be changed or revised when a better understanding of the problem dictates. In any event, the assumptions should be made explicit and be subject to the critical review by all concerned throughout the conduct of the study.

The scope and duration of the study must be established at its outset. In some cases, the time and resources available are dictated by higher levels, and the scope and duration of the study must be tailored to meet these needs. When time is not a limiting factor, availability of systems analysis resources may influence the study scope, plan, and schedule. Every effort must be made to keep the study simple and limited.

In all cases, reports should be in terms of the commander's frame of reference, not the analyst's; and the commander's time should not be wasted with extraneous and complicated technical detail. If difficult technical representations cannot be avoided, they must be fully explained in terms of a common reference, indicating methods and techniques used and the steps taken to attain results. At the very minimum, the commander should be sufficiently apprised of the significance of the material so that he may seek qualified and competent counsel on its implications.

The preparation of the study objectives, assumptions, and study guidance is an iterative process conducted between the systems analyst and the commander and his staff. The initial statements may be prepared by the systems analyst based on general guidance, and after review by the commander may be revised to reflect his desires more specifically. The initial review will assist in establishing the probable resource requirements and time necessary to conduct the study. Modifications in study objectives, assumptions, and depth of analysis may be required to limit the study to the time and resources available.

The formal study directive may be prepared by the systems analyst who will conduct the study (based on discussion with the decision maker and his staff) or may be prepared by the senior systems analyst reporting to the commander. In either event, the commander should review and approve the directive to ensure that it includes his guidance. The study directive should provide for a series of review points to insure the periodic participation of all who can contribute to its successful completion. While some of these reviews will be internal to the systems analysis staff itself, the commander should provide leadership and encouragement by informal reviews and by scheduled reviews at key project milestones.

7-4 CONSIDERATIONS DURING THE ANALYSIS

Once the study is under way, the commander should stay aloof for a period and let analysis operations proceed with little or no interference. This is not to say that he does not observe in general how

the study is proceeding, but he must provide a free rein for the analysts to attack the problem properly and complete the solution.

The commander's major and initial role has been played in assisting with the formulation of the study. During the search, explanation, and interpretation phases, his participation is relatively slight. When problems arise, or progress is stalled, he may be called upon to provide additional guidance or resources necessary for proper program prosecution and control. Normally, however, his next significant action will be to review the tentative results to see if they meet his needs. In the interim review process, he should determine the extent and depth of study coordination that has taken place with lateral, subordinate, and higher commands. He should also obtain the advice of his civilian and military advisors who are knowledgeable of the issues at his decision level to assist in the review. All should strive to ensure the logic, consistency, accuracy, and integrity of the report. They should strive also to eliminate any:

1. Deviation of the study from specified goals
2. Inappropriate, overly complicated, or overemphasized models of mathematical or statistical techniques
3. Unbalanced emphasis on one portion of a problem to the neglect of others of equal or even greater importance
4. Overexpansion of the scope of study that results from trying to do too big a job
5. Parroting results which reflect what is thought to be the "party line". (This is of considerable importance.)
6. Discrepancies in calculations, programming logic, or technical design.

During the review, the analyst(s) responsible for the major portion of the material being reviewed should be present to answer questions, explain methods used, justify approaches taken, and explain how results were derived. Such discussions often divulge the strengths and weakness of the study and provide the opportunity to clarify and strengthen those points the commander feels will be of most importance to commanders at higher levels. Should additional effort be desirable, specific guidance given to those who will perform the work will avert misunderstanding as to the intent and desired emphasis.

An excellent account of systems analysis and policy planning with applications in defense is covered in Refs. 2 and 3. The latter reference also gives a survey of the nature, aims, and limitations of systems analysis in current defense planning.

Ref. 4 is also of special interest to both the decision maker and the weapon system analyst, for the so-called "Red Teams" are used to answer questions and help in the milestone reviews covered by the Army Systems Acquisition Review Council/Defense Systems Acquisition Review Council (ASARC/DSARC) procedures in AR 15-14 (Ref 4). In this connection, it is believed that the weapon systems analyst and the decision maker better serve the Army by always taking the impartial approach to systems evaluation problems. Indeed the analyst, as we have seen in Chapter 6, should not be either on the "Blue" or "Red" side, so to speak, as he is not the advocate for a particular weapon system. On the other hand, the Army decision maker can and does indeed make good use of Red Teams in connection with ASARC studies. A Red Team, consisting of some 4 or 5 people, can be invaluable and perform first-class staff work in helping ASARC become fully informed on the issues by investigating all of the different points of view. Red team members are not the devil's advocate, they are not an Inspector General (IG) team, and do not act as an audit compliance group. Rather they provide the checks and balances needed for ASARC deliberations.

The duration of Red Team studies may be about 90-120 days; they select the issues to be studied and seek out their proper relationships. The Red Team leader is most often a colonel or a GS-15 civilian.

Red Teams have already been used to great advantage in connection with the Heavy Lift Helicopter, TACFIRE, BUSHMASTER, COPPERHEAD, HELLFIRE, PATRIOT, and other weapon studies.

Some of the lessons learned are:

1. The Red Team Chief is critical.
2. Red Teams should hold open meetings.
3. The attitude of the Project Manager is very important.
4. Red Teams must have free access to data.
5. They should narrow the issues as early as possible.
6. The nature of their contributions may vary widely.

In summary, Red Teams can be of considerable value to the decision maker during the analysis phase, or in connection with all ASARC deliberations.

7-5 REVIEWING THE ANALYSIS

In reviewing the draft of the final report of the analysis, the decision maker must realize that the analysis does not always provide a clear-cut decision which he can approve and forward to the next level. Almost without exception, there are a multitude of nontechnical considerations which will have to be resolved purely on the basis of his own expert intuition and judgment. The basic results of the analysis should reinforce his intuition and sharpen his judgment by providing credible and valid information which will answer basic questions. In fact, some of the key review questions for assessing the credibility and validity of the study are:

1. Has the problem been defined adequately?
2. Has the scope of the problem been defined properly?
3. Are the assumptions unduly restrictive?
4. Are the criteria selected the proper criteria?
5. Have any feasible and significant alternatives been omitted?
6. Are the facts correct as stated?
7. Is the study adequately documented?
8. To what extent is the study biased?
9. Are the cost estimates relevant and reasonable?
10. Are incremental costs considered?
11. Is an amortized cost used?
12. Are the models adequately identified and explained?
13. Are the models intuitively acceptable?
14. Is the effectiveness measure appropriate to the function or mission?
15. If quantitative measures of effectiveness are unattainable, are qualitative comparisons feasible?
16. What are the unknowns and the risks involved?
17. Are measures of effectiveness sensitive to changes in assumptions?
18. Are the criteria consistent with higher echelon objectives?
19. Have the significant ramifications been considered in arriving at the conclusions and recommendations offered?
20. Are the conclusions and recommendations intuitively satisfying?

A more expanded and complete list of review questions is available in Appendix D of AMCP 706-191 (Ref. 5). Those included here are considered of key importance.

The analyst, too, should realize that the most significant decisions involve complex considerations that cannot be resolved by quantitative analysis alone. Therefore, he should place the problem in

proper perspective and review all the reasonable alternatives. The study results should provide a thorough analysis of the problem, a reasonable resolution of some of the subproblem areas, the identification of significant variables, frank statements as to the problems that were not resolved, and any other limitations of the study. The report should be condensed into an executive summary that contains the problem and objectives, the basic assumptions, a condensation of the discussion and conclusions, and a clear presentation of results. The results must highlight unresolved problems as well as those results that can be supported by the particular study undertaken.

The commander should insure that the study is reviewed by all who are concerned with its results, and then personally consider all comments made. Such comments will help to identify any significant gaps and limitations, and may shed new light on considerations that have not been fully developed. Thus, the commander should welcome both concurrence and nonconcurrence with the results of the study. On the one hand, constructive comments will assist in his understanding of the significant variables. Conversely, negative comments may provide useful insights into opposing views which might be encountered at higher echelons.

During the final review process, the commander must decide what his recommendations to the next echelon will be and how the study will be used to support such recommendations. The study and review process should identify those areas where the commander's judgment and additional supporting rationale must augment the study results. If the study is sufficient to support the commander's views, a simple cover letter highlighting its conclusions will suffice. When extensive additional comments are necessary, the study should become a supporting tab for those points it supports clearly. In some cases, the study will be used only as an aid to developing the command position. In such cases, it may be retained and become part of the reference file.

The completed study and comments from all echelons, particularly those from higher echelons, should become part of the system analysis reference files. The compilation of completed studies forms a useful data bank to support future studies, or answer questions which may arise, while comments provide guidance that may improve the effectiveness of future studies.

Landmark studies should be distributed to interested personnel and used as case studies in the training of new analysts. The data developed in the course of the study should be introduced into appropriate data banks. An administrative summary sheet should identify the circumstances of the study, including the names of participating personnel as well as the number of man-hours and amount of resources expended otherwise.

Command interest in systems analysis will inevitably result in continuous improvement in the contribution that systems analyses can make to improve command decisions. Recognition of the difficulties which confront systems analysts and participation in systems analysis studies by the commander and his key staff members will encourage the professional growth of the systems analysis staff. While good management of systems analysis effort is necessary, command leadership is the spark that develops systems analysts, and military and technical professionals as well into an imaginative team capable of substantially improving overall military effectiveness.

The references that follow should be helpful to the decision maker in the system analysis process. Ref. 6 contains valuable reading and background for the decision maker as well as for the systems analyst.

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CHAPTER 8

THE SPHERE OF CONFLICT

The types of war, intensities of conflict, and levels of commitment; Army combat functions, objectives, operations, and trends; and the Army combat organizations are discussed in this chapter.

8-1 TYPES OF WAR, INTENSITIES OF CONFLICT, AND LEVELS OF COMMITMENT

8-1.1 GENERAL

Throughout the past several decades there have been numerous definitions of the types and levels of conflict, either in the context of forecast or prediction, or in the interpretation of on-going combat action. While the establishment of a "level" of conflict is convenient as a reference on which to base weapon systems analytical procedures, it may be inappropriate to assume that well-defined bounds determine a certain conflict of known and measurable characteristics for which predictable actions and reactions can be forecast. In fact, the range of combat intensity may be more or less continuous rather than a series of discrete levels. Typically, several levels of conflict could be in progress simultaneously. It therefore may be inappropriate, or at least incomplete, to analyze a weapon system only with respect to an expected level of conflict. Nevertheless, we address such subjects to some extent here.

8-1.2 TYPES OF WAR AS DEFINED BY THE JOINT CHIEFS OF STAFF

The types of war, intensities of conflict, levels of commitment, and details of related subjects are officially defined by the Joint Chiefs of Staff in a series of classified publications not citable as references in this Handbook. They represent current doctrine as of the date of publication and include response positions and reactions to be taken at various conflict levels. Such doctrine is too fluid to be considered as fixed bases for analysis of weapon systems but rather should be considered as vital current inputs to the nature of the system or particular weapon being examined. The analyst should review current doctrine and definitions when he uses this Handbook since definitions and criteria change over time. His supervisor or the sponsor of the analysis can guide him to the necessary references. Forecast periods for both the intensities of conflict and the availability of the weapon systems must enter into the analysis. The Joint Chiefs of Staff have divided the conflict spectrum into three types of war (see AR 310-25, Ref. 1), namely;

1. General War. General war is defined as armed conflict between major powers in which the total resources of the belligerents are employed, and the national survival of a major belligerent is in jeopardy. It is characterized by the application of the most modern military technology in intelligence, mobility, and firepower, to include the use of nuclear and other advanced weapons.

2. Limited War. Armed conflict short of general war, exclusive of incidents, involving the overt engagement of military forces of two or more nations is termed limited war. It also may be thought of as "conventional war", as conflict in which the weaponry is limited, or as conflict for a limited objective.

3. Cold War. Cold War is a state of international tension wherein political, economic, technological, sociological, psychological, paramilitary, and military measures short of overt armed conflict involving regular military forces are employed to achieve national objectives.

Stability operations cover military activity that is of lesser intensity than limited war, such as the presence of US Forces as part of a treaty agreement or the assignment of advisors, even though the advisors may be engaged in intermittent contact with the enemy and participate in exchanges of fire.

8-1.3 INTENSITIES OF CONFLICT AND LEVELS OF COMMITMENT

The US Army Training and Doctrine Command has defined intensities of conflict and levels of commitment as follows:

1. Intensities of Conflict:

a. High Intensity Conflict. War between two or more nations and their respective allies, if any, in which the belligerents employ the most modern technology and all resources in intelligence; mobility; firepower (to include nuclear, chemical, and biological weapons); command, control and communications; and combat service support.

b. Mid Intensity Conflict. War between two or more nations and their respective allies, if any, in which the belligerents employ the most modern technology and all resources in intelligence; mobility; firepower (excluding nuclear, chemical, and biological weapons); command, control, and communications; and combat service support; for limited objectives under definitive policy limitations as to the extent of destructive power that can be employed or the extent of geographic area that might be involved.

c. Low Intensity Conflict (Type A). Internal defense and development assistance operations involving *actions by US combat forces* to establish, regain, or maintain control of specific land areas threatened by guerilla warfare, revolution, subversion, or other tactics aimed at internal seizure of power.

d. Low Intensity Conflict (Type B). Internal defense and development assistance operations *involving US advice, combat support* and *combat service support* for indigenous or allied forces engaged in establishing, regaining, or maintaining control of specific land areas threatened by guerilla warfare, revolution, subversion, or other tactics aimed at internal seizure of power.

2. Levels of Commitment.

a. Heavy Level of Commitment. Friendly force operations involving more than 60% of all force maneuver echelons and all fire support means engaged in all-out combat demanding total strength application over a period of time to include possible commitment of next higher echelon resources to assure accomplishment of the friendly force mission.

b. Moderate Level of Commitment. Friendly force operations involving 30-60% of all force maneuver echelons and over 50% of all fire support means engaged in continuous combat over a period of time, during which commitment of next higher echelon resources to assure accomplishment of the friendly force mission is not anticipated.

c. Light Level of Commitment. Friendly force operations involving less than 30% of all force maneuver echelons and less than 50% of fire support means engaged in sporadic combat over a period of time during which commitment of next higher echelon resources to assure accomplishment of the friendly force mission will not be required.

d. Reserve. Portion of a body of troops which is kept to the rear, or withheld from action at the beginning of an engagement, available for decisive movement.

e. Noncommitted Force. A friendly force not yet committed to combat or one withdrawn from a higher level of commitment for one or more reasons such as excessive losses, need for retaining, or requirement for reassignment. Such a force may require replacement of personnel and equipment to bring it to the appropriate authorized level of organization (ALO).

8-2 SUMMARY OF OBJECTIVES, OPERATIONS, AND TRENDS IN THE US ARMY

In order to give the weapon systems analyst some pertinent background and orient his thinking properly, we cover some of the highlights of FM 100-5, *Operations* (Ref. 2). This particular field manual sets forth the basic concepts of US Army doctrine; it is the capstone of the Army system of field manuals and treats the relationships among the operations described in detail in many other manuals.

FM 100-5 covers topics on:

1. US Army objectives
2. Modern weapons on the modern battlefield
3. How to fight
4. Offense
5. Defense
6. Retrograde
7. Intelligence
8. The air-land battle
9. Electronic warfare
10. Tactical nuclear operations
11. Chemical operations
12. Combat service support
13. Operations within NATO
14. Special environments.

8-2.1 BACKGROUND INFORMATION

The weapon systems analyst should be cognizant of the following:

1. The Army's primary objective is to *win the land battle* — to fight and win in battles, large or small, against whatever foe.

2. US Army *combat development* seeks to increase the Army's ability to fight decisively by searching combat experience, experiments, tests, and technology for ways to provide weapon systems, organizations, tactics, and techniques.

3. Battlefield Dynamics. To win a battle, four prerequisites must be met:

a. Adequate forces and weapons must be concentrated at critical times and places. The combination is known as combat power.

b. The battle must be controlled and directed so that maximum effect is obtained from both firepower and maneuver.

c. The battle must be fought using cover, concealment, suppression, and combined arms teamwork to "maximize" the effectiveness of our weapons and to "minimize" the effectiveness of enemy weapons.

d. Our teams and crews must be trained to use the maximum capabilities of their weapons.

4. *Infantry* can destroy or suppress enemy infantry with antitank weapons, small arms, machine guns, mortars, and, in the case of mechanized infantry, the weapons mounted on their fighting vehicles.

5. *Field artillery* can destroy or suppress infantry at short ranges, antitank guided missiles (ATGM) at medium ranges, and enemy artillery or air defense weapons at long ranges. Suppression, of course, gives a high probability of destruction of enemy weapons if their gunners or crews fail to take evasive or protective action. Artillery can cause enemy tanks to lose 50% of their effectiveness by forcing them to

button-up, and can destroy light-armored vehicles. Artillery can place smoke around enemy tanks and ATGM's, and thus render their fires ineffective.

6. *Tanks* can kill or suppress infantry with their machine guns and kill or suppress enemy tanks with their main guns.

7. *Attack helicopers* can destroy enemy tanks.

8. *Air Force aircraft* can destroy or suppress ATGM's, tanks and armored vehicles, field artillery, and air defense artillery.

9. *Air Defense weapons* can destroy or suppress fighter aircraft.

10. *Offense.* Offensive operations are undertaken to:

a. Destroy enemy forces.
b. Secure key terrain.
c. Deprive the enemy of resources, demoralize him, and destroy his will to continue the battle.
d. Deceive and divert the enemy.
e. Develop intelligence.

11. *Defense.* Defensive operations are undertaken to:

a. Cause an enemy attack to fail.
b. Preserve forces, facilities, installations, activities.
c. Retain tactical, strategic, or political objectives.
d. Gain time.
e. Concentrate forces elsewhere.
f. Wear down enemy forces as a prelude to offensive operations.
g. Control terrain essential to friendly force mission.
h. Force the enemy to mass so that he is more vulnerable to our firepower.

12. *Retrograde Operations.* During the course of any combat operation, it may be necessary for a command to move to the rear, or away from the enemy (Retrograde).

a. Generally, a command may conduct retrograde when:

(1) There are insufficient forces to defend or attack, making it necessary to give up space in exchange for time.
(2) The command is to be deployed elsewhere, or in a better position.
(3) Continuation of the ongoing operation no longer promises success.
(4) The purpose of the ongoing operations has been achieved.

b. There are three types of retrograde operations:

(1) Delay — in which a force trades space for time.
(2) Withdrawal — in which a force disengages from enemy contact.
(3) Retirement — in which a force moves away from an area without enemy pressure.

13. *Intelligence* is a prerequisite to winning the first battle. What can be seen or heard can be hit; what can be hit can be killed. Intelligence is the basis of tactical decision. In the defense, the corps and division commanders must ascertain the location and direction of the enemy's main or breakthrough effort as soon as possible. Only the corps and division have the resources to "see" into the enemy rear sufficiently to detect his major thrust before it overwhelms the initial defenses.

14. *Division of responsibilities*:

a. Generals commanding corps and divisions allocate and *concentrate the forces* as required.
b. Colonels and lieutenant colonels of brigades and battalions *control and direct the battle.*
c. Captains and their companies, troops, and batteries *fight the battle.*

15. As a rule of thumb, forces deployed in defense should seek not to be outweighed more than 3 to 1 in terms of combat power.

16. For the attack, the commander must concentrate overwhelming combat power.

8-2.2 TRENDS IN DOCTRINE

The analyst will find the current Army trends in doctrine of interest:

1. Tanks. All great armies rest their land combat power upon the tank. The tank, with its cross-country mobility, its protective armor, and its formidable firepower has been and is likely to remain the single most important weapon for fighting the land battle.
2. Infantry. Infantry remains a versatile fighting force. It can inflict heavy losses on armored forces at short and long ranges. Now infantry is much more mobile than formerly and can keep up with tanks.
3. Field Artillery. Artillery caused more than half the casualties in World War II. Artillery type weapons are constantly being improved and are becoming more versatile. Field artillery will continue to be one of the main arms in future wars.
4. Air Defense Artillery. In recent years, many significant advances have been made in air defense weapons. They will be needed and widely used for their key role in any future conflict.
5. Battle Concept. Airmobility is now being stressed. The "airmobile" concept is the most dramatic organizational advance in the US Army.
6. Air-Land Battles. Modern battles are fought and won by air and land forces working together. Such close coordination must continue.
7. Military Operations in Special Environments. The vital national interest of the United States requires that the Army be prepared to operate in any environment anywhere in the world. Commanders must turn the environment to their own advantage, use combined arms appropriately, and apply the fundamentals of battlefield dynamics properly.
8. Command. The commander who employs his weapons at or near their full effectiveness; who reduces his vulnerability by using cover, concealment, and suppression; and thus moves decisively on the battlefield to accomplish his mission, has mastered the command of combined arms teams.
9. Combat Service Support. Effective combat service support keeps fielded systems operating. The general must ensure that his combat forces have the wherewithal to fight effectively, and he must therefore:
   a. Arm the systems — ammunition
   b. Fuel the systems — POL to move the forces
   c. Fix the systems — maintenance and repair parts
   d. Man the systems — troop replacements.

8-3 ARMY COMBAT FUNCTIONS

The basic Army land combat functions are:

1. Intelligence
2. Mobility
3. Command, control, and communications
4. Firepower
5. Combat service support.

It is, at times, difficult to separate each of these interrelated functions as they are applied in combat. However, each has a distinct part to play in the basic combat scenario:

1. Intelligence results in finding and identifying the target, and determining its location.
2. Mobility brings the source of firepower to the point from which it may be deployed most effectively.

3. Command gives the order to engage the target, which is transmitted by communications. Control is exercised by means of coordination and communications to insure the optimum placement of firepower.
4. Firepower seeks to destroy the target.
5. Intelligence determines the effect of the engagement.
6. Command evaluates the effectiveness and efficiency of the actions taken.
7. Combat service support provides the wherewithal to permit all other functions to operate.

A typical combat scenario may contain many additional functions and subfunctions, but the pattern contained in the previous simplified statements will be evident. These functions represent the prime elements of the Army in combat or in a potential conflict situation. Each of the functions is examined in detail in the paragraphs that follow.

8-3.1 INTELLIGENCE

Military intelligence generally is divided into two major areas — combat intelligence and strategic intelligence. Although the Army's major interest is in combat intelligence, it obviously has certain interests in strategic intelligence. Official definitions of the two categories of intelligence are given in FM 30-5 (Ref. 3):

"Combat intelligence is that knowledge of the enemy, weather, and geographical features required by a commander in the planning and conduct of tactical operations. It may be obtained from within his own command, or from higher, lower, or adjacent headquarters. Combat intelligence is derived from the evaluation of information on the enemy (both his capabilities and his vulnerabilities) and the environment. The objective of combat intelligence is to minimize the uncertainty concerning the effects of these factors on the accomplishment of the mission. The commander employs combat intelligence to determine how best to use available resources in accomplishing the mission and maintaining the security of his command. In noncombat command, combat intelligence provides a basis for security measures, for decisions as to the best use of the area of operations in accomplishing the mission, and for determining or anticipating future support requirements."

"Strategic intelligence is intelligence which is required for the formulation of policy and military plans at national and international levels. Oriented on national objectives, it assists in determining feasible national objectives and in furnishing a basis for planning methods of accomplishing them. Factors which influence the military capabilities, vulnerabilities, and probable courses of action of nations are considered components of strategic intelligence."

While combat intelligence and strategic intelligence are treated as separate components of military intelligence, they have much in common. There are several sources from which combat intelligence and strategic intelligence are derived. Technical intelligence (of interest to the weapon systems analyst) is one of these sources. Similarly, many elements of strategic intelligence can be applied to combat requirements.

In the field of technical intelligence, the weapon systems analyst should use intelligence inputs from the strategic and combat levels. For example:

1. What weapons is the enemy likely to possess and employ?
2. How effective is his employment of such weaponry likely to be at certain levels in the spectrum of conflict, in certain geographic areas, and in the hands of troops of certain skill levels?
3. What counterweaponry is required by US Forces in these circumstances?
4. How effective is the counterweaponry likely to be?
5. What changes or adjustments should be made in the use of existing US weaponry or in operational tactics to minimize enemy use of his weapons?

Intelligence must be timely. Intelligence collection elements of the future will produce data with such speed and volume that manual methods of filing and collation will be inadequate. At the higher levels, therefore, modern intelligence procedures will depend on multi-echeloned automated data processing and communications. Elements of such an intelligence system must be linked by reliable, dedicated, and rapid communication means, and the entire intelligence load must be assigned appropriately to the organizations and personnel involved. Concerning some of the key personnel involved in combat operations, the intelligence required by generals, colonels, and captains is quite different for each, as indicated in Table 8-1.

8-3.2 MOBILITY

Mobility, which is the capability of a military force to move from place to place while retaining the ability to perform its primary mission, is an essential element of combat. Improvement in unit and individual mobility has been a desired objective since the inception of warfare.

Considerable advances have been made in local tactical mobility in which the classic role of the division is to combine fire and maneuver. Maneuver is basically the tactical employment of mobility — the delivering of men and their associated firepower to accomplish a tactical objective by application of combat power at a decisive point and time. Certain types of mobility are applied when the tactical scenario is most responsive to the elements of surprise — such as the use of amphibious, airmobile, or airborne capabilities. Certain conditions of terrain (desert, mountains, swamps, marshes, or arctic)

TABLE 8-1. INTELLIGENCE REQUIREMENTS OF CERTAIN ARMY PERSONNEL

OPERATION	Generals	Colonels	Captains
OFFENSE	Where is the enemy main concentration? What will he use to reinforce? How fast? From what location? Enemy vulnerabilities? Enemy's major intention?	Avenues of approach to objective Type, size, number, and location of maneuver and fire support units Units capable of reinforcing by maneuver and fire Location of obstacles	Terrain analysis Location and type of units at point of contact Defending forces at the objective Fire support — direct and crew served
DEFENSE	Where will the main thrust come? Where are tank and artillery concentrations? Supporting and reinforcing fires Size of zone of attack Security of friendly flanks	Avenues of approach into defended area Composition and size of attacking force Scheme of maneuver and and fire support	Avenues of approach to defensive positions Type of attacking force Fire support — tanks
RETROGRADE	Where can he disengage with the least risk? How can I deceive and blind the enemy? Where to move? What key terrain to occupy? Enemy capability to exploit	Units in contact — size, type, number Reinforcing maneuver and fire support Intention and capability to to exploit Enemy reconnaissance	Routes of departure Key terrain Type, size, and number of units in pursuit Fire support Rear and flank security

impose great constraints on mobility. However, the possessor of mobility holds a considerable tactical advantage over a less mobile opponent. FM 61-100 (Ref. 4) emphasizes this point:

"Surprise is always sought. It may be gained by choosing an unexpected time, place, direction, form of maneuver, or strength of attack. It is enhanced by cover and deception operations."

The mobility of a force often may depend on the mobility of its slowest component. Initial tactical advantage gained by highly mobile forces may be lost if the supporting forces do not have sufficient mobility to exploit the initial thrust, penetration, or envelopment.

Examples of the system constraints placed on mobility may be found in historical examples of tank warfare in which the slower moving fuel supplies could not keep pace with the advancing armored columns. Another example is that of helicopter-borne attacks which must be supported by ammunition and firepower moved by conventional trucks. In Vietnam, for example, only the advent of the Medium Lift Helicopter, the CH-47, permitted essential firepower (a 105-mm Howitzer, crew, and a basic load of ammunition) to be deployed by helicopter at the same pace as the airlifted troops.

Requirements for air mobility place additional constraints upon the weapon systems designer. He must build in the proper compromises between weight and durability, and, in the case of armored vehicles, good protection. The advent of high capacity, long-range assault transport aircraft has relaxed weight penalties so that fairly heavy equipment may be "air transportable" but limitations still exist. Weapon systems introduced in the 1960's were designed with air transportability as a very significant parameter. Thus, new lightweight field artillery pieces made their appearance at the same time the CH-47 helicopter was placed in the field in quantity. The ability to airlift troops and their associated support firepower simultaneously aided the tactical mobility of the US, Vietnamese, and Allied Forces in Vietnam to a great extent during this period.

The very large cargo aircraft has been introduced to enhance the strategic mobility of the US Army and the US Marine Corps by providing a capability to transport not only the men, but also the arms and support equipment to give a vital initial staying capability.

Tactical mobility has improved markedly with the advent of the helicopter. Many tactical targets are fleeting in nature and should be exploited immediately by maximum use of tactical mobility. A weapon system which does not possess either its own means of mobility or compatibility with its battlefield transport system fails to give the Army in the field the great advantages of battlefield mobility.

8-3.3 COMMAND, CONTROL, AND COMMUNICATIONS

Communications is the means through which command and control are applied on the battlefield. Effective employment of control, via communications, is the only practical method by which a commander may execute the authority of command and carry out his planned tactical operations. Without operating communications, there may be nominal "command" but little "control" except in the immediate vicinity of the commander. The three elements of command, control, and communications are strongly interrelated, interdependent, and require appropriate integration, especially thorough proper coordination.

8-3.3.1 COMMAND

Command is a combination of authority vested in an individual to direct, coordinate, and control military forces and the individual leadership by which the commander exerts his authority. In the discharge of his command responsibility, the commander exerts his authority to direct actions and to establish standards. In the exercise of command leadership, the commander must devise means to project his character and personality to create a positive impression on his units. This may be accomplished by personal inspiration and direction and by proper use of subordinate commanders and their

capabilities. Each subordinate commander and staff member is a potentially effective instrument of command and control in the hands of the commander.

The successive commanders through which command actions are channeled form the chain of command. The chain of command extends downward from superior to subordinate and upward from subordinate to superior. Effective military operations demand strict adherence to the chain of command. Violation of the chain of command usurps the prerogatives of the intermediate commander concerned and abrogates his authority without a commensurate lessening of his responsibility.

8-3.3.2 CONTROL AND COMMUNICATIONS

The commander, through communication and coordination, exerts control over his organization via the chain of command. "Control" is the system of techniques the commander uses to insure that his policies and orders are being carried out, and that the battle is progressing according to his plan. Communications and coordination are tools used by the commander to obtain necessary information for control. Battlefield communication nets are structured on the premise that control will be executed through the chain of command. Numerous small-unit, tactical communication nets are established at the brigade forward levels. Such nets are required so that the commanders at the foremost echelons may acquire the proper information to manage their units and exercise their command responsibilities. The same situation prevails at the battalion-to-brigade level. It is incumbent upon each comander to pass the appropriate information to his superiors at the next higher headquarters.

As the chain of command is ascended, the quantity of the information increases unless it is filtered by staff and subordinate commanders. Senior commanders require a constant input of pertinent and timely information in order to make proper decisions. In order that the decisions made may be correctly and rapidly implemented, commanders must communicate with a large number of individuals, units, and forces. Computerized information management systems therefore are being developed for field units. Automatic data processing is intended to improve the commander's ability to make prompt decisions.

Semiautomated command and control systems which incorporate sophisticated communications and automated data processing are features of air defense systems; only modern computing technology and highly dependable communications can meet the time requirements of contemporary air and missile warfare. The land battlefield, however, still relies to a greater extent upon voice communication through the traditional chain of command, although constant improvements in both tactical and strategic communications have strengthened Army capabilities in command and control.

8-3.4 FIREPOWER

All weapon systems used in combat have one objective: the delivery of effective firepower on each target. The desired effects may be to neutralize, destroy, deter, or even decoy the target in question. The choice of weapon, selection of target, fuzing accuracy, target location, and a host of additional factors which may enter the scenario may either reduce or enhance the effects of the firepower applied. However, whether the firepower is delivered from the rifle of an infantryman or the launcher of a sophisticated guided missile, its purpose is the same — i.e., to deliver the shock of impact or the explosive warhead properly to the target in order to accomplish the tactical mission.

The Army has the responsibility for delivery of firepower to land, sea, and aerial targets. The following are some examples:

1. A major constitutent of land combat is land-based firepower delivered to land-based targets, as by field artillery and mortar fire.
2. The attack helicopter is employed to deliver air-to-ground firepower.

3. In its air defense mission, the Army employs both guns and missile (including nuclear) air defense weapons.

4. The traditional Army mission of defense against seaborne attack is now related primarily to the defense of small tactical units in a coastal environment.

In battle, fire and maneuver are combined. The two elements are clearly complementary, must be jointly used, and their combined effects jointly exploited to gain success on the battlefield.

8-3.5 SUPPORT ELEMENTS

Supporting elements of the Army in combat are divided into combat support and combat service support.

8-3.5.1 Combat Support Elements

Combat support elements organic to the division in combat are:

1. Field artillery
2. Aviation
3. Air defense artillery
4. Engineer
5. Signal
6. Military police.

Additional elements and reinforcements in these combat support areas may be attached from higher headquarters when the tactical situation demands. Similarly, if brigades are operating independently of their parent division, combat support units — both organic to the division and attached from other resources — support the combat role of the brigade.

8-3.5.2 Combat Service Support Elements

Combat service support elements organic to the division in combat are:

1. Support Command, including administration, supply, maintenance, transportation, and medical services
2. Other combat service support functions contained within the combat organizations, such as chaplain, legal, and civil affairs.

8-3.5.3 Use of Support Elements

Attachment of combat support and combat service support elements is made commensurate with the combat mission of the division. Functional support is furnished in the following forms:

1. Fire support
2. Air defense (missile and gun)
3. Aviation: transport, reconnaissance, and fire support
4. Intelligence: counterintelligence, signal intelligence, technical intelligence, combat surveillance, and target acquisition
5. Intelligence support of deception and cover operations
6. Chemical and radiological detection and monitoring (under general war conditions)
7. Engineer: river crossing, amphibious operations, mine emplacement and clearance, obstacles, assistance in preparing positions
8. Signal: extension of communication capabilities when needed
9. Military police
10. Special chemical support (if needed)

11. Additional ground transportation capabilities
12. Psychological warfare
13. Medical
14. Civil affairs.

These support functions typically are contained within the organic capabilities of the division, and many of them for the separate brigade. Additional support would be furnished by higher headquarters in their traditional tactical and administrative roles.

8-4 GEOGRAPHIC CONSIDERATIONS

Geography is a major factor in the effective functioning of any army and its associated weaponry. However, the basic Army division or brigade is not configured for battle in a specific geographic environment.

During World War II, our "standard" infantry division operated in the deserts of Africa, the mountains of Italy, the coral islands of the Pacific, and the jungles of New Guinea. Gradually by experience, equipment and clothing were developed to suit the environment, but as stated by Major General Willard Pearson (Ref. 5), the concept of the specialized division was not accepted by the Army. General Pearson wrote:

"The Infantry School queried major commanders on the need, past, present and future, for forces trained, equipped and manned, to perform their mission under a very specific set of circumstances of terrain or climate. Reaction was mixed, but the general tenor of the replies indicated that the infantry division which emerged from World War II as the work horse of the field army had demonstrated its ability to fight in any environment and in any type of specialized operation: amphibious, mountain, desert, forest, cold weather, or tropic. Units picked up their specialized training on the march. The thinking ran 'as long as we have infantry divisions, there would be no need for specialized units.' Besides, it was too wasteful of manpower to organize units to fight solely in one type of environment."

8-5 WEATHER AND WAR

As is well known, weather conditions prior to and during battle can be and often amount to one of the most critical factors determining the outcome of a conflict. An excellent example is that of Hitler's invasion of Russia in World War II. The weather not only blunted the Nazi blitzkrieg tactics, but also the German army was ill prepared to fight in a Russian winter and failed miserably. Commanders have, of course, been aware of the importance of weather and consequently have planned or programmed engagements with their enemies so that favorable weather conditions would likely be on their side. A good historical account of weather and its effect on wars is given in Ref. 6. The subject will be discussed more fully in Chapter 9, "The Physical Environment".

8-6 ARMY COMBAT UNITS

8-6.1 GENERAL

US Army combat units must be designed and given the best weapons to fight all types of conflict, and especially any possible major threat. Thus, depending on possible or probable international situations, it seems reasonable to study and keep abreast of ground force developments by certain major countries of the world. As an example, the weapon systems analyst might profit by studying FM 30-40, *Handbook on Soviet Ground Forces* (Ref. 7), which covers many topics on doctrine and tactics for a competitive country.

The weapon systems analyst should also be familiar with our basic combat organizations. The smallest fire and maneuver element in a combat organization is the squad. In an infantry unit, the platoon, which is composed of three rifle squads, is the basis for the formation of companies, which are in turn the basis for the formation of battalions. The same is true of the platoon with its five tanks in an armor unit. The battalion is the smallest combat unit which contains enough support elements to allow independent operation. This is accomplished by the addition of a headquarters unit, usually a company, to support three or more combat companies, and artillery support. An infantry battalion configured for use in an infantry division is shown on Fig. 8-1 (Ref. 8). The symbols used are the standard map symbols for an infantry unit. Symbols for some of the various types of units are shown in Fig. 8-2, and the symbols designating unit size are illustrated in Fig. 8-3. For a complete list of military symbols, see Ref. 9.

8-6.2 THE INFANTRY BATTALION

Infantry battalions organized basically as shown in Fig. 8-1 are found in all types of Army divisions. Thus, infantry battalions are found in the infantry division, the mechanized division, the armored division, the airborne division, and the air assault division.

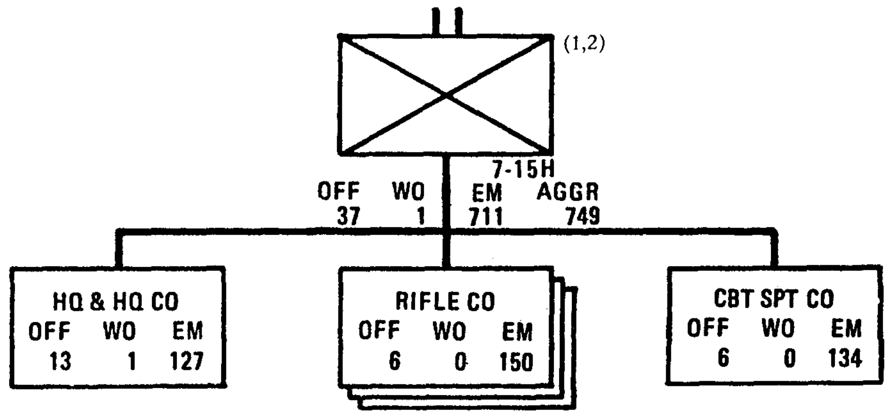
The various types of infantry battalions provide:

1. A base of fire and maneuver elements
2. The means to seize and hold terrain
3. The capability to conduct independent operations on a limited scale
4. Limited antitank protection
5. Indirect fire support for organic and attached units
6. Long-range patrolling when properly equipped
7. A force that can participate in motorized, mechanized, and joint airborne operations when provided with appropriate transportation
8. A force that can participate in air assault operations when provided with sufficient air transport.
9. The capability to maneuver in various types of terrain under all climatic conditions.

8-6.3 OTHER TYPES OF BATTALIONS

The other types of battalions which are concerned primarily with the employment of weapon systems are the tank battalion, the air cavalry squadron, the armored cavalry squadron, the field artillery battalion, and the air defense artillery battalion. Several terms are introduced by the last four types of units to designate units of battalion or company size. Within air and armored cavalry units, a battalion-size unit is called a squadron, while a company-size unit is called a troop (an inheritance from the days of horse-mounted cavalry). The artillery designates its company-size units as batteries, as shown on Fig. 8-3. The organization of these battalions is similar to that of the infantry battalion, having three or four company-size combat units and a headquarters unit of company size to provide administration and support. They do not have the combat support company found in the infantry battalion.

The organization of combat service support battalions follows no regular pattern from one unit type to another. However, these units are not charged with the employment of weapon systems. Their primary mission is to provide combat service support. Accordingly, the weapon systems analyst may not be concerned with these units in any context other than their ability to support the weapon system being analyzed.



1. MISSION

To close with the enemy by means of fire and maneuver in order to destroy or capture him or to repel his assault by fire, close combat, and counterattack.

2. ASSIGNMENT

Organic to infantry division, TOE 7, and separate infantry brigade, TOE 7–100.

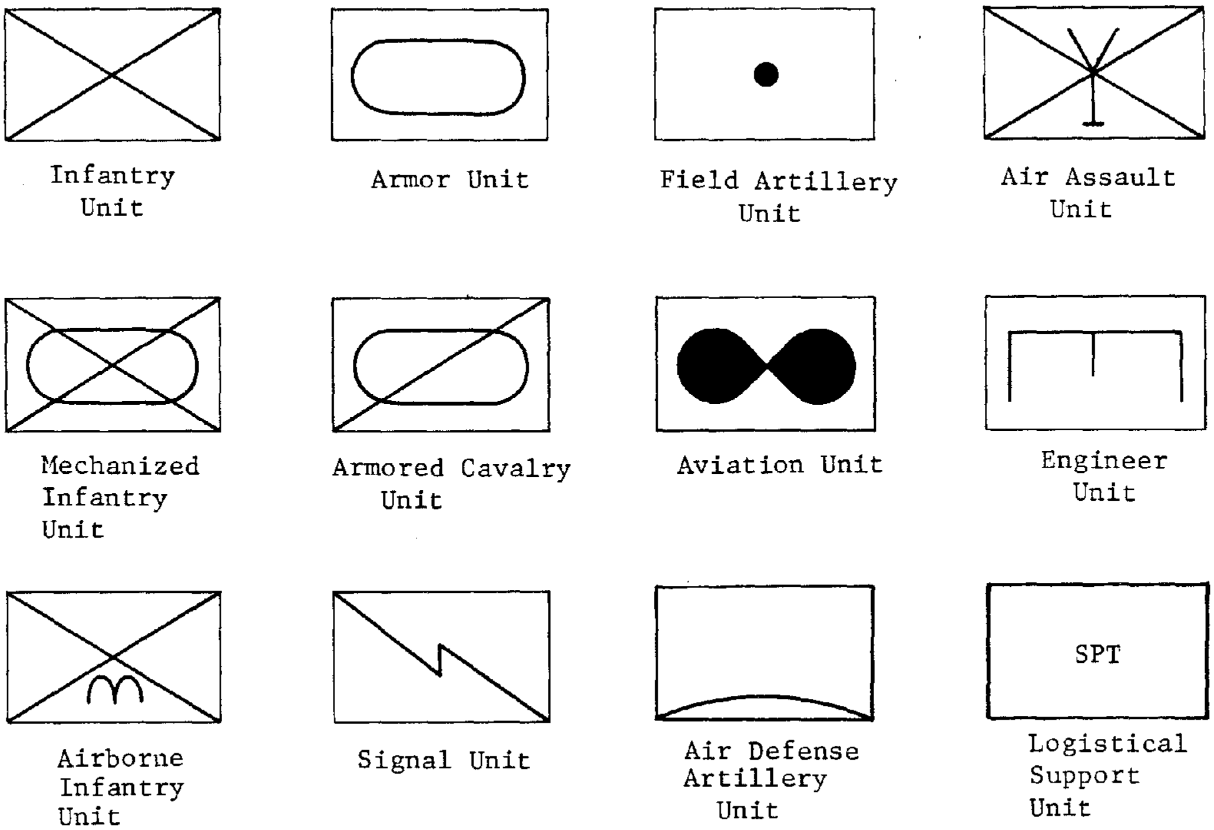
3. CAPABILITIES

a. A base of fire and maneuver elements
b. The means to secure and hold terrain
c. The capability to conduct independent operations on a limited scale
d. Limited antitank protection
e. Indirect fire support for organic and attached units
f. Long-range patrolling when properly equipped
g. A force that can participate in motorized, mechanized, and joint airborne operations when provided with sufficient transportation
h. A force that can participate in airmobile operations when provided with sufficient air transport
i. The capability to maneuver in all suitable types of terrain under all climatic conditions
j. Participates in amphibious operations
k. Provides limited air defense
l. Participates in counterinsurgency operations as elements of brigade-sized backup forces.

[1]Equipped with 18 TOW's and 27 Dragons.

[2]Infantry battalion of airborne division equipped with 12 TOW's and 30 Dragons.

Figure 8-1. Infantry Battalion



Note: Basic symbols may be combined.

Figure 8-2. Selected Map Symbols Indicating Unit Type

8-6.4 LARGER COMBAT UNITS

Using the combat battalions as basic building blocks, the Army has constructed a series of brigade and division size units for combat in the field. These units are, in turn, tied together for operational command and control into still larger forces, specifically tailored to the combat mission, the terrain and geography, and the intensity of the conflict involved.

8-6.4.1 THE DIVISION

There are five basic types of divisions (Ref. 4). Their organizations are similar, containing the basic combat units described earlier, along with the necessary combat support and combat service support elements to facilitate the designated missions. A brief description of each type follows. Additional information, including the weapons and equipment characteristic of each type, may be found in Refs. 4 and 10.

1. The Infantry Division. The organization of an infantry division is depicted in Figs. 8-4 (along with armored and mechanized divisions) and 8-5. The basic combat units are the infantry battalions (usually eight), a mechanized battalion, and a tank battalion. These are given command and control by the three brigade headquarters and headquarters companies. The combat battalions are assigned to the brigades in accordance with the mission given the brigade commander and may be tailored as "infantry heavy" or "armor heavy". Wheeled vehicles organic to the infantry division are the primary means of mobility.

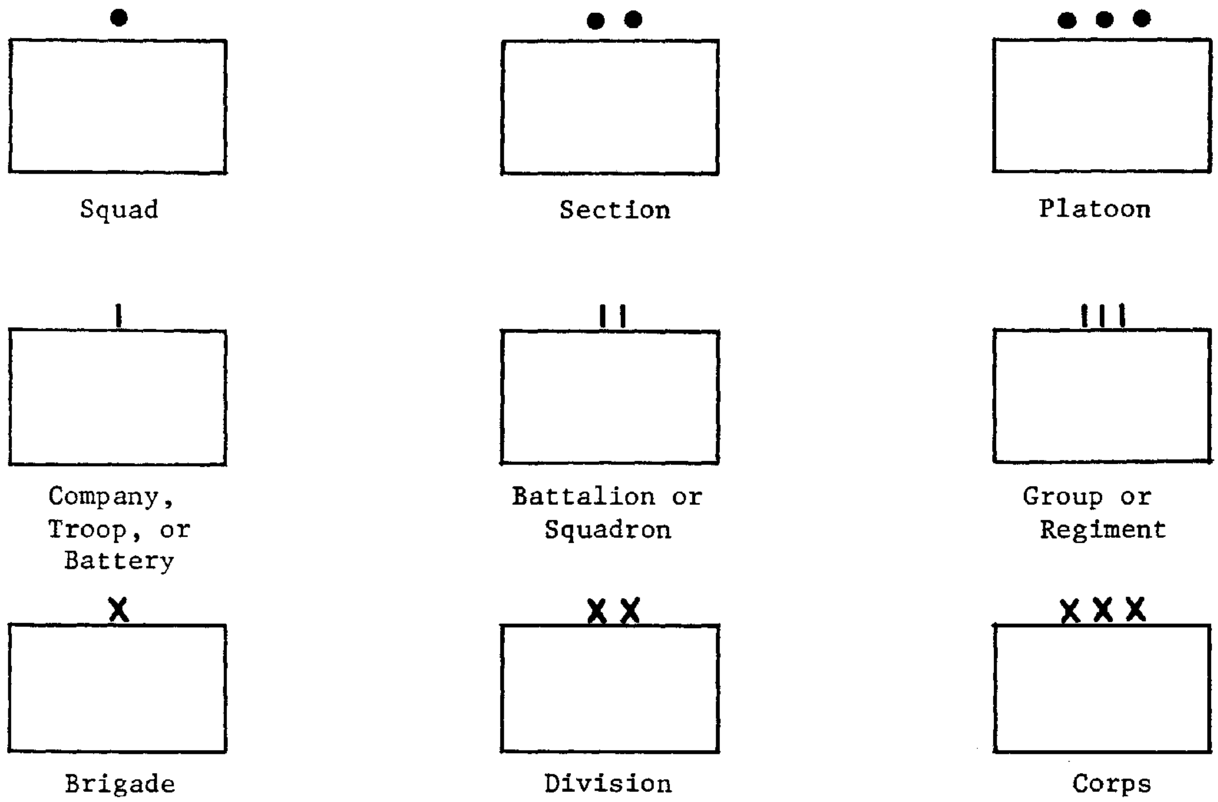


Figure 8-3. Selected Map Symbols Indicating Unit Size

2. The Mechanized Division (Figs. 8-4 and 8-6). The salient difference between this unit and the infantry division is the increase in mobility from that offered by wheeled vehicles to that of tracked vehicles. The combat battalions are still predominantly infantry (usually six mechanized battalions and four tank battalions). The division artillery has self-propelled weapons rather than the towed weapons found in the infantry division.

3. The Armored Division (Figs. 8-4 and 8-7). Within the armored division, the mobility of the mechanized division is retained but the firepower balance is shifted to the armament of the modern tank. Combat battalions are predominantly tank (usually six) rather than mechanized infantry (usually five).

4. The Airborne Division (Figs. 8-8 and 8-9). The airborne division is best characterized by its light weight, allowing all elements to be air dropped. Organic transportation is minimal, with the division depending on aircraft of the US Air Force for its long-range tactical mobility. The brigades are generally "infantry heavy" although the division does have one airborne armor battalion. The number of airborne divisions is rapidly being decreased in favor of the more mobile air assault division. An example is the conversion of the 101st Airborne Division to an air assault division during 1968.

5. The Air Assault Division (Figs. 8-10 and 8-11). The air assault division is by far the most mobile of the combat divisions. Its maneuver elements are airmobile infantry battalions, and it is characterized by the addition of an aviation group to provide organic helicopter mobility. The division artillery is equipped with lightweight, air transportable weapons which allow movement of the fire support in conjunction with the maneuver elements. Even the engineer construction equipment organic to the air assault division is transportable with the organic aircraft of the division.

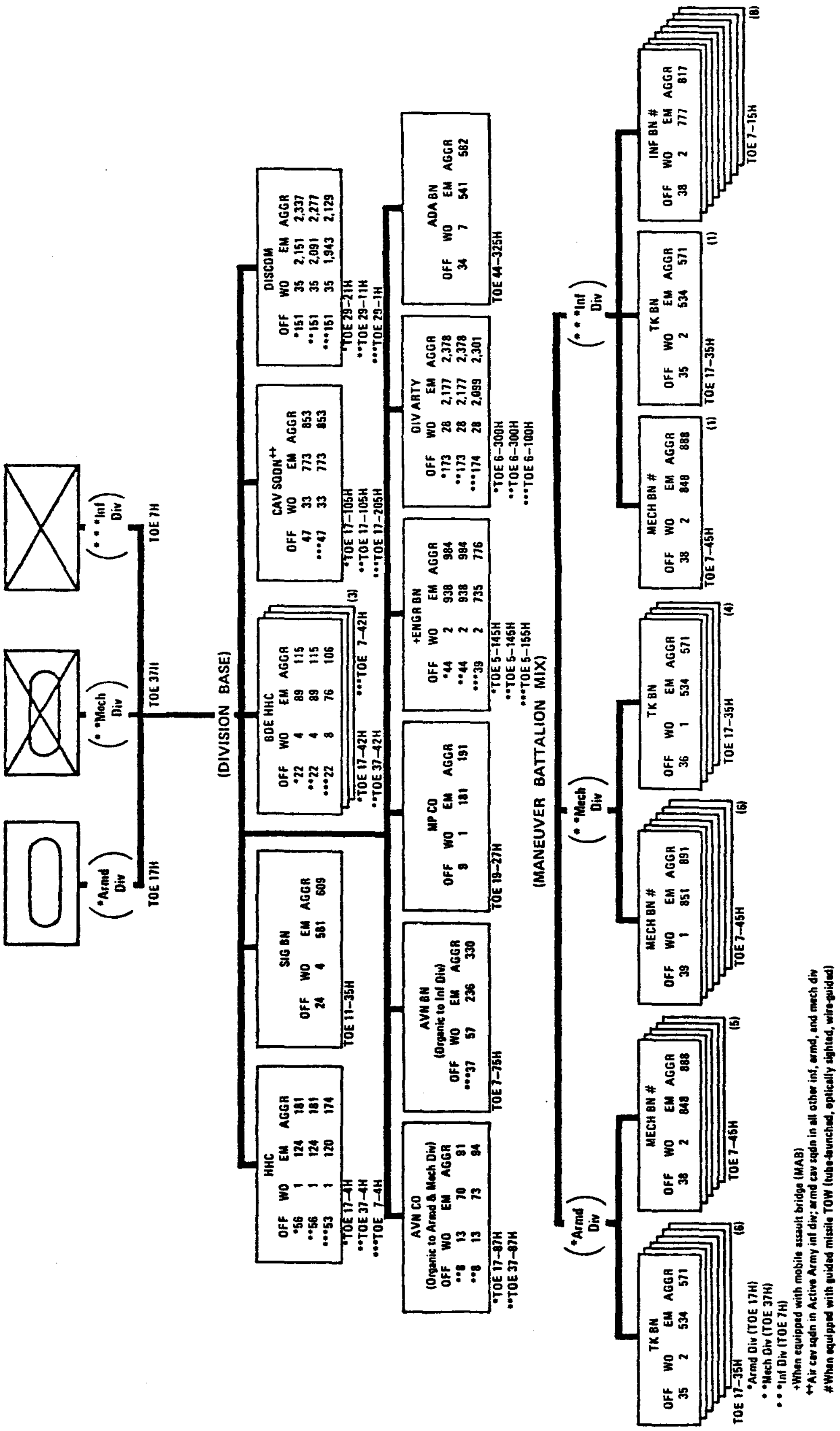
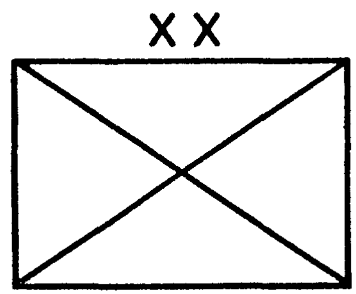


Figure 8-4. Basic Infantry, Mechanized, and Armored Divisions



	OFF	WO	EM	AGGR
DIV BASE	660	166	7,819	8,645
8 INF BN	304	16	6,216	6,536
1 MECH BN	38	2	848	888
1 TK BN	35	2	534	571
DIV TOTAL	1,037	186	15,417	16,640

1. MISSION

To destroy enemy armed forces and to control land areas, including populations and resources.

2. ASSIGNMENT

To corps.

3. CAPABILITIES

a. This unit has the following capabilities:

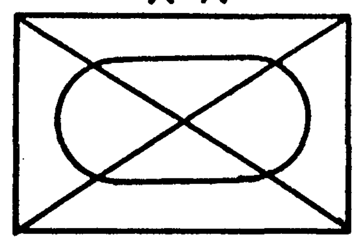
(1) Conducts sustained combat operations
(2) Conducts operations in difficult weather or terrain
(3) Participates in airmobile operations
(4) Operates as part of a joint amphibious task force
(5) Operates as part of a joint airborne force
(6) Conducts riverine operations
(7) Operates with austere logistic support
(8) Provides control and administration of up to 15 maneuver battalions
(9) Organizes and conducts defensive operations
(10) Provides air defense against low-altitude hostile aircraft.

b. This unit has the following limitations:

(1) Limited airlift capability
(2) Limited ground vehicular mobility
(3) Limited protection against armor
(4) Limited protection against artillery and nuclear fires.

TOE 7H

Figure 8-5. Infantry Division



	OFF	WO	EM	AGGR
DIV BASE	614	131	7,778	8,523
6 MECH BN	234	6	5,106	5,346
4 TANK BN	144	4	2,136	2,284
DIV TOTAL	992	141	15,020	16,153

1. MISSION

To destroy enemy armed forces and to control land areas, including populations and resources.

2. ASSIGNMENT

To corps.

3. CAPABILITIES

a. This unit has the following capabilities:

(1) Conducts sustained combat operations

(2) Provides rapid movement, deep penetration, and pursuit

(3) Disperses and concentrates rapidly over great distances

(4) Exploits successes, including effects of nuclear, nonnuclear, and chemical fires

(5) Conducts covering force operations and operates as a mobile counterattack force

(6) Conducts defensive operations

(7) Conducts limited airmobile operations

(8) Provides air defense against low-altitude hostile aircraft

(9) Provides control and administration of up to 15 maneuver battalions.

b. This unit has the following limitations:

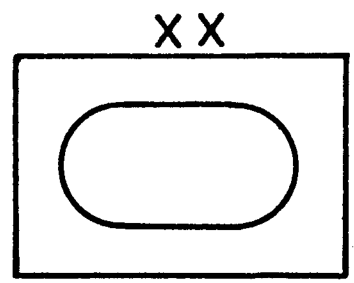
(1) When employed in airmobile operations it loses much of its striking power and ground mobility

(2) Vehicular mobility is restricted by jungle, dense forest, steeply rugged terrain, and water obstacles

(3) Requires heavy logistic support. This includes rail or highway transport of tracked vehicles for long administrative moves.

TOE 37H

Figure 8-6. Mechanized Division



	OFF	WO	EM	AGGR
DIV BASE	614	131	7,835	8,580
5 MECH BN	195	5	4,255	4,455
6 TK BN	216	6	3,204	3,426
DIV TOTAL	1,025	142	15,294	16,461

1. MISSION

To destroy enemy armed forces and to control land areas, including populations and resources.

2. ASSIGNMENT

To corps.

3. CAPABILITIES

a. This unit has the following capabilities:

(1) Conducts sustained combat operations

(2) Provides rapid movement, deep penetration, and pursuit

(3) Disperses and concentrates rapidly over great distances

(4) Exploits successes, including effects of nuclear, nonnuclear, and chemical fires

(5) Conducts covering force operations

(6) Conducts defensive operations and operates as a mobile counter-attack force

(7) Provides air defense against low-altitude hostile aircraft

(8) Provides optimum protection against antitank, artillery, and nuclear effects

(9) Provides control and administration of up to 15 maneuver battalions.

b. This unit has the following limitations:

(1) Heavy equipment cannot be lifted by Army aircraft

(2) Mobility is restricted by jungle, dense forest, steeply rugged terrain, and water obstacles

(3) Requires heavy logistic support. This includes rail or highway transport of tracked vehicles for long administrative moves.

TOE 17H

Figure 8-7. Armored Division

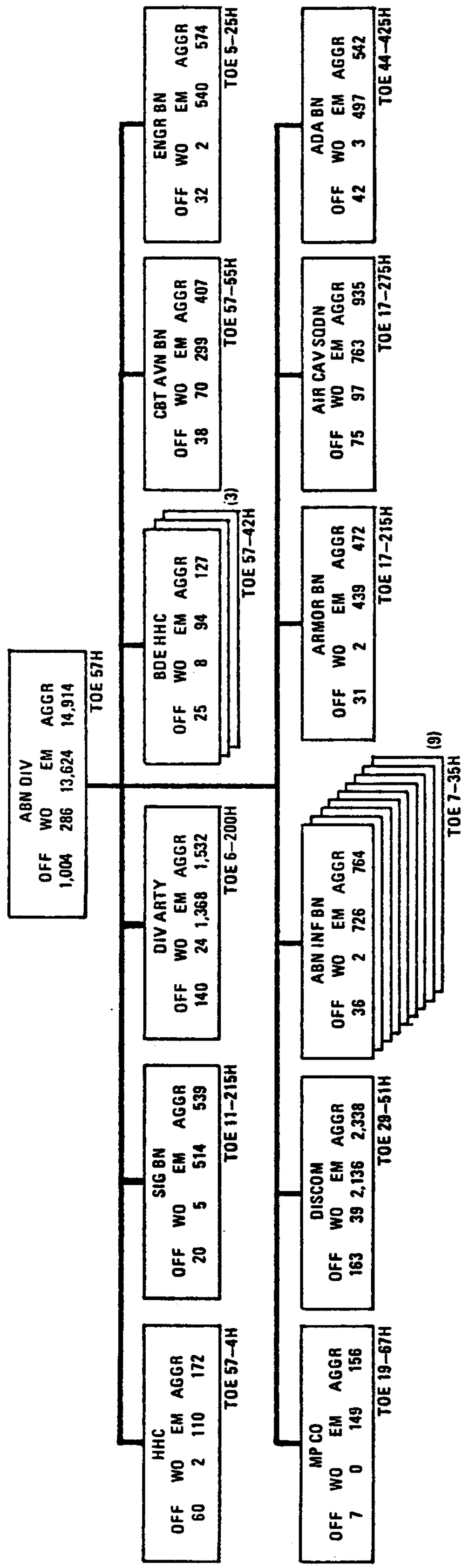
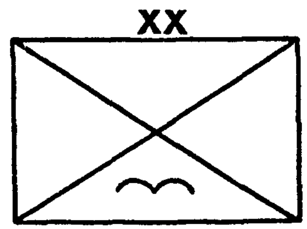


Figure 8-8. Airborne Division—Organization



	OFF	WO	EM	AGGR
DIV BASE	649	266	6,651	7,566
9 ABN INF BN	324	18	6,534	6,876
1 TK BN	31	2	439	472
DIV TOTAL	1,004	286	13,624	14,914

1. MISSION

a. Movement by air and by airborne assault to secure and hold assigned objectives, to close with the enemy, and to destroy or capture him in an unsophisticated or semisophisticated environment until ground linkup can be accomplished or until reinforced by air or surface landing.

b. Movement by air and by airborne assault, *when reinforced,* to secure and hold assigned objectives in sophisticated operational environments

c. Execution of small-scale airborne commando-type operations to perform selected missions.

d. Movement by air on short notice to any oversea land areas as a deterrent or resistance force in any threatened area.

TOE 57H

2. ASSIGNMENT

To theater or corps.

3. CAPABILITIES

a. Executes airborne assault by means of parachute drop or airlanding

b. Closes with the enemy to destroy or capture him, utilizing fire, maneuver, and close combat.

c. Conducts sustained ground operations when augmented by additional combat, combat support, and combat service support units

d. Acts alone or as part of a larger force

e. Conducts airmobile operations.

Figure 8-9. Airborne Division—Mission and Capabilities

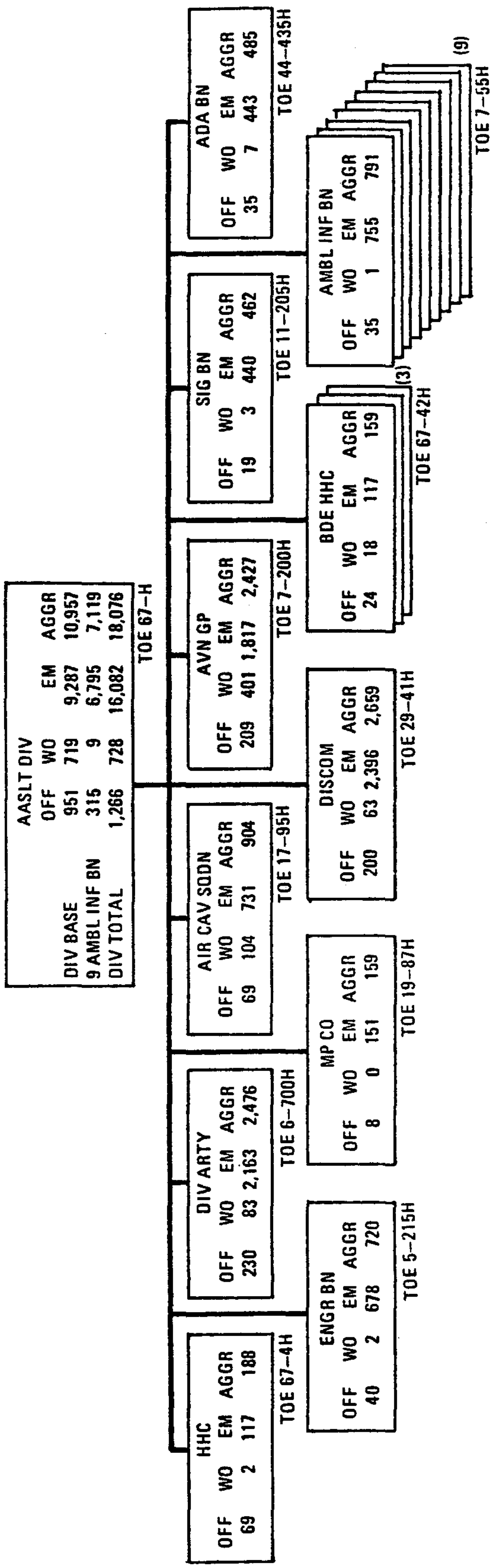
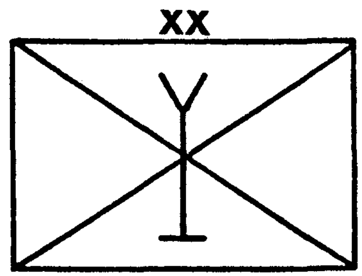


Figure 8-10. Air Assault Division—Organization



	OFF	WO	EM	AGGR
DIV BASE	951	719	9,287	10,957
9 AMBL INF BN	315	9	6,795	7,119
DIV TOTAL	1,266	728	16,082	18,076

1. MISSION

To destroy enemy armed forces and to control land area including populations and resources.

2. ASSIGNMENT

To corps or separate task force.

3. CAPABILITIES

a. Operates over a wide frontage in all types of terrain, locates and maintains contact with the enemy or friendly forces

b. Responds immediately and maneuvers rapidly to influence the tempo and frequency of engagements

c. Provides combat elements for economy of force roles, retrograde screening, rear area security, antiairborne and counterguerrilla operations, raids, and exploitation

d. Recycles combat forces for immediate use in other areas by vertical entry into and recovery of units from the battlefield

e. Can act as a highly responsive mobile reserve force

f. Possesses limited air defense capability against low-flying aircraft

g. Provides nuclear fires, and provides nuclear demolitions when engineer battalion is augmented by ADM platoon,

h. Provides control and administration for all organic and attached units.

i. Provides a parachute-qualified brigade and supporting elements capable of independent operations when organized under SRC 67000H120.

4. MOBILITY

For mobility of the components of this division, see each TOE.

TOE 67H

Figure 8-11. Air Assault Division—Mission and Capabilities

8-6.4.2 Separate Combat Brigades

In addition to the brigades already covered as organic parts of the divisions, separate brigades have been organized for maximum flexibility of operation and tailoring of forces for differing combat situations. Separate brigades have been created for independent operations like divisions. The separate combat bridges in use within the Army as of 1977 are:

1. The separate infantry brigade
2. The separate light infantry brigade
3. The separate mechanized brigade
4. The separate airborne brigade
5. The separate armored brigade
6. The air cavalry combat brigade.

The organization of the six types of brigades is shown in Figs. 8-12 through 8-17. Except for the air cavalry combat brigade, the number of maneuver battalions may vary between brigades of the same type. The air cavalry combat brigade is designated a light or heavy brigade depending on whether it has one or two attack helicopter battalions. A major difference between the separate brigades and the brigades within the divisions is the addition of combat support and combat service support elements in the separate brigades.

8-6.4.3 THE CORPS

The corps is the next level of command, control, and support over the division. The corps has both tactical and logistical functions and is structured with an organic corps support command (COSCOM) to provide logistical support to the forces within the corps.

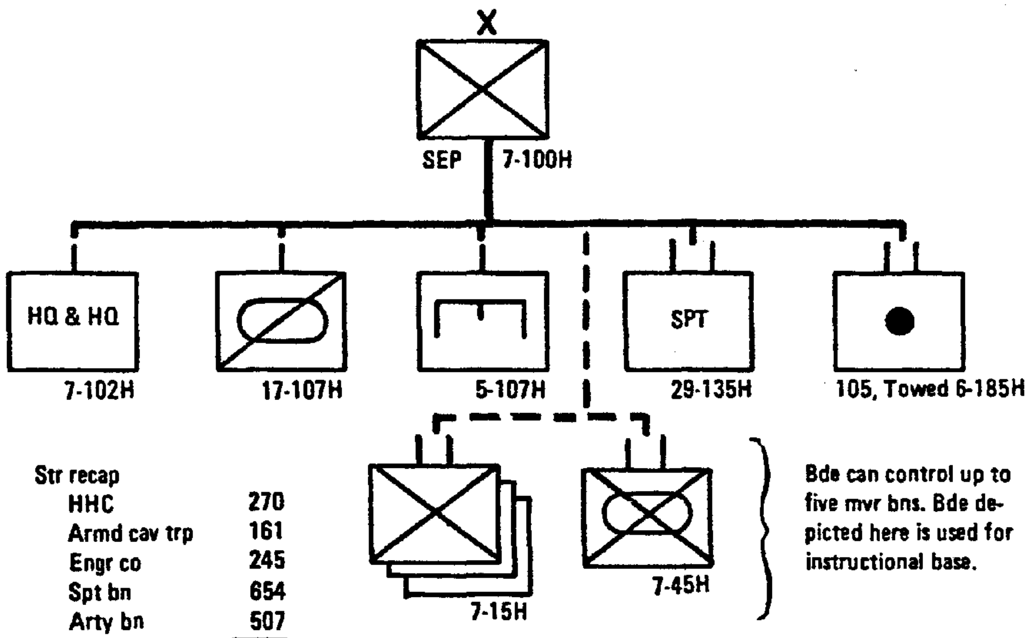
The composition of the corps is not fixed but rather tailored for the mission. Fig. 8-18 depicts an illustrative corps organization. The elements of the corps are depicted by type, not size, to highlight the flexibility of the organization.

8-6.4.4 Command Echelons Above the Corps

There is no established echelon above the corps to which it will habitually report. The next echelon above the corps depends on the size of the conflict and the organization for combat.

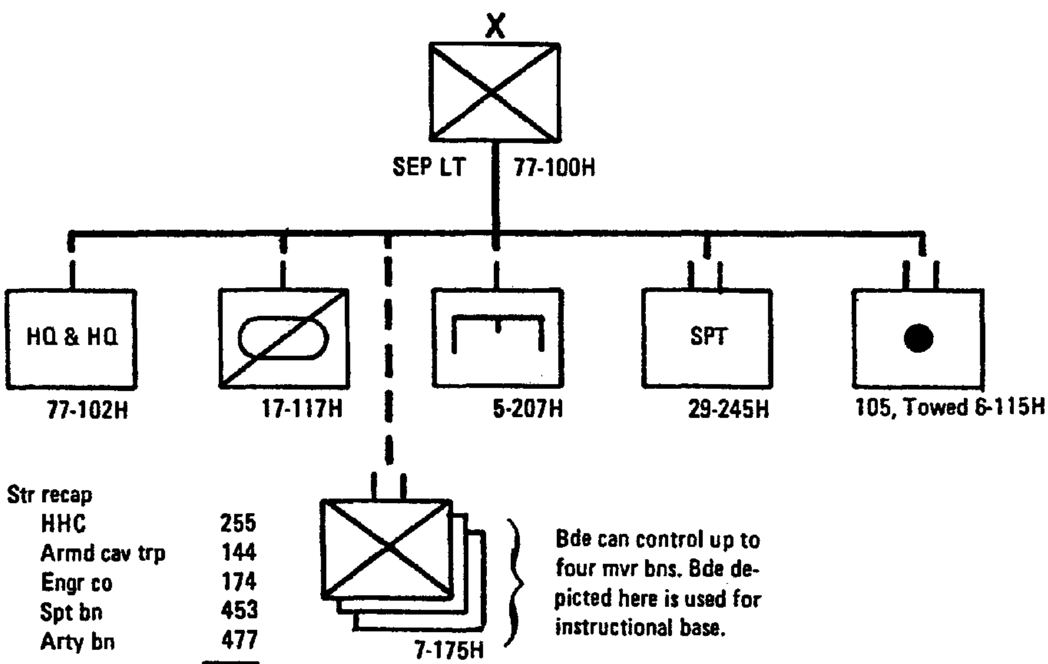
In some situations, the corps, or maybe even a reinforced division, may be the only Army organization within a type of command called a joint task force which includes not only Army but also maybe Navy, Air Force, and Marines. In this case, the joint task force commander will have tactical control of the Army organization.

A "theater" in a military sense encompasses a major geographic area such as a continent or an ocean basin. In World War II, US combat forces were organized in such area groups as Pacific Theater, European Theater, and China-Burma-India Theater. The composition of forces within a theater will depend on the geography and mission. Nevertheless, all US forces (Army, Navy, etc.) within the theater are controlled by the theater commander. A corps or several corps could be controlled directly by the theater commander who would have only tactical control over them. A theater army is established when it is advantageous to combine all army elements under a single command. The theater army provides both tactical control and logistical support to the Army organizations in the theater. The organization and size of the theater army will depend upon the location of the theater and the mission assigned to it. Should the number of corps within the theater army become too large to be managed properly by it, numbered field armies may be established to exercise tactical control over the corps.



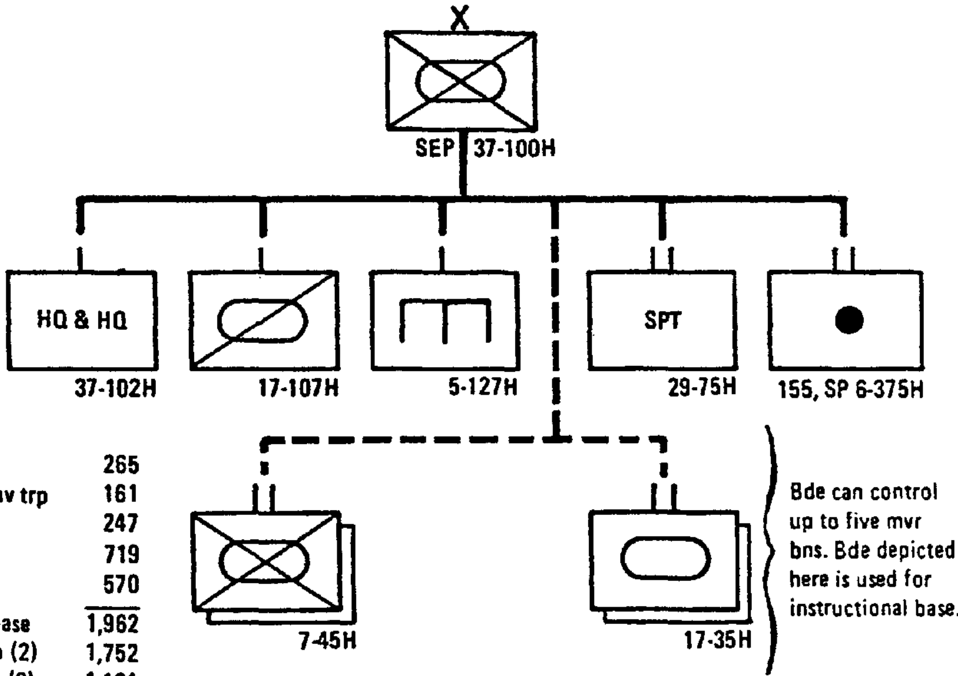
Str recap	
HHC	270
Armd cav trp	161
Engr co	245
Spt bn	654
Arty bn	507
Total bde base	1,837
Inf bn (3)	2,394
Mech bn (1)	876
Total bde	5,107

Figure 8-12. Separate Infantry Brigade



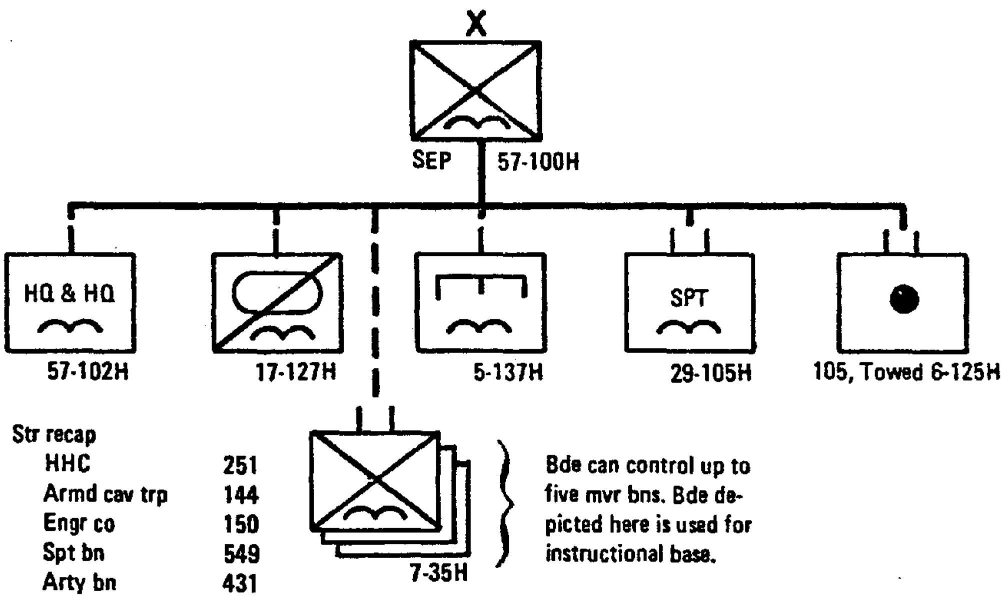
Str recap	
HHC	255
Armd cav trp	144
Engr co	174
Spt bn	453
Arty bn	477
Total bde base	1,503
Inf bn (3)	2,355
Total bde	3,858

Figure 8-13. Separate Light Infantry Brigade



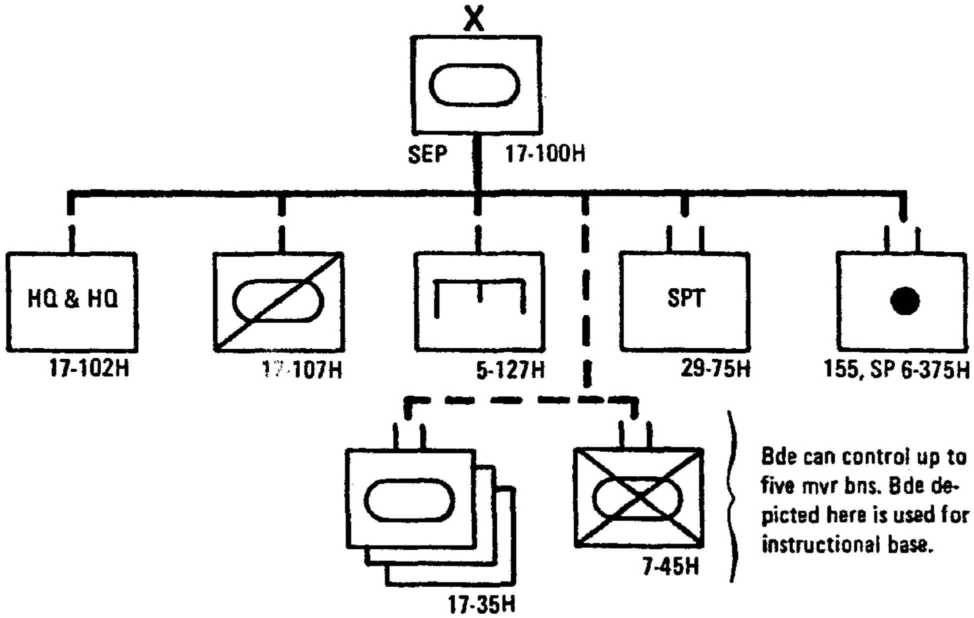
Str recap	
HHC	265
Armd cav trp	161
Engr co	247
Spt bn	719
Arty bn	570
Total bde base	1,962
Mech bn (2)	1,752
Tank bn (2)	1,104
Total type bde	4,818

Figure 8-14. Separate Mechanized Brigade



Str recap	
HHC	251
Armd cav trp	144
Engr co	150
Spt bn	549
Arty bn	431
Total bde base	1,525
Abn bn (3)	2,292
Total bde	3,817

Figure 8-15. Separate Airborne Brigade



Str recap	
HHC	264
Armd cav trp	161
Engr co	247
Spt bn	719
Arty bn	570
Total bde base	1,961
Tk bn (3)	1,656
Mech bn (1)	876
Total bde	4,493

Figure 8-16. Separate Armored Brigade

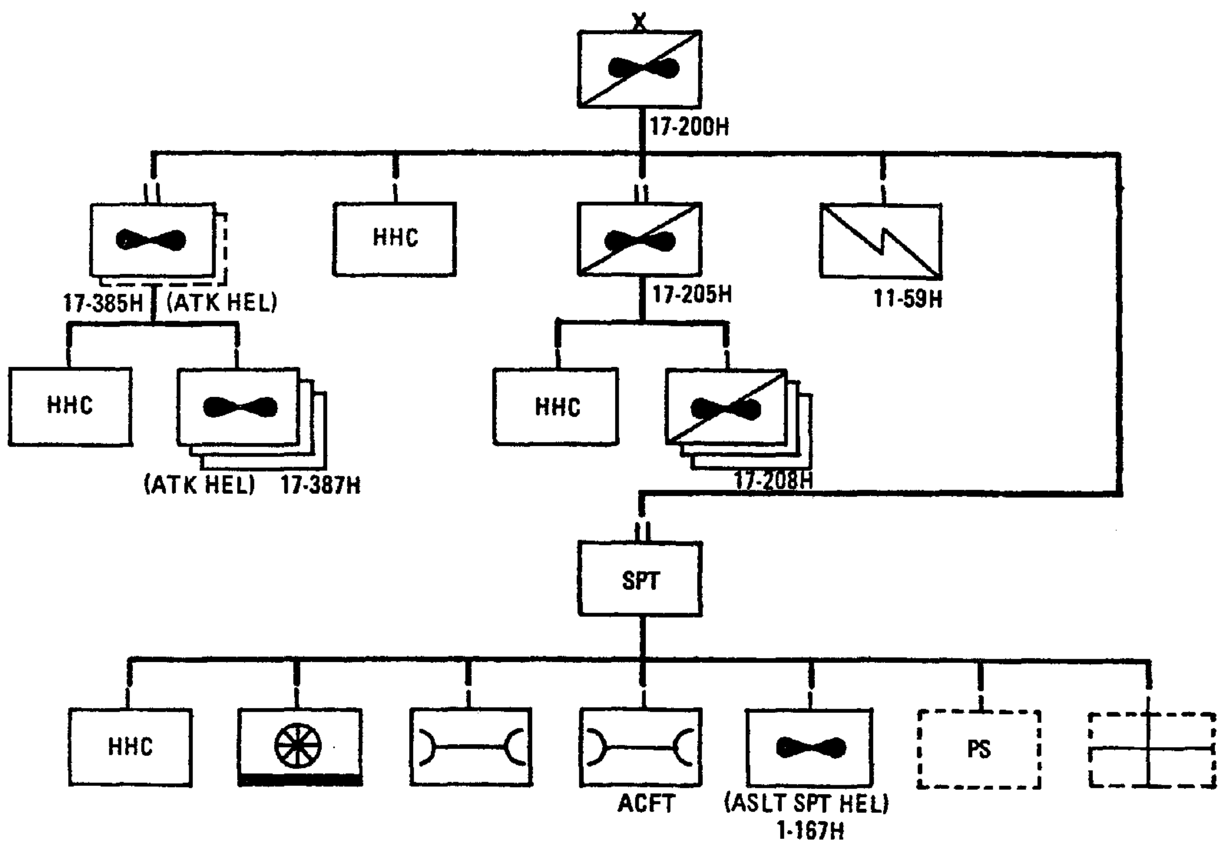
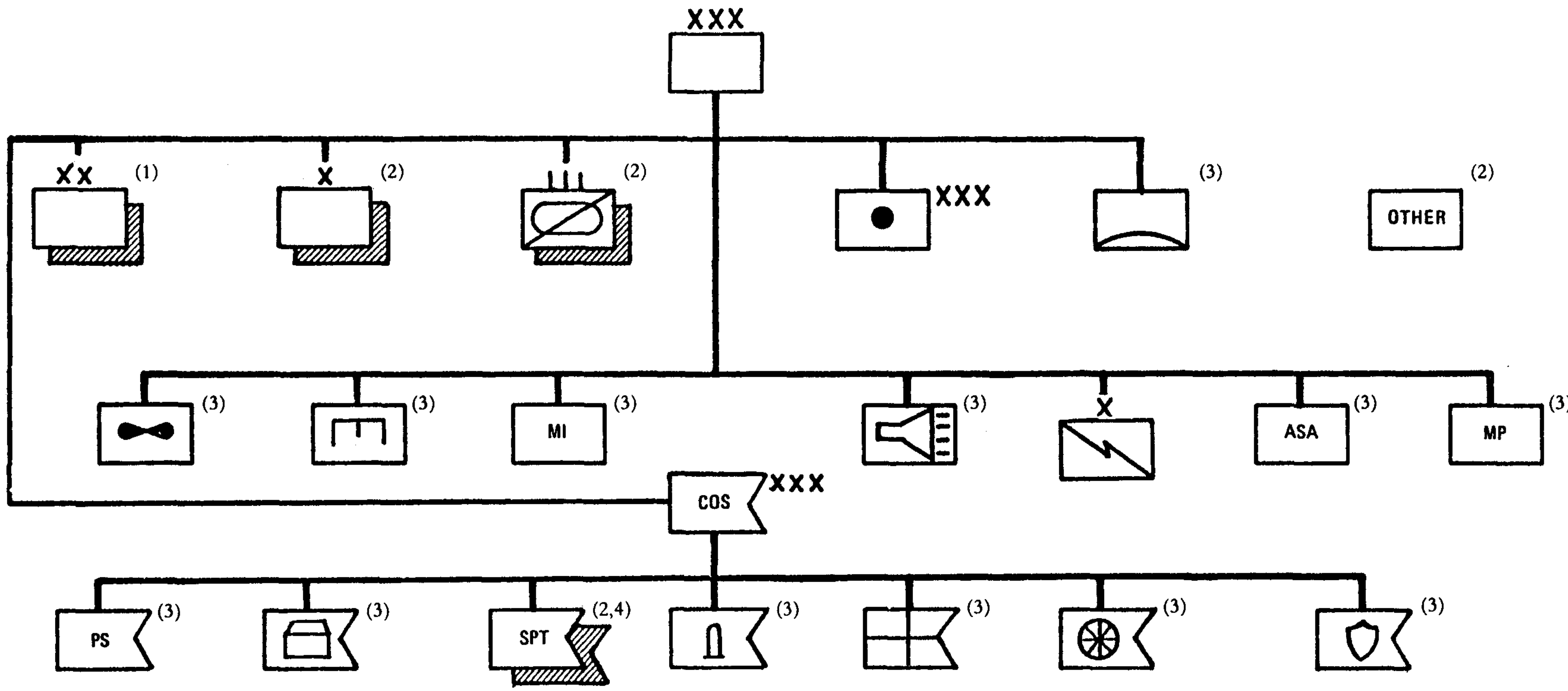


Figure 8-17. Separate Air Cavalry Combat Brigade



[1]Five divisions, more or less.

[2]Numbers and types of units will vary with requirements.

[3]The size of the command and control headquarters will depend on the scope and magnitude of its mission to include the number of subordinate units assigned.

[4]Provides DS supply and maintenance to nondivisional units and GS supply and maintenance in support of the entire corps.

NOTE: When performing combat service support missions, combat support units may be attached to either corps headquarters or COSCOM, depending on mission requirements and other considerations.

Figure 8-18. Illustrative Corps in an Established Theater of Operations

8-7 SUMMARY AND CONCLUSIONS

This chapter has covered the spectrum and types of conflicts in which the US Army may expect to become engaged and has indicated the roles and missions of various combat elements. Conflict is a very complex subject; consequently, the preceding paragraphs have only highlighted certain significant parameters of Army combat organizations and the differing levels of conflict for the weapon systems analyst. Much emphasis in the cited field manuals is placed upon activities on the nuclear battlefield. Yet, historically, the role of the Army in nonnuclear combat has been the main factor in shaping organizational structures and in determining the combat characteristics of weapon systems. It is essential that the weapon systems analyst keep in mind that the Army must be capable of multiple functions. It must be capable of conducting conventional and low-level combat (even with localized scenarios of high-intensity), and must therefore not trade off its combat effectiveness to "move, shoot, and communicate" in low-level conflicts (such as Vietnam) for combat effectiveness only on the nuclear battlefield. Often the trade-offs in combat effectiveness are very difficult to identify and even more difficult to quantify analytically. In devising his scenarios, the analyst must recognize the potential "operational environments" of his weapon systems in order to enable him to assess fairly the overall combat effectiveness of the using unit or combat element under varying levels of conflict.

Finally, we remark that the reader might wonder just how the various types of divisions and combat organizations discussed were arrived at, especially with the current types of weapons, personnel, and their numbers. In fact, just what has the field of weapon systems evaluations accomplished to aid in the current establishment of our combat units and the weaponry involved? Moreover, the size of Soviet Divisions in numbers of personnel currently run about one-third less than corresponding US divisions; for other units, such as companies, the size of Soviet units may be as low as one-half that of similar US units. This in itself also would seem to indicate that there is much opportunity for further study and optimization, perhaps through the means of operations research type studies. Therefore, it is believed that one of the important future problems for research in the field of weapon systems analysis and military operations research would seem to involve that of quantifying or modeling in some way the overall effectiveness, firepower, and composition of an infantry division as compared to an armored division, or air assault division, etc., in a given combat environment. Clearly, there is considerable room for serious developments in such a complex combined arms type of study and the so-called "firepower index concept" (Ref. 11) leaves much to be desired.

For the information of the analyst, a list of the "How to Fight" manuals may be found in the Bibliography.

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1. AR 310-25, *Dictionary of United States Army Terms.*
2. FM 100-5, *Operations.*
3. FM 30-5, *Combat Intelligence.*
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5. William Pearson (Brigadier General), "Fit to Fight Where?", *Army Magazine,* p. 54 (June 1966).
6. J. H. Fuller, *Weather and War,* Military Airlift Command, US Air Force, Scott Air Force Base, IL, December 1974.
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8. USACGSC RB 101-1, *Reference Book on Organizational Data for the Army in the Field.*
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10. FM 101-10-1, Staff Officers' Field Manual, Organization, *Technical and Logistical Data, Unclassified Data*.
11. J. A. Stockfisch, *Models, Data, and War: A Critique of the Study of Conventional Forces,* Project RAND Report R-1526-PR, March 1975.

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FM	TITLE
	CAPSTONE
100-5	Operations
100-5A	Electronic Warfare (SECRET)
100-5-1	Conventional-Nuclear Operations
101-5	Command and Control of Combat Operations
71-100	Brigade and Division Operations (Mechanized and Armor)
71-101	Brigade and Division Operations (Infantry/Airborne/Air Assault)
	COMBAT
71-1	The Tank and Mechanized Infantry Company Team
71-2	The Tank and Mechanized Battalion
7-7	The Mechanized Infantry Platoon/Squad
7-8	The Light Infantry Platoon/Squad
7-10	The Rifle Company
7-20	The Battalion (Infantry/Airborne/Airmobile/Ranger)
7-85	Ranger Operations
17-12	Tank Gunnery
17-50	Attack Helicopter Operations
17-47	Air Cavalry Combat Brigade
17-95	Cavalry
100-999	Air/Land Operations
	COMBAT SUPPORT
5-100	Engineer Combat Operations
6-20	Fire Support for Combined Arms Operations
11-50	Communications in the Division
11-92	Corps Signal Communications
21-40	NBC Defense
24-1	Tactical Communications Doctrine
44-1	Air Defense Artillery Employment
44-3	Chaparral/Vulcan Employment
44-23	Redeye Employment
44-90	Hawk Employment
90-1	Employment of Army Aviation Units in a High Threat Environment

BIBLIOGRAPHY (cont'd)

"How to Fight" Manuals

FM	TITLE
	SPECIAL OPERATIONS
90-2	Tactical Deception
90-3	Desert Operations
90-4	Air Assault Operations
90-5	Jungle Operations
90-6	Mountain Operations
90-7	Denial Operations and Barriers
90-8	Counter Guerrilla Operations
90-10	Military Operations in Built-Up Areas (MOBA)
90-11	Northern Operations
90-12	Airborne Operations
90-13	River Crossing Operations

CHAPTER 9

THE PHYSICAL ENVIRONMENT

The nature and effect of the physical environment in combat on general weapon employment is discussed in this chapter.

9-1 INTRODUCTION

Throughout the entire process of evaluation the weapon systems analyst should keep firmly in mind that the weapon system must be employed effectively in a combat or operational environment under conditions far removed from the environment of the laboratory, engineering facility, test facility, or proving ground. The *physical* environment must be considered as a vital component of the overall *operational* environment in which the weapon system will eventually be employed, and the evaluation made with realism. A vital element in any effective and meaningful weapon systems analysis is therefore the selection, quantification, and inclusion of significant parameters of the physical environment. (In this chapter, the terms *operational environment* and *combat environment* will be used more or less synonymously. A combat environment may or may not actually involve exchange of fire with the enemy, particularly in a limited war or counterinsurgency situation.)

9-2 BACKGROUND

9-2.1 GENERAL

The weapon systems of the major nations of the world have been designed traditionally for operation in temperate climates; their application in environmental extremes often results in some failures or less than optimum performance. Historically, it may be shown that failure to give sufficient attention to the physical environment in an operation or campaign has resulted either in military defeat or excessive casualties. In the Russian campaigns of Napoleon and Hitler, armies equipped with weapons and transport designed for temperate zone operations advanced into a subarctic environment with disastrous results for the invader (Ref. 1). Yet, even the somewhat arctic-oriented Russians were soundly defeated by the Finns in several significant engagements during World War II since the Finns truly optimized their weapons and transport for most effective use in a more nearly arctic environment.

In World War II, despite potential combat zones being otherwise known, the temperate-zone orientation prevailed in the design of US military equipment, and US forces were obliged to fight in the nontemperate deserts of North Africa, the jungles of New Guinea, and the mountains and tundra of the Aleutians. Planning went on to encompass combat in the mountains and glaciers of Norway and Greenland. Belatedly, the U.S. manned and trained a "mountain division" for this contingency, and developed special equipment for it as well. (The M29 "Weasel", for example, a tracked cargo carrier, was one of the results of this program.) Equally belatedly, but in reaction to the effects of the physical environment upon military equipment and its operators, the desert training areas in the American Southwest were developed. Even from experiences with combat operations during the winter of 1945, the U.S. still in the early 1950's had not adequately taken into account the effects of the physical environment in the design and employment of its weapon systems. This is evidenced by the record of weather-caused incapacitating casualties experienced during the winters of the Korean conflict.

9-2.2 DEVELOPMENTS IN THE EARLY 1960'S

In the 1960's involvement came in active conflict in the wet/dry tropical environment of Vietnam. That conflict highlights several examples of not taking into account the effects of the physical environment in weapon systems design and employment. Major environmental factors, such as the degradation of helicopter lift capabilities as a function of temperature and humidity, were well known and considered, but other factors were not considered and critical problems occasionally resulted. Examples are:

1. The very high frequency-frequency modulated (VHF-FM) radio was the mainstay of tactical communications in Vietnam, particularly in the vital "manpack" role, in which the forward infantryman communicated with his artillery, his air support, and his base. The effects of terrain masking on so called "line-of-sight-dependent" communications were well known. The fact that "vegetation" attenuates a radio signal, particularly in the heavily used VHF-FM band, had been known for decades. However, the extent of dense tropical vegetation effects, particularly in such areas as rubber plantations, were inadequately considered. As a result, numerous critical tactical situations occurred because of the inability of small units to communicate in such terrain.
2. The capabilities of the M113 personnel carrier to traverse soils of known weight bearing capabilities and to surmount obstacles of known dimensions and configuration were well known and had been quantified through an extensive test program. Unfortunately, the ranges and effects of the tides in the canals of the Mekong Delta of Vietnam were neither known nor considered in operational planning. Several serious tactical situations resulted in which M113's were trapped in steep-walled tidal canals of the Delta. The canals had been used as avenues of approach a few hours earlier and, at high tide, had been traversed with ease. A crash development program of vehicle-recovery kits was instituted as an after-the-fact operation. Only upon receipt of this equipment, much of which was hastily conceived and not overly effective, could the excellent mobility potential of the vehicles be employed to the fullest extent.

Lessons to be learned were not limited to the tropical environment of Southeast Asia. Serious shortcomings in equipment design, which were not foreseen in the weapons systems analysis procedures of the 1950's, were brought to light when the equipment was used by troops on field maneuvers in Alaska. For example:

1. When tanks traversed muskeg and the melted surfaces of ground underlain by permafrost, they would make deep ruts. When the ruts became frozen, tanks became entrapped in them, unable to extricate themselves. Even when "zero-length steering" was used, the tracks would simply slide in the frozen ruts without obtaining traction.
2. Ice fog from vehicle engines not only produced local whiteouts, but also gave an "intelligence signal" detectable for miles.
3. Steel Bailey Bridges, erected in sub-zero temperatures over still-flowing arctic streams, served as collectors of ice vapor until they accumulated a burden of rime exceeding their bearing capabilities.

These examples illustrate an operational interface between an element of a weapon system and the physical environment.

9-3 ENVIRONMENTAL ANALYSIS

9-3.1 REQUIREMENTS

The examples given in par. 9-2.2 demonstrate it is essential that the *significant* parameters of the natural or physical environment be identified, that they be quantified wherever possible, and that they

be properly considered in any weapon systems analysis. A partial list of natural and induced environmental parameters that should be considered is given in Table 9-1. Induced parameters are those brought about by man (Ref. 2). A valuable treatment of characteristics, effects, and quantification of some environmental parameters is given in Refs. 2-6 (Environmental Series Handbooks).

The weapon systems analyst usually is not able to conduct an environmental analysis, but he should incorporate pertinent environmental factors in his evaluation. Numerous data sources, guidelines, and professional resources are available and are discussed in the paragraphs that follow. Basic guidelines are provided in Chapter 8, "The Sphere of Conflict". The levels of conflict anticipated may provide some guidance as to the geographic areas in which weapon systems might logically be employed. Operational requirements—based upon forecast Army functions, posture, and organization in contingency conflict planning—should provide valuable inputs to the weapon systems analyst. The contingency plans thus provide geographic insight into the employment of a given weapon system in a specified area of the world. The systems analyst must not consider "requirements" given to him as final, requiring no further investigation or amplification. He must consider the requirements of the eventual user in the field, requirements which may not have been identified in detail prior to the start of the analysis. The weapon system must, above all, be combat effective in a spectrum of environments.

The guidelines for an environmental analysis are clear. The *mission* to be performed determines the *requirements* for the weapon system. The weapon system, in order to perform the mission, must operate in a spectrum of physical *environments* commensurate with the nature of the conflict, discussed in Chapter 8. The weapon system must also operate in an *operational environment* generated by the interaction of

TABLE 9-1. SOME NATURAL AND INDUCED ENVIRONMENTAL ANALYSIS PARAMETERS

Natural		Induced
Asteroids	Meteroroids	Acceleration
Clouds	Ozone	Acoustics
Cosmic radiation	Pressure, air	Aerodynamic heating
Density	Rain	Explosive atmosphere
Dew	Salt spray	Gases, dissociated
Electricity, atmosphere	Sand and dust	Gases, ionized
Fog	Sleet	Magnetic fields
Frost	Snow	Moisture
Fungi	Solar radiation	Nuclear radiation
Gases, dissociated	Spores	Shock, mechanical
Gases, ionized	Temperature	Temperature
Geomagnetism	Temperature shock	Temperature shock
Gravity	Turbulence	Vibration
Hail	Vacuum	Zero gravity
Humidity	Winds and gusts	
Ice	Wind shear	
Insects		

friendly and enemy forces. Effective operation in the combination of physical and operational environments is the true measure of *combat effectiveness*. If a weapon systems analysis does not consider fully these criteria, the analyst has not fulfilled his role properly, even though the system may be functioning satisfactorily in test, laboratory, or proving ground environments. The systems analyst, therefore, must be able to model the significant parameters in the employment of weapons and evaluate them accordingly.

9-3.2 INTERACTIONS BETWEEN THE PHYSICAL ENVIRONMENT AND WEAPON SYSTEMS

It is obviously impossible for the weapon systems analyst to incorporate all the variables of all physical environments in which a given weapon system may be employed. He may, however, first identify the major environmental parameters, examine them for significance *vis a vis* the weapon system being analyzed, and finally incorporate appropriate environmental elements in his weapon systems analysis.

The general characteristics of the weapon system being analyzed will provide the initial elements of guidance in the determination of the significant environmental parameters to be examined, for example:

1. The systems analysis of a ground vehicle should consider carefully the terrain, which affects mobility, in the intended operational area. Specifically, he must consider:
    a. Topography or configuration of the surface
    b. Vegetation
    c. Soil and rock
    d. State of the ground by season
    e. Hydrographic features.

2. The analysis of a radar detection system should be concerned with similar aspects of the physical environment. If the system were to be employed in ground surveillance, the configuration of the surface and the presence of "shadow zones" would be important elements of the analysis. If, on the other hand, the system were to be incorporated into an air defense missile system, not only must the geography of the site be considered as a fundamental element of the environment in order to avoid radar shadowing by nearby mountain ranges, but also climatic factors would play an important part in that there may be signal attenuation caused by rain, snow, fog, or clouds.

3. A radar-guided antitank missile systems analysis must consider multiple constraints imposed by terrain. Terrain affects ranges of engagement, siting of the weapon system, possible paths of target vehicles, and possible attenuation of either radar detection signal or missile control signal by vegetation. Also, effects of the vegetation on either blast or the probability of premature detonation, and the effect of the climate on the operator's ability to see and track the target by optical means must be considered.

Table 9-2 summarizes some characteristics of the physical environment which could affect the operation of Army weapons and equipment. It is presented as a representative sampling of various types and categories of weapons and equipment in their operational interface with the natural environment.

Development of weapon systems and equipment is a dynamic process, with new and revised items constantly being introduced into the inventory, or proposed for introduction. Evaluation of competitive weapons and optimization of weapon design are based, in part, on proper measures of their effectiveness — e.g., the probability of a hit or kill, or a lethal area. Effectiveness is dependent on warhead

TABLE 9-2. REPRESENTATIVE EFFECTS OF MAJOR ELEMENTS OF THE TERRAIN UPON SELECTED TYPES OF MILITARY EQUIPMENT AND WEAPONS

Equipment or Weapon	Topography	Vegetation	Hydrography	Surface Materials	Cultural Features	Weather	Climate	Remarks
Tracked combat Vehicles	1	2	1	2	2	2	2	Tanks and self-propelled artillery using tank hulls have ground pressures of from 12-15 psi. Size and weight of vehicles interact with topography, soil condition, local relief features and, to a lesser extent, vegetation to inhibit movement. Bridges impose weight restrictions; configuration of towns and villages also restrict movement.
Tracked special purpose vehicles	1	1	2	2	2	2	2	Special-purpose vehicles are lighter, smaller, of lower ground pressure (8 psi for the M113 APC, 1.5 psi for the M29C "Weasel"). Vehicles are generally amphibious.
Wheeled vehicles	1	1	1	1	2	2	1	Wheeled vehicles have higher ground pressures, with the exception of GOER-type vehicles, normally road-bound by topography and state of ground. Mostly nonamphibious, they must rely on bridges or fords. Less restricted than tracked vehicles in built-up areas.
Mortars	2	1	-	2	2	2	2	Mortars are high-trajectory weapons, portable with considerable flexibility in siting. Saturated soils would limit dug-in positions. Minimal effects caused by cultural features in siting and employment. Weather may limit visibility by observers. Terminal effects may be affected.
Recoilless rifles	2	1	-	-	2	2	2	Recoilless rifles are mainly man-carried, flat-trajectory weapons. Mobility of weapon is offset by requirements for extensive open fields of fire. Weather and visibility limitations are important. Vehicle-mounted recoilless rifles subject to restrictions of carrying vehicles.
Field Artillery	1	1	-	2	1	2	2	Field artillery mobility is limited by type of prime mover, truck, tractor, or helicopter, and state of ground at emplacement. Vehicle-towed F.A. essentially limited by terrain, state of ground, and vegetation. Premature burst effects due to vegetation must also be considered. Use limited in built-up areas. Type of piece reflects terrain constraints — e.g. 105 howitzer more "flexible" than 175 gun when type of trajectory and weight of piece are considered.
Surveillance radar	1	1	-	-	2	2	2	Surveillance radar is limited by shadowing effects of topography and vegetation. Latter may either attenuate signal or reflect as clutter. Multipath reflections in built-up areas also a problem. Local animal life, wild or domestic, cause of false alarms. Design should reflect climate environment for arctic or tropic use.

(continued)

TABLE 9-2. (cont'd)

Equipment or Weapon	Topography	Vegetation	Hydrography	Surface Materials	Cultural Features	Weather	Climate	Remarks
Wire-guided antitank missiles	2	1	-	-	2	2	2	Topography is less constraining on the employment of this weapon than is vegetation which imposes constraints on visibility essential for guidance. Also imposes visibility restrictions. Mobility of weapon is based on the nature of the platform (manpack, vehicle, or helicopter). Climate requires consideration in design of components for arctic or tropical extremes.
VHF-FM Tactical radios	1	1	-	-	2	2	2	Very-High Frequency, Frequency Modulation (VHF-FM) radios are shielded by topography and are either attenuated or completely shielded by vegetation. Tropical vegetation has serious effects on signal propagation of such line-of-sight dependent electronic equipment. Weather effects are marginal, climate effects on component life and battery life must be considered.
HF tactical radios	2	2	2	2	-	2	2	High Frequencies (HF), Tactical Radios are less constrained by terrain and vegetation. They do, however, have a ground-wave component, responsive to the electronic conductivity and dielectric constant of the surface and subsurface material. Effects of the topography increase with frequency. This affects their performance in the short-range, tactical mode.
Strategic communications (Tropo-scatter)	2	-	-	-	-	2	2	Troposcatter communication equipment, if sited properly, clear of shielding vegetation and topography, is essentially terrain-independent. Climate effects should influence design for tropic or arctic use.
Tactical battlefield missile systems Guidance	2	2	-	-	-	2	2	Missile guidance systems are subject to essentially the same constraints as surveillance radar. Topography and vegetation restrict deployment capabilities. System design should reflect possible climatic extremes as above.
Missile	2	-	-	-	-	-	-	If sited properly, the performance of the missile itself should be only minimally affected by terrain, subject primarily to the limitations on mobility of the carrying vehicle. Performance of warhead of surface-to-surface missiles may be affected by the environment in the terminal portion of the trajectory to include the target environment.

(continued)

TABLE 9-2. (cont'd)

Equipment or Weapon	Topography	Vegetation	Hydrography	Surface Materials	Cultural Features	Weather	Climate	Remarks
Air defense missile systems Guidance	2	-	-	-	-	-	2	Air defense missile guidance systems are subject to general constraints of topography in siting for most effective coverage. Guidance electronics must be designed responsive to climatic effects on components.
Missile	2	-	-	-	-	-	-	Air defense missile operation must be responsive, primarily in siting geography to topographic and cultural features. Other effects of the physical environment are minimal.
Fixed-wing Army aircraft	1	2	2	1	2	1	1	Primary features of Army fixed-wing aircraft have always been capabilities for operation from primitive airstrips and a basic STOL capability. Terrain and climate have extensive inputs in determining design requirements. The state of the ground determines the "foot print" of the aircraft and other elements of "rough field" operation. Vegetation and topography also impose limits, primarily in the siting of airstrips. These elements of the terrain may, however, be altered by engineer effort (and typically are). Climatic extremes determine such problems as fungus contamination of jet fuels and deterioration of material in the tropics, and the numerous operational and maintenance problems encountered in the arctic.
Helicopters	2	2	2	1	2	1	1	Army helicopters are constrained by the configuration of the topography to a very limited extent except in actual basing requirements. Their operation is affected by vegetation, weather, climate and most particularly by the state of the ground. The bearing power of the soil will affect dimensions of landing skids or wheels. The tendency of the soil to become dusty will affect rotor design to minimize down-wash and possible avionics requirements for "blind" landings. The dust factor also is significant in the design of engine intakes (the "ingestion" problem). Degradation of performance in arctic and tropical environments imposes additional constraints. In the tropics, fuel contamination and loss of lift in temperature and humidity extremes are additional significant elements of the environment. In the arctic the effects of low temperatures on metals, resulting in premature fatigue; flow problems with lubricants; the induced ice-fogs generated by discharge of hot exhausts into super-cooled air; and the blinding of pilots by down-wash induced snow (similar to the dust problem) are but a few of the problems in operation of rotary-wing air-craft.

LEGEND

1 Item definitely affects equipment design and operation.
2 Item may affect equipment design and operation.
- Not applicable

performance which includes such factors as height of burst (for fuzed warheads), terminal velocity, yaw, and dispersion, all of which may be affected by the terrain environment in which the weapon is employed. The degrading effects of terrain on these performance characteristics must be modeled. A Degradation Effects Program was established as a continuing triservice effort to estimate the entire range of environmental effects on the terminal performance of standard and development items (Refs. 7-10).

If the designer fails to consider major environmental parameters and their interaction with the system in an operational environment, failure or costly retrofits later in the operational life of the system may result. The consequences of inadequate attention to environmental factors may be illustrated by the following:

1. A combat surveillance radar was tested in 1957. The equipment had an effective range of 30,000 yd. Analysis, using topographic maps (particularly of Western Europe), showed that if the radar were to be emplaced in "tactically logical" terrain (e.g., sited for the best possible coverage of roads and trails)—because of topographic masking—it would need its 30,000-yd range capabilities less than two percent of the time, its 20,000-yd capabilities less than ten percent of the time, and only would be capable of continuous surveillance of significant roads in the 3,000 to 5,000-yd range. Ground surveillance radars capable of these ranges, which were already in the inventory, were lighter, more mobile, less complex, and considerably cheaper. Consequently, the proposed project was curtailed, but only after considerable development effort.

2. The magnitude of the dust problem in Vietnam, an element of "induced environmental change", was not given proper consideration in helicopter design and testing. A research and development program for an effective dust palliative was instituted on a priority basis. Environmental analysis (Ref. 2) would have shown that establishment of clearings in lateritic soil and removal of covering vegetation would result in a dust problem of severe proportions.

3. Frequent reports of enemy "jamming" of VHF-FM communications nets in Vietnam were traced to improperly waterproofed radio handsets. When immersed in water, a common occurrence in Vietnam, the set would "key" to the "on" mode, thus effectively "capturing" the entire net of radios in the vicinity which were tuned to the same frequency.

9-3.3 SIGNIFICANCE OF EFFECTS OF THE PHYSICAL ENVIRONMENT ON WEAPON SYSTEMS ANALYSIS

The weapon systems analyst must make certain pragmatic decisions in assessing the possible effects of various elements of the physical environment on the weapon system being analyzed. As a minimum, he must consider the questions that follow:

1. Is the weapon system, in its present form, limited to operations in temperate climates?
2. How do the following elements of the physical geographic environment affect system performance:
   a. Topography, physiography, and landform surface
   b. Vegetation
   c. Surface materials
   d. Hydrographic features
   e. Induced environmental features?
3. Do potential tactical scenarios postulate the use of the weapon system being analyzed in:
   a. A worldwide environment
   b. Arctic environments

c. Tropical environments
d. Temperate climates with seasonal aspects of arctic or tropical climates
e. Desert environment
f. Mountain environments?

4. To what extent would the performance of the weapon system be restricted or enhanced under the various environments cited in 3a through f?

5. If system deficiencies are anticipated in operation to nontemperate climates, should the system be subject to:
a. Basic redesign for the designated environment
b. Modification of the weapon system in the continental United States (CONUS)
c. Fitting of "modification kits" in CONUS
d. Modification by "kits" shipped to the field?

6. What qualitative and quantitative data or tests are required in order to design and analyze a weapon system which will be responsive to the major parameters of the physical environment?

9-4 BASIC ENVIRONMENTAL ANALYSIS METHODOLOGY

9-4.1 GENERAL

Having determined that the weapon system being analyzed is eligible for worldwide deployment, the systems analyst must next examine the technical and operational constraints which would be imposed by a spectrum of geographic environments. Climate* immediately emerges as the dominating geographic factor since it affects vegetation, soil, hydrography, local weather patterns, and in some areas, the configuration of the terrain. Terrain, at the same time, has a major effect on the climate. Precipitation is increased by air flow up the windward slopes of mountains and decreased by downward flow on the lee side.

Extensive sources of climatic and topological information, either in raw or evaluated form, are available to the analyst (See Refs. 11 and 12 for example).

9-4.2 CLIMATIC TYPES

Meterologists have divided the world's surface into climatic-type regions, as shown in Table 9-3 and Fig. 9-1 (reproduced from Ref. 11). Specific climatic criteria to be considered in the research, development, test, and evaluation of a weapon system are delineated in Ref. 11. Table 9-4, extracted from MIL-STD-210 (Ref. 12), summarizes the probable extreme climatic conditions to which military equipment may be exposed. This MIL-STD establishes uniform limits not to be exceeded in normal design requirements of military equipment.

The weapon systems analyst, by reference to Tables 9-3 and 9-4 and Fig. 9-1, could, with confidence, establish the climatic bounds in which his weapon system would be expected to operate. If he chooses to go into greater detail, extensive climatic records are available through the Air Weather Service of the US Air Force (USAF), both as basic records and as compiled tabular presentations. Further refinements of climatic data for many areas throughout the world have been compiled and are available in the National Intelligence Survey. Climate data inputs to these studies have been prepared by the Air Weather Service of the USAF.

*Climate is defined as the long-term patterns of wind, temperature, humidity, and precipitation that are characteristic of a given region. Weather is defined as the day-to-day changes in these elements. Climate also may be defined as the composite of weather over a period of time.

TABLE 9-3. WORLDWIDE CLIMATIC TYPES AND REGIONS

Climatic Type	Climatic Region	Sample Area	Basic Climatic Description
Tropical Rainy	Rain Forest	Sumatra, Congo, Amazon	Constant dampness and heat, dense vegetation. Heavy rains throughout year.
Tropical Rainy	Savanna (also called "monsoon" or "wet/dry tropic")	Vietnam, Kenya	Monsoon-controlled wet and dry seasons. Extensive grassland areas interposed with dense and sparse tree growth.
Dry	Steppe	Western US Plains States, N.E. China, S. Soviet	Evaporation in excess of rainfall. Artificial irrigation required. Meager, natural vegetation, grasslands predominate.
Dry	Desert	Sahara, Saudi Arabia, US Southwest	Hot days, cool nights varying with altitude. Environmental ranges reflected in sample areas. May be "no rainfall" or may be interposed by "cloudburst type" rains of US southwest.
Warm Temperate Rainy	Mediterranean (Dry Winters)	Southern Calif., Mediterranean basin, Spain	Hot, dry summers and mild winters. Vegetation sparse to moderate.
Warm Temperate Rainy	Dry Summers	US Gulf States, Argentina, Japan, N. Laos, E. Central China	Moderate to high rainfall, high humidity. Coastal areas subject to less seasonal variation than inland areas. Vegetation varies with land use and soil conditions.
Marine	Wet Summers	Northwest Europe, UK, New Zealand	Abundant rainfall, general lack of temperature extremes. Mild winters, cool summers. Influenced by local terrain, particularly mountains.
Subarctic	Continental (Hot summers. Wet winters)	Central US Plains States (Dakotas), Great Lakes Region, S. Central Canada, Central Europe and Western USSR. Balkans	Extensive temperature ranges by season. Very hot summers in coastal areas. High humidity. Cold snowy winters with blizzards common in interior areas.
Subarctic	Cold summers, cold winters	Bulk of N. and Central USSR, Finland, bulk of N. and Cent. Canada	Also called "Polar Continental." Long cold winters, severe winter temperatures, brief summers. Light precipitation. Permafrost in northern sectors. Timber present, generally coniferous.
Polar	Tundra	Alaska North Slope, "Lapland", Northern Siberian and USSR coast	Warmest month. 50°F. Light precipitation. Grassland and brush only. Trees not present. Temperatures to -40°F common in winter months.
Polar	Icecap (Perpetual Frost)	Greenland, Antarctica	Polar temperature extremes are combined with altitude causing winter temperatures below -50°F. Light precipitation, strong winds, and gales in icecap marginal areas.

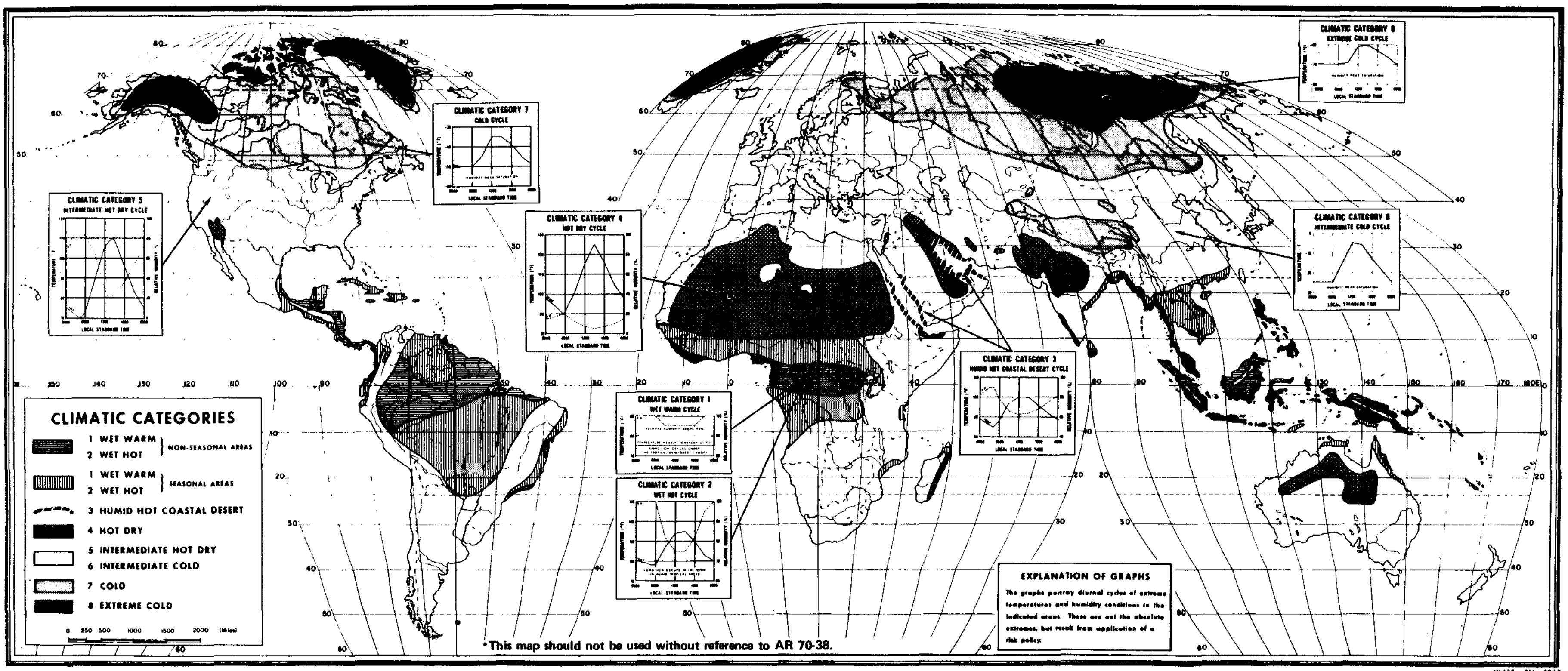

Figure 9-1. Areas of Occurrence of Indicated Climatic Categories

9-11

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TABLE 9-4. CLIMATIC EXTREMES FOR MILITARY EQUIPMENT

Extreme stress conditions		Environmental factors	Operation: Ground, exposed: Worldwide				Arctic winter	Moist tropics				Hot desert	Shipboard (worldwide)				Short-term storage and transit, land-sea-air (worldwide)			
Thermal	Hot	Duration, hr Air temperature, °F Radiation, W ft^{-2} UV < 4000 Å, % IR > 7000 Å, % Windspeed, mph	10 90 0 0 0 7	5 ↗ ↗ 6 50 7	4 125 105 6 50 7	5 ↘ ↘ 6 50 7		10 75 0 0 0 4	5 ↗ ↗ 4.5 51 4	4 95 90 4.5 51 4	5 ↘ ↘ 4.5 51 4	W	10 90 0 0 0 6	5 ↗ ↗ 4.5 51 6	4 100 90 4.5 51 6	5 ↘ ↘ 4.5 51 6	10 90 0 0 0 0	5 ↗ 0 0 0 0	4 160 0 0 0 0	5 ↘ 0 0 0 0
Thermal	Cold	Duration, hr Air temperature, °F Sky temperature, °F Windspeed	Equilibrium -40		72 -65 -80 5 mph		W					W	24 -20 -45 40 kt				Equilibrium -40 0		24 -80 0	
Humidity		ABS Humid, gr ft^{-3}	13				2.1	W				W	W				W			
Humidity	High	Duration, hr Relative humidity, % Air temperature, °F	20 93 to 97 80 to 85		4 100 w/cond 75 to 80		98 to 100 before freezing	W					W				W			
Humidity		ABS Humid, gr ft^{-3}	0.01				W					0.5	0.17				W			
Humidity	Low	Duration, hr Relative humidity, % Air temperature, °F	10 15 90	5 ↘ ↗	4 5 125	5 ↗ ↘						W					10 15 90	5 ↘ ↗	4 2 160	5 ↗ ↘
Precipitation	Rain	Duration, hr: min Amount, in. Drop diam. {Mean, mm; Std. Dev., mm} Air and water temp, °F Wind, mph	11:55 12 2.25 0.77 70	00:05 2 4.00 1.08 70	11:00 11 2.25 0.77 70 40	01:00 7 3.20 1.10 70		W					W				P			
Precipitation	Snow	Expectancy, days Snow load, lb ft^{-2}	1 10	3 20	150 40		W													
Wind	10 ft above surface	Expectancy, yr Ordinary {Steady, mph; Gusts, mph} Exceptional {Steady, mph; Gusts, mph}	2 40 60 60 90	5 50 75 70 105	25 65 100 80 120		W	W				W	85 (75 kt) 115 (100 kt)				P			
Penetration and abrasion	Blowing snow	Flake diameter, mm Wind, mph Air temperature, °F	1 to 3 40 0				W						W				P			
Penetration and abrasion	Blowing sand	Grain diameter, mm Wind at 5 ft, mph Air temperature, °F	0.18 to 0.30 40 100									W	W				W			
Penetration and abrasion	Blowing dust	Grain diameter, mm Density, gr cm^{-3} Wind at 5 ft, mph Air temperature, °F	0.0001 to 0.01 6×10^{-9} 40 70									W	W				W			
Pressure	Maximum		1,060 mb = 31.30 in. Hg 15.40 lb in.$^{-2}$				W	W				W	W				W			
Pressure	Minimum		505 mb = 14.94 in. Hg 7.35 lb in.$^{-2}$				W	844 mb=24.85 in. Hg=12.20 lb in.$^{-2}$				844 mb	887 mb = 26.18 kt = 12.90 lb in.$^{-2}$				375 mb = 11.1 in. Hg = 5.45 lb in.$^{-2}$			

W = same as ground, worldwide
P = same as ground, worldwide, for packaging only
↘ = decrease at uniform rate
↗ = increase at uniform rate

Climatic effects, primarily of temperature and humidity, on one type of military equipment may be illustrated by the example that follows.

Radios were deployed throughout Vietnam and Thailand for village and hamlet security. The radios were powered by a conventional lead-acid storage battery. According to published data, a fully-charged battery, for use in the tropics, has an electolyte with a specific gravity of 1.2300. A battery is considered fully discharged when the battery electrolyte drops to a specific gravity of 1.0450. These data are translated into the environment of Thailand and Vietnam in Table 9-5 and Fig. 9-2, which illustrate the lead-acid storage batteries will retain their charge, in storage, about twice as long in a temperate climate as in a hot, humid climate. As a consequence of the climate in Southeast Asia, twice as many batteries were required in the supply pipeline as would be required for a temperate climate. This type of information on equipment life expectancy therefore is necessary to the system designer, analyst, and logistician.

9-4.3 TERRAIN CHARACTERIZATION, DESCRIPTION, ANALYSIS, AND STUDIES

9-4.3.1 Terrain Characterization and Description

Numerous systems are used to categorize terrain. For the sake of standardization of approach, those employed by Department of the Army Field Manual, FM 30-10 (Ref. 13), are cited herein. Ref. 13 includes man-made or induced cultural features under "terrain". Terrain is defined in Ref. 13 as "a consideration of topography in terms of military significance." Major elements considered are:

1. Landforms
2. Relief
3. Drainage (rivers, streams, lakes, swamps, and marshes)
4. Surface materials (soil and rock)
5. Special physical phenomena (permafrost, volcanism, landslide or avalanche hazards, chinook or foehn winds, or similar phenomena)
6. Man-made objects (roads, bridges, dams, tunnels, etc.).

Each of the stated elements is categorized in greater detail in Ref. 13. The characterization and ranking are based primarily upon military operational requirements. Much material is not germane to the problems of the military weapon systems analyst in either the establishment of an appropriate terrain analogue or in the inclusion of significant terrain parameters in his analysis. The analyst should consider only those elements of topography which are pertinent to the mission requirements of the weapon system in the areas where the system is likely to be deployed. Selection of the significant elements of the physical environment also should be guided by the level of conflict in which the system

TABLE 9-5. STANDING LOSS IN A GOOD BATTERY PER DAY

Temperature, °F	Standing Loss Per Day	
	Specific Gravity	Percentage of Full Charge
100	0.0025	1.35
80	0.0010	0.54
50	0.0003	0.16

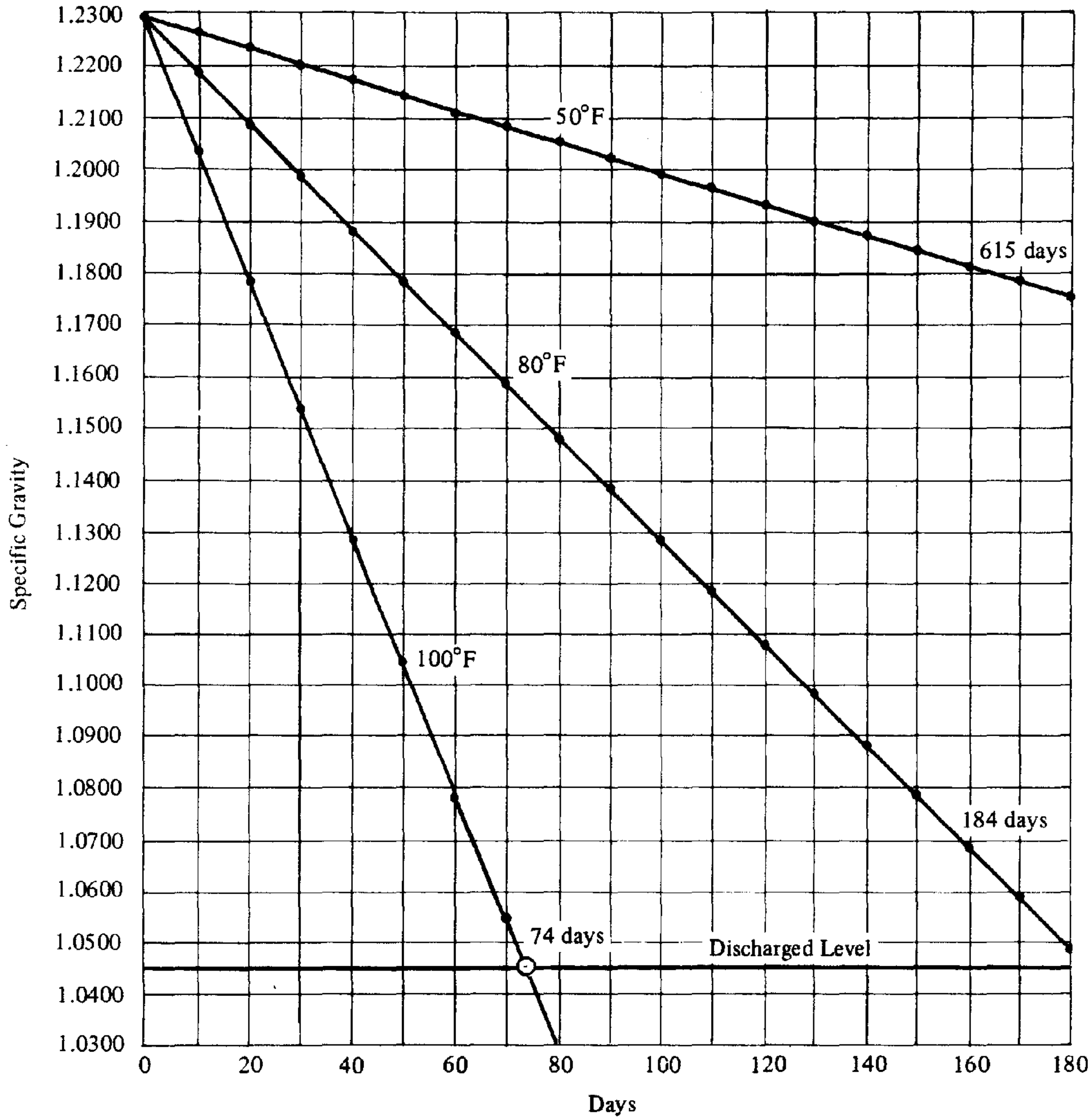


Figure 9-2. Extrapolated Standing Loss in a Lead-Acid Storage Battery Under Differing Temperature Conditions

is likely to be used, and the type of personnel (US military, sophisticated allied, unsophisticated allied, and paramilitary) who will actually be operating the system in the field.

9-4.3.2 Terrain Data Sources

9-4.3.2.1 General

Extensive professional sources capable of providing the appropriate terrain data exist within the US Government. The topographic map, discussed in par. 9-4.3.2.2, is a source which contains a wealth of

terrain and environmental information which could be of value to the weapon systems analyst. Table 9-6 lists US Army published references. On the local level, US Army Terrain Intelligence Teams are attached to most unified, specified, or theater commands.

Aerial and satellite photography is a source supplying extensive quantities of environmental data. The Army Topographical Command, US Air Force, US Navy, National Aeronautics and Space Administration, and the Environmental Science Services Administration are constantly acquiring new and updated photographic data which should be used when appropriate.

Additional sources of evaluated and synthesized terrain and environmental information may be made available to the weapon systems analyst within the Department of Defense. These additional sources exist in the Military Geology Branch of the United States Geological Survey, Department of Interior, the World Soils Group of the Department of Agriculture, and the Central Intelligence Agency.

Ideally, the systems analyst should be able to ascertain those areas of the world in which the weapon system is likely to be employed as well as the expected levels or spectrum of conflict. In most cases, and for most critical areas of the world, terrain intelligence studies already have been published and should be available. In special cases, additional studies can be undertaken and additional data gathered as needed. Table 9-7 lists those agencies formally chartered to conduct terrain intelligence studies on the national level. These agencies should be able to supply the analyst with synthesized terrain information or analyses commensurate with his needs.

TABLE 9-6. LIST OF US ARMY PUBLICATIONS IN THE AREA OF BASIC AND APPLIED TERRAIN AND ENVIRONMENTAL SUBJECTS

1. Field Manuals	
FM 5-15	Field Fortifications
FM 5-20	Camouflage
FM 5-30	Engineer Intelligence
FM 5-35	Engineer's Reference and Logistical Data
FM 5-36	Route Reconnaissance and Classification
FM 30-10A (C)	Special Applications of Terrain Intelligence (U)
FM 30-16	Technical Intelligence
FM 31-10	Denial Operations and Barriers
FM 31-25	Desert Operations
FM 31-50	Combat in Fortified and Built-Up Areas
FM 31-60	River Crossing Operations
FM 31-70	Basic Cold Weather Manual
FM 31-71	Northern Operations
FM 31-72	Mountain Operations
FM 55-8	Transportation Intelligence
FM 57-35	Airmobile Operations
2. Technical Manuals	
TM 5-248	Foreign Maps
TM 5-297	Well Drilling Operations
TM 5-312	Military Fixed Bridges
TM 5-330	Planning and Design of Roads, Airbases, and Heliports in the Theater of Operations
TM 5-530	Materials Testing
TM 5-545	Geology
TM 5-700	Field Water Supply
TM 30-245	Image Interpretation Handbook
TM 30-246	Tactical Interpretation of Air Photos

TABLE 9-7. SOURCES OF TERRAIN INTELLIGENCE AND TERRAIN ANALYSIS—NATIONAL LEVEL

Agency	Product
Department of the Army Corps of Engineers US Army Topographic Command* Engineer Strategic Studies Group	Topographic maps Engineer strategic studies Environmental studies
Central Intelligency Agency	National Intelligence Survey Chapter II-Military Geography (Done on strategic areas)
Defense Intelligency Agency	Special Geographic Area Studies Environmental and terrain Studies Collated and synthesized regional climatic data
Department of the Army Training and Doctrine Command Environmental Sciences Group	Terrain analysis and environmental studies of strategic areas
Department of the Army Corps of Engineers Engineer Topographic Laboratories	Strategic studies – emphasis on remote area sensing technology and application
Department of the Army Corps of Engineers Engineer Agency for Resources Inventory	Geologic studies with emphasis on economic mineral deposits
US Department of the Interior US Geological Survey, Military Geology Branch	Military geology and terrain Intelligence studies on both strategic and tactical levels Support to Advanced Research Projects Agency in Vela Programs
US Department of Agriculture World Soils Group NASA	Military mobility and vehicular trafficability studies and meteorology topography from satellite photos
Waterways Experiment Station Vicksburg, Mississippi	Terrain studies, mobility maps, etc.
Cold Regions Research and Engineering Laboratories	Information on cold regions, snow, ice, etc.

*Formerly Army Map Service

9-4.3.2.2 Topographic Maps

The Army Topographic Command of the US Army Corps of Engineers (formerly the Army Map Service) publishes detailed large scale topographic maps covering most of the world. Their catalogues and indexes are upgraded frequently as new coverage becomes available. Current catalogues always include "forecast" coverage of map sheets in preparation. The most commonly used "large-scale" topographic map is at a scale of 1:50,000. It is the basic tactical operational document available to military planners and combat forces.

The 1:50,000 scale maps are produced from US-generated survey and geodetic data, from foreign data, from aerial photography, or from a combination of all available sources. In many instances, the United States merely reprints foreign maps for its own use, employing conventional military signs, symbols, and abbreviations. In other cases, foreign maps are used as basic data which are corrected and upgraded for US military use. Reliability of the coverage, dates of survey, ground checks, and aerial photography usually are indicated on the margins of the maps.

A standard US Army topographic map contains a wealth of terrain intelligence. Such a map, reinforced by a modicum of general geographic background about the country, often contains sufficient information for the systems analyst to develop appropriate environmental inputs for his evaluations.

As an example, Fig. 9-3 shows a portion of a standard US Army topographic map (1:50,000 scale) of Uijongbu, Korea, a strategic junction town approximately 25 mi north of Seoul.

By examination of the dual-language legend of the map (Fig. 9-4), the extent of the detail which is routinely presented on this map and similar maps (prepared to similar standards by the US Army Topographic Command of the Corps of Engineers) may be seen.

From this basic, uninterpreted map, the following data are directly available: the general physical geography, the relief, drainage, vegetation (type and density), and the presence or absence of such man-made features as roads and railroads, villages, towns, dams, embankments, and bridges. From this, information such as that which follows may be extracted:

1. Suitability of the area for the operation of tracked and wheeled vehicles
2. Areas of vegetative cover and concealment
3. Suitability for employment of flat trajectory weapons or line-of-sight-dependent electronic equipment
4. Availability of natural organic engineer construction materials
5. Potential barriers and restrictions affecting mobility.

If the reader of the map either is familiar with the basic geography of Korea or integrates available climatic data into his analysis, he may add the followng facts:

1. The presence of numerous dikes and embankments indicate a flooding hazard.
2. Rainfall data indicate extremely heavy rains in July and August; therefore, operations will be primarily roadbound during these months.
3. Farmers grow rice as a single crop in this part of Korea. Paddies are flooded in the late spring, heavily inundated during the summer rains, drained in the fall, and hard frozen during the period November-March. Dry crops are grown on soils of greater bearing pressure, which are less likely to be flooded in the summer.
4. Railroad bridges will carry tanks (railroads are 4 ft 8.5 in. US standard gauge). Hence, clearance of bridges, platforms, and tunnels will allow operations of the M48 and M60 tanks independent of the highway pattern.
5. The town of Uijongbu, however, is representative of older settlement patterns of Japan and Korea. Narrow streets and overhanging eaves of buildings at intersections probably will preclude main battle tank operation in the town, which must therefore be bypassed.

Although the bulk of the data presented on the map cited as an example is of primary value to the planner of tactical operations, those individuals concerned with the conceptual design, analysis, and detailed design of a weapon system which may be used in this area also would have to consider many of the same parameters. The vehicle designer would be concerned with the limiting widths of structures, weight limitations on bridges, capabilities of bypass obstacles, embankment materials and gradients, stream depth, current bank and bottom conditions, and soil bearing and loading capabilities by season.

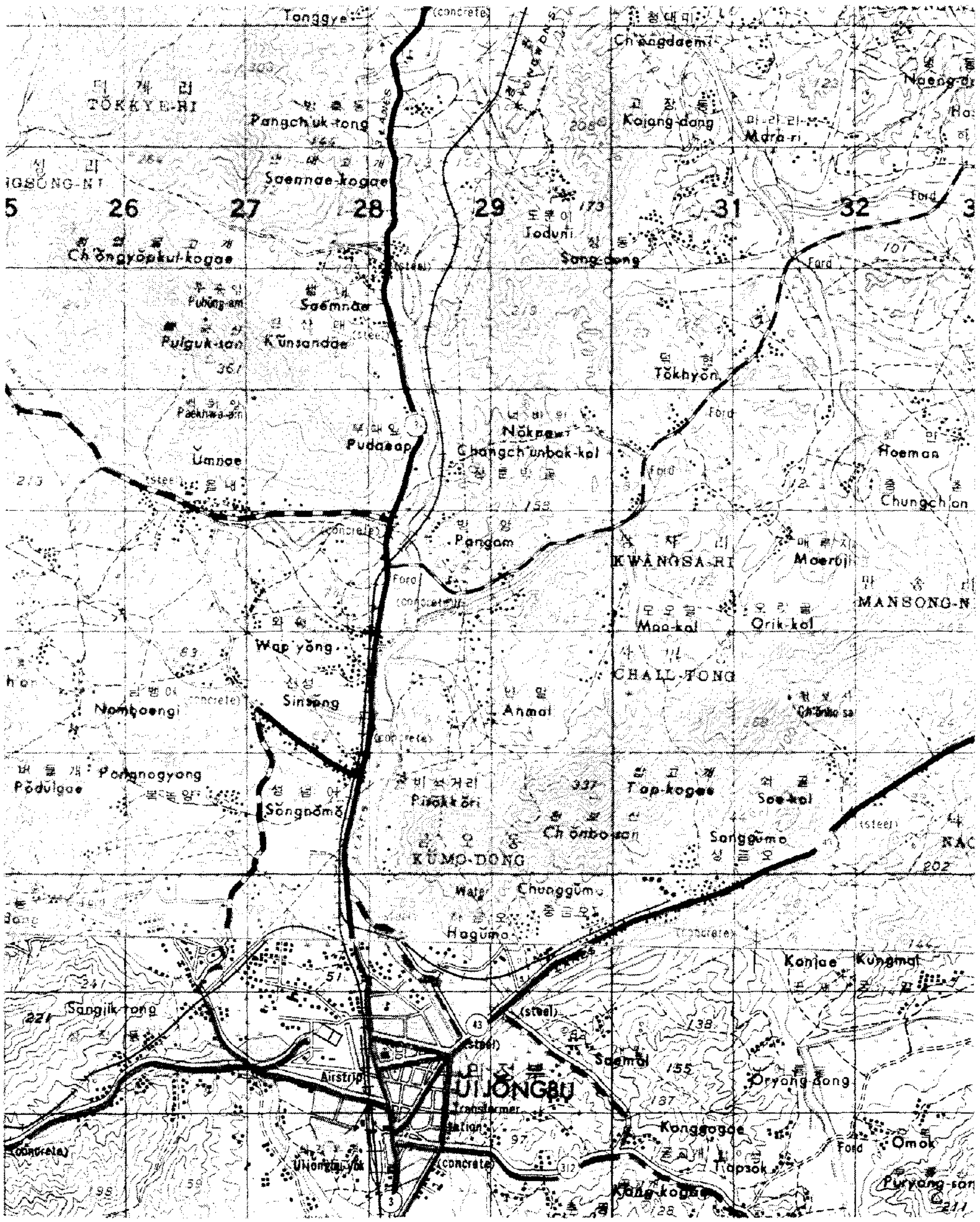


Figure 9-3. Portion of Edition 1-KAMS Series L752 Sheets 1 and 2

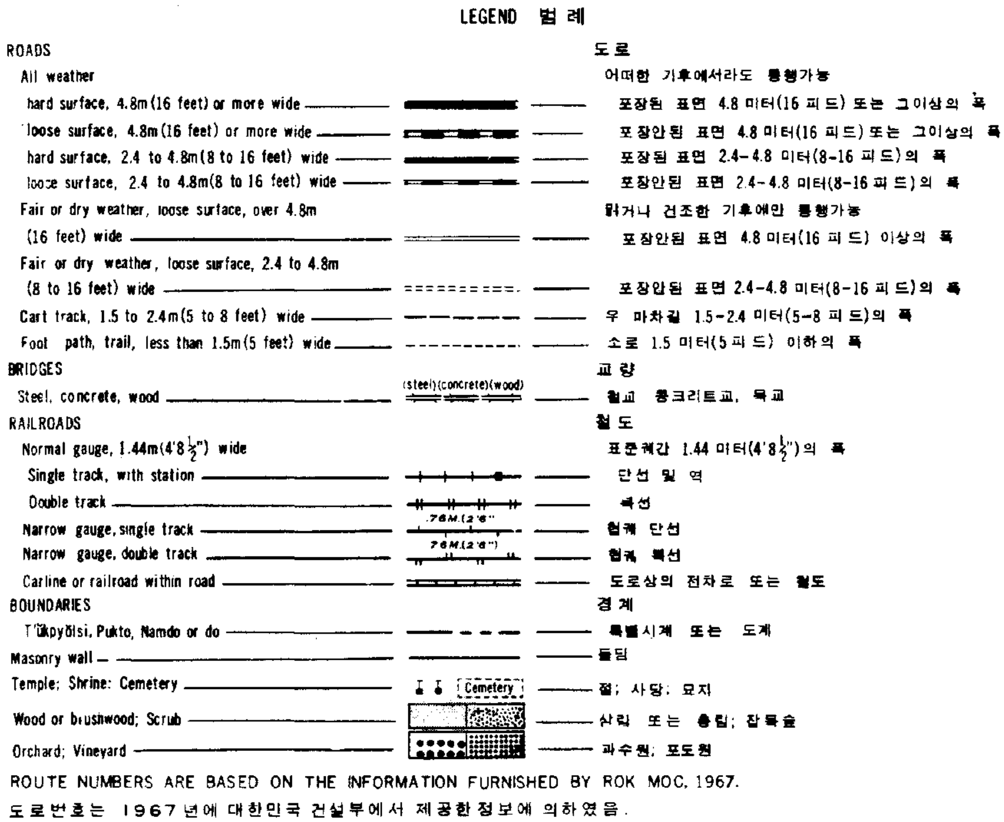


Figure 9-4. Legend of Series L752 Maps as Produced by the US Army Topographic Command (US Army Map Service), the Army Map Service (Far East), and the Republic of Korea Army Map Service

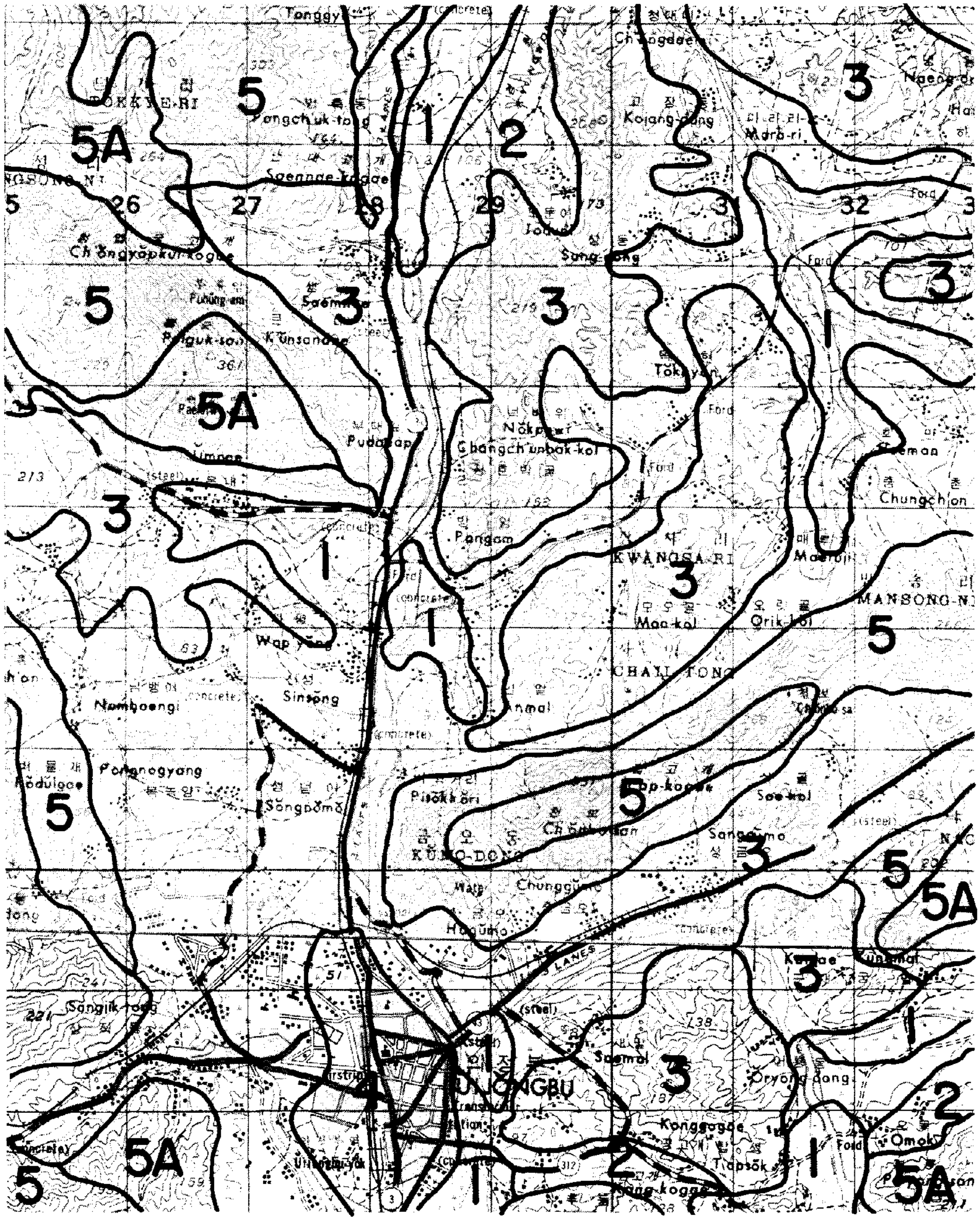
9-4.3.3 Terrain Analysis and Terrain Studies

Fig. 9-5 represents the same area after a basic terrain analysis has been performed. The interaction of soil, slope, and vegetation has been combined in a series of "map units". Map units are established so that the combined effects of the terrain upon military operations are synthesized for the tactical military planner.

By integration of the combined effects of these terrain and climatic elements, not only may the map units shown in Fig. 9-5 be appropriately delineated but an interpretation of the effects of the terrain encompassed within a particular map unit may be made. Such an interpretation, by map unit, is made in Figs. 9-6 through 9-9 and in Table 9-8 in which the effects of terrain on the operational mobility of three typical military vehicles are presented. The terrain features (map unit numbers identified on Fig. 9-5) are:

1. Rice Paddies. Generally flat, diked to heights of less than 1 m in flat areas, up to 2 m in upper stream valleys. Alluvial soils floored by impervious clays. Depths of water in growing season 15 to 20 cm irrigation by canal system 1 to 2 m wide;

2. Dry Crop Land. Generally sandy and coarse soils, well drained. Undulating to rolling topography. Areas cut by steepsided embankments up to 9 m in height, containing streams and controlling irrigation.



(For Map Unit type, see Table 9-10)

Figure 9-5. Sample of Terrain Analysis

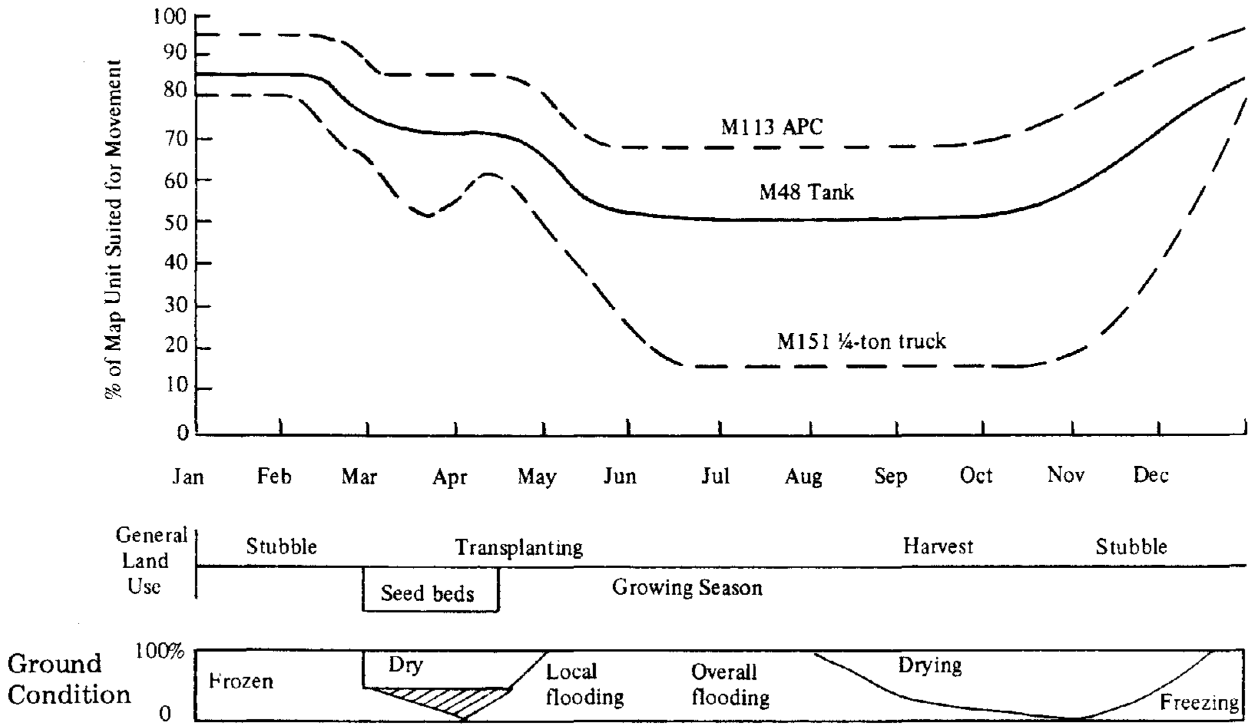


Figure 9-6. Off-Road Movement Characteristics of Map Unit 1 by State of Ground for Representative Military Vehicles

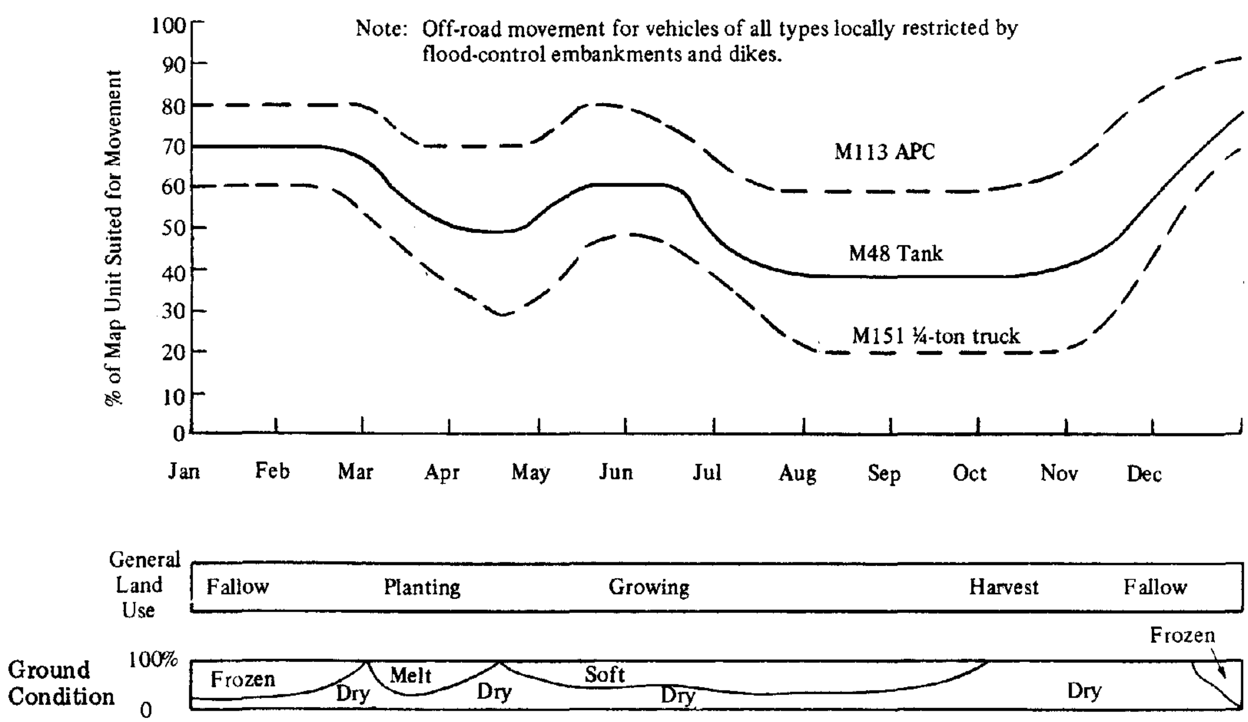


Figure 9-7. Off-Road Movement Characteristics of Map Unit 2 by State of Ground for Representative Military Vehicles

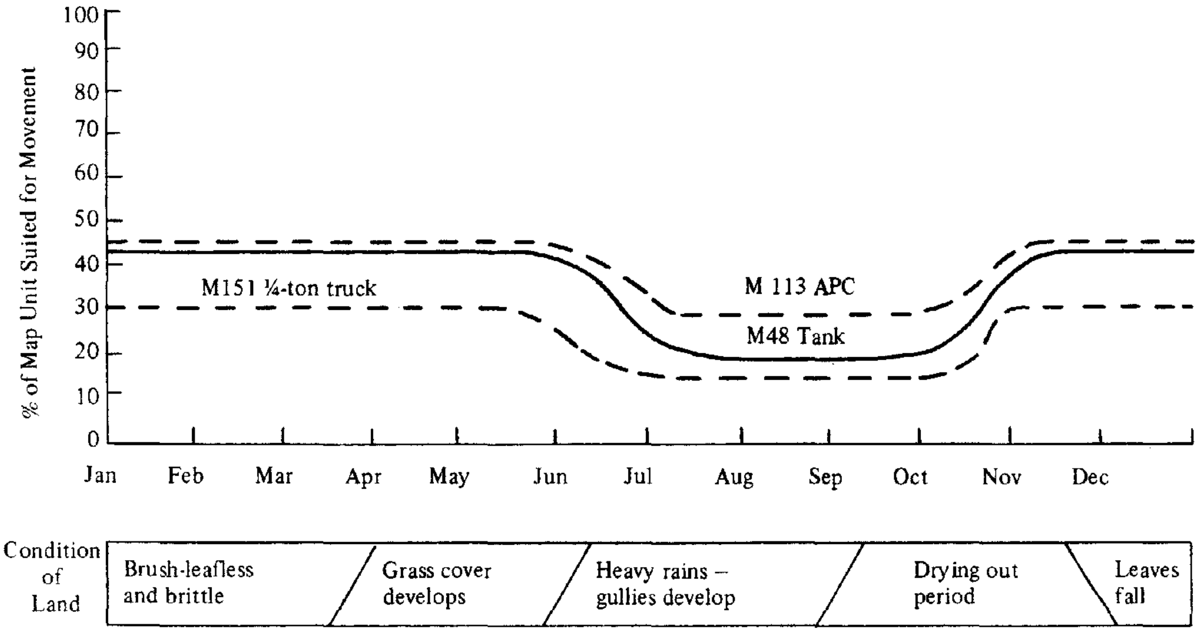


Figure 9-8. Off-Road Mobility Conditions in Map Unit 3 by Season

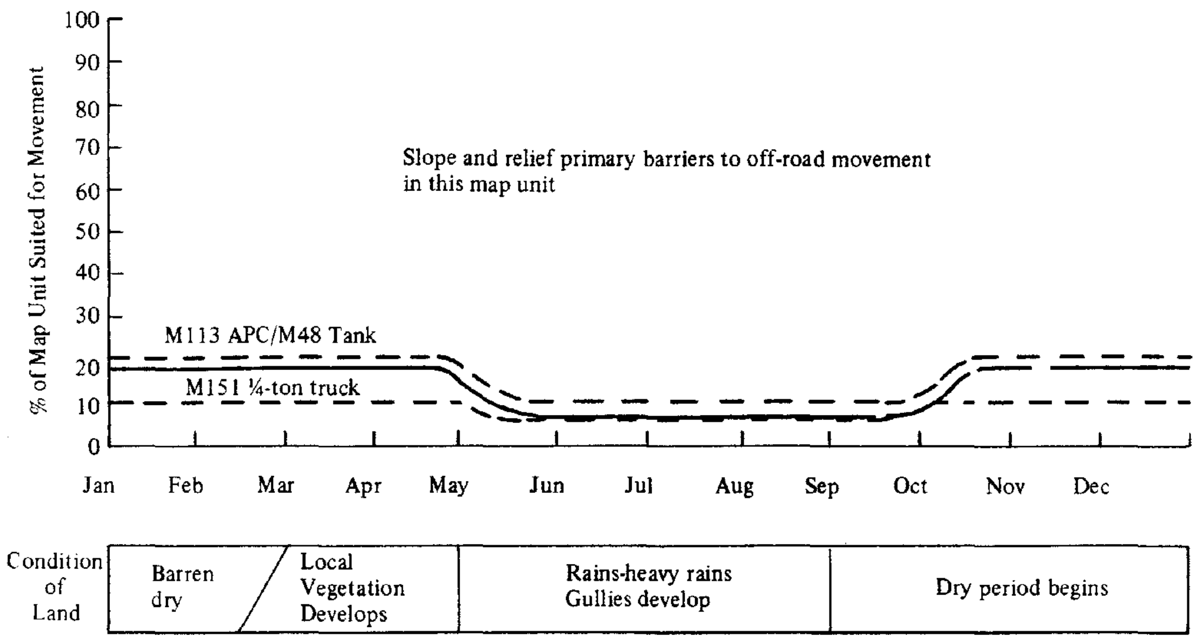


Figure 9-9. Off-Road Mobility Conditions by Map Unit 5 and 5A by Season

TABLE 9-8. MILITARY VEHICLE OPERATIONS IN MAP UNIT 4 (SEE FIG. 9-5), BUILT-UP TOWNS AND VILLAGES

Generally not affected by season, although locally roads may be flooded in late June, July, August, and early September. Local roads muddy but trafficable.

M48 Tank. Locally restricted in villages by structures, narrow roads, and overhanging eaves of buildings.

M113 APC. May pass through villages and towns with little or no restrictions at all seasons.

M151 ¼-ton truck. May have difficulty on local roads under following circumstances:

Rainy season	– Multiple passes deepen ruts until ground clearance becomes problem. Will have ground clearance problem if follows convoy of heavy tracked vehicles which has caused ruts in secondary roads.
Dry season	– No problems.
Winter season	– Occasional problem in frozen ruts, particularly those well worn in roads during rainy season.

3. Rolling Barren Hills. Little or no tree growth, scrub and grassland predominate. Relative relief ranges from 50-100 m above surrounding lowlands. Hills rounded, gullies limited in extent and depth. Map unit also includes foothills of mountains carried as map unit 5. Physiography of foothills essentially similar to isolated hill masses.

4. Built-Up Area. Towns and villages of sufficient size to form barrier to movement. Few ferroconcrete buildings, majority of structures wood and thatched roofs. Buildings closely spaced, impeding movement, particularly at intersections.

5. Hills and Mountains. Steep and rugged hills in this region, spurs on mountain ranges to the north and east. Lightly vegetated and heavily eroded. Primarily granitic material which crumbles and decomposes when surfaces are exposed to moisture. Relative relief 200 to 400 m above surrounding valleys in this region.

5A. Barren, Heavily Eroded Hills. Similar to map unit 5 except that tree growth is completely lacking, caused by centuries of cutting for timber and charcoal.

In determining the ability of certain vehicles to traverse these terrain features successfully, the data which follow, for example, are pertinent:

1. The M48 tank with a nominal ground pressure of 12 psi.
2. The M113 armored personnel carrier with a ground pressure of approximately 8 psi.
3. The M151 1/4-ton jeep with a ground pressure of up to 40 psi (depending on tire pressure).

Ref. 11 presents, in operational fashion, these data in more technical and engineering detail, using the "Vehicle Cone Index". The cone index reflects the relationship of soil bearing capabilities to vehicle ground pressure and is described in Ref. 14 as follows:

"A numerical indication of the carrying ability (resistance to penetration by wheels and tracks of vehicles) of a soil obtained with the cone penetrometer, a dimensionless number representing resistance to penetration into the soil of a 30 degree cone with 1/2 sq inch base area (actual load in pounds of cone base area in square inches)."

A vehicle cone index is defined in Ref. 13 as:

"The index assigned to a given vehicle which indicates that minimum soil strength in terms of rating cone index required to permit 50 passes of the vehicle."

9-5 VEHICLES BY MOBILITY CHARACTERISTICS

The terrain analysis and the map units presented in Fig. 9-5 show, however, that "trafficability" is only one of the factors affecting movement. Slope, vegetation, topography (particularly microrelief features), and state of the ground have considerable impact on operational movement or maneuverability of military vehicles. For example, a vehicle often is rated and evaluated on its grade-climbing ability. A vehicle which can climb a 60% slope will not necessarily, however, be able to turn, maneuver, or even back down the same slope. Small elements of brush and tree vegetation which the vehicle could traverse with ease on the level may become a sufficient barrier to halt movement on grades. Grass as well as soil has a "slipperiness factor" which must be considered. Thus, the interaction of all terrain elements must be considered in the overall examination of mobility or trafficability of vehicles.

The terrain analysis and associated map units also may provide the systems analyst with additional input information in expected weapon systems performance. For example, Tables 9-3 and 9-8, using Fig. 9-3 as a datum, present further operational interpretation of the terrain.

Table 9-9 gives a ranking of military vehicles by mobility characteristics.

Table 9-10 has been prepared as a sample of the application of terrain analysis to military operational problems. In systems analyses, such detail seldom is needed. The military weapon systems analyst should, however, be aware of the operational implications of the natural environmental factors and their interface with a weapon systems design and employment.

TABLE 9-9. RANKING OF MILITARY VEHICLES BY MOBILITY CHARACTERISTICS

Category	Vehicle cone index range	Vehicle types
1	20-29	The M29 "Weasel", the M76 "Otter" and the Canadian snowmobiles are the only vehicles currently in this category – except experimental units not in TOE inventory.
2	30-49	Engineer and high-speed tractors, with special wide tracks and low ground-contact pressures.
3	50-59	The M113 APC and tractors with "average" (sic) contact pressures. Tanks with low contact pressures, and some trailed vehicles with very low contact pressures.
4	60-69	Most medium tanks, artillery prime-mover tractors, and all-wheel drive trucks with low-pressure tires.
5	70-79	Bulk of current inventory of all-wheel drive trucks, most trailed vehicles, heavy tanks.
6	80-99	Heavy all-wheel drive trucks, rear-wheel drive military trucks intended primarily for highway use. Some heavy trailed vehicles.
7	100 or greater	Most rear-wheel drive and commercial trucks not expected to be used in off-road operations.

NOTE: This table is adapted from FM 30-10 (Ref. 13) and does not contain latest vehicles in inventory nor experimental articulated off-road vehicles.

TABLE 9-10. OPERATIONAL ASPECTS OF THE TERRAIN

Map Unit * (By Season)	Foot Troops Operation	Weapon Employment
1-Wet	Movement slow, impeded by heavy sticky mud and clay. Paddies flooded to 18 in. depth. Canals and ditches full, local barriers, capability to dig foxholes and bunkers limited by high water table.	Visibility poor for optically aimed weapons. Terrain good for flat trajectory weapons. Mobility and emplacement of heavy weapons and mortars difficult because of mobility restrictions. Artillery roadbound without tracked prime movers.
-Dry	Mobility good; locally, poorly drained all year, concealment good during pre-harvest period.	Ground generally stable for emplacement of mortars and heavy weapons. Visibility limited in pre-harvest by stalks of rice 24-48 in. tall. Artillery deployment limited locally by muddy soils.
-Frozen	Mobility excellent. Digging of foxholes and field fortifications difficult.	Visibility for hand-held weapons emplacement excellent. Weapon emplacement excellent.
2-All seasons	Mobility locally restricted in wet season, but generally good throughout year. Capability for establishment of bunkers and foxholes limited in frozen season, and locally by flooding.	Visibility for employment of hand-held weapons limited only in rain. Soil bearing power suited for emplacement of heavy weapons and artillery.
3-All seasons	Little restrictions on personal mobility throughout year. Suited for excavations except locally in frozen season throughout year. Limited rock areas also restrict digging.	Locally, slope inhibits use of heavy weapons. Area generally good for their employment. Artillery limited to vicinity of roads unless drawn by tracked prime movers.
4-All seasons	Mobility limited to existing roads in towns. Some ferro-concrete buildings suited for firing positions. Excellent concealment, fair cover.	Structures limit use of flat trajectory weapons except at short ranges. Primarily suited for hand weapon combat. Local mortar positions available in open spaces.
5 and 5A All seasons	Steepness of slopes and seasonal slipperiness of soils are primary limiting factors on foot-soldier mobility. Approximately 85% of Map Unit delineated in sample suited to foot troops in tactical operations.	Numerous excellent firing positions for hand-held and heavy weapons employment. Numerous defiladed firing positions. Vegetative concealment poor, cover fair. Field artillery towed, and self-propelled weapons limited to roads and trails throughout most of area.

Map Unit * (By Season)	Helicopter Operations	Close Air Support
1-Wet	Poor: Unstable saturated soils and poor visibility.	Poor: Visibility limitations. Many areas of paddy lands in narrow valleys. Air support under marginal visibility conditions difficult.

*See Fig. 9—5.

(continued)

TABLE 9-10. (cont'd)

-Dry	Good to excellent: soils stable. Grain not high enough to interfere with operations. Local wet, poorly drained areas.	Good to excellent: Local terrain of hills flanking this map unit only inhibition on aircraft operation.
-Frozen	Excellent: Good visibility. Stable soils.	Excellent: Good visibility conditions. Frequent light snow cover aids in target identification.
2-All seasons	Fair to excellent: Visibility restricted in rainy season.	Good to excellent: Locally, terrain precludes full effectiveness of weapons. Such areas limited in extent. Seriously degraded in rainy season by visibility limitations.
3-All seasons	Fair to good: Slope limits operational areas. Local areas of brush.	Good to excellent: Comments above apply. Limited in wet season as above.
4-All seasons	Poor: Limited to parks and school areas.	Poor to fair: Effectiveness of air delivered weapons in town environment requires excellent ground/air coordination and weapon delivery accuracy.
5	Poor during rainy season in which visibility in hills and mountains is extremely limited. Good to excellent on extensive open slopes. Poor in narrow valleys flanked by steep slopes.	Comments pertaining to helicopter operations apply in general conditions. Locally, combinations of poor visibility and steep mountains make close air support infeasible except with the aid of advanced avionics. In circumstances where close air support is feasible in marginal conditions of visibility, stand-off ranges between aircraft and troops being supported must be increased.
5A	Conditions essentially similar to Map Unit 5 with the additional restriction of brush and tree patches. Locally, reforestation has precluded helicopter operations without clearing activities.	Restrictions essentially similar to Map Unit 5. Enemy forces could be concealed in vegetation in limited areas.

9-6 CONCLUDING REMARKS

By way of conclusion, the analyst is cautioned to make his evaluations as realistically combat oriented as possible, especially taking proper account of the effects of the physical environment and highly variable weather conditions on weapon and crew performance. We now know that weather conditions in Vietnam had some very unusual and undesirable effects on our operations there, especially air support and bombing operations to such an extent that Admiral Thomas H. Moorer, then Chairman of the Joint Chiefs of Staff, stated before Congress that "weather programs play a greater part with respect to the military activities in South Vietnam than in any other part of the world" (Ref. 1). Thus, the great importance of the environment and accompanying weather variations on the effectiveness of our equipment and combat operations may be of a critical character. In fact, it can be said that there is a need for much additional progress in analytical methodology concerning the physical environment and its effect on the actual performance of weapon systems.

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12. MIL-STD-210, *Climatic Extremes for Military Equipment.*
13. FM 30-10, *Military Geographic Intelligence (Terrain).*
14. 61 Joint Technical Coordination Group for Munition Effectiveness (JTCG/ME) 70-5, *Single Fragment Performance in Grass and Jungle Tangle Environments,* September 1970 (available from AMSAA).
15. MIL-STD-1165, *Glossary of Environmental Terms (Terrestrial).*

CHAPTER 10

SOME FUNDAMENTALS OF OFFENSE, DEFENSE, AND TARGET DAMAGE ASSESSMENT

An introduction to offensive actions, defensive actions, and target damage assessment is given to enlighten the analyst in such phases of combat. The "shoot-look-shoot" tactic and methodology are discussed also.

10-0 LIST OF SYMBOLS

$E(x)$ = expected value of x
$f(x;r,N)$ = negative binomial frequency
N = number target elements
$p(k|h)$ = conditional chance that a hit is a kill
p_h = single shot hit probability
$p_k = p$ = $p_h \cdot p(k|h)$ = kill probability per round
q = $1-p$
R_D = radius of damage of a warhead
r = number of target elements out of N which will cause withdrawal
x = 0, 1, 2, ... = a discrete random variable
$\sigma^2(x)$ = variance of x = σ^2 (as it is often abbreviated)

10-1 INTRODUCTION

The purposes of and some limited background information on offensive, defensive, and retrograde operations were given in par. 8-2.1. In this chapter, we expand such information so that the analyst will have some further appreciation of the possible scope of realistic weapon systems evaluations in combat. No attempt is made here to explore fully the variations in offensive and defensive actions which may be dictated by extremes of climate or terrain such as desert, arctic, jungle, or mountains (Chapter 9); levels of conflict intensity (Chapter 8); or the need to execute special operations such as amphibious, airborne, or air assault. Certain basic principles applicable to the classic offensive and defensive procedures employed by the Army are presented, with particular emphasis on the interplay among intelligence collection, fire support means, and troop maneuver. An understanding of how the effects of fire are exploited by maneuver or how they may adversely affect the accomplishment of the mission is important to the analyst's base of knowledge. Accordingly, this chapter discusses offensive operations, exploitation of the attack, some undesirable effects of the attack, some defensive operations and introduces the related and important subject of damage assessment in combat operations.

10-2 OFFENSIVE ACTION FUNDAMENTALS

10-2.1 GENERAL

Combat power in the offense is achieved by organizing responsive, combined arms forces that can move rapidly, deliver accurate fire, and maintain continuous communications. The attack is planned carefully and executed aggressively. Once the attack is launched, the commander exploits all available means to gain the objective in the shortest possible time. Every effort is made to disrupt and neutralize enemy support and reinforcement actions. Successful offensive action requires the concentration of superior combat power at the decisive point and time (Ref. 1).

Fire superiority must be gained early and maintained throughout the attack to permit freedom of maneuver without prohibitive loss. The attacker maneuvers to exploit the effects of his fire and to close with and destroy the enemy by assault. Maneuver may force the enemy to fight on unfavorable terrain or may lure him into creating a target suitable for destruction by fire.

Plans must provide for the exploitation of any favorable advantage that develops during the attack. This may require the commander to retain a mobile reserve of troops and fire support to exploit successes. When an opportunity for decisive action presents itself, the commander commits all necessary resources and demands the ultimate from his troops. Pressure applied day and night against a weakening enemy denies him respite from battle, the chance to recoup losses, and the opportunity to gain the initiative. Failure to capitalize on opportunities results in slow, indecisive attacks in which the attacker usually suffers heavy losses.

Terrain is important in offensive combat and often provides advantages that can be exploited. Operations often are directed toward the early control or neutralization of key terrain features. This is done to gain an advantage in observation, provide cover and concealment, obtain better fields of fire, enhance maneuver and support, secure routes used by friendly forces, allow freedom of movement, afford additional security, and gain control of routes useful to the enemy.

After the enemy has been located, there are three principal tasks in the attack: holding the enemy in position, maneuvering to gain an advantage, and delivering an overwhelming attack at the decisive time. Surprise is always sought. It can be gained by deceiving the enemy defense and by choosing an unexpected time, place, direction, and form of maneuver. Cover and deception operations aid in achieving surprise. An aggressive attack inherently provides security. The division commander insures that the attacks of his subordinate units are coordinated and contribute to the division's mission by assigning tasks, allocating means, and applying other controls.

Forces are dispersed to reduce vulnerability to attack but only to the extent that the performance of the mission is not impaired. Offensive operation plans must provide for adequate combat support and combat service support to sustain the attack. Electronic warfare is an integral part of operations planning.

Although the discussion of the attack in this chapter concerns primarily conventional offensive operations, the analyst should keep in mind that battles might escalate to the nuclear type of attack if so directed.

10-2.2 EXPLOITING THE ATTACK

While the basic characteristics of an offensive action are the timely applications of fire and maneuver, the techniques and tactics employed by a force commander are varied in style and influenced by many factors to exploit the attack. Exploitation is the following up of gains to take full advantage of success in battle. It is a phase of the offensive that destroys the enemy's ability to reconstitute an organized defense or to conduct an orderly withdrawal in the face of threatened destruction or capture. The exploitation is initiated when an enemy force is having recognizable difficulty in maintaining its position (Ref. 2).

Analysis of the terrain, enemy, weather, and disposition of his own forces provides the commander with pertinent data upon which to base his decision for committing the combat power under his control. The weapon systems analyst should be aware of the different formations and maneuvers which an attacking force may employ in order to gain a better appreciation of the multiple roles which might be assigned to specific supporting weapons (see Ref. 2). For example, the requirement for mobile systems is more critical in the envelopment and turning movement than for the penetration or frontal assault.

Night attacks may require battlefield illumination to facilitate the employment of supporting fire. If nuclear weapons are to be used, fire support coordination takes on added significance. The advance to contact, which precedes deployment of a main force, requires supporting fire and, in the exploitation and pursuit phases, highly mobile airborne and air assault elements may be essential to success.

In the attack, the most decisive results usually are obtained by strong, mobile exploiting forces. (A nuclear environment in particular favors the use of small, highly mobile forces moving on the ground, through the air, or both.) Every effort is made to maintain the forward momentum by destroying enemy forces by fire, bypassing or containing them, and, only where necessary, reducing their number by close combat. The plan of attack is either to divide the enemy force and defeat it in detail, or to concentrate it to an extent where it can be destroyed by available weapons. Should it become necessary for the commander to concentrate his force, he does so only at a decisive point, in close proximity to the enemy, and for the shortest practicable time.

To insure rapid and forceful execution of the attack, the commander exploits all means of tactical mobility available to him. He selects the appropriate combination of necessary ground vehicles and aircraft from resources available to provide the desired degree of flexibility for his scheme of maneuver. He reacts quickly to initiatives taken by opposing forces by integrating properly the five basic functions of combat: intelligence; firepower; mobility; command, control, and communications; and combat service support. For further details, the analyst should read the references and the "How to Fight" manuals in the Bibliography of Chapter 8.

10-2.3 CONDUCT OF THE ATTACK

The flow of intelligence through the cycle of collection, processing, production, and dissemination is continuous.. The collection effort, however, is intensified just prior to and during the attack with emphasis being directed toward the identification, acquisition, size, and composition of targets, including suitable nuclear targets. Information is sought on hostile unit identifications and their ground dispositions, the location and extent of obstacles, artillery and mortar positions, nuclear storage and delivery sites, locations of headquarters installations and reserves, and avenues of approach into the hostile positions (Ref. 2).

The successful execution of the attack centers on the coordinated employment of combined arms teams made up of infantry, field artillery, and armor. Immediately preceding the attack, preparatory fire employing conventional (or even nuclear) warheads may be delivered to weaken and demoralize the defender, limit his reaction capability, and cover the movement of attacking units. Known and suspected enemy positions, gun emplacements, communication centers, supply dumps, and numerous other targets are attacked by "flat" and high trajectory fire. Smoke may be used to reduce the effectiveness of enemy observation. Field artillery forward observers, aerial and ground observation posts, forward air controllers, and front line units adjust fire, report fire effects, and identify new targets. (If a nuclear strike is used in the preparation, time may be required for tactical-damage evaluation and the issuance of necessary modifying orders before the general attack is launched. It is desirable, however, to exploit the effects of nuclear strikes as rapidly as possible so as not to lose the physical and psychological advantages gained over the enemy. Nuclear weapons employment is discussed in Refs. 3, 4, and 5.)

At a prearranged time or on signal, attacking units move rapidly from dispersed locations, under cover of preparatory fire and fire in support of the attack. The advance may be a single rapid approach to the final assault position from which the objective is seized, neutralized, destroyed, or overrun; or it may be a series of rapid advances and assaults to obtain such results. Between areas of opposition, attacking forces move in a partly deployed formation; infantry and tanks may move forward separately,

together, or one may lead the other; mechanized infantry may remain in their carriers until forced to dismount by hostile fire or obstacles (Ref. 2). As enemy resistance is encountered, the attacking echelons converge, closely following their supporting fire until they are within assaulting distance of the hostile position. The principal purpose throughout the attack is to gain and maintain fire superiority. Fire superiority limits the enemy's capability for fire and maneuver. It is gained by causing greater target damage than can the enemy, and by sustaining the firing longer than he can maintain his. If the preparatory and supporting fires have been effective in neutralizing antitank opposition, the tanks normally lead the assault, overrun the objective, and take up overwatching positions on the perimeter while the following infantry clears pockets of resistance. If antitank opposition remains strong, the infantry leads the final assault, with the tanks supporting by direct fire until their fire is masked. (Nuclear strikes may make the assault unnecessary or greatly reduce the number of friendly casualties sustained during the assault.)

The assault phase of a successful attack is a short, well-coordinated effort which overruns or destroys the objective. Supporting fires on the objective continue to the moment of assault and then are shifted to the flanks and rear of the enemy position. Following the assault, attacking units disperse rapidly to avoid offering lucrative targets for hostile fire and continue the attack or prepare for subsequent operations, including possible enemy counterattack. Supporting weapons move forward, usually by echelon displacement to insure that continuous fire support is available during the critical consolidation period following seizure of the objective.

If the attack is to be continued, minimum forces retain control of the key terrain while ground mobile and/or airmobile units maintain contact with the enemy, keep him off balance, and gather information on his tactical dispositions. Tactical Air Force aircraft in day and night visual, photo, and electronic reconnaissance missions augment the efforts of Army aircraft to detect the movement of enemy reserves into the area, to report damage to critical target areas, and to provide information on the nature of the terrain to be traversed. Frequently, the attack is continued with fresh troops moving ahead or relieving the attacking unit, or by an airmobile exploitation using a portion of the unit's reserve.

Replenishment of ammunition, fuel, and equipment often will govern the timing of the continuation of the attack (Refs. 6 and 7).

Forces on the offensive may be designated as the main attack, the supporting attack, or the reserve. The main attack is directed to secure the objective, or objectives, that contribute mostly to the accomplishment of the mission. The supporting attack contributes to the success of the main attack by controlling terrain that enhances the maneuver of the main attack, destroying enemy forces that hinder the main attack, confining enemy forces to selected terrain features, deceiving the enemy as to location of the main attack, and preventing reinforcement against the main attack. Reserves are retained to be committed at a decisive time and place to exploit success and insure accomplishment of the mission.

Security of offensive forces is of great importance. The purpose of security in the offense is to avoid unexpected interference by enemy forces, to maintain the integrity of the formation, and to gain and maintain freedom of action. The violence and speed of the attack frequently offer the best security by keeping the enemy off balance. Also, the retention of a reserve enhances the security of the command in its endeavors.

The basic forms of maneuver include the penetration, the frontal attack, and the envelopment. In the penetration, the attacking force passes through the enemy's principal defensive position and ruptures it completely—he destroys or neutralizes forces, installations, and controls means—securing objectives that break up the continuity of the defense. This maneuver divides the enemy force and results in it being defeated in detail (Ref. 2).

ERRATA SHEET

DARCOM-P 706-101 ARMY WEAPON SYSTEMS ANALYSIS, PART ONE HANDBOOK

1. Page 20-1. Line 17 should read:

   A_{VP} = volley damage pattern area.

2. Page 20-4. Eq. 20-1 should read:

   $p_h = 1 - \exp[-R_T^2/(2\sigma_x^2)]$.

3. Page 24-3. Line 29 should read:

   ... is used to describe or characterize terminal... .

10.5

The frontal attack, using the most direct route, strikes the enemy all along his front. It is employed to overrun and destroy or capture a weaker enemy in position or to fix an enemy force in position to support another form of maneuver. The frontal attack may be used by the division in the exploitation, but normally this form of maneuver is appropriate only for corps and higher levels of command.

In the envelopment, the enveloping force attempts to avoid the enemy's main defensive strength by passing around or over his principal defensive positions to secure objectives in the rear that cut his escape routes, disrupt his communications and support, and subject him to destruction in place. Supporting attacks hold the enemy in place during the advance of the enveloping attack. The envelopment forces the enemy to fight in two or more directions simultaneously to meet the converging attacks.

A "double envelopment" is executed by two enveloping forces and a supporting attack force. It requires great combat power and is difficult to control. Nuclear and chemical munitions may be a significant part of the required combat power. The force executing a double envelopment must be able to deploy on a broad front against an enemy who is on a narrower front or who has limited mobility.

Finally, in a turning movement, which is a variation of the envelopment, the attacking force passes around or over the enemy's main force to secure objectives deep in his rear, forcing him to abandon his position or to divert major forces to meet the threat. The enemy is then destroyed on ground of the attacker's choosing. The turning force normally is out of supporting distance of any other ground attacking force.

The analyst easily may see that the attack places very great demands on all kinds of fire delivery means, whether it be infantry weapons, tanks, or artillery. The attack also requires that the commander use great skill in the employment of the combined arms under his control. Accuracy and efficiency of supporting fires become mandatory.

10-3 DEFENSIVE OPERATIONS

The defense is a temporary measure adopted until a force can assume or resume the offensive. Defensive operations prevent, resist, repulse, or destroy enemy attacks. The defender undertakes the defense to develop more favorable conditions for subsequent offensive operations, to economize forces in one area in order to apply decisive force elsewhere, to compel an enemy force to mass, to destroy or trap a hostile force, to deny an enemy entrance to an area, or to reduce the enemy's combat power with minimum losses to friendly forces (Ref. 1).

The concepts and details of defensive operations are presently undergoing extensive revision. Therefore, the reader is encouraged to refer to Field Manuals 71-100 and 71-101, when they become available for additional information regarding defensive operations.

10-4 DAMAGE ASSESSMENT

The attack and defense place heavy demands on the employment of weapons and ammunition requirements; accordingly, good damage assessment during engagements may lead to considerable savings. Damage assessment is defined in the *Dictionary of United States Army Terms* (Ref. 8) as "The determination of the effect of attacks on targets". While every attempt is made to evaluate the nature and extent of damage caused by a weapon system, it is not always possible, however. As a general rule, those intelligence collection resources which acquired the target location, configuration, and makeup initially, report the damage effects as well. For conventional fires, these would include the surveillance, target acquisition, and night observation equipment, and personnel organic to, or in support of, the attacking force. (For nuclear fires, more sophisticated means such as high-performance aircraft, light rotary and fixed-wing aircraft, and other sensor platforms are used in post-strike analyses, due to

its importance and more deliberate planning for such type of attack.) Some of the means available to Army unit commanders for assessing damage are listed in Table 10-1.

Damage assessment reports must be accurate and timely to be of operational value to the commander concerned. The fastest collection capabilities organic to the Army are airplanes, helicopters, and drones. Aerial observers, employing optical and electronic sensors, collect target information out to the operational limits of the aircraft or sensors. Present collection capabilities include visual observations, photography, radar, infrared imagery, and electronic surveillance. Other services, principally the Air Force but also the Navy and Marine Corps, when included as part of a joint operation, provide aerial collection means which may be required to increase the area of coverage beyond the limits of organic aerial collection means. While the primary role of these collection assets is to fulfill the intelligence requirements of the unit commanders and staff, their secondary function of determining the type and extent of damage in the field is also very significant. This especially is true of imagery production and imagery interpretation, wherein proper evaluation of damage may have a great effect on the tactical situation.

Other means available on the ground to collect and report on the effects of fire include a combination of sensory devices and human intelligence resources. New equipment entering the Army inventory is destined to play a major role in maintaining 24-h surveillance over the battlefield. Though limited in range, these versatile sensor systems will augment such standard intelligence resources as patrols, agents, prisoners of war, and observation posts to report damage effects direct to the requesting agency or to the intelligence processing center.

An important element of the overall intelligence collection effort is the Army Security Agency's (ASA) signal intelligence activities. Dedicated to gaining information on enemy communications/electronics, the location and type of noncommunication electromagnetic radiations and their radiating emitters, the ASA can complete, confirm, or refute damage assessment reports from other sources.

TABLE 10-1. MEANS AVAILABLE FOR DAMAGE ASSESSMENT

1. Surveillance Intelligence (SURVINT) Air and Ground
   a. Aircraft (Manned/Unmanned)
      (1) Visual Observation
      (2) Radar
      (3) Photo
      (4) IR
      (5) Radiological
   b. Ground Sensors
      (1) Portable Radars
      (2) Acoustic Sensors
      (3) Seismic Sensors
      (4) Optical and Other Sensors
2. Human Intelligence (HUMINT)
   a. Agents
   b. Patrols
   c. Front Line Units
   d. POW's
3. Signal Intelligence (SIGINT)
   a. Communications Intelligence (COMINT)
   b. Electronic Intelligence (ELINT)

The focus of damage assessment in a truly scientific sense is applied specifically to the employment of nuclear weapons. The implications arising from their offensive use are so complex and of such magnitude that detailed studies have been prepared on every facet of their effects. The results, however, have been obtained largely from controlled tests and not from combat experience. It is, therefore, necessary to accept assigned effects criteria at face value and to examine a few basic conclusions reached in damage estimation which may facilitate the understanding of nuclear damage assessment.

Two specific types of information pertaining to the use of weapons have been developed through weapon tests:

1. The thermal and/or nuclear radiation levels and blast effects required to cause a particular degree of damage to materiel or personnel targets
2. The distance to which the required levels will extend from the ground zero of a given weapon.

By knowing the approximate damage each weapon will cause under a given set of circumstances, a weapon employment officer is able to select the most appropriate weapon for attacking a target. The same terminology and definitions used to estimate the degrees of damage expected from the detonation of a weapon are used to assess the damage resulting from the strike. Damage to materiel is classified as light, moderate, or severe:

1. *Light damage* does not prevent the immediate use of equipment or installations for which it was intended. Some repair by the user may be required to make full use of the equipment or installations.
2. *Moderate damage* prevents the use of equipment or installations until extensive repairs are made.
3. *Severe damage* prevents use of equipment or installations permanently.

Moderate damage is usually all that is required to deny the use of equipment. In most situations, this degree of damage to enemy equipment will be sufficient to support friendly tactical operations. There may be situations, such as the attack of a fortified position, in which only severe damage will produce the desired results.

Personnel casualties (combat ineffectiveness) unlike damage, are not classified as to degree. Personnel who cannot perform their duties are considered casualties. Some personnel will be effective immediately following attack, but later will become combat ineffective because of the delayed effects of the attack. In most tactical situations, it is desirable to base target analysis on personnel casualties rather than materiel damage. Exceptions are targets such as missile launchers, bridges, and other key structures.

A parameter used in estimating damage to the target is the radius of damage R_D (Ref. 8) which is that distance from ground zero at which a particular target has a 50% probability of sustaining a specified level of damage (light, moderate, or severe). Every chemical and nuclear explosion produces a damage radius for each associated target element and degree of damage. For example, the detonation of one size of warhead will produce one damage radius R_D for moderate damage to wheeled vehicles, another R_D for severe damage to wheeled vehicles, and still another R_D for casualties to protected personnel. Tactical plans are based on the condition of the target area predicted in the target analysis. If a commander decides that the desired effects cannot be achieved without undue risk to friendly personnel or that the burst will create obstacles which will impede the accomplishment of the mission, he modifies his plan by adopting one or more of the following trade-offs:

1. Accepting less damage to the target
2. Accepting a higher degree of risk for friendly troops
3. Delaying the attack to permit friendly troops to acquire greater personnel protection
4. Accepting the possibility of obstacles (or induced radiological contamination in certain areas after a nuclear strike)

5. Accepting the possibility of damage to industrial complexes, structures, materiel or objects, which he would prefer left undamaged
6. Accepting a higher probability of fallout from a nuclear strike.

Faulty damage assessment in past conflicts has been either so conservative as to delay or obstruct the advance unnecessarily or so optimistic as to cause additional casualties to attacking troops. Perhaps one of the best examples of the latter case was the US campaign in Italy during World War II. In the spring of 1944, the Germans made a last stand in the monastery of Monte Cassino, situated on a strategic hilltop above the Rapido River Valley. Allied artillery and air strikes had almost demolished the town of Cassino and the monastery had been reduced to rubble. Confident that very little resistance remained on the objective, US forces assaulted the town while ghurkas moved up the mountain to the monastery. Both attack elements met tenacious and almost impregnable defenses. The destructive effects of the bombing and artillery fire had worked in favor of the defenders by creating better cover and ideal firing positions among the debris and rubble from which the Germans continued to deliver accurate fire. It took the allies several more days to secure the objective—at great cost in killed and wounded.

Target kills cannot be measured by total tonnage of delivered ammunition alone, no matter how overwhelming the amount might be. Combat experience during the Pacific island campaigns of World War II, the Korean conflict, and especially the Vietnam fighting, demonstrated in numerous instances how a preponderant application of firepower on dug-in enemy positions did not assure their neutralization or destruction. On the contrary, often the tenacity of the defenders would increase and the defense would stiffen, thus prolonging the engagement. Unfortunately, the exact nature and degree of damage sustained by a target still cannot be ascertained by the employment of any single damage assessment system. Rather a combination of methods and agencies must be employed to gain as complete a picture as possible, even at the expense of overlap and redundancy.

10-5 UNDESIRABLE EFFECTS OF THE ATTACK

The application of both conventional and nuclear fires in support of the attack often will produce side effects which may adversely influence tactical operations. The greater the destructive power of the particular munition employed, the greater is the risk of creating obstacles which could limit or deny completely the use of terrain essential to the attacker's scheme of maneuver. These undesirable effects may come about through incomplete target intelligence, a natural tendency to overkill targets, improper selection of the appropriate weapon system to engage a target, faulty calculations, or simply by accident. Mass destruction fire involving nuclear weapons holds the greatest potential for contaminating vast areas outside the intended kill zones and thereby may delay or restrict friendly passage through, or occupancy of, critical areas. Nuclear damage phenomena—such as tree blowdown, fallout, and other residual effects—and unexploded ammunition, minefields, fires, rubble, and other hazards are the byproducts of offensive operations which must be anticipated and planned for by any commander.

The attacker's employment of nuclear munitions may intentionally produce radioactive fallout as the principal desired effect and thus enlarge the lethal area of a weapon in a predetermined pattern. This may be accomplished by the surface or subsurface burst of a nuclear weapon or by the use of atomic demolition munitions (ADM) (Ref. 9). Fallout prediction procedures are discussed adequately in Ref. 10 and are not repeated here. Because of weapon unreliability and the possibilities of error, inadvertent surface bursts will produce undesired fallout. Since radioactive fallout can produce contamination and personnel casualties over large areas of terrain, it can have a major influence on tactical operations and greatly increase logistical problems. When exploiting our own surface bursts, or

when the enemy employs nuclear strikes, troop protection from residual contamination is of paramount importance. However, enemy action, obstacles, or the attainment of significant tactical advantage may compel a commander to move his unit through the area of fallout. The problems which arise from this course of action include the determination of the fallout pattern, calculation of the average radiation dose rate, and the question of what acute total radiation dosage will be received by his troops as they pass through areas of varying radiation intensities for specific periods of time (Ref. 11). While the technical data can be determined from nomograms, charts, and equations, standard procedures require adherence to a certain set of principles. Chief among these is the need to keep the total accumulated radiation dose to a minimum by keeping the stay time as short as possible, by delaying the entry time as long as possible, and by providing shielding. The route that can be traversed most quickly (consonant with the operational situation) provides the minimum stay time. Routes through areas of lowest radiation dose rates are selected provided radiological surveys are able to provide this information. The use of tanks, armored vehicles, or sandbags on the floors of vehicles provides a measure of shielding and will reduce the total radiation dose received by personnel. Areas of higher radiation are avoided through continuous radiation monitoring. The advantages which accrue to the party employing the nuclear weapon result from knowledge of the location of planned ground zero, the information obtained from the preparation of a fallout prediction chart when post-burst analysis indicates that fallout will probably occur, and the availability of data essential to the maintenance of a radiological contamination overlay. The lack of precise information on wind structure and the fallout phenomenon makes the exact fallout pattern most difficult to predict.

Other obstacles created by the use of nuclear weapons, which may extend to distances considerably beyond the primary target area, are tree blowdown, rubble in built-up areas, and forest and urban fires. These effects are measurable, and can be predicted by the use of templates and nomograms constructed for this purpose. Since the importance of the primary target will dictate the degree of risk acceptable with respect to the production of side effects, often the commander consciously will permit the creation of obstacles even though they may restrict his freedom of maneuver and impose additional burdens on his combat service support elements, especially the engineers.

The use of mines, demolitions, booby traps, and other preset explosive charges on the battlefield is intended either to produce military casualties or to serve as obstacles to harass, canalize, divert, restrict, or delay, or stop enemy troop movement (Ref. 12). Unfortunately, they are great hazards to friendly forces also, even though locations and patterns are known and ostensibly recorded. Unlike nuclear contamination, which will decay with time and become ineffective, minefields remain potent until the mines are removed or disarmed. Consequently, uncleared minefields can cause problems even after a war is over. While no statistics are available, it is a matter of record that many civilians have been killed or wounded by uncleared mines in such places as France, Korea, Italy, and Vietnam. Duds and other unexploded ammunition also have taken their toll among civilian populations. Requirements for munitions designed in the U.S. since 1970 have stressed the importance of "sterilization" or "self-destruction" to avoid such post-war civilian casualties and to prevent the enemy from making use of our dud munitions.

10-6 THE SHOOT-LOOK-SHOOT TACTIC IN DAMAGE ASSESSMENT

We finally round out the subject of damage assessment by associating it with some methodology that may be used to evaluate the expected performance of weapons. The technique discussed here has been called the "shoot-look-shoot" philosophy, for it refers to the practice of being able to observe the

effect of fire and using information on damage to enemy targets in order to reduce the number of firings, launchings, etc., or hold ammunition expenditures to a minimum. The shoot-look-shoot philosophy may be important or even required for expensive items such as missiles.

Suppose that a target consists of N elements. The N elements might represent the number of enemy personnel in a given target area, or N might be the number of enemy tanks attacking a friendly position, or the number of enemy aircraft employed in a support role that friendly forces should defend against, etc. In this connection, the weapon systems analyst often will use some criterion for defeat of enemy force or targets. For example, if a third of the enemy troops are incapacitated in a battle, the enemy might withdraw; or the defeat of 40% of the enemy tanks in an attacking force would be severe enough for withdrawal. In any of these cases, or applications, the analyst could specify that the loss of r targets ($r \leq N$) means defeat of the enemy force. We therefore have the problem of assessing damage in combat until r target elements have been killed, or put out of action. This leads us directly to the application of the negative binomial distribution, and its relation to target damage assessment.

The basic parameter involved in this analysis is the average or single shot kill probability of a weapon. In Chapter 14 we discuss the chance p_h of hitting a target for an individual round, and in Chapter 15 we discuss the vulnerability of a target, or target element, and in particular the conditional probability that a hit results in a kill, i.e., $p(k|h)$. The product of these two probabilities is known as the single shot kill probability p_k or simply p for our analysis here:

$$p = p_k = p_h \bullet p(k|h) . \tag{10-1}$$

With this basic quantity, we may now compute the expected number of rounds fired from a weapon in order to kill r of N targets or target elements.

Consider a number of random trials denoted by $x = 0, 1, 2, 3, \ldots$. Now in order for r successes (kills) to occur on the xth trial or shot, then $(r - 1)$ kills must have occurred in the preceding $(x - 1)$ shots, and the chance or probability for this is clearly

$$\binom{x-1}{r-1} p^{r-1} \bullet q^{x-r} \tag{10-2}$$

where the notation $\binom{M}{m}$ means a combination of M things taken m at a time and $q = 1 - p$ is the target "survival" probability for a shot. Finally, multiplying Eq. 10-2 by the chance of success or kill on the xth shot (which is p), then we obtain the negative binomial distribution (or Pascal frequency function)

$$f(x;r,N) = \binom{x-1}{r-1} p^r \bullet q^{x-r} \tag{10-3}$$

where now $x = r, r + 1, \ldots, \infty$ since at least r shots will be required. This frequency function may be seen to be the general term in the expansion of $p^r(1 - q)^{-r}$, which incidentally shows that

$$\sum_{x=r}^{\infty} f(x;r,N) = 1 \tag{10-4}$$

so that it is a proper probability distribution.

It can be shown that the mean or expected number of trials (shots) $E(x)$ to obtain r kills is simply

$$E(x) = r/p \tag{10-5}$$

and that the variance σ^2 of the number of trials is

$$\sigma^2(x) = rq/p^2 \ . \tag{10-6}$$

The negative binomial distribution (Eq. 10-3) is also called the binomial waiting-time distribution, and has application to many problems in the evaluation of weapons, although it often has been overlooked. We give an example.

Example:

Twenty enemy tanks are approaching our defensive position. It is known that under average conditions of the ranges of engagement the single shot kill probability of our antitank weapons is $p = 0.25$, but if 40% of the tanks can be killed the enemy will be defeated and must withdraw. Then, what is the expected number of rounds we must fire to rout the enemy tank attack, and what is the standard deviation of this number?

Now the battle plan involves firing and observation of damage assessment until $r = (0.4)(20) = 8$ enemy tanks are put out of action. Hence, the expected number of rounds required is simply

$$E(x) = r/p = 8/(0.25) = 32$$

and the standard deviation of the number of required rounds is

$$\sigma(x) = (rq)^{1/2}/p = [(8)(0.75)]^{1/2}/(0.25) = 9.8 \ .$$

We note, incidentally, that the standard deviation of the number of required rounds (9.8) may seem somewhat large, but that is the nature of the stochastic problem involved here. For high assurance, therefore, the number of rounds needed may be as great as the mean value plus two or three sigmas, i.e., 52, or perhaps even 61 rounds, nearly double the mean value!

REFERENCES

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2. FM 61-100, *The Division* (Being replaced by two new manuals, FM 71-100 and FM 17-101).
3. FM 101-31-1, *Staff Officers' Field Manual: Nuclear Weapons Employment, Doctrine, and Procedures*.
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CHAPTER 11

FACTORS AFFECTING TARGET SELECTION

Some of the problems of detecting, acquiring, locating, and engaging enemy targets by friendly weapon systems are discussed. Also, problems relating to target analysis, worth, assignment, reaction, and recovery are introduced. The scope of the chapter is rather elementary and introductory since the overall target acquisition problem is of wide scope and must be covered in more detailed analyses elsewhere. The importance of timely detection, acquisition, and engagement of enemy targets cannot be stressed too much, however, since the efficient utilization of friendly weapons is crtically dependent on target selection and engagement. Target detection chances are introduced.

11-0 LIST OF SYMBOLS

E_{ij} = expected damage for the ith weapon against the jth target
$E(n)$ = expected number of trials for a target detection
$E(t)$ = expected time to target detection
$F(t)$ = cumulative probability function
$f(t)$ = probability density function
G_n = chance of not detecting a target during the first $(n - 1)$ glimpses but detecting a target on the nth glimpse
g_i = chance of target detection on ith glimpse
$MTBD$ = mean time between detections
$MTTD$ = mean trials to detection; mean time to detection
n = number glimpses or trials
$P_{k,ij}$ = kill probability for ith weapon against jth target
P_n = probability of at least one detection in n trials or glimpses
$p(t)$ = probability of detection during time t of continuous search
Q_n = $1 - P_n$ = chance of no detections in n trials
$q(t)$ = probability of failure of detection during time t of continuous search
$R(n)$ = reliability of detection in n trials
t = time
Δt = short interval of time
V_j = value of jth target
θ = mean number of trials to the detection of a target; or the mean time to the detection of a target
ν = $\nu(t)$ = instantaneous probability density of detection

11-1 INTRODUCTION

Typical targets might consist of enemy personnel (either dug-in or stationary, under casual cover, or moving on foot), command, control, and communication centers, forward observation posts, trucks, tanks, missile launchers, artillery batteries, assembly areas, choke points, antitank guided missile launchers, air defense systems, aircraft, and helicopters. It can be seen that some targets are easily defined, acquired, located, and engaged; whereas, on the other hand, because of natural ground cover, camouflage, etc., other targets, such as heterogeneous or composite types, present considerable difficulties to intelligence gathering activities. In fact, only some of the elements of a potential target may

be visible or accidentally detected, and, consequently, there exists an involved problem in extrapolating from the target elements that are identifiable to the size, type, and location of the overall target itself. There are problems in estimating target or target element identity, in estimating target radius, and locating target centers. Also, target movement, duration in each location, the density of targets, weapon availability to engage attackable targets, and friendly weapon attrition are all important. In addition, there is the problem of assigning or allocating weapons to appropriate targets, so that the available fire support is the best possible and response times are adequate.

Surveillance and acquisition of enemy targets are carried out with forward observers, telescopes, radars or moving target indicators, drones, aircraft, various ground sensors, ranging devices, and other means available. In fact, a major Army problem is that of developing sensors for reliable and timely detection, acquisition, identification, and location of enemy targets or threats. It is necessary to keep abreast of intelligence reports and scenario documents, or other available means, to predict accurately the types of units, personnel, and materiel any potential enemies might be expected to place on the battlefield.

We cannot emphasize too much the great importance of the timeliness of acquiring targets, the prompt transmission of such information back through intelligence and command channels, and the timely engagement of enemy targets by appropriate friendly weapon systems.

The most important elements of a land combat force are its physical assets—i.e., its personnel and materiel, its doctrine and procedures, the will of its officers and men to win, and also the support of the indigenous population. Damage to any one of these will degrade the effectiveness of the force. Physical assets are the most obviously vulnerable to attack; consequently, a land combat force structures its major capability to destroy the physical assets of opposing forces. Each element of the opposing force's physical assets is a potential target. If a sufficient number of these targets are destroyed, the force as a whole will be defeated. (A potential target becomes a target when it actually is taken under fire.)

The principal factors affecting target selection other than detection and acquisition can be grouped under the classifications of:

1. Target definition
2. Target assignment
3. Target engagement
4. Target reaction.

Target definition requires specification of the number of dimensions it possesses, its overall shape, its location, its composition including defensive armament, and preferably vulnerability. Target definition will be discussed qualitatively in par. 11-2 to avoid duplication of the specific quantitative target models which are developed later. *Target assignment* involves the allocation of targets to weapon systems for engagement in a manner to defeat the targets with an economical use of men and other combat resources such as ammunition. Target assignment and the relationships of tasks and targets are discussed in pars. 11-3 and 11-4. *Target engagement* involves the many facets of identifying elements of the enemy force as targets and locating them in time and space with sufficient precision to engage and destroy them. This is discussed in par. 11-5. *Target reaction* pertains to the response of the target as a result of actual or expected damage. It includes reaction time, tactics and effectiveness, and recovery time. It is discussed in par. 11-6. The subject of target analysis is treated briefly in par. 11-7.

11-2 TARGET DEFINITION

The manner in which the basic item (element) of enemy personnel, equipment, or real property is grouped for engagement or destruction will determine whether or not the target will be classified as a

point, area, or linear target. A linear target, for example, is a column of tanks or trucks, and often would be moving, of course. Classifications such as "hard" and "soft" are considered insufficient since more detail is needed on the characteristics of targets or target elements. An attempt is made to describe targets as accurately as possible since this is important in evaluating and establishing the most appropriate weapon requirements.

11-2.1 POINT AND AREA TARGETS

A point target is usually one for which the single target element occupies a single location. This classification results from the assumption that the dimensions of the target are small in comparison with either the range between the weapon and target or the effective radius of the warhead damage mechanism. An enemy tank is an example of a point target. A long bridge also could be a "point" target nevertheless.

If a target is described as two-dimensional, it is classed as an area target, i.e., simply a collection of target elements distributed over an area. Ground targets dealt with mostly are points, circles, or lines, although for evaluation purposes circles, ellipses, rectangles, or some other more complex shape may be used. An enemy infantry company might be considered an area target. The distinction between a point target and an area target depends, among other things, on the number of elements and the arrangements of these elements within a given area. This distinction may be system dependent since what may be categorized as a point target for one weapon system may constitute an area target for another weapon system.

Concerning volume targets, an aircraft might be considered to fall in this classification, although aerial targets are ordinarily evaluated from the standpoint of presented area or radar cross section, and may be attacked by blast volumes from a warhead.

The vulnerability of an area target to attack is determined by an analysis of specific target elements in a specific target description. The vulnerability analysis must begin with a detailed target description. An area target, whose grouping of target elements is considered to be attacked collectively, may be further categorized as simple or complex.

11-2.1.1 Simple Targets

Simple targets are those whose elements are functionally independent. Consequently, the kill effects on simple targets are cardinal, i.e., each element in the target must be killed in order to kill the entire target.

The elements of simple area targets may be similar or dissimilar. This fact leads to a further refinement of the definition of simple area targets as to target homogeneity. Since simple targets may be fixed or mobile, they may be further categorized as to their spatial relationship with one another.

11-2.1.1.1 Homogeneous Targets

Homogeneous targets are those simple area targets in which all elements are of the same type. For example, a parked fleet of helicopters would constitute a simple, homogeneous target, as would also the trucks in a truck park or a deployed company of infantry.

11-2.1.1.2 Nonhomogeneous Targets

A nonhomogeneous target would be one having elements of different types. A composite infantry-tank force whose target elements would be the individual infantrymen and the individual tanks is a nonhomogeneous, simple area target. A supply depot with both hard and soft targets and a division command post are examples of this category.

11-2.1.2 Complex Targets

Elements having a functional relationship as well as a spatial relationship constitute a complex target. An example of target elements which are functionally dependent but might well be spatially independent would be a missile site where the missile launcher, though undamaged, could be prevented from firing a missile by the disabling of the fire control center or by damaging the power supply. (Warheads with large damage radii, such as nuclear weapons, represent natural choices for many complex targets—or by their nature damage different types of targets elements over a wide area.)

11-2.2 GROUND AND AERIAL TARGETS

The location of the target affixes its categorization as either a ground or aerial target. Ground targets may be further categorized as either point, line,, or area targets, in which case the target description must be modified by the terminology given in par. 11-2.1.

The various types of weapon fire used to attack ground or aerial targets may be classified by establishing the relationship of the location of the weapon launcher to the location of the target. Thus, the various types of weapon fire may then be classified as:

1. Surface-to-surface
2. Surface-to-air
3. Air-to-surface
4. Air-to-air.

The classification of surface-to-surface weapon fire includes all munitions that are fired from a weapon or launcher located on the ground with the intent of destroying a target also located on the surface of the earth. The air-to-surface category includes all munitions fired from a aircraft whose purpose is to destroy a target on the ground. The destructive intent of these two categories of fire is directed in defense and attack against stationary targets and moving targets.

11-2.2.1 Stationary Target

A stationary ground target is one that is fixed or is not subject to movement while under attack. In this case, only the present position of the target need be known and the generation of the target model may be relatively uncomplicated.

11-2.2.2 Moving Target

A ground target that possesses some nonzero velocity vector is classed as a moving target. Moving targets necessitate the generation of a time dependent target path model of attack.

11-2.2.3 Aerial Targets

If the target is not located on or under the surface of the earth, it is classed as an aerial target. Thus, surface-to-air weapon fire includes any munition fired from a weapon or launcher located on the surface of the earth whose purpose is to destroy a target in the air. The air-to-air classification includes any munition fired from an aircraft whose purpose is to destroy an aerial target.

The destructive intent of surface-to-air and air-to-air fire is directed in defense and attack against two basic categories of targets, single and multiple targets.

11-2.2.3.1 Single Targets

Single aerial targets are those units of fixed- or rotary-wing aircraft, missiles, or rockets that are in motion somewhere within or above the earth's sensible atmosphere.

11-2.2.3.2 Multiple Targets

Multiple aerial targets are those flights of a group of single fixed- or rotary-wing aircraft, missiles, or rockets, which are significantly separated in time and/or position.

11-3 TARGET ASSIGNMENT

A target is considered "acquired" when it is detected, identified, and located in sufficient detail and with sufficient accuracy to permit effective employment of weapons against it.

Targets must be allocated to the weapon systems of the force on a rational basis that gives reasonable expectation of success of the mission at a reasonable cost. The critical concepts in target assignment are target worth and target priority.

11-3.1 TARGET WORTH

Target worth is important primarily to target assignment in its connection with measuring the combat effectiveness of a force. Which, for example, is more combat effective or valuable: a force that can destroy 30 tanks and 20 howitzers or a force that can destroy 20 tanks and 30 howitzers? The answer depends on the relative values of opposing tanks and howitzers; the first is more effective only if for the particular battle the destruction of opposing tanks is worth more than the destruction of opposing howitzers.

There are two approaches to establishing target worth. On the one hand, the worth of a target may be construed as its economic value or the value of resources consumed to create the target. On the other hand, the worth of a target can be related to its utility in combat. The concept of "utility", expressed as a number which lies between zero and unity—zero standing for useless, and unity standing for maximum usefulness attainable in a given situation—is borrowed from the discipline of economics (Ref. 1).

In the long-term view, both approaches to target worth may be equivalent, since it is to a nation's advantage to allocate its defense resources roughly in proportion to the utility of its combat elements. In the short-term view, however, the approaches may be quite different. Utility values may be completely out of proportion to economic values. For example, a reinforced rifle squad may be considered more useful than a tank battalion when the squad is successfully defending a road block on an opposing division's route of advance, and the tank battalion is immobilized by soft soil or mud.

Target worth, in the utility sense, is a measure of its usefulness to the enemy as a threat to friendly forces (i.e., the potential damage or inconvenience it represents to the opposing force). Thus, an enemy weapon system in a covered position with good fields of fire is a valuable target since it potentially can destroy or delay a large number of friendly combat elements.

The economic approach to establishing target worth is very valuable in studies of a broad scope with respect to time and/or geographical scale. However, in general, the user of this Handbook will be engaged in the analyses of individual Army weapon systems whose employment will be in combat situations. Therefore, this Handbook emphasizes the utility approach to target worth in weapon systems analysis.

Defining a quantitative measure of target worth is not a simple task. If a model exists to compute measures of effectiveness of opposing forces in terms of numbers and types of force elements, then a systematic variation of force compositions between runs of the model would provide a basis for computing target values. Unfortunately, this is not a practical course in most situations.

An important aspect of target worth is the change in the value of a target during the course of a battle. The tank platoon that penetrates the opposing main line of resistance suddenly has a quantum increase in its value because it is threatening to annihilate the opposing command and control center.

Expert opinion is a possible source of judgment as to target worth. Of course, biased judgment cannot be eliminated entirely no matter how objectively the assessment is made. A consistent method tending to reduce human biases and simultaneously detemine weapon values of opposing forces in a combat situation evolved from a cost-effectiveness study, "The Tank Antitank Assault Weapon Systems (TATAWS) Study, Phase III", conducted by the Armor Agency of the former US Army Combat Developments Command (Refs. 2, 3, and 4). (See also Chapter 30.)

11-3.2 TARGET PRIORITY

Target priorities can be approached in two ways. First, target priorities can be imbedded in decision mechanisms that assign targets to weapon systems for engagement. Second, target priorities can regulate the volume of fire to be delivered on each type of target. In a broad scope, both approaches are equivalent since repeated application of decision mechanisms will determine relative volumes of fire by type of target.

With a basis for determining the relative values of targets, optimum target assignments can be made to a force's weapon systems. This optimum should be such as to maximize the total worth of all the targets subsequently destroyed. It also must consider the probabilities of target destruction, since targets that are invulnerable to a particular type of weapon should not be engaged by it. Although the field commander does not have a capability to compute these optimum assignments, some simple but competent analyses during weapon development are needed to provide the field commander with the appropriate mix of weapon capabilities, so that his priorities can be accomplished.

Analyses of weapon systems effectiveness often involve building and exercising a combat simulation model or war game of some kind. Target priorities are an extremely important input to these models. They usually can be expressed as ordinal numbers. For example, an artillery battery may assign the following priorities to these targets:

1. Infantry platoon — 1st priority
2. Bridge over unfordable river — 2nd priority
3. Tank platoon in overwatch — 3rd priority.

The artillery battery would engage an infantry platoon before engaging a bridge, and a bridge before engaging a tank platoon which is well protected against the effects of artillery.

The effectiveness of a weapon system, measured by a combat simulation or a war game, is highly sensitive to the target priorities used in the decision mechanisms to assign targets for engagement. This suggests that target priorities may be equally important in determining the outcomes of real battles. It is a good practice to make special runs of the model to measure the sensitivity of final effectiveness measures to the input target priorities.

11-4 RELATIONSHIPS OF TASKS AND TARGETS

Par. 11-3.1 noted the changes in values of targets during combat. The relationship of tasks, such as attack and defend, to targets is allied closely to the concept of target worth; i.e., certain types of targets are important to specific tasks.

Tasks are military operations intended to accomplish specific objectives. An example is a combined infantry-armor attack against an enemy located on high ground overlooking a rail center with the objective of seizing and holding the ground in order to deny the enemy the use of the rail center.

Descriptions of tasks include the objective (attack, defend, reconnoiter, etc.) and the conditions (visibility, weather, terrain, fortifications, etc.). Considering all of the relevant variations and combinations possible, one could structure a virtually infinite number of unique tasks to which targets could be related. In the interests of brevity, this paragraph discusses several representative tasks in a manner to

give the reader a basis for relating tasks and targets in whatever situation may interest him. They are categorized as offensive tasks and defensive tasks. (See also Chapters 8 and 10.)

11-4.1 OFFENSIVE TASKS

Offensive tasks have the general objective of seizing a specific terrain feature and/or destroying the enemy's forces which are occupying it. These tasks depend on the maneuver of combat forces to the point of decision at which they destroy the enemy and seize the objective. This maneuver is executed under the protection of supporting fire which destroys or suppresses enemy targets that threaten movement to the point of decision.

Targets that are important in the offense, therefore, are:

1. Enemy weapons that are capable of slowing or stopping the advance of the maneuver element; an example is an enemy minefield protected by enemy tanks and infantry.
2. Enemy weapons that are capable of inflicting damage on the fire support element; an example is an enemy artillery battalion that fires on the attacker's supporting artillery. Destruction of the attacker's supporting artillery eliminates his capability to "soften up" the objective for a final assault.
3. Enemy command, control, and communication centers which link together the elements defending the objective; an example is an enemy area or complex housing a major enemy headquarters and communication center.
4. Enemy reconnaissance and target acquisition agencies; an example is an enemy observation helicopter directing artillery fire on the attacking force.
5. Enemy combat service support (logistics) centers; an example is an ammunition supply point. The importance of this type of target depends on the scope of the offensive. They always are important as part of a large-scale campaign, lasting weeks or months. For limited-scale attacks, such as a company-sized action, they may not usually be important, although such a target may be an objective for a small unit operating as part of a larger force.

To a large extent, targets for offensive tasks are relatively fixed in time and space. A large intelligence effort is usually necessary to detect and locate targets whose destruction can be programmed in the plan of attack.

11-4.2 DEFENSIVE TASKS

Defensive tasks are actions to prevent, resist, repulse, or destroy enemy attacks. These tasks depend on the delivery of accurate, lethal fire everywhere in the battle area that the enemy can possibly appear. If the enemy is mobile, targets cannot be acquired very far in advance, and the defensive force must deliver fire rapidly on targets of opportunity once the battle is joined. Hence, impediments to enemy mobility (such as minefields) must be protected by direct fire weapons artillery and mortars. Targets slowed by the obstacle should be engaged.

The important targets in defense are both the enemy's fire support force and maneuver force. Early in the enemy attack, the enemy's fire support forces are very important because destruction of friendly targets by that force facilitates enemy advances. If the enemy succeeds in closing on the defensive position, then targets in his maneuver force dominate all others in importance, since they threaten the defensive force's viability.

It always is important to acquire targets in the enemy's attacking force at as great a range as possible. When targets can be acquired at a long range, the defending force can deliver fire on them over a longer period of time, thereby increasing the probability of destroying them. However, sensor accuracy, in most cases, does depend indeed on range to target.

11-5 RANGE ESTIMATION

Range estimation is a very important part of the target acquisition process. Visual estimation with the unaided eye may be subject to a standard error of perhaps 20-25%. Coincidence range finders often have suitable accuracy, although now the laser type range finder is coming into much prominence because of its great accuracy. Survey by triangulation is also of use.

11-6 TARGET ENGAGEMENT

Each assignment of a target to a weapon system can be associated with some expected damage. To maximize the total expected damage, then, is one basis for target allocation available to the field commander. As a practical matter, these allocations are reflected in the target priorities previously discussed. A detailed examination of the target engagement process will reveal the rationale supporting target priorities.

For the ith weapon system engaging the jth target, one measure of the expected damage E_{ij} is

$$E_{ij} = P_{k,ij} V_j \tag{11-1}$$

where

$P_{k,ij}$ = probability of the ith weapon destroying the jth target

V_j = value of the jth target.

The dimension of E_{ij} and V_j is the unit of value assigned in the analysis.

The parameter $P_{k,ij}$ depends upon the probabilities of:

1. Locating the target
2. The weapon system having ammunition and being operable
3. The probability of hitting the target
4. The probability of destroying the target if it is hit.

Fig. 11-1 gives a typical functional flow diagram for target search, detection, identification, location, and engagement for a weapon system.

11-6.1 TARGET DETECTION

Potential targets usually occupy a very small fraction of the total space available in a combat area. Except by accident, therefore, it is nearly impossible to destroy undetected targets. (An undetected target is one for which no elements are seen.) Each combatant strives to make detection of his own weapon systems as difficult as possible. He does this by movement, cover and concealment, maintenance of radio silence, design of equipment with minimum silhouettes, camouflage, and other means of counter-surveillance such as noise and heat (thermal) suppression.

Detection of a target is aided by discovering its signature. Common signatures are:

1. *Trajectory.* This signature is created by the path of a projectile in flight. It is detectable by radars and radar-type sensors. This signature is associated with artillery and mortar weapons, and is of importance in counterbattery operations.

2. *Silhouette.* The visible configuration of a target is an obvious signature that is common to all targets (uncamouflaged). The probability of detection is degraded by poor visibility and masking by terrain and vegetation.

3. *Heat.* Infrared (IR) energy emitted by a target may be detected by IR sensors. This signature is associated with artillery and mortar (indirect fire) weapons; tank and antitank (direct fire) weapons; and especially internal combustion engines. Heat signatures are affected by all atmospheric conditions except darkness.

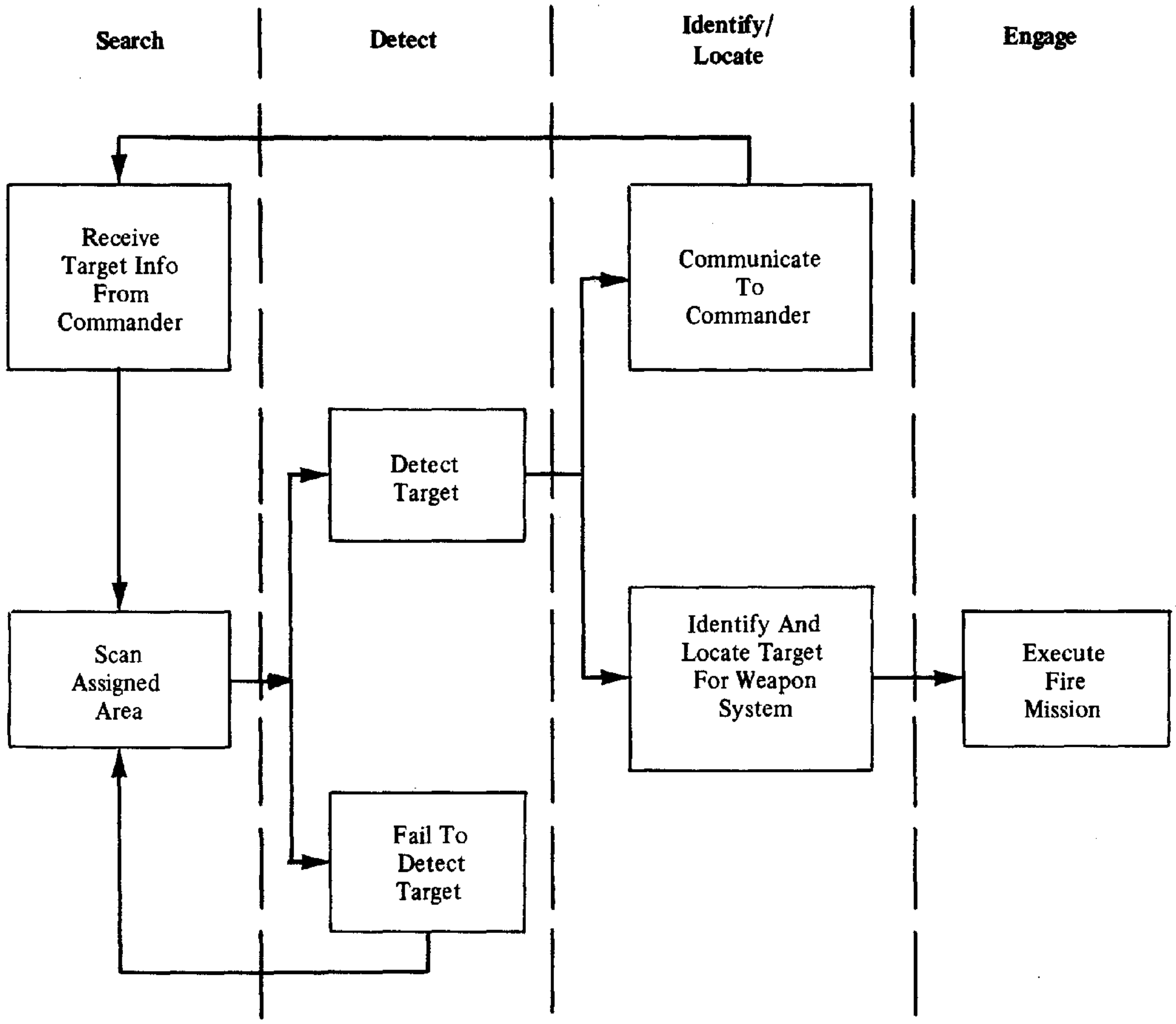

Figure 11-1. Functional Flow Diagram of Target Acquisition Information for an Individual Weapon System

4. *Flash.* This signature is equivalent to heat, but it is transmitted over the visible portion of the electromagnetic spectrum. It is associated with direct and indirect fire weapons.

5. *Smoke/Dust.* Smoke and dust often are raised when a weapon fires or when vehicles move. Smoke and dust are observable during daylight hours.

6. *Sound.* Virtually all targets emit sound. The sounds of weapons firing (cannon reports) are detectable by sound-ranging sensors.

7. *Motion.* An object, such as a vehicle in motion, is detectable by radar and radar-type sensors, as well as by the human eye. The visual signature is degraded by poor visibility, terrain and vegetation masking, and distance. The accuracy of location is further degraded by the motion itself.

11-6.2 TARGET LOCATION

After a target has been detected, it must be located with sufficient accuracy so it can be engaged and destroyed. In addition to range (par. 11-5), an estimate of the three spatial coordinates of the target

may be needed at the instant a projectile will be delivered. Time dependency is an important consideration for moving targets and for targets that are changing in hardness, such as troops digging fox holes.

As indicated in par. 11-5, range is determined with the aid of such sophisticated equipment as laser range finders, stereoscopic range finders, and sound-ranging sets. In all cases one must know the position of the sensor plus direction and distance from the sensor to the target. Target-location error is an important determinant of the hit probability. Target location often is characterized or expressed in terms of the circular probable error (CEP). (See Chapter 13.)

11-6.3 COMMUNICATIONS

While targets can be located by a weapon system, it often is done by special means such as an artillery forward observer or a reconnaissance aircraft. Within a weapon system, the target location element may be physically separated from the weapon itself. These possibilities, together with the needs of commanders at all levels to exercise control over all weapon systems, dictate an important communications requirement.

Communications are especially important in command and control. Where a target appears to be lucrative to several weapon systems, a command decision must be made to allocate it. In real life, this allocation is transmitted via the communication system. (The allocation may be by rules discussed before the battle starts as earlier indicated.)

11-7 TARGET REACTION

The goal of a land combat force is to impose its will on the opposing force. This usually is accomplished by destroying, or threatening to destroy, the enemy's materiel and personnel. The types of damage that can be inflicted on a target are categorized and discussed in Ref. 2, for example. The choice of target reactions to avoid or minimize damage and recovery time are discussed next.

11-7.1 CHOICE OF REACTIONS

A target to one force in combat usually is a weapon system, or component thereof, to the opposing force. Targets therefore are dynamic with respect to the opposing force's actions. The dynamics that must be anticipated in system analysis can be categorized broadly as precautionary reactions, protective reactions, and corrective reactions.

11-7.1.1 Precautionary Reactions

Precautionary reactions are taken before a target is fired on to avert its destruction. It may be camouflaged to prevent detection and location. A target may be hardened to reduce its vulnerability, even if it is located and fired upon; examples are "buttoning up" actions of armored vehicles and "digging in" actions of infantry. Dispersion of a force's elements over the widest possible area is a key principle of doctrine to minimize the damage to any target by a single weapon. An aggressive precautionary reaction is the delivery of a high rate of suppressive fire that inhibits the opposing force's efforts to locate and engage targets; an example is the use of "coaxial" or secondary machine guns on a tank to inhibit opposing infantry antitank weapons from delivering lethal fire on the tank. Movement is an important precautionary reaction, including both continuous motion to increase the opposing force's location error and random change of position to upset the opposing force's detection and acquisition efforts.

11-7.1.2 Protective Reactions

Protective reactions are taken by a target after it has been fired on or when it is certain hostile fire is imminent. A target may take evasive measures to avoid being hit, e.g., an infantry platoon sprinting forward to remove itself from an artillery impact area. A target may return fire in order to suppress the opposing forces. In addition, a target may take all of the precautionary reactions discussed in par. 11-7.1.1.

11-7.1.3 Corrective Reactions

Corrective reactions are taken by a force over a protracted period of time to reduce its vulnerability based on its experience with and assessment of the opposing forces' capabilities. A force can upgrade system capabilities to match or exceed those of the enemy. An example would be to install more powerful engines in all vehicles of a type which may increase mobility and thus improve survivability. Another example is to increase the weapon effects when it is learned that the enemy has upgraded his protection. The evolution of armored weapons is a dramatic example of how opposing forces alternatively increase their firepower with larger caliber weapons and reduce their vulnerability with heavier armor protection and greater speed.

Another corrective reaction is to alter the mix of weapon systems in the total force. An historical example is given by Germany in World War II. As their offensives were blunted and the Allied armies began to close in on two fronts, German production of tanks was curtailed in favor of the production of inexpensive antitank weapons which were relatively effective in retrograde and defensive operations.

In addition to altering the mix of weapon systems in the total force, the force can be increased in size to give a more favorable force ratio. This is a proven technique that has been applied successfully in every war. In application, then, the reaction to an improved weapon system may be such an increase in the number of effective weapons on the other side that it can be overwhelmed, for example, only by sheer numbers. (Lanchester type combat theory will be covered in Chapters 28 and 29.)

In summary, corrective reactions of targets can take the form of improved system capabilities, altered mix of weapons in the force, and increased size of the force. Combination of these reactions should be considered as plausible as any one of them taken separately.

11-7.1.4 Accounting for Reactions

Systems analyses must take these reactions into account for the study results to be meaningful. Failure to do so can lead to highly biased data and misleading conclusions. The example that follows will illustrate.

Suppose a particular study ignored the fact that targets can be concealed by camouflage and other means. This could be implemented by assuming all weapons systems to be targeted instantly by the weapon systems of the opposing side. A simulation of this unreal combat would terminate very rapidly while the opposing forces were still a great distance apart. The outcome would be biased in favor of the side with superior long-range capabilities; yet the short-range capabilities of the other side could be decisive, and they might be the winner in a real battle.

11-7.2 RECOVERY TIME

Recovery time can vary from a few seconds in the case of a soldier avoiding incoming fire, to days in the case of major field repairs on a vehicle, or even to years in the case of natural decay of radiation effects from a nuclear weapon. The length of the recovery time is important to maneuver considerations because targets usually can be maneuvered against while they are in the process of recovering. If the

maneuver is executed in a timely manner, so that the weight of the force's combat power can be brought to bear decisively, the value of the recovery of the target is seriously lessened. Conversely, if a target can recover before the maneuver against it is completed, the recovery of the target has considerable value since it may be combat effective again. It should be remembered in this regard that maneuver covers the range from such simple actions as a soldier darting from tree to rock, to such complex actions as the deployment of several hundred thousand men and associated materiel over months or even years. Thus, recovery and maneuver times are relative.

11-8 TARGET ANALYSIS

The weapon systems analyst must consider the target in terms of its importance to the strategist or the tactician. Before the analyst can evaluate the effectiveness of the weapon system under study, the potential target must be analyzed to determine, as applicable, its characteristic size, shape, structure, motive power, path of motion, type of guidance, maneuverability, and payload. The technical aspects of producing target damage by defined mechanisms must include methods and criteria against which specific effects on targets can be evaluated. These damage effects are dependent on the target characteristics and the target position on the battlefield. Defensive measures against attack, involving the same target characteristics, also must be considered in the analysis.

In addition to the target characteristics enumerated, the vulnerability of the target may lie in its associated personnel, the type of control employed, or even its lines of supply. Target intelligence must include target vulnerability studies which are comprehensive processes in themselves. For instance, armor is evaluated in terms of mobility, armor protection, main and secondary weapon accuracy, rate of fire, tactical employment, reliability, availability, maintainability, and survivability. Tank vulnerability studies indicate not only the best method of killing enemy tanks, but also vulnerability of our own tanks.

Similarly, if the study addresses the problem of the human target, it should extend beyond the consideration of the effect of one round of ammunition delivered against an enemy soldier. Incapacitation of enemy troops requires wound ballistic studies which include vulnerability of the human body, effects of body armor, and armament of friendly troops in terms of weight of the principal weapon, weight of ammunition carried, weapon accuracy, training time required to reach proficiency with the weapon, and logistical requirements. The optimization of these parameters should indicate which combination of small arms and personal protective devices will give the infantryman the greatest combat effectiveness.

The probability of locating the target is a function of its characteristic size, shape, structure, and maneuverability. Target characteristics also enter into a determination of hit and kill probabilities which is covered in later chapters. Consequently, the weapon systems analyst must give consideration to potential target characteristics before he can perform the task levied on him. These considerations must be included in the scenario and the resultant war game as well.

The reader should consult Refs. 5, 6, and 7 for allied and supplementary material regarding target selection problems. Refs. 8 and 9 also are recommended for further overall orientation relative to the general subject.

11-9 INTRODUCTORY ACCOUNT OF TARGET DETECTION PROBABILITIES

It should be realized and this chapter would not be complete unless we warned that the mechanics of the problem of target selection involves a stochastic process; i.e., probabilities of detection, probabilities of acquisition, etc., must be considered. Such a treatment is covered in Ref. 10, and we give here some of the more elementary concepts of the probabilistic characterizations of target detections.

To introduce the subject, suppose that an observer is searching for a target, either visually or with the aid of some sensor—the unaided eye, radar, etc. In scanning for a target, the procedure may be in the form of a series of brief, disconnected glimpses, or the observer may be looking continuously. We will examine both methods of scanning for possible targets here.

11-9.1 TARGET DETECTION BY GLIMPSES

For distinct "glimpses", the chance of detecting a target may vary from trial to trial. Suppose the chance of detecting a target on the ith trial or attempt is g_i, i.e.,

$$g_i = \text{chance of detection on the } i\text{th glimpse} .$$

Then for n glimpses, the chance P_n of detecting a target (and one might say "at least once") will be

$$P_n = Pr(\text{detection}) = 1 - \prod_{i=1}^{n} (1 - g_i) \tag{11-2}$$

where the chance of no detections in n trials is subtracted from unity to obtain the chance of at least one detection. It is clear that for nonzero values of g_i, then P_n increases steadily. For the case of constant probability of detection from glimpse to glimpse, then we have $g_i = g$ and

$$P_n = 1 - (1 - g)^n \approx 1 - \exp(-ng) \tag{11-3}$$

if g is sufficiently small.

Note that the chance of not detecting a target in n glimpses is $Q_n = \exp(-ng)$.

Looking further, the chance G_n of not detecting a target in any of the first $(n-1)$ glimpses—or with the first $(n-1)$ sensors—but detecting it on the nth glimpse is clearly

$$G_n = (1 - g)^{n-1} g . \tag{11-4}$$

One might then ask the question, "What is the expected number $E(n)$ of glimpses for the detection of a target?" To determine $E(n)$ we have only to multiply G_n by the number of the trial n and sum from 1 to ∞. Thus,

$$\begin{aligned} E(n) &= \sum_{n=1}^{\infty} n(1 - g)^{n-1} g \\ &= g + 2g(1 - g) + 3g(1 - g)^2 + 4g(1 - g)^3 + \cdots \\ &= -g \frac{d}{dg}[1 + (1 - g) + (1 - g)^2 + (1 - g)^3 + \cdots] \\ &= -g \frac{d}{dg}\left[\frac{1}{1 - (1 - g)}\right] = -g \frac{d}{dg}\left(\frac{1}{g}\right) = 1/g \end{aligned}$$

or

$$E(n) = 1/g . \tag{11-5}$$

Thus, the expected number of trials until a detection is the reciprocal of the chance of detection on a single trial (i.e., a special case of the negative binomial distribution of par. 10-6).

We are now in a position, incidentally, to note the similarity of this analysis with that of the field of reliability and life-testing. The quantity n is a random variable describing the (random) number of trials or glimpses until a target is detected, the mean number of trials to a detection being $E(n) = 1/g$. Now looking at the formula for the chance of at least one detection P_n in n trials, we note from Eq. 11-3 that

$$P_n = 1 - (1 - g)^n \approx 1 - \exp(-ng) = 1 - \exp(-n/\theta) \tag{11-6}$$

where

$\theta = E(n) = 1/g =$ the mean number of trials to the detection of a target.

Then the probability of detection or the "reliability" $R(n)$ of detection in n trials is

$$P_n = 1 - \exp[-n/(MTTD)] \tag{11-7}$$

where

$MTTD$ = mean (number) trials to detection,

$R(n)$ = "reliability" of detection in n trials with the sensors used.

11-9.2 CONTINUOUS SEARCH

For the case of continuous scanning, we speak of detection, or rather the chance of detection, during a short interval of time Δt. Suppose that $\nu(t)$ is the instantaneous probability density of detection. The probability density $\nu(t)$ may well depend on the time itself, although here we will assume for the time being that it is constant, i.e., $\nu(t) = \nu =$ a constant. If we let $p(t)$ be the chance of detection in time t and $q(t) = 1 - p(t)$ be the probability of failure of detection during time t, then, the chance of failure to detect during the accumulated time $t + \Delta t$ will be denoted by $q(t + \Delta t)$. Further, $q(t + \Delta t)$ is clearly the product of the chance of failing to detect a target in time t and the chance of failure to detect during the additional interval Δt. That is to say

$$q(t + \Delta t) = q(t)[1 - \nu\Delta t] = q(t) - \nu q(t)\Delta t\,.$$

Thus,

$$\frac{q(t + \Delta t) - q(t)}{\Delta t} = -\nu q(t)$$

and

$$\frac{dq(t)}{dt} = \lim_{\Delta t \to 0} \frac{q(t + \Delta t) - q(t)}{\Delta t} = -\nu q(t)$$

or

$$\ln q(t) = -\nu t$$

and

$$q(t) = \exp(-\nu t) \tag{11-8}$$

where

$$q(0) = 1 .$$

Finally

$$p(t) = 1 - \exp(-\nu t) \tag{11-9}$$

which is the probability of detecting a target in time t when the instantaneous chance of detection is νdt.

The quantity $p(t)$ is a cumulative probability of detection and could be denoted by the statistical symbol $F(t)$ as in statistical literature. The probability density function $f(t)$ is given by

$$F'(t) = f(t) = p'(t) = -\exp(-\nu t)(-\nu) = \nu \exp(-\nu t) \tag{11-10}$$

where the primed functions indicate the derivative with respect to time t.

We may easily find the mean or expected time to detection for the *exponential* distribution. We first observe that the probability of detection between time t and $(t + dt)$ is the product of the probability of no detection up to time t and the chance of detection during the next small increment of time dt. Thus, this chance is clearly

$$\exp(-\nu t)\nu dt = f(t)dt .$$

Therefore, the mean of t or $E(t)$ is

$$E(t) = \int_0^\infty t \exp(-\nu t)\nu\, dt \tag{11-11}$$

where

$E(t)$ = expected time to target detection

$\theta = MTTD$ = mean time *to* detection (beginning from time zero)

or the parameter $\nu = 1/E(t) = 1/\theta$.

On the other hand, the concept of the mean time *between* detections $MTBD$ may be very useful also. Indeed, the time between detections is a random variable and its distribution usually may also be described by an exponential distribution.

In summary, the chance of detecting a target within time t is $F(t) = 1 - \exp[-t/(MTTD)]$, and the chance of a detection within intervals as a function of time may be given by $F(t) = 1 - \exp[-t/(MTBD)]$.

Note that in the equation

$$p(t) = 1 - e^{-\nu t}$$

we may put $\nu = -\ln(1 - g)$ and $t = n$, i.e., for "glimpses" $n = 1, 2, 3, \ldots$ Then we have

$$\begin{aligned} p(t) &= 1 - e^{t[\ln(1-g)]} \\ &= 1 - [e^{\ln(1-g)}]^n \\ &= 1 - (1 - g)^n \end{aligned}$$

as before for the case of detection by glimpses, i.e., Eq. 11-3.

Finally, we remark that if one target element is detected, the chance is good for detecting other elements, and the process extends to the usual case where several or many elements must be detected to identify the target.

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CHAPTER 12

THE SCENARIO

The use of the scenario is examined as an important tool in the evaluation of weapon systems. The chapter explains how study objectives, assumptions, limitations, and specific guidance received from the sponsor of the study, in addition to operational factors, are used to simulate realistic conflict situations.

12-1 GENERAL

In the context of weapon systems analysis, a scenario is a narrative description of a hypothetical conflict containing all of the important elements in the analysis of the given system. The effectiveness of the system is measured against the parameters established by the scenario. The assumptions and background data obtained from the intelligence community to make such parameters realistic provide important input to the study. A partial list of broad topics which can be studied by means of a scenario is shown in Table 12-1. Scenarios for combat analysis models are covered in Chapter 40.

12-2 PURPOSE

For a variety of political and technical reasons, the military planner and analyst must consider a range of realistic alternative assumptions and contingencies in any overall evaluation. The scenario has been developed as an analyst's tool or technique to manipulate all of the factors to be considered in a simulation and to provide the outline of the "plot". In fact, it creates the environment in which alternative courses of action may be explored and establishes parameters for measurement of the effectiveness of the system being evaluated. To be effective, the scenario must present a situation which will illuminate critical factors upon which attention should be focused. This permits selective analyses of tactics, doctrine, weapon systems parameters, command and control procedures, logistical requirements, and other military operational aspects, in addition to the consideration of political and socio-legal factors if required in the evaluation.

The scenario plot postulates a situation which defines the enemy, establishes the forces to be employed, and selects the environment of projected operations. It acts as a control mechanism in that it contains instructions to insure that certain actions occur in their proper sequence in the evaluation. The matter of the amount of control is important since control measures provide realism to the events or items being studied and, in the end, to the validity of the information being sought. Thus, the scenario can serve as a useful measurement device of constraint parameters to assist in system development, especially in the research and development phases. In practice, the situation, the events, and the parameters of the new system to be evaluated in the environment frequently are programmed so that a computer-played game will yield useful outputs for further analysis.

12-3 SCOPE

The typical scenario accentuates certain environmental or situation factors which may affect the solution of the problem by permitting selected developmental approaches. If weapon systems are to be evaluated properly, the specific details associated with their deployment need to be identified, but other portions of the scenario may be simplified. However, scenario simplification should not have adverse effects on the completeness of the overall evaluation. An oversimplified scenario can lead to a requirement for additional follow-on scenarios to develop the necessary detail. Often, techniques used in

TABLE 12-1. STUDY AREAS AMENABLE TO SCENARIO APPLICATION

1. Threat Analysis

2. US Objectives
   a. Programs
   b. Policies
   c. Political
   d. Military
   e. Foreign
   f. Domestic

3. Essential Elements of Intelligence

4. Terrain Appreciation
   a. Mobility Factors
   b. Surveillance
   c. Fire Support
   d. Target Acquisition
   e. Logistical Support
   f. Communications

5. Meteorlogical Factors
   a. Mobility Factors
   b. Air Operations
   c. Night Observation
   d. Logistical Support

6. Political Factors
   a. Civil Government
   b. Religious Constraints
   c. Political Constraints

7. Opposing Forces
   a. Deployment Patterns
   b. Force Composition
   c. Weapon Systems
   d. Reinforcement Capability
   e. Resupply Capability
   f. Offensive Doctrine
   g. Defensive Doctrine
   h. Effectiveness of Weapons

8. Environmental Factors
   a. Medical, Public Health Factors
   b. Epidemiological Environment
   c. Casualty Production
   d. Casualty Care
   e. Casualty Evacuation
   f. Preventive Medicine Factors

9. Logistics
   a. Supply Resources Onsite
   b. Transportation Requirements
   c. Port and Air Terminals
   d. Lines of Communication
   e. Sealift and Airlift Demands
   f. Logistic Pipeline Problems

10. Command and Control
   a. External Communications
   b. Internal Communications
   c. Command Structure
   d. Command Interfaces with Allies
   e. Command Interfaces with Indigenous Forces
   f. Command Equipment Compatibility
   g. Electronic Countermeasures Environment

11. Time Phasing
   a. Climatic Environmental Implication
   b. Tempo of Conflict
   c. Continuous Operations
   d. Escalation Factors

12. Personnel Requirements
   a. Overall Troop Structure
   b. Special Forces
   c. Inter-Service Support
   d. US Civilians in Area
   e. Allied Relationships
   f. Handling of Indigenous Leaders
   g. Medical Posture
   h. Military-Civil Government

13. Air Situation

the development of scenarios are telescopic in nature because they establish three levels of detail of the information presented, namely:

1. Set forth the generalized assumptions.
2. Describe briefly the general situation.
3. Describe in greater detail the military encounter.

Examples of these levels follow:

1. A general assumption might be, "No strategic nuclear weapons will be employed by either side."
2. The general situation might state, "The increased tension on the part of both the Blue Allies and the Green Pact countries, due to renewed pressures with regard to the access rights to City XYZ, has brought both sides to an increased readiness posture."

3. The specific situation, which can take into consideration any level of command, might examine a tank company, "The UN Ninth Tank Company occupied defensive positions along the interzonal border with the mission to delay the enemy west of the T River for at least 24 h until D plus 1. At 0400 hours on 10 September 19XX, enemy motorized rifle elements penetrated the border in three places in the company's sector of responsibility and continued west to seize the village of HDG".

This telescopic technique permits an all-purpose approach to scenario preparation. It allows the incorporation of necessary detail at each echelon and can be written to bring out the specific subject matter to be examined, usually in the specific situation section.

Besides the narrow military-type scenario, previously discussed, scenarios can be written to permit examination of a wide range of topical material (see Table 12-1). A broad military/political type scenario would be applicable for use at a relatively high level. For example, the highest military body in the fictional Blue Alliance, the Military Group, could be involved in a scenario which could be based on a series of deliberate military actions. At each phase in the military situation, the military players would simulate political decision making by the presentation of briefings, followed by recommendations and specific requests to the Group. A typical briefing might state, "Pact forces are advancing across the entire front and Blue military forces are conducting delaying actions at predetermined lines." At each defensive line, the military representative could trigger the political play by requesting the release of tactical nuclear weapons in an effort to stop the enemy surge into allied territory. In addition, on the military side, players might be obliged to consider the problem of timely allied reinforcement. Other factors, such as the use of tactical aircraft in conventional attacks against Pact lines of communication and the matter of increased military pressures on the flanks of the enemy, also could be introduced into a scenario of this type. The military/political plot allows for a dynamic interplay between policy makers and senior military commanders and should provide some insight into the type of problems that would likely confront combined leadership in a real wartime situation.

In general, weapon systems analysts, unless they are evaluating a worldwide strategic system, probably will work with a lower level scenario. As an example of this, consider the scenario to support the evaluation of an improved small-arms weapon system. Such an evaluation probably would require a completely instrumented target array, programmed to portray behavior representative of a localized combat situation. Sophisticated layouts, such as this, provide the opportunity to repeat a given scenario for successive groups in various combat modes and to record the results instantly. Instrumented range facilities would allow scenarios to be developed which depict a variety of tactical squad-level firing situations. The situations might include: assault against a fixed defense, attack against a delaying action, and a squad in the defense against attack.

The assault against a fixed defense could provide for three modes of squad action, for example:

1. Employment of a marching force.
2. Attack using the support fires from another squad deployed at a greater range.
3. Use of an automatic weapons squad in support of an attack.

The scenario against the delaying action could include the squad:

1. In a sweep action interdicted by ambushes
2. With the supporting fires of the second rifle squad
3. Action moving with the fires of the automatic weapons squad in support.

The defensive scenario could provide for both day and night operations against an enemy squad. An evaluation program such as this would permit the use of a variety of scenarios, all of which were cast at a low level, but with the capability of realistically scrutinizing squad combat operations in isolation against a fixed enemy target array.

These few examples, the telescoping military small unit action, the political/military plan, and the single weapon system evaluation program suggest the wide scope of situations in which a scenario may be employed.

The schematic diagram in Fig. 12-1 outlines the steps in the formulation of a scenario which, if followed, should assist in the production of reasonable and realistic scenarios. This especially is true if the original guidance is clear and objectives are well defined for the writer. Table 12-2 presents a brief checklist which will be of assistance to the scenario writer, especially if used in conjunction with Fig. 12-1.

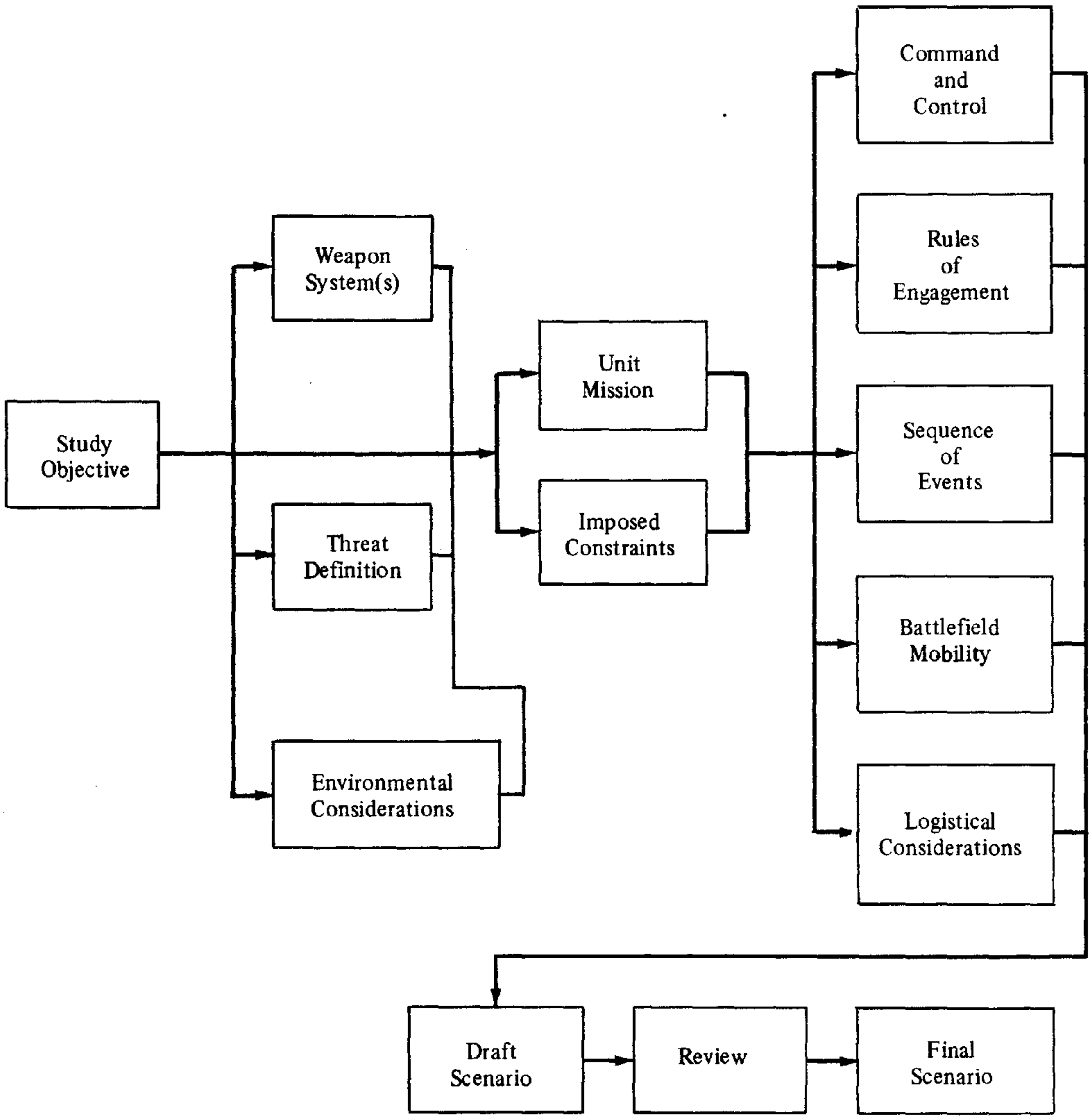


Figure 12-1. Simplified Flow Chart of Scenario Production

TABLE 12-2. A CHECK LIST FOR SCENARIO PRODUCTION

1. *Define the Objective.* Are the results to be hardware, unit, or people oriented?
2. *Weapon Oriented Results.* Is a reasonable mix of weapons being employed? What is the level of command being considered? Is the target selection adequate?
3. *Unit or Individual Oriented Results.* What is the level of command? Is the threat realistic? What are the command and control arrangements?
4. *Identify the Locale.* Be as definitive as the objective of the study demands. Use properly scaled maps.
5. *Determine the Time Frame.* Is it a now situation, in the immediate future, or in the long-range future?
6. *Consider Self-Imposed Constraints.* What are the limitations placed on the command? On the use of the weapon?
7. *Describe the Level of Conflict.* Is it a small unit action against a guerrilla force or a reinforced battalion opposed by a regular army force of like size?
8. *State the Major Assumptions.* Do not assume to the degree that there is no longer any object to the study.
9. *Define the Threat.* Endeavor to use an accepted threat provided by the intelligence community if possible. It may require modification to fit the scenario.
10. *Examine Environmental Constraints.* They may have a significant impact on the entire effort but may not be evident at first review.
11. *State the Rules of Engagement.* At what ranges do weapons fire? When does an individual open fire with his weapon? How does the action commence?
12. *Factors to be Derived.* What data are expected or desired from the evaluation or test? How will they be collected and recorded?
13. *Credible Data.* Use input data that are assembled from credible sources and document the sources.
14. *Demand Realism in All Aspects.* Day and night actions, time sequencing of events, rates of fire and movement, reinforcement, and logistical input all demand a high degree of realism.
15. *Logistical Constraints.* This factor becomes critical when weapons employing high rates of ammunition expenditure are to be evaluated.
16. *Prepare First Draft.* Some study situations require a set of scenarios to be used in different phases of the study.
17. *Obtain Comments.* Circulation of the first draft to other interested staff personnel may bring about improvement of the content.
18. *Consideration of Comments.* Accept or disregard comments in accordance with command guidance or the prerogatives of the writer.
19. *Prepare Final Draft.* This should be a command-approved document acceptable for use throughout the program.
20. *Possible Scenario Alteration.* If data are not being produced as expected, the scenario may require revision.

12-4 COMMON PITFALLS

The realism of the scenario must be developed through a careful synthesis of real-world facts and the adoption of credible combat situations that offers a useful basis for a sound systems analysis. Unfortunately, scenarios which deal with actual countries and their problems can become sensitive, i.e., must be classified, a factor which may limit the use of the scenario in open analytical work. However, regardless of this aspect, it is essential to achieve an objective scenario presentation in order to obtain realism from every point of view. The scenario, when it is fully developed, should reflect a real locale, occupied by the normal population, and complete with such natural and man-made objects as rivers, buildings, and trees. The tendency to enlarge a scenario to a point where it becomes unmanageable or insensitive to key output information should be avoided.

There are numerous other pitfalls in the construction of meaningful scenarios. Perhaps the most common is the failure of the writer to keep the scope of the scenario in line with the precise objectives of the study. Other weaknesses result from imbalance in detail, lack of realism, incompleteness, ignoring certain nonquantifiable conditions, and introduction of bias. Each of these shortcomings is discussed in greater detail in the paragraphs that follow.

12-4.1 UNSURE OBJECTIVE

To construct a workable scenario, the writer must have a clear understanding of the objective. With the specific goal of evaluation in mind, a situation must be developed to fit the overall needs and to bring out the correct and desired data. A scenario which is written to support the evaluation of a new

weapon system should be developed in a different manner than a scenario designed to highlight infantry small unit tactics. Similarly, a scenario to support a cost-effectiveness study should be careful to present total capabilities of the competing systems in a standard fixed scenario. This type of scenario should feature each of the strong points or weaknesses of the systems under nearly similar circumstances. Care also must be exercised in the correct design of the threat, if a threat is applicable to the problem. For example, meaningful conclusions cannot be drawn if small units are played against armies or if the system to be evaluated is played against a duplicate system to the exclusion of other threat factors. Tanks should not be arrayed exclusively against tanks, but against *all* tank destroyers; i.e., antitank missiles, rockets, grenades, close support artillery, and air-to-ground missile systems.

12-4.2 UNNECESSARY DETAIL

In the preparation of a scenario, sufficient detail is required to insure that proper results are achieved, but irrelevant detail is to be avoided. An evaluation of the effectiveness of a particular artillery weapon, for example, requires such detail as the characteristics of the weapon, the density of weapons, the deployment pattern, the type of mission assigned to the pieces, and availability of needed supplies. However, minute details pertaining to the supported forces may not enhance the scenario. If logistical data are to be analyzed, the overall logistical posture needs to be mentioned, but only as a framework for the details of the study. Excessive detail leads to difficulty of control and to the tendency on the part of the players to overcontrol, which brings about unwarranted subjective judgments, thereby possibly biasing the study exercise. In practice, there is a limit to the program size and available computer time for playing a situation game. Also, excessive output data may impede evaluation. At the other end of the spectrum, the free-wheeling open situation with very little attention given to the more significant details of the scenario brings about equally unproductive results. This lack of detail seems to engender a lack of control and the risk of similarly unrealistic results. Balanced degrees of detail and control, therefore, are essential to a well constructed scenario. In addition to reasonable detail, scenarios should incorporate enough meaningful assumptions to make the detail acceptable to the users. The assumptions and details, however, should not be so numerous that they detract from the main objective of the study.

12-4.3 LACK OF REALISM

There are instances in which scenario writers quite unintentionally lose the realism connected with certain aspects of the scenario. For example, realism is lost in a scenario that evaluates tanks only against opposition tanks, especially if both sides are stationary.

Tanks can be evaluated realistically only by a scenario that highlights tank mobility, one of the important capabilities of the weapon. Realism is also lost in a scenario which is so narrow in scope that it does not permit examination of an item in all phases of its potential. A scenario that limits tank action to the penetration of an integrated defensive system and does not continue the action to allow the tank to operate in the exploitation and pursuit phases of the action does a disservice to the overall evaluation and puts the entire systems analysis approach under suspicion. Realism of the scenario suffers also in the acceptance and misuse of maximum or minimum parameter values. This manifests itself as a tendency to maximize system capabilities, rather than to normalize such factors. For example, if the movement rate of a particular vehicle is expressed as "up to x number of miles per hour", this does not mean that the vehicle moves at that speed at all times. Units are often (erroneously) given movement rates which permit them to move 24 h per day without any regard to rest and refueling time.

12-4.4 INCOMPLETE SCENARIO

Another pitfall is the study of a weapon system in isolation. Its capability should be measured not only by its own performance but also in its normal battlefield mix. The assessment of the effectiveness of an artillery piece only in division artillery, for example, without consideration of other artillery support, may lead to distorted results. Similarly, as mentioned earlier, a tank does not operate in isolation against a single enemy tank, but it operates as part of a combined-arms team functioning against the opposition's combined-arms team. A worthwhile scenario will take this into consideration and reflect the effect of the total system, and not only depict individual weapon duels. In World War I, for example, tanks completely overpowered enemy machine gun positions, but this did not lead to the elimination of the machine gun as an infantry weapon. Admittedly, the complexity of the total weapons approach taxes analytic abilities but the systems analyst must face up to the challenge and consider a multiplicity of competing weapons on the modern battlefield, especially through simulations or war games. More advanced scenarios also will take into account not only weapon systems but also complicated electronic devices which, if not countered, can render some weapons inoperative.

12-4.5 NONQUANTIFIABLE HUMAN ASPECTS

Competent systems analysts realize that there are some factors which do not lend themselves to numerical evaluation. These factors, nevertheless, can be vital to the evaluation problem. At times, their omission stems from a lack or awareness of all of the ramifications of military conflict, and in other instances their absence may be due to institutional bias. However, even though factors—such as leadership, morale, training, esprit de corps, and the value of human life—cannot be quantified, they do play a role in the results and can be judged in a subjective manner by qualified observers. The scenario should include these factors, at least in the form of narrative description. Even though these factors cannot be put into the computer, their use in the scenario can serve as a vehicle to permit consideration of their effects. Just how these nonquantifiable factors affect the evaluation is difficult to assess. The introduction of nonquantifiable factors adds to the complexity of the problem in a field where package solutions are popular, but to ignore them will lead to the presentation of only half-truths and may cause a decision to be both incorrect and costly in lives and dollars.

12-4.6 NONQUANTIFIABLE PARAMETERS

In addition to human characteristics, certain military or conditional factors may be difficult or impossible to quantify. If such is the case, the analyst must resort to some ordinal or nominal ranking scheme to structure the material for analysis and presentation in the study. For example, it may be necessary to describe cross-country mobility using such ordinal descriptions as "excellent", "good", "fair", and "poor" in place of numerical values.

12-4.7 BIAS

In preparing a scenario, the analyst must constantly strive to eliminate bias or he will influence a favored outcome for a study by biasing the input scenario. Should this happen, the analyst is not uncovering a truth or adding an insight; he is merely reinforcing his own prejudices.

12-5 DOCUMENTATION

It is most important to the systems analyst, and especially to the writer of the scenario, that the credibility of the scenario be supported by thorough documentation. A concentrated effort should be made to collect and develop up-to-date, relevant, primary sources. Scenario credibility, even though

established by use of recognized data sources such as Army regulations, field manuals, and technical manuals, is subject to criticism, particularly when the writer is dealing in future events. The details of a 1985 scenario would be extremely difficult to formulate and usually have their foundations in assumptions. Who is to say what will be the outcome of future events which may affect the study? Any assumptions concerning the future are susceptible to criticism because they generally lack factual data to support them. When reasonable assumptions are used in the scenario, they must be logically arrived at and adequately documented so that their use will not jeopardize the usefulness of the scenario as an analytical device for systems analysis.

12-6 TRADOC STANDARD SCENARIOS FOR COMBAT DEVELOPMENTS

Often the comparison of results of systems analyses is hampered because different scenarios were used for each analysis. To preclude this situation, the US Army Training and Doctrine Command (TRADOC) established the TRADOC standard scenarios for combat developments. These scenarios provide the combat developments community with a common framework of selected situations and real world conditions in which specified US forces are deployed. They permit a common visualization and integration of effort by the combat developments community in current and future efforts to:

1. Assess the capabilities of current forces under specific conditions.
2. Identify recommended improvements to doctrine, organization, and materiel of current Army forces.
3. Facilitate a rapid assessment of proposed concepts and changes to Army forces.

The TRADOC Standard Scenarios form the basis of the Scenario Oriented Recurring Evaluation Systems (SCORES). SCORES provides the combat development community with an evaluation technique and a framework for identifying required improvements and addressing questions concerning organization, doctrine, training, and materiel. The major steps in this system include scenario development, phase I (force) evaluation, and phase II (detailed) evaluation. A key ingredient of the evaluations is a computer assisted manual war game.

Additional information concerning the TRADOC standard scenarios and SCORES may be obtained from Refs. 1 and 2.

REFERENCES

1. TRADOC Regulation No. 71-4, *TRADOC Standard Scenarios for Combat Developments*.
2. TRADOC Regulation No. 71-5, *Scenario Oriented Recurring Evaluation System (SCORES)*.

CHAPTER 13

WEAPON DELIVERY ERROR CHARACTERISTICS AND DISTRIBUTIONS

Described herein are delivery error distributions for the impacts of rounds fired from a weapon which are commonly used in evaluations, along with the concepts of probable error (PE), circular probable error (CEP), and some preliminary coverage of the probability of hitting. The problem of estimation of parameters of delivery error distributions also is considered briefly.

13-0 LIST OF SYMBOLS

CEP = circular probable error = radius of circle about the center of impact (C of I) containing 50% of the shots or impacts = 1.1774σ

$E(r)$ = mean value of the radial error

ES = extreme spread (see Eq. 13-17)

FOM = figure of merit = $(R_x + R_y)/2$

$f(x)$ = probability density function of the random variable x

$f(y)$ = probability density function of the random variable y

MR = mean radius (see Eq. 13-16)

n = number of rounds fired

PE = probable error = deviation about the mean of a univariate normal distribution which includes 50% of the population = 0.6745σ

P_H = probability of a hit

R = radius of target

RSD = radial standard deviation = $\sqrt{s_x^2 + s_y^2}$

R_x = range (maximum dispersion) in x-direction

R_y = range (maximum dispersion) in y-direction

r = radial deviation

S_x^2 = $\Sigma(x_i - \bar{x})^2/(n-1)$ = sample variance of the x's based on $(n-1)$ degrees of freedom

S_y^2 = $\Sigma(y_i - \bar{y})^2/(n-1)$ = sample variance of the y's based on $(n-1)$ degrees of freedom

s_x = $\sqrt{\Sigma(x_i - \bar{x})^2/n}$ = sample standard deviation of the x's

s_y = $\sqrt{\Sigma(y_i - \bar{y})^2/n}$ = sample standard deviation of the y's

s_x^2 = $\Sigma(x_i - \bar{x})^2/n$ = sample variance of the x's

s_y^2 = $\Sigma(y_i - \bar{y})^2/n$ = sample variance of the y's

(x,y) = coordinates of the impact of a round

$\bar{x}$ = sample mean of the x's

$\bar{y}$ = sample mean of the y's

z = a third coordinate or variate

μ = x-coordinate of the true center of impact

ν = y-coordinate of the true center of impact

σ = round-to-round standard deviation used when $\sigma_x = \sigma_y = \sigma$

σ_x = population standard deviation of x

σ_y = population standard deviation of y

χ^2 = chi-square variate

13-1 GENERAL

With our discussions of what systems analysis is, how it is used, the role of the decision maker, the role of the systems analyst, the types of conflict, and the scenario; we are now in a position to develop models for the evaluation of weapons and weapon systems. The reader will appreciate that our problem in this connection is that of describing realistically the characteristics of weapons in probability or statistical terms and evaluating the effectiveness of the weapon warheads employed against various types of targets. Of course, evaluations in terms of costs or other measures of effort or resources are also part of the problem. Our aim is to quantify the performance of weapons against targets of different types, sizes, shapes, and vulnerability as they appear on the battlefield. The evaluations should be as realistic as possible in order that recommendations to the decision maker will be sufficiently accurate and military appropriations properly spent for development and production.

It is to be expected that some of the methodology of evaluation will be quite general and apply properly to all or a large number of weapon systems analyses, while on the other hand certain of the weapons used in combat will require their own particular analyses for accurate evaluation. We will start with and develop the more general methods first, and then later cover the particular ones which apply specifically to the weapons requiring such analyses.

In order that we be able to analyze the effectiveness of a weapon system, we must select the more appropriate analytical models and substitute therein weapon and target characteristics or parameters. Such characteristics or parameters will include impact delivery errors of weapons, target sizes, shapes, and characteristics and later the vulnerability of targets to various types of attack. In view of the overall importance of delivery error distributions and especially the bivariate normal probability distribution, we will concentrate on this first, going into the problems of estimation of parameters—the *CEP* and the *PE*. Then we will cover weapon aiming errors and probability of hitting targets of various sizes and shapes for both individual and multiple rounds in later, more appropriate chapters. The reader not familiar with many of the elementary topics in probability and statistics should consult any good standard text on the subject as a required introduction.

13-2 WEAPON DELIVERY ERROR DISTRIBUTIONS

In the firing of rifles or antitank weapons at vertical targets, or guns or missiles for ground impact, there results a two-dimensional pattern of shots or impact points which exhibits an amount of scatter depending on the round-to-round aiming error and the ordinary "ballistic" dispersion. The two-dimensional pattern of shots gives rise to various measures of dispersion which are used by analysts and ballisticians to summarize the "accuracy" of the pattern of shots. The measures of dispersion usually employed by ballisticians consist of the (sample) standard deviation in each direction, the extreme horizontal dispersion, the extreme vertical dispersion, the figure of merit, the mean horizontal deviation, the mean vertical deviation, the mean radius, the extreme spread, the radial standard deviation, the diameter of the covering circle, and the "diagonal" of the pattern. Thus, there exist many different measures or ways of measuring the scatter of shot impacts. A point of considerable importance we record here is that the mean or expected values of all of the various measures of dispersion depend on the sample size or number of rounds, some depending very markedly on the sample size. All of these measures of dispersion require statistical analysis, for they are really random variables from one firing group to another, especially since the individual impacts themselves are randomly scattered about the target area.

We should keep in mind that a group of several shots fired from a weapon represents a sample of rounds from a lot of ammunition, or a "population"—or a "universe" as described in statistical

terms—so that the scatter pattern will vary from one group to another. It is this random variation which we will have to model in order to estimate probabilities of hitting targets or "hit probabilities", as often called. We will first discuss measures of dispersion of the sample groups and then enlarge our thinking to the parameters of the lot or population sampled.

13-2.1 EXAMPLE OF DISPERSION PATTERN

Now consider, for example, a group of 10 rounds (bullets) fired from a rifle at a vertical target. The horizontal and vertical (or x and y) coordinates of the 10 impacts, in the order the bullets were fired from a rifle are: (1,2), (−3,−1), (1,−1), (2,4), (3,0), (4,3), (−1,3), (2,−2), (−2,1) and (3,1).* The mean of the x's or horizontal locations of impacts is $\bar{x} = 1$ and the mean of the y's or vertical positions is $\bar{y} = 1$. Thus, the sample (or observed) center of impact (C of I) often called mean point of impact (MPI) of the 10 rounds is located at (1,1). Note that the observed or sample C of I is at the radial distance of $\sqrt{1 + 1} = \sqrt{2} = 1.414$ from the origin, 0, or the aim point.

If another group of 10 rounds were fired at the target, the pattern of impact points would be somewhat different and the new sample or group C of I of the rounds would be located at a point different from the previous C of I, i.e., (1,1). Thus, the pattern of shots on the target would vary from group to group in a random manner. Moreover the observed C of I and all of the measures of dispersion previously mentioned also would vary randomly due to the random round-to-round ballistic dispersion and the small sample size.

The round-to-round ballistic dispersion and the movement of the center of impact during the firing of a group of rounds will affect the probability of hitting a target, which is of great importance in evaluating the effectiveness of a weapon. As far as is known, the round-to-round ballistic dispersion remains relatively stable although the true C of I of the group of rounds may vary in some unpredictable manner from one group of shots to another, or occasion to occasion, or even from one round to the next, especially if some re-aiming is necessary. For a single round, the total aiming error is easily handled. However, for multiple rounds fired from a weapon, or rounds from several weapons, it is the shift in C of I that causes complex analytical problems as we will see in the sequel. Thus, a rather complete analysis of the dispersion patterns of weapons is one of the key factors to be considered. For the case of rifles fired at vertical targets, the pattern of shots or impacts is nearly circular; whereas for the case of artillery weapons fired for ground impact, the pattern of shots on the ground elongates in the direction of range, so that the dispersion in the range direction may be considerably greater than that in deflection. The pattern of shots or impacts have an effect on probability of hitting a target and probability of target damage. These two cases often are referred to as the "circular" and "noncircular" cases, respectively.

In discussing measures of the dispersion characteristics of patterns of shots we will deal with distances on the target, although the distances on the target could be converted to angles in mils if we know the range to the target. Indeed, for small arms fire the dispersion in mils is nearly constant as a function of the range to the target, and angular data therefore would represent a more general treatment. Since the conversion from distances on the target to angular data is straightforward, however, and our illustrations are served better by use of bullet impact positions on the target; we will approach the subject matter by using distances.

*Statistically speaking, these points may be considered to be a random sample of size 10 from a bivariate normal population.

We will start with locations and deviations of the projections of impact points on the horizontal and vertical axes, since the (univariate) measures of scatter for such point projections are of basic importance. Although the standard errors or standard deviations we will discuss first are somewhat complex computationally, they nevertheless are very efficient and of a fundamental character.

Precision of the shots, as used here, will refer to the dispersion of the bullets about their own mean or C of I, whereas accuracy includes not only the round-to-round precision but also the closeness of the mean, of the C of I, to the aim point on the target.

13-2.2 ESTIMATES OF DISPERSION

If the sum of the squares of the deviations of the x-coordinates (i.e., projections of points on a horizontal axis) from their mean is divided by $n = 10$, the number of points, we obtain the sample "variance" of the horizontal or x-coordinates. Thus, for the 10 impact points of par. 13-2.1 where $\bar{x} = 1$, the sample variance s_x^2 is given by

$$s_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{10}[(1-1)^2 + (-3-1)^2 + (1-1)^2 + \cdots + (3-1)^2] = 4.8 \,. \qquad (13\text{-}1)$$

This value may best be calculated generally on a computer without the accumulation of rounding errors by means of the formula

$$s_x^2 = \frac{1}{n^2}A_{xx} = \frac{1}{n^2}[n\Sigma x^2 - (\Sigma x)^2] \qquad (13\text{-}2)$$

where

$A_{xx} = n\Sigma x^2 - (\Sigma x)^2$.

Note that the sum and sum of squares may be computed simultaneously on a calculator. That is to say

$$s_x^2 = \frac{1}{100}\{10[1^2 + (-3)^2 + 1^2 + 2^2 + \cdots + 3^2] - [1 - 3 + 1 + 2 + \cdots + 3]^2\}$$

$$= \frac{1}{100}[10(58) - (10)^2] = 480/100 = 4.8 \,. \qquad (13\text{-}3)$$

The square root of the variance is known as the standard deviation s_x. Thus

$$s_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} = \sqrt{4.8} = 2.19 \,. \qquad (13\text{-}4)$$

In a like manner, we find the sample variance s_y^2 and standard deviation s_y of the y's or the projections of the impact points on the vertical axis. Thus, we have

$$s_y^2 = \frac{1}{n^2} A_{yy} = \frac{1}{n^2} [n\Sigma y^2 - (\Sigma y)^2] \tag{13-5}$$

$$= \frac{1}{100} \{10[2^2 + (-1)^2 + (-1)^2 + \cdots + 1^2] - [2 - 1 - 1 + \cdots + 1]^2\}$$

$$= 360/100 = 3.6$$

and

$$s_y = \sqrt{3.6} = 1.90 \ .$$

For this sample of 10 rounds, we get $s_x = 2.19$ and $s_y = 1.90$. For another firing of 10 rounds we might get, e.g., $s_x = 3.03$ and $s_y = 2.10$, etc., so that the sample standard errors s_x and s_y vary in a random manner from group to group.

Had there been a very large number of shots or impacts on the target, then the sample standard deviations, s_x and s_y, would approach their large sample or "true" values which we will call σ_x and σ_y, respectively. The true standard deviations σ_x and σ_y for many, many thousands of shots are called or are known as the *population standard deviations* of the randomly varying x- and y-projections, as compared to the small sample or group values, s_x and s_y. The population values represent the true round-to-round standard deviations in the horizontal and vertical directions for huge lots of ammunition and the weapon characteristics, for example. For samples of 10 rounds or for other small sample sizes, we find, as already mentioned, that the sample standard deviations, s_x and s_y, vary from sample to sample of rounds fired at the target. Thus, for small samples or small numbers of shots, s_x varies in a random manner about the true population standard deviation σ_x, and the amount of variation depends on the sample size or number of rounds n. The quantity σ_x, as seen from its definition, is a measure of the round-to-round variation of *individual* impact points in the horizontal direction. In an analogous manner to the description just given for an individual x, we could express the variation of s_x from sample to sample by means of the standard deviation of a large number of such values themselves for a fixed sample size. On the average, s_x for a given small sample size n does not exactly equal the population standard deviation σ_x. That is to say, for small samples s_x is a *biased* estimate of the population σ_x. This bias is due to the fact that the computation of each s_x involves deviations about the sample mean and not the (unknown) true population mean, so that on the average s_x is somewhat less than σ_x. The amount of bias depends on the sample size n, but approaches zero for large n (Ref. 1).

It is known from statistical theory that the sample variance computed as

$$S_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = A_{xx}/[n(n-1)], \text{ (Note that } S_x^2 = \left(\frac{n}{n-1}\right) s_x^2) \tag{13-6}$$

which is based on $(n - 1)$ "degrees of freedom" (df)—one degree of freedom being used in the calculation of the sample mean $\bar{x}$ as an estimate of the population mean—is on the average equal to the population variance σ_x^2. The sample variance computed from $s_x^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$, based on the whole sample size n, is on the average equal to $(n - 1)\sigma_x^2/n$, and is thus biased by the amount $-\sigma_x^2/n$, so that this correction is needed. It might seem curious to the reader, but it is a fact that S_x—based on using the square root in Eq. 13-6—is not an unbiased estimate of σ_x for small sample sizes (see Ref. 1). Similar statements could be made and apply, of course, to s_y and S_y.

The sample means and standard deviations describe only the location of the center of impact and the "internal" or ballistic dispersion of shots. These two quantities do not, however, describe or measure the shape characteristics of the overall distributions of shot impacts or ammunition lots we sample. Thus, in estimating probabilities of hitting, we will have to make some assumptions of reasonable practical value about the population sampled.

To summarize a bit, the firing of groups of rounds will result in a random scatter of shots on the target area, and the sample or group means and the sample standard deviations will vary from group to group. For very large numbers of shots, the sample center of impact $(\bar{x},\bar{y})$ will converge to the true or unknown C of I which we might refer to in location as (μ,ν) to distinguish it from small sample values; and the sample or group standard deviations, s_x and s_y, will converge to the population or lot true (but unknown) standard deviations or measures of dispersion, σ_x and σ_y, respectively. For a fixed center of impact, the quantities σ_x and σ_y will serve as measures of ballistic dispersion, whereas if there is a movement in the C of I from round to round, then the total dispersion of the shots will be enlarged by the properly measured amount of the variation due to the changing center of impact (μ,ν). Appropriate treatment of the previous two components of variation will be very important in the process of evaluating weapons, but for the present we should approach the subject by first assuming that the C of I is fixed or stationary.

Having seen that the sample groups of rounds indeed come from a much larger category or lot of such rounds, we must concentrate on the nature of the population or universe from which such groups might possibly come. In fact, the characteristics of shots from the overall lot or population, or the shape of that population will indeed have an effect on probability of hitting. In ballistics work, it is widely assumed, and also verified sufficiently, that the distribution of rounds is approximately normal or Gaussian in character. Thus, the probability density function (pdf) of rounds in the x-direction may be described by

$$f(x) = [1/(\sqrt{2\pi}\ \sigma_x)]\exp[-(x-\mu)^2/(2\sigma_x^2)] \tag{13-7}$$

and that of the dispersion in the y-direction by

$$f(y) = [1/(\sqrt{2\pi}\ \sigma_y)]\exp[-(y-\nu)^2/(2\sigma_y^2)] \tag{13-8}$$

where (μ,ν) represents the coordinates of the true C of I, and σ_x and σ_y are the population standard deviations in the x and y, or range and deflection directions, respectively. The normal distributions Eqs. 13-7 and 13-8 are the symmetric, bell-shaped distributions, depending on only two parameters—the mean and standard deviation. If the true C of I is located at the origin or center of the target, i.e., there is no aiming error, then $\mu = \nu = 0$ and the pdf depends only on the standard deviations (σ_x and σ_y).

The assumption of normal or Gaussian distributions in weapon systems analysis is a common practice because the model applies well and proof of the lack of normality would require very large numbers of shots. Accordingly, the effect of a more realistic assumption on evaluations would be rather slight. We will, of course, make other more proper assumptions wherever necessary.

The pdf's, Eqs. 13-7 and 13-8, represent univariate or one-directional distributions, whereas the dispersion we observe in firing weapons is of a bivariate character. If x and y are independent in the statistical sense, and there is no aiming error ($\mu = \nu = 0$), the appropriate bivariate normal density function would be

$$f(x,y) = f(x)f(y) = [1/(2\pi\sigma_x\sigma_y)]\exp\{-[x^2/(2\sigma_x^2) + y^2/(2\sigma_y^2)]\}\ . \tag{13-9}$$

If $\sigma_x \neq \sigma_y$, we call this the "noncircular" case; and if the dispersions in the two directions are equal, i.e., if $\sigma_x = \sigma_y = \sigma$, we speak of the "circular" bivariate normal distribution.

The reader will appreciate that estimation of the parameters—μ, ν, σ_x, and σ_y (or σ)—involves statistical considerations, not necessarily an essential part of our account here of weapon systems analysis, although some considerations of estimation are given in par. 13-4.

13-3 THE CIRCULAR PROBABLE ERROR (*CEP*) AND THE PROBABLE ERROR (*PE*)

A measure of dispersion widely used to describe weapon delivery accuracy is the *CEP*. As shown in Fig. 13-1 the *CEP* is defined as the radius of the circle, usually about the true C of I of the rounds, but also sometimes about the point of aim, which includes one-half or 50% of the shots, considering a very large number of rounds fired onto the target area under stable firing conditions.

The *CEP* has been developed primarily for the "circular" normal distribution and is easily derived. Given the circular normal density function

$$f(x,y) = [1/(2\pi\sigma^2)]\exp[-(x^2 + y^2)/(2\sigma^2)] \tag{13-10}$$

with C of I at the origin, we integrate this over a circular target with center at the origin and equate the result to one-half, i.e.,

$$\iint_{x^2 + y^2 \leq R_{0.50}^2} f(x,y)dxdy = 0.5 \tag{13-11}$$

where the *CEP* is equal to the "fifty percent" radius $R_{0.50}$. By making the transformation of variables

$$x = r\cos\theta$$

$$y = r\sin\theta \quad , \quad 0 \leq \theta \leq 2\pi$$

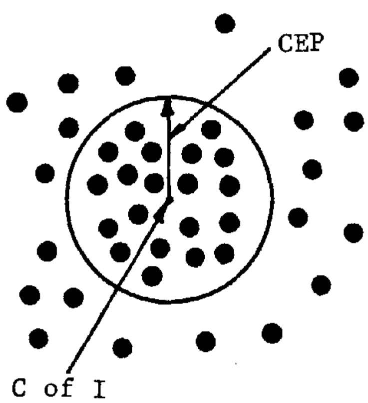


Figure 13-1. Circular Probable Error

we obtain that Eq. 13-11 is equal to

$$[1/(2\pi\sigma^2)]\int_0^{R_{0.50}}\int_0^{2\pi} d\theta \exp[-r^2/(2\sigma^2)]rdr = 1 - \exp[-R_{0.50}^2/(2\sigma^2)] = 0.5$$

or $\exp[-R_{0.50}^2/(2\sigma^2)] = 0.5$, so that the desired radius R, i.e., $R_{0.50}$, or the *CEP*, is

$$CEP = R_{0.50} = \sqrt{2\ln 2}\ \sigma = 1.1774\sigma\ . \tag{13-12}$$

Hence, the *CEP* is 1.1774 times the round-to-round standard deviation in one direction. Note that the standard deviation σ is really a univariate or one-directional measure of dispersion, whereas the *CEP*, which depends on the coefficient of 1.1774, is often used and understood as a measure of bivariate or two-dimensional scatter of weapon shots, even though it depends on the same standard deviation in either direction.

We are reminded that the standard deviation σ is a very large sample or population value, obtained from extensive firings or experience with a given weapon system. An estimate of σ based on limited firings, therefore, will be subject to sampling error.

There is another interpretation of the *CEP* in terms of probability of hitting. Indeed, if we have a weapon which has a delivery error dispersion equal to the standard deviation σ, and if that weapon is fired at a circular target of radius $R = 1.1774\sigma$ whose center is located at the C of I of the rounds, then the probability of hitting that particular target is 0.5, i.e., it is expected that half the rounds would hit and half miss the target, as the *CEP* is defined.

If all of the shots are projected onto the x-axis (or y-axis), the interval about both sides of the mean which includes 50% of the shots is called the *PE*. As shown in Fig. 13-2, the interval from the (true) mean minus the *PE* to the mean plus the *PE* contains 50% of the shots in the x- (or in the y-) direction, considering a very large number of shots. The *PE* is 0.6745 σ, i.e., $PE = 0.6745\ \sigma$. The *PE* is a one-directional or univariate measure of dispersion, and the one commonly used for range and deflection precision in Firing Tables.

For the two-dimensional case and for unequal standard deviations ($\sigma_x \neq \sigma_y$) in the x- and y-directions, the *CEP* is still often used. A suitable approximation for this case is discussed in Ref. 1 and Chapter 14.

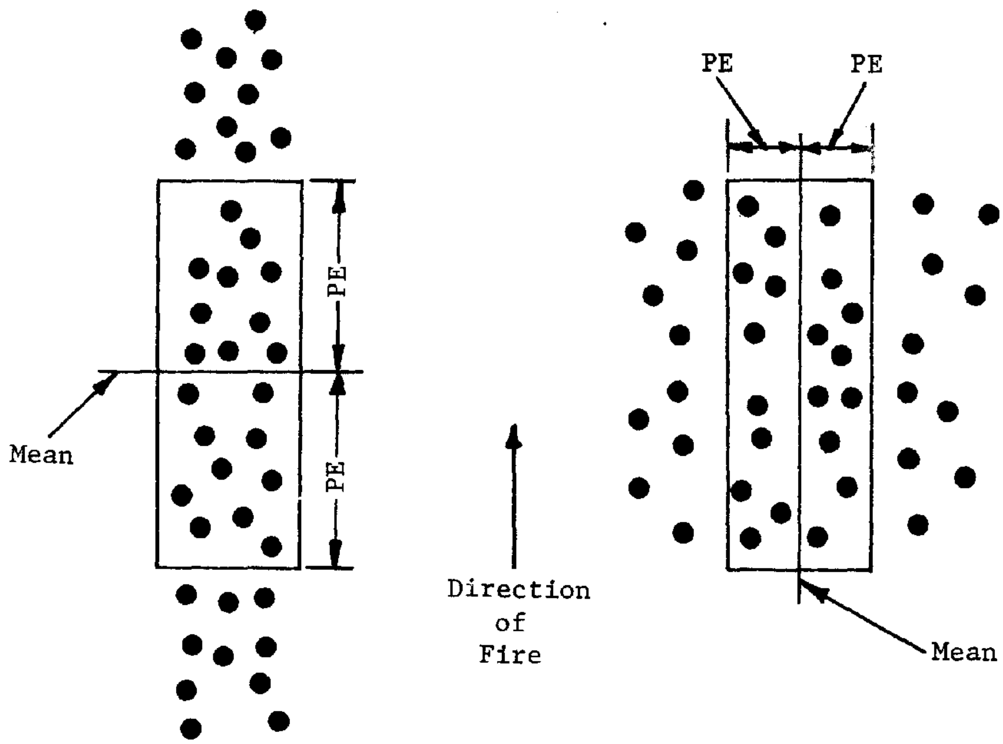
13-4 THE RADIAL ERROR

For the circular normal distribution of shots with zero means, Eq. 13-10, the quantity

$$r = \sqrt{x^2 + y^2} \tag{13-13}$$

is called the radial error. The radial error could be defined for nonzero means, or about an arbitrary target reference point, but with some complications as discussed later. (See, for example, Ref. 1 or Ref. 2).

The radial error is a measure of scatter or dispersion for an individual round or shot, and its probability distribution is easily obtained. In fact, since x and y each follow a univariate Gaussian or normal distribution with zero mean and variance σ^2, i.e., $\mathcal{N}(0,\sigma^2)$, the x^2/σ^2 and y^2/σ^2 each follow the well-known chi-square (χ^2) distribution with one degree of freedom, so that $(x^2 + y^2)/\sigma^2$ is distributed in



(A) Range Probable Error (B) Deflection Probable Error

Figure 13-2. Probable Error

probability as chi-square with two degrees of freedom. Hence, r/σ is the corresponding chi-variate for 2 df.

The mean value of the radial error r is $E(r) = \sqrt{\pi/2}\sigma = 1.2533\sigma$, and its standard deviation is $(2 - \pi/2)^{1/2}\sigma = 0.6551\sigma$ which gives a measure of its dispersion.

The radial error is defined also for three dimensional shot patterns. In this case

$$r = (x^2 + y^2 + z^2)^{1/2} \tag{13-14}$$

and its mean value is $2\sqrt{2/\pi}\sigma = 1.5958\sigma$ and its standard deviation is $(3 - 8/\pi)^{1/2}\sigma = 0.6734\sigma$.

The median values of the radial error for one, two, and three dimensions turn out to be, respectively, the probable error ($PE = 0.6745\sigma$), the circular probable error ($CEP = 1.1774\sigma$), and the spherical probable error ($SEP = 1.5382\sigma$), respectively. (See, for example, Ref. 3).

13-5 ESTIMATION OF PARAMETERS

From the preceding discussion, we begin to see that the parameters σ_x, σ_y (or σ) for the measures of scatter, and the location of the true C of I (μ,ν), are of fundamental importance in evaluating probabilities of hitting or weapon performance. For the noncircular cases of Eqs. 13-7, 13-8, and 13-9 efficient estimates of μ and ν, which locate the C of I, are of course $\bar{x}$ and $\bar{y}$, respectively; and moreover s_x or S_x, corrected for bias, as discussed in Ref. 1, gives an efficient estimate of σ_x. Similar considerations apply to σ_y.

For the circular case $\sigma_x = \sigma_y = \sigma$, there is an advantage in using the joint measures of dispersion for the two-directions (Ref. 1). Some of the principal bivariate measures of scatter of shots are as follows:

1. The Radial Standard Deviation (*RSD*)

$$RSD = \sqrt{s_x^2 + s_y^2} \; . \qquad (13\text{-}15)$$

The *RSD* for the group of 10 shots of par. 13-2.1 is 2.90.

2. The Mean Radius (*MR*)

$$MR = \frac{1}{n}\sum_{i=1}^{n}\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2} \qquad (13\text{-}16)$$

which is simply the average of the radial distances from the observed C of I. For the group of 10 shots of par. 13-2.1, the *MR* is 2.75.

3. The Extreme Spread (*ES*)

$$ES = \max \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad , \quad i \neq j = 1, 2, \ldots, n \; . \qquad (13\text{-}17)$$

Thus, the extreme spread is the largest of the distances between all *pairs of points* or impacts on the target plane, and is hence a rather simple measure of bivariate dispersion.

The extreme spread for the group of 10 shots, where the candidate pairs of points are $(-3,-1)$ and $(4,-3)$, is

$$ES = \sqrt{(-3 - 4)^2 + [-1 - (-3)]^2} = 8.06 \; .$$

4. The Figure of Merit (*FOM*)

$$FOM = (R_x + R_y)/2 \qquad (13\text{-}18)$$

where R_x and R_y are the ranges or maximum distances between points projected onto the x-and-y axes, respectively.

The *FOM* for the 10 shots is

$$FOM = \frac{[4 - (-3)] + [4 - (-2)]}{2} = 6.5 \; .$$

Such measures of bivariate dispersion and their efficiency are thoroughly covered in Ref. 1. Moranda (Ref. 4) also gives some comparisons. The *RSD* is discussed in Ref. 5, and Taylor and Grubbs (Ref. 6) give approximate distributions for the extreme spread. As previously indicated, each measure is a random variable from one group of shots to another. Also, the mean values of the measures of scatter depend on the sample size and generally are biased, so that to estimate σ properly corrections for bias are needed. For example, referring to the 10-shot group of par. 13-2.1, the estimated value of σ using the *RSD* is $2.90/1.323 = 2.19$ (see Table 4 of Ref. 1 for the factor 1.323), and the estimated σ using the *ES* is $8.06/3.805 = 2.12$ (see Table 6 of Ref. 1).

There exists some very significant differences in efficiency of estimation also. For example, the *RSD* for a group of 11 shots is slightly more efficient than the *ES* for a group of 16 shots. The reader should consult Ref. 1 for a rather complete discussion of such statistical properties.

13-6 PRELIMINARY COMMENT ON PROBABILITY OF HITTING A CIRCULAR TARGET (SINGLE ROUND)

If we return to the derivation of the *CEP* in par. 13-3, it is easy to see that the chance P_H of hitting a circular target of radius R centered at the true C of I, or aim point of the rounds, for the case $\sigma_x = \sigma_y = \sigma$ is simply

$$P_H = [1/(2\pi\sigma^2)] \int_0^R \int_0^{2\pi} \exp[-r^2/(2\sigma^2)]\, r dr d\theta = 1 - \exp[-R^2/(2\sigma^2)] \, . \tag{13-19}$$

For example, suppose the underlying σ is 2 and we desire the chance of hitting a circular target of radius 3 for these conditions. Then

$$P_H = 1 - \exp\{-3^2/[2(2)^2]\} = 0.68.$$

The reader should note for later reference and discussion that if x and y are normally distributed with zero means and variance equal to σ^2, then the chance of hitting is also the probability that $x^2 + y^2 \leq R^2$. That is to say, the chance of hitting may be found from the probability distribution of the quadratic form, $x^2 + y^2$, in normal random variables.

13-7 ADDITIONAL COMMENTS

The reader will note that we have been careful to distinguish between the ordinary round-to-round ballistic dispersion and the movement of the true C of I (or MPI) due to changes in aiming error, wind, or other meteorological conditions. The dispersion among rounds for a stable C of I often has been referred to as the "precision" of shots, or "internal" dispersion, and the standard deviations in the two directions will be designated by σ_x and σ_y, respectively. This is to be distinguished clearly from the measure of dispersion describing the movement of the center of impact from one occasion of firing to another, or even from round to round if such changes in aim and position of the C of I are involved. These represent two very different components of variance, or variation, and there is often so much confusion concerning them that any resulting analysis could be faulty.

In order to distinguish the round-to-round ballistic dispersion expressed in terms of σ_x and σ_y from the dispersion of the center of impact, we will use the components of variation designated by σ_μ and σ_ν for the C of I. These components of variation may be estimated by use of analysis of variance techniques applied to firing data. These sigmas, describing the movement of the true C of I in each of the two directions are also very large sample values, or population "parameters", of course. In order to clarify the concept somewhat, the true C of I on the ith occasion may be located at (μ_i,ν_i), and at (μ_j,ν_j) for the jth firing occasion. Thus, σ_μ represents the standard deviation for the varying μ's, and σ_ν the standard deviation of the ν's. Also, the movement of the point (μ_i,ν_i) may be described in terms of a "*CEP*" for the centers of impacts. For preliminary thinking, σ_x and σ_y may be approximately one-quarter to several mils, whereas σ_μ and σ_ν may be several or many times σ_x and σ_y in combat situations.

To compute probabilities of hitting, the components of variance (true parameters) must be determined accurately and their values, which do not depend on sample size or number of rounds, substituted into the appropriate model.

We have gone into perhaps a considerable amount of detail in this chapter concerning the more elementary or basic concepts of weapon firing, so that hopefully the new weapon systems analyst will gain some proper appreciation for visualizing what happens stochastically in the process of firing weapons.

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CHAPTER 14

PROBABILITY OF HITTING FOR SINGLE ROUNDS (SINGLE SHOT HIT PROBABILITIES)

A description is given of the methods of calculating the chances of hitting targets of different shapes for the case of single or individual rounds. The methodology includes both exact and approximate techniques for detemining hit probabilities for the cases of the centered aim point and offset aim point.

14-0 LIST OF SYMBOLS

A = area

A_i = deviation in standard units

A_v = vulnerable area of a target

a = semilength of a rectangular target

b = semiwidth of a rectangular target

b_i = ith component of a constant

CEP = circular probable error

c = semiaxis of an ellipse

c_i = ith component of constant c

D = radial offset of the C of I

d = semiaxis of an ellipse

d = radius of a circle Germond's notation (see Eq. 14-14)

I = exact value of an integral

$k = \sqrt{A/2\pi}$ = constant depending on an area A

m = mean or expected value of Q

n = number of rounds

$P(r,\rho)$ = chance that a sample point will fall on or within a circle of radius r with the C of I offset from the origin by a distance ρ

$p(a)$ = Polya-Williams approximation

$p(h)$ = hit probability

p_k = kill probability

$p(k|h)$ = conditional probability that a hit is a kill

$p(R,r)$ = circular coverage function (Germond) where an integral is taken over an offset circle of radius R with center at (a,b), i.e., $(x-a)^2 + (y-b)^2 = R^2$ and here $r^2 = a^2 + b^2$

$Q = Q(x,y)$ = quadratic form in two random variables x and y

$q(a,b)$ = elliptic coverage function

$q(c/\sigma,d/\sigma)$ = difference of two circular coverage functions

R = radius of circular target

R_D = damage radius

r = radius of a circle

r_d = damage radius

$S(R,r)$ = the integral of $2\pi r p(R,r)$ = expected overlap of two circles

t = a(unit) standard normal variable

u = x/σ_x = deviation in standard units
u_i = unit standard normal variate for ith component
V_i = σ_i^2/c_i^2
v = y/σ_y = deviation in standard units
v = variance of Q
x = random variable describing a delivery variation
y = random variable describing a delivery variation
(μ,ν) = coordinates of center of impact (C of I)
ρ = offset distance of the C of I from the origin
σ = one directional round-to-round delivery standard deviation for the case $\sigma_x = \sigma_y = \sigma$
σ_x = round-to-round standard deviation in x-direction
σ_y = round-to-round standard deviation in y-direction
σ_T^2 = $\sigma_x^2 + \sigma_y^2$
$\chi^2(\nu)$ = chi-square for ν degrees of freedom (a statistical variate)

14-1 INTRODUCTION

The probability of killing or incapacitating a target for many problems in the analyses of weapon systems may be expressed as the product of the chance of hitting a target and the conditional probability that a hit will result in a "kill" or incapacitation. Thus,

$$\begin{aligned} p_k = Pr(\text{kill}) &= p(\text{hit})\cdot p(\text{kill if hit}) \\ &= p(h)\cdot p(k\,|\,h)\,. \end{aligned} \tag{14-1}$$

The conditional probability that a kill is obtained, given a target hit, i.e., $p(k|h)$, depends on the vulnerability of the target to attack and the damage mechanism of the projectile or warhead. Terminal ballistic effects from which the conditional probability $p(k|h)$ is derived depend on the particular application involved for the "ordnance" or warhead versus target conditions and is treated appropriately as a field in its own right, i.e., "vulnerability". Vulnerability will be treated in later chapters and a handbook, DARCOM-P 706-163, is devoted to the subject. This chapter, therefore, is devoted to hit probability analyses.

In general, the chance of hitting a target will depend upon the error in locating the target, meteorological conditions, weapon cant, jump, the size and shape of the target, the range to the target, the aiming error (and its distribution from occasion to occasion), and the round-to-round ballistic distribution. Such sources of error often are divided into two classes: random and systematic variations, or random and "variable bias" errors. The round-to-round dispersion usually is assumed to follow a normal or "Gaussian" distribution as explained in Chapter 13. The total aiming error is actually the amount of offset of the center of impact (C of I) of the rounds from the target center. The target itself often is located with error, so that the location of the target center does indeed have a "distribution" for location error. For the time being, we will ignore this source of error and concentrate on determining the probability of hitting a target which is quite accurately located.

In connection with the previously described errors, we should mention another type of error, and it is the sighting error. We defined the aiming error as the total error which results from our inability to place the center of impact on the center of the target or the aim point. A component of this error is the sighting error, which we define as the error resulting from our inability to place the cross-hairs of a

sighting system exactly on the target center or the part of the target we are particularly interested in. There is also the "zeroing" error, which occurs by not properly aligning the sighting system with the bore of the weapon. Thus, the sighting error and the zeroing error are both components of the aiming error, although the aiming error includes other and often much larger sources of error such as those due to meteorological conditions, human error, and improper leveling of the weapon.

In Chapter 13, we learned that rounds fired from a weapon are randomly scattered about a true (unknown) C of I, and that due to aiming errors (or factors that affect aiming error) the C of I of the weapon was subject to variation, such dispersion often being as large or larger than the round-to-round residual or ballistic dispersion. Therefore, the major problem in obtaining effective fire on a target involves that of adjusting the C of I onto the desired target center or desired aim point. (There is no aim error when the C of I of rounds is at the target center or desired aim point.) Methods of adjusting fire are covered in FM 6-40 (Ref. 1), and optimum corrections for adjusting fire are given in Refs. 2, 3, and 4. Ref. 3 shows that for quantitative adjustment a full correction for the observed deviation in range from the desired aim point is made for the first round, one-half the observed deviation for the second round, one-third for the third round, etc. Similar considerations for optimum corrections apply to adjusting deflection errors to move properly the C of I to the desired aim point.

There exists a very extensive amount of literature on the general subject of this chapter, so that references listed at the end of this chapter include mostly a selection of the key or useful techniques for the weapon systems analyst. The curious reader or analyst may wish to extend his knowledge by studying all of the individual papers or reports listed in the bibliography and references.

14-2 PROBABILITY OF HITTING A CIRCULAR TARGET WHEN THE AIM ERROR EQUALS ZERO

For a circular target of radius R with center at the origin (no aim error, or the C of I at the origin) and a circular normal delivery distribution for the weapon as given by Eq. 13-10, then the chance of a round hitting the target is simply

$$p(h) = p(\text{hit}) = 1 - \exp[-R^2/(2\sigma^2)]^* \tag{14-2}$$

where σ is the total standard deviation of weapon delivery errors.

Example 14-1:

Fire from an enemy bunker is holding up the advance of friendly troops. If an artillery weapon with a warhead "damage" radius of 30 m and delivery CEP of 20 m is used, what is the chance of destroying the bunker?

Since the $CEP = 20 = 1.1774\sigma$ (see Eq. 13-12), then $\sigma = 17$, and the chance of killing the point target may be found from the chance of a round falling on or within the radius of 30 m, i.e., from Eq. 14-2

$$p(h) = 1 - \exp\{-(30)^2/[2(17)]^2\} = 0.79\,.$$

Alternatively, noting that the exponent of Eq. 14-2 is

$$R^2/(2\sigma^2) = [R^2/(CEP)^2][(CEP)^2/(2\sigma^2)]$$

*This probability is also the chance that chi-square with 2 degrees of freedom, i.e., $\chi^2(2)$, does not exceed R^2/σ^2.

and also that $\exp[-(CEP)^2/(2\sigma^2)] = 1/2$, then

$$p(h) = 1 - (1/2)^{(R/CEP)^2} = 1 - (1/2)^{(1.5)^2} = 0.79 \text{ also.}$$

(Such calculations are easily made with many pocket calculators.)

14-3 PROBABILITY OF HITTING A RECTANGULAR OR SQUARE TARGET CENTERED AT THE ORIGIN

Suppose we have a rectangular target centered at the origin, i.e., zero aim error, as indicated in Fig. 14-1. The chance of hitting this target for a circular normal delivery distribution of rounds involves integrating x over the interval $-a \leq x \leq a$, and y over the interval $-b \leq y \leq b$; i.e., over the area of the rectangle. Thus, the probability of hitting is obtained from

$$p(h) = [1/(2\pi\sigma^2)] \int_{-a}^{a} \int_{-b}^{b} \exp[-(x^2 + y^2)/(2\sigma^2)]dx\,dy$$

or, in so-called standard form,

$$= \left[(1/\sqrt{2\pi}) \int_{-a/\sigma}^{a/\sigma} \exp(-x^2/2)dx\right] \left[(1/\sqrt{2\pi}) \int_{-b/\sigma}^{b/\sigma} \exp(-y^2/2)dy\right] \qquad (14\text{-}3)$$

which involves the product of two quantities found from tables of the standard normal distribution.

For a square target, i.e., $b = a$, the chance of a hit becomes

$$p(h) = \left[(1/\sqrt{2\pi}) \int_{-a/\sigma}^{a/\sigma} \exp(-x^2/2)dx\right]^2. \qquad (14\text{-}4)$$

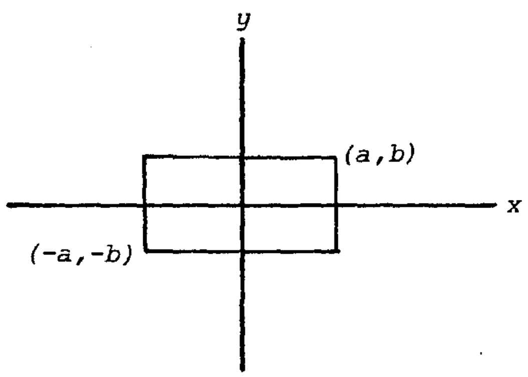

Figure 14-1. Rectangular Target Centered at the Origin

Example 14-2.

Suppose we replace the circular "target" (damage radius) of Example 14-1 by a square target of the same area in order to compare hit probabilities.

In this case,

$$\pi R^2 = 4a^2$$

or

$$\pi(30)^2 = 4a^2$$

so that

$$a = 26.59\,.$$

As before $\sigma = 17$, and hence $a/\sigma = 1.56$. From a table of the normal integral, we find

$$(1/\sqrt{2\pi})\int_{-1.56}^{1.56}\exp(-t^2/2)dt = 0.8812, \text{ and } (0.8812)^2 = 0.78\,.$$

Hence, $p(h) = 0.78$, so that there is very little difference between the chance of hitting the circular target and a square target of the same area.

This brings us to the Polya-Williams approximation (Refs. 5 and 6).

14-4 THE POLYA-WILLIAMS APPROXIMATION

By referring to Eq. 14-3, we see that the chance of hitting a rectangular target centered at the origin as in Fig. 14-1 is actually the probability content cutoff of the normal distribution in the x-direction times the probability content cutoff in the y-direction. However, the numerical values of the integrals have to be obtained from tables of the normal integral. We can, however, relate the actual probability content of the normal distribution to an exponential function by comparing probabilities of hitting a *square* target with that of a circular target of the same area as follows.

Define I as

$$I = (1/\sqrt{2\pi})\int_{-a}^{a}\exp(-x^2/2)dx\,.$$

Then

$$\begin{aligned} I^2 &= [1/(2\pi)]\int_{-a}^{a}\int_{-a}^{a}\exp[-(x^2+y^2)/2]dx\,dy \\ &\le [1/(2\pi)]\int_{A}\int\exp[-(x^2+y^2)/2]dx\,dy \end{aligned}$$

where the region A is taken as the circular area, $x^2 + y^2 \leq 4a^2/\pi$, which is the same as for the square. But we know from Eq. 13-19 that since $R = 2a/\sqrt{\pi}$ here, then

$$[p(a)]^2 = 1 - \exp(-2a^2/\pi) .$$

Therefore,

$$I = (1/\sqrt{2\pi}) \int_{-a}^{a} \exp(-x^2/2)dx \leq [1 - \exp(-2a^2/\pi)]^{1/2} = p(a) . \quad (14\text{-}5)$$

The quantity $p(a)$ is known as the Polya-Williams approximation (Refs. 5 and 6) to the truncated normal integral I, and the relative error of $p(a)$ is never more than 0.0075 or three fourths of 1%!! The Polya-Williams (exponential) approximation is never less than and generally just slightly larger than the exact normal integral I and may be calculated on many pocket calculators without looking into any tables. Table 14-1 gives an indication of the error.

Therefore, we can see that the exact chance of hitting a square target ($2a \times 2a$) centered at the origin, and for weapon delivery standard deviation σ, is from Eq. 14-5 just slightly less than the (standardized) Polya-Williams approximation given by

$$p(h) = 1 - \exp[-2a^2/(\pi\sigma^2)] . \quad (14\text{-}6)$$

Furthermore, for equal areas of the circle and square, i.e., $\pi R^2 = 4a^2$, or $a = \sqrt{\pi}R/2$, then Eq. 14-6 may be used to obtain the chance of hitting a circular target, which is $1 - \exp[-R^2/(2\sigma^2)]$ and is therefore exact.

For the rectangular target of Fig. 14-1, it is seen that the chance of hitting given by Eq. 14-3 may be well approximated by

$$p(h) \approx \Big[\{1 - \exp[-2a^2/(\pi\sigma^2)]\}\{1 - \exp[-2b^2/(\pi\sigma^2)]\}\Big]^{1/2} . \quad (14\text{-}7)$$

(We will compare rectangular and elliptical target hit probabilities in par. 14-5.)

TABLE 14-1. ERROR IN POLYA-WILLIAMS (P-W) APPROXIMATION

Limit of Integral I, a	P-W Approximation $p(a)$	Exact Value of Integral, I	Error in P-W, $p(a) - I$
0.2	0.1586	0.1585	0.0001
0.4	0.3112	0.3108	0.0004
0.6	0.4526	0.4515	0.0011
0.8	0.5785	0.5763	0.0022
1.0	0.6862	0.6827	0.0035
1.2	0.7747	0.7699	0.0048
1.4	0.8443	0.8385	0.0058
1.6	0.8967	0.8904	0.0063
1.8	0.9343	0.9281	0.0062
2.0	0.9600	0.9545	0.0055

Example 14-3:

If a rocket has a round to round sigma in each direction of 3 mils and the aiming error in each direction amounts to a sigma of 4 mils, then what is the chance of hitting a tank target (on the first round) which is approximately rectangular in shape and subtends angles of 4 and 6 mils, respectively.

Now the total delivery standard deviation is $\sigma = (3^2 + 4^2)^{1/2} = 5$ mils for a round. The limits on the integrals are $\pm 2/5 = \pm 0.4$ and $\pm 3/5 = \pm 0.6$. Hence, the exact probability of a first round hit from Eq. 14-3, using Table 14-1, is

$$p(h) = \left[(1/\sqrt{2\pi})\int_{-0.4}^{0.4} \exp(-x^2/2)dx\right]\left[(1/\sqrt{2\pi})\int_{-0.6}^{0.6} \exp(-y^2/2)dy\right]$$

$$= (0.3108)(0.4515) = 0.140 .$$

The Polya-Williams approximation Eq. 14-7 gives

$$p(h) = (0.3112)(0.4526) = 0.141, \text{ a very close value.}$$

14-5 NONCIRCULAR NORMAL OR ELLIPTICAL GAUSSIAN DELIVERY ERRORS

For the case of unequal delivery standard deviations (or variances) in the x- and y-directions, we are dealing with the noncircular or elliptical bivariate normal distribution. It follows immediately from Eqs. 13-9 and 14-3 that the chance of hitting a rectangular target, with sides parallel to the axes and center at the origin and unequal delivery standard deviations is given by

$$p(h) = \left\{ \left[1/(\sqrt{2\pi})\right] \int_{-a/\sigma_x}^{a/\sigma_x} \exp(-x^2/2)dx\right\}\left\{ \left[1/(\sqrt{2\pi})\right] \int_{-b/\sigma_y}^{b/\sigma_y} \exp(-y^2/2)dy\right\} \tag{14-8}$$

which as before is the product of the probability contents for the two directions, so that only the table of the standardized univariate normal distribution is needed.

It also follows immediately that the chance of hitting, given by Eq. 14-8, may be calculated accurately by the Polya-Williams approximation, i.e.,

$$p(h) \approx \left[\{1 - \exp[-2a^2/(\pi\sigma_x^2)]\}\{1 - \exp[2b^2/(\pi\sigma_y^2)]\}\right]^{1/2}. \tag{14-9}$$

Again, such a calculation is carried out quickly on many pocket calculators.

For a circular target of radius r centered at the origin, the probability of hitting for the stated conditions is

$$\left.\begin{aligned} p(h) &= [1/(2\pi\sigma_x\sigma_y)] \iint_{x^2 + y^2 \le r^2} \exp[-x^2/(2\sigma_x^2) - y^2/(2\sigma_y^2)]dxdy \\ &= [1/(2\pi)] \iint_{\sigma_x^2u^2 + \sigma_y^2v^2 \le r^2} \exp[-(u^2 + v^2)/2]dudv \end{aligned}\right\} \tag{14-10}$$

where

$u = x/\sigma_x$

$v = y/\sigma_y$.

We see therefore that the chance of hitting a circular target when there are elliptical normal delivery errors may be transformed easily to the chance of hitting an elliptically shaped target when there is a circular normal (standardized) delivery error distribution.

Methods for evaluating the exact and approximate chances of hitting given by Eq. 14-10 are covered in Refs. 7-18, and in the general statistical literature. Such computations invariably involve the use of tables or complex series expansions. However, a rather quick and useful technique for many weapon system analyses may be developed using the Polya-Williams approximation. This involves replacing the ellipse by a rectangle of the same area. Thus, if the semiaxes of the ellipse are c and d, and the sides of the rectangle are represented by $2a$ and $2b$, we equate the areas

$$\pi cd = (2a)(2b) \tag{14-11}$$

and use $a = c\sqrt{\pi}/2$ and $b = d\sqrt{\pi}/2$ in Eq. 14-9, where the cutoff limit a is the same as that of Eq. 14-5, i.e., specifically for the rectangle. Thus, the probability of a hit on an elliptically shaped target of semiaxes c and d, and noncircular normal distribution is very nearly

$$p(h) = \left[\{1 - \exp[-c^2/(2\sigma_x^2)]\}\{1 - \exp[-d^2/(2\sigma_y^2)]\}\right]^{1/2} . \tag{14-12}$$

Note that for a circular target, $c = d = R$, and circular normal delivery error σ, Eq. 14-12, reduces to the exact value given in Eq. 14-2. Thus, in arriving at the rather general Eq. 14-12, we have in effect split up Eq. 14-2 into two parts (the two directions) and taken square roots to use the parameters in the x- and y-directions for the ellipse.

In making such transformations of the variables involved, the reader should also note for the second double integral of Eq. 14-10 that the standard form of the ellipse is

$$u^2/(r^2/\sigma_x^2) + v^2/(r^2/\sigma_y^2) = 1$$

so that in this case $c = r/\sigma_x$, $d = r/\sigma_y$, $a = r\sqrt{\pi}/(2\sigma_x)$, $b = r\sqrt{\pi}/(2\sigma_y)$, and the probability of a hit given by Eq. 14-10 for the circular target is through the use of the Polya-Williams approximation

$$p(h) = \left[\{1 - \exp[-r^2/(2\sigma_x^2)]\}\{1 - \exp[-r^2/(2\sigma_y^2)]\}\right]^{1/2} . \tag{14-13}$$

Finally, to go to an elliptical target and elliptical normal delivery errors the radius r of Eq. 14-13 is replaced by the semiaxis c in the first parenthesis and the semiaxis d in the second, giving Eq. 14-12 as before. The key is to use the exponent of the form $-r^2/(2\sigma_x^2)$ for circles or ellipses and the exponent form $-2a^2/(\pi\sigma_x^2)$ for squares or rectangles.

We next take up the "circular coverage function" or "offset circle probabilities" which cover the case where the aim point, or C of I, is not located at the target center, and delivery errors are equal.

14-6 THE CIRCULAR COVERAGE FUNCTION OR OFFSET CIRCLE PROBABILITIES

The integral of the circular Gaussian probability density Eq. 13-10—but in standard units, i.e., $\sigma = 1$, $x/\sigma = u$, and $y/\sigma = v$—over a circle of radius R, which is offset from the origin at a radial distance of r (in any direction), is known as the Circular Coverage Function. H. H. Germond (Ref. 19) and others at the RAND Corporation made extensive studies of this problem in the early 1950's, as

this integral is the probability of hitting a circular target which is offset from the aim point. (Note that the circular target could be centered at the origin and the aim point offset at distance r—the same problem.) At the risk of slight confusion, we will use the notation of Germond (Ref. 19) to define the chance of hitting (in standard units, or distance divided by σ). We start with the chance of a hit for offset C of I given by

$$\left.\begin{aligned} p(h) &= [1/(2\pi\sigma^2)] \iint_{x^2+y^2\le d^2} \exp[-(x-\mu)^2/(2\sigma^2) - (y-\nu)^2/(2\sigma^2)]dxdy \\ &= [1/(2\pi)] \iint_A \exp[-(u^2+v^2)/2]du\,dv \end{aligned}\right\} \quad (14\text{-}14)$$

where

$u = (x - \mu)/\sigma$

$v = (y - \nu)/\sigma$

(μ,ν) = coordinates of C of I

where this latter integral or region A is taken over the circle,

$$(u + \mu/\sigma)^2 + (v + \nu/\sigma)^2 \le d^2/\sigma^2 \quad (14\text{-}15)$$

where

d = radius of the circle.

By putting $R = d/\sigma$, $a = -\mu/\sigma$ and $b = -\nu/\sigma$, which are in standard units, we have the Germond circular coverage function $p(R,r)$,

$$p(h) = p(R,r) = [1/(2\pi)]\iint \exp[-(u^2 + v^2)/2]du\,dv \quad (14\text{-}16)$$

this integral being taken over the area bounded by the offset circle $(x - a)^2 + (y - b)^2 = R^2$, for which $r^2 = a^2 + b^2$, the square of the radial offset distance.

Some tables of this function are given in Refs. 19 and 20; hence, such chances of hitting may be found easily.

Example 14-4:

Return to Example 14-1; suppose that an aiming error of 8.5 m is made. Then, what is the chance of destroying the bunker?

We have $\sigma = 17$, $R = 30/17 = 1.76$, and $r = 8.5/17 = 0.50$, and by entering Table 14-2, we find $p(1.76,0.50)$ by interpolation so that the chance of damage is approximately 0.746, the amount of offset being only $\sigma/2$. (The original chance of damage of 0.79 also may be found in Table 14-2.)

In using the RAND tables, the reader should note that $p(R,r)$ in Ref. 19 is $1 - Q(r_d - D/\sigma, D/\sigma)$ of Ref. 20, where D = radial offset of the C of I, r_d = damage radius, $R = r_d/\sigma$, and $r = D/\sigma$. Thus, for example, $p(1.8,1.1) = 0.6210$ from Table 14-2 for the circular coverage function, and from tables of the offset circle probabilities of Ref. 20, we find $1 - Q(r_d - D/\sigma, D/\sigma) = 1 - Q(0.7,1.1) = 1 - 0.379 = 0.621$.

For possible use, we point out by reference to Eq. 14-16 that $p(R,0) = 1 - \exp(R^2/2)$, $p(0,r) = 0$, and $p(R,r)$ approaches $(1/\sqrt{2\pi})\int_r^\infty \exp(-t^2/2)dt$ as R increases. Also, if R is 0.4 or less, then the

TABLE 14-2. OFFSET CIRCLE PROBABILITY VALUES FOR $1 \leq R \leq 2$ (Ref. 19)

$p(R,r)$ for R = 1.0(0.1)2.0, r = 0.0(0.1)4.0

r \ R	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.0	.3935	.4539	.5132	.5704	.6247	.6753	.7220	.7643	.8021	.8355	.8647
0.1	.3920	.4523	.5115	.5686	.6229	.6735	.7202	.7626	.8005	.8340	.8633
0.2	.3874	.4474	.5063	.5632	.6174	.6681	.7149	.7575	.7957	.8296	.8593
0.3	.3801	.4393	.4977	.5543	.6083	.6591	.7061	.7490	.7877	.8222	.8525
0.4	.3699	.4282	.4859	.5421	.5959	.6466	.6939	.7373	.7767	.8119	.8430
0.5	.3573	.4144	.4712	.5267	.5802	.6309	.6785	.7224	.7625	.7987	.8309
0.6	.3424	.3980	.4537	.5084	.5615	.6122	.6600	.7045	.7454	.7826	.8160
0.7	.3256	.3795	.4338	.4876	.5400	.5906	.6386	.6837	.7255	.7638	.7986
0.8	.3072	.3592	.4119	.4645	.5162	.5665	.6146	.6602	.7029	.7424	.7735
0.9	.2876	.3374	.3883	.4396	.4904	.5401	.5883	.6343	.6778	.7184	.7559
1.0	.2671	.3146	.3635	.4132	.4623	.5120	.5599	.6062	.6504	.6921	.7310
1.1	.2461	.2911	.3378	.3857	.4340	.4823	.5299	.5763	.6210	.6636	.7038
1.2	.2250	.2673	.3117	.3575	.4043	.4515	.4985	.5447	.5898	.6332	.6745
1.3	.2040	.2436	.2855	.3291	.3741	.4200	.4661	.5120	.5571	.6010	.6433
1.4	.1836	.2203	.2595	.3008	.3438	.3881	.4331	.4783	.5233	.5675	.6105
1.5	.1638	.1976	.2341	.2730	.3138	.3563	.3999	.4442	.4887	.5329	.5763
1.6	.1450	.1759	.2096	.2459	.2844	.3249	.3668	.4099	.4537	.4975	.5411
1.7	.1272	.1553	.1863	.2198	.2559	.2942	.3343	.3759	.4185	.4613	.5052
1.8	.1108	.1361	.1642	.1951	.2286	.2645	.3025	.3424	.3837	.4260	.4689
1.9	.0956	.1183	.1436	.1718	.2027	.2361	.2719	.3098	.3494	.3905	.4325
2.0	.0819	.1019	.1247	.1501	.1784	.2092	.2426	.2784	.3161	.3556	.3965
2.1	.0695	.0871	.1073	.1302	.1558	.1840	.2150	.2484	.2840	.3217	.3611
2.2	.0585	.0739	.0917	.1120	.1350	.1607	.1890	.2200	.2534	.2891	.3267
2.3	.0489	.0621	.0776	.0956	.1161	.1392	.1650	.1935	.2245	.2579	.2936
2.4	.0404	.0518	.0652	.0809	.0990	.1197	.1429	.1688	.1974	.2285	.2620
2.5	.0332	.0428	.0543	.0679	.0838	.1021	.1228	.1463	.1723	.2010	.2321
2.6	.0270	.0351	.0449	.0566	.0703	.0863	.1048	.1257	.1492	.1754	.2042
2.7	.0218	.0285	.0368	.0467	.0586	.0725	.0886	.1072	.1283	.1520	.1782
2.8	.0174	.0230	.0299	.0383	.0483	.0603	.0744	.0907	.1094	.1306	.1544
2.9	.0138	.0184	.0241	.0311	.0396	.0498	.0619	.0761	.0926	.1114	.1328
3.0	.0108	.0145	.0192	.0250	.0321	.0408	.0511	.0634	.0777	.0943	.1133
3.1	.0084	.0114	.0152	.0200	.0259	.0331	.0419	.0523	.0647	.0792	.0959
3.2	.0065	.0089	.0119	.0158	.0207	.0267	.0340	.0428	.0534	.0659	.0805
3.3	.0050	.0068	.0093	.0124	.0163	.0213	.0274	.0348	.0437	.0544	.0670
3.4	.0038	.0052	.0072	.0096	.0128	.0168	.0218	.0280	.0355	.0446	.0554
3.5	.0028	.0040	.0055	.0074	.0100	.0132	.0173	.0224	.0286	.0362	.0454
3.6	.0021	.0030	.0041	.0057	.0077	.0103	.0136	.0177	.0228	.0292	.0368
3.7	.0016	.0022	.0031	.0043	.0059	.0079	.0105	.0139	.0181	.0233	.0297
3.8	.0011	.0016	.0023	.0032	.0044	.0060	.0081	.0108	.0142	.0184	.0237
3.9	.0008	.0012	.0017	.0024	.0033	.0046	.0062	.0093	.0110	.0145	.0188
4.0	.0006	.0009	.0012	.0018	.0025	.0034	.0047	.0064	.0085	.0113	.0147

values of $p(R,r)$ as given by the Carlton approximation

$$p(R,r) \approx [R^2/(2 + R^2/2)]\exp[-r^2/(2 + R^2/2)]$$

are good to four decimals.

The RAND tables of RM330 and R234 (Refs. 19 and 20) may be used to find the chances of hitting a circular target of radius R centered at the origin for equal delivery errors σ in x and y, of course, this being $p(R/\sigma,0)$.

The polar integral of $p(R,r)$ with respect to r, or

$$S(R,r) = \int_0^r 2\pi x p(R,x)dx$$

is called the circular coverage function, and geometrically it is the mean or expected overlap of a circle of radius r dropped onto a circle of radius R for unit bivariate normal delivery error, Thus, $S(R,r)$ is of considerable interest in target damage problems discussed in later chapters, and RM330 includes a table of this function.

We now show a further use of the RAND tables in connection with elliptical targets or elliptical delivery errors.

14-7 THE ELLIPTIC COVERAGE FUNCTION AND SOME OF ITS USES

We first complete and round out the discussion involving approximate equations such as Eq. 14-12 for ellipses. For the case of an elliptically shaped target centered at the origin, with major and minor axes ($2c$ and $2d$, respectively) parallel to the x- and y-coordinate axes, and circular normal delivery errors, then the single shot hit probability is given by

$$\left.\begin{aligned} p(h) &= [1/(2\pi\sigma^2)] \iint_{\frac{x^2}{c^2}+\frac{y^2}{d^2}\leq 1} \exp[-(x^2+y^2)/(2\sigma^2)]dxdy \\ &= 1/(2\pi) \iint_A \exp[-(u^2+v^2)/2]dudv \end{aligned}\right\} \quad (14\text{-}17)$$

where the region of integration A is now

$$u^2/(c/\sigma)^2 + v^2/(d/\sigma)^2 \leq 1\,. \quad (14\text{-}18)$$

Note that we have transformed the x's and y's to u's and v's, which are in standard, dimensionless units of the delivery error sigma. Such a transformation to standard units is always a good practice.

An equivalent problem is that of finding the hit probability for noncircular normal delivery errors when firing at a circular target. As already indicated, even for an elliptical target and noncircular normal delivery errors as approximated by Eq. 14-12, the exact problem reduces to that of determining the integral of the bivariate unit or standard normal distribution over an ellipse in standard form such as Eq. 14-18. In fact, the elliptical region generally becomes

$$u^2/(c/\sigma_x)^2 + v^2/(d/\sigma_y)^2 \leq 1 \quad (14\text{-}19)$$

in standard form for a target of unit radius.

Hence, these are all equivalent problems in probabilities of hitting, and we notice immediately that the Polya-Williams approximation as given in Eq. 14-5 may be used with

$$a = \sqrt{\pi}\, c/(2\sigma_x) \quad \text{and} \quad b = \sqrt{\pi}\, d/(2\sigma_y)$$

to obtain the elliptical form of Eq. 14-9, which turns out to be Eq. 14-12. For $\sigma_x = \sigma_y = \sigma$ in Eq. 14-12, we have the approximate chance of hitting the elliptical target for circular delivery errors as in the first expression of Eq. 14-17.

The exact probabilities for the previous cases may be obtained by use of the circular coverage function of par. 14-6. In fact, Snow (Ref. 18) and others have shown that the chance of hitting given by Eq. 14-17 may be expressed in terms of an "elliptic coverage function", $q(c/\sigma,d/\sigma)$, which is the difference of two offset circle probabilities, i.e.,

$$q(c/\sigma,d/\sigma) = p[(c+d)/(2\sigma),(c-d)/(2\sigma)] - p[(c-d)/(2\sigma),(c+d)/(2\sigma)] \quad (14\text{-}20)$$

where c/σ is taken as the major semiaxis of the ellipse. (In Snow's Theorem 1, page 5 of Ref. 18, his divisors should be 4 and not 2 for his A and B, the major and minor axes, respectively.)

Example 14-5:

An infantryman with rifle delivery accuracy for a bullet of about $\sigma = 1.27$ mils notices the silhouette of an enemy soldier at 100 m, which appears elliptical in shape and about 4 ft in the vertical direction and 1 ft in the horizontal direction. What is the chance of a first-round hit?

With these data, we obtain $c/\sigma = (48/2)\text{in.}/[(1.27)(3.937)\text{in.}] = 4.80$, so that the probability content in the vertical direction is practically one, and $d/\sigma = (12/2)\text{in.}/[(1.27)(3.937)\text{in.}] = 1.20$. Then, using Eq. 14-20 and Tables 14-2 and 14-3, we find the exact value,

$$p(h) = p(3,1.8) - p(1.8,3) = 0.8365 - 0.0777 = 0.759.$$

The Polya-Williams approximation of Eq. 14-12 gives a chance of hitting of 0.716 for the rather long and thin elliptical target, which is in effect converted to a rectangle to obtain the approximate probability of hitting. Other inputs to the analysis of a weapon system, such as target vulnerability, may be subject to an error of at least this amount.

A brief table of probability values for the elliptic coverage function $q(a,b)$ is given in Table 14-4; Fig. 14-2 is a graph of the probabilities, showing the effect of noncircularity of delivery errors. These are taken from Snow (Ref. 18). For noncircular delivery standard deviations ($\sigma_x \geq \sigma_y$), a circular target of radius R and no offset, the probability of a hit is given by $q(a,b)$, where $a = R/\sigma_y$ and $b = R/\sigma_x$. To illustrate use Example 14-5 where $a = 4.8$ and $b = 1.2$, so that $q(4.8,1.2)$ is read either from Table 14-3 or from Fig. 14-2 as 0.759. Eq. 14-19 always may be used as the standard form if the "a" of $q(a,b)$ is taken as the semimajor axis. When a and b are equal, then we see from Eqs. 14-20 and 14-16 that

$$q(a,a) = p(a,0) - p(0,a) = p(a,0) = 1 - \exp(-a^2/2)$$

which is a special case of the circular coverage function.

There is no offset for the elliptic coverage function, and the reader is so warned. Nevertheless, in par. 14-9 we give a very suitable approximation for the chance of hitting an elliptical target, with axes coinciding with the x- and y-axes, and an offset noncircular normal delivery distribution. The procedure is known as the "approximate chi-square" method. Next, however, we will discuss a long-known, widely used, and rather simple approximation.

14-8 THE von NEUMANN-CARLTON DIFFUSE TARGET CONCEPT

The so-called "diffuse target" concept of von Neumann and Carlton (Ref. 21) involves the use of the normal or Gaussian distribution function over infinite limits to replace and "diffuse" the target,

TABLE 14-3. OFFSET CIRCLE PROBABILITY VALUES FOR 2 ≤ R ≤ 3 (Ref. 19)

$p(R, r)$ for R = 2.0(0.1)3.0, $r - R$ = −3.0(0.1)3.0

$r-R$ \ R	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
−3.0											.9889
−2.9										.9851	.9886
−2.8									.9802	.9848	.9879
−2.7								.9739	.9798	.9838	.9866
−2.6							.9660	.9734	.9786	.9822	.9847
−2.5						.9561	.9654	.9720	.9766	.9798	.9822
−2.4					.9439	.9554	.9636	.9695	.9737	.9767	.9789
−2.3				.9290	.9431	.9533	.9607	.9660	.9699	.9727	.9748
−2.2			.9111	.9281	.9406	.9498	.9565	.9614	.9650	.9677	.9697
−2.1		.8897	.9100	.9252	.9365	.9448	.9510	.9555	.9589	.9616	.9636
−2.0	.8647	.8885	.9068	.9205	.9307	.9383	.9440	.9483	.9516	.9542	.9563
−1.9	.8633	.8849	.9014	.9138	.9231	.9302	.9355	.9396	.9429	.9455	.9476
−1.8	.8593	.8788	.8937	.9051	.9137	.9203	.9254	.9294	.9326	.9352	.9374
−1.7	.8525	.8702	.8839	.8943	.9023	.9085	.9134	.9174	.9206	.9232	.9255
−1.6	.8430	.8592	.8717	.8814	.8889	.8949	.8996	.9035	.9067	.9095	.9118
−1.5	.8309	.8457	.8572	.8663	.8734	.8791	.8838	.8877	.8910	.8938	.8962
−1.4	.8160	.8297	.8404	.8489	.8557	.8613	.8659	.8698	.8731	.8760	.8785
−1.3	.7986	.8112	.8212	.8293	.8358	.8413	.8458	.8497	.8531	.8560	.8586
−1.2	.7785	.7902	.7997	.8074	.8137	.8190	.8236	.8275	.8309	.8339	.8365
−1.1	.7559	.7669	.7758	.7832	.7893	.7945	.7990	.8030	.8064	.8095	.8122
−1.0	.7310	.7412	.7497	.7568	.7627	.7679	.7723	.7763	.7797	.7828	.7856
−0.9	.7038	.7134	.7214	.7282	.7340	.7391	.7435	.7474	.7509	.7541	.7569
−0.8	.6745	.6835	.6912	.6977	.7034	.7083	.7127	.7166	.7201	.7232	.7261
−0.7	.6433	.6518	.6591	.6654	.6709	.6757	.6800	.6839	.6873	.6905	.6934
−0.6	.6105	.6185	.6255	.6315	.6368	.6415	.6457	.6495	.6529	.6561	.6589
−0.5	.5763	.5839	.5905	.5962	.6013	.6059	.6100	.6137	.6171	.6202	.6230
−0.4	.5411	.5482	.5544	.5599	.5648	.5692	.5732	.5768	.5801	.5831	.5859
−0.3	.5052	.5118	.5177	.5229	.5276	.5318	.5356	.5391	.5423	.5453	.5480
−0.2	.4689	.4751	.4806	.4855	.4899	.4940	.4976	.5010	.5041	.5069	.5095
−0.1	.4325	.4383	.4434	.4481	.4523	.4561	.4595	.4627	.4657	.4684	.4709
0	.3965	.4018	.4066	.4109	.4149	.4184	.4217	.4247	.4275	.4301	.4325
+0.1	.3611	.3660	.3705	.3745	.3781	.3815	.3845	.3874	.3900	.3924	.3947
+0.2	.3267	.3312	.3353	.3390	.3423	.3454	.3483	.3509	.3534	.3557	.3578
+0.3	.2936	.2977	.3014	.3048	.3079	.3107	.3133	.3158	.3180	.3202	.3221
+0.4	.2620	.2657	.2690	.2721	.2749	.2775	.2799	.2822	.2842	.2862	.2880
+0.5	.2321	.2354	.2384	.2412	.2437	.2461	.2483	.2503	.2522	.2540	.2557
+0.6	.2042	.2071	.2098	.2123	.2145	.2167	.2186	.2205	.2222	.2238	.2253
+0.7	.1782	.1808	.1832	.1854	.1874	.1893	.1911	.1927	.1943	.1957	.1970
+0.8	.1544	.1567	.1588	.1607	.1625	.1642	.1657	.1672	.1686	.1698	.1710
+0.9	.1328	.1348	.1366	.1383	.1398	.1413	.1427	.1439	.1451	.1463	.1473
+1.0	.1133	.1150	.1166	.1180	.1194	.1206	.1218	.1229	.1240	.1250	.1259
+1.1	.0959	.0973	.0987	.0999	.1011	.1022	.1032	.1042	.1051	.1059	.1067
+1.2	.0805	.0817	.0829	.0839	.0849	.0859	.0867	.0876	.0883	.0891	.0898
+1.3	.0670	.0681	.0690	.0699	.0708	.0716	.0723	.0730	.0737	.0743	.0749
+1.4	.0554	.0562	.0570	.0578	.0585	.0592	.0598	.0604	.0609	.0614	.0619
+1.5	.0454	.0461	.0467	.0474	.0480	.0485	.0490	.0495	.0500	.0504	.0508
+1.6	.0368	.0374	.0380	.0385	.0390	.0394	.0399	.0403	.0406	.0410	.0413
+1.7	.0297	.0301	.0306	.0310	.0314	.0318	.0321	.0325	.0328	.0331	.0333
+1.8	.0237	.0241	.0244	.0248	.0251	.0254	.0257	.0259	.0262	.0264	.0267
+1.9	.0188	.0191	.0194	.0196	.0199	.0201	.0203	.0206	.0208	.0210	.0211
+2.0	.0147	.0150	.0152	.0154	.0156	.0158	.0160	.0162	.0163	.0165	.0166
+2.1	.0115	.0116	.0118	.0120	.0122	.0123	.0124	.0126	.0127	.0128	.0129
+2.2	.0088	.0090	.0091	.0093	.0094	.0095	.0096	.0097	.0098	.0099	.0100
+2.3	.0068	.0069	.0070	.0071	.0072	.0073	.0073	.0074	.0075	.0076	.0077
+2.4	.0051	.0052	.0053	.0054	.0054	.0055	.0056	.0056	.0057	.0058	.0053
+2.5	.0038	.0039	.0040	.0040	.0041	.0041	.0042	.0042	.0043	.0043	.0044
+2.6	.0029	.0029	.0030	.0030	.0030	.0031	.0031	.0032	.0032	.0032	.0033
+2.7	.0021	.0021	.0022	.0022	.0022	.0023	.0023	.0023	.0024	.0024	.0024
+2.8	.0015	.0016	.0016	.0016	.0016	.0017	.0017	.0017	.0017	.0017	.0018
+2.9	.0011	.0011	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0013	.0013
+3.0	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0009	.0009	.0009	.0009

TABLE 14-4
BRIEF TABLE OF ELLIPTIC NORMAL COVERAGE FUNCTION OR PROBABILITY
$q(a,b)$

	$\rho = b/a$ $(b \leq a)$						
b	0	0.2	0.25	0.33	0.5	0.67	1
0	0	0	0	0	0	0	0
0.2	0.159	0.088	0.074	0.057	0.039	0.030	0.020
0.4	0.311	0.264	0.238	0.199	0.145	0.113	0.077
0.6	0.451	0.435	0.407	0.369	0.292	0.234	0.165
0.8	0.576	0.560	0.549	0.523	0.448	0.375	0.274
1.0	0.683	0.673	0.666	0.649	0.590	0.505	0.393
1.2	0.770	0.763	0.759	0.748	0.707	0.637	0.513
1.4	0.838	0.834	0.832	0.825	0.799	0.748	0.625
1.6	0.890	0.888	0.886	0.882	0.866	0.832	0.722
1.8	0.928	0.926	0.925	0.923	0.913	0.891	0.802
2.0	0.955	0.953	0.952	0.951	0.945	0.932	0.865
2.2	0.972	0.971	0.971	–	0.966	0.959	0.911
2.4	0.983	0.983	0.983	–	0.981	0.976	0.944
2.6	0.991	–	0.991	–	0.989	0.986	0.966
2.8	0.995	–	0.995	–	0.994	0.993	0.980

(This table is taken from RM-2765-PR (Ref. 18)

For unequal delivery errors $(\sigma_x \geq \sigma_y)$ and no offset, the chance of hitting a circular target of radius R is $q(R/\sigma_y, R/\sigma_x)$.
(The quantities a and b are in units of the standard deviations of delivery errors.)

thereby avoiding the complication of truncating the normal integral. The recommended technique is derived and explained in the Appendix (by John von Neumann) of BRL Report 241 (Ref. 21).

Suppose we consider a target, e.g., a square one, of area A on one hand and then on the other a negative square exponential fall-off function of the Gaussian form which is to be integrated over infinite limits to give the area A. That is, the elementary area, $dxdy$, is weighted by such a function and then integrated. By equating the area A of the (square) target to the area for the integral, we have

$$A = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp[-(x^2 + y^2)/(2k^2)]dx\,dy \tag{14-21}$$

where k is a constant to be determined. We find immediately that

$$A = 2\pi k^2 \quad \text{or} \quad k = \sqrt{A/2\pi} \ . \tag{14-22}$$

Hence, the function which "diffuses" over infinite limits to give the desired target area A is

$$\exp[-\pi(x^2 + y^2)/A], \quad -\infty \leq x, y \leq \infty \ .$$

This function is unity at the target center, $x = y = 0$, and decreases to zero as the values of x, or y, or both, increase beyond bounds. Then, for a circular normal delivery distribution, the probability of hitting the "target" becomes

$$p(\text{hit}) = [1/(2\pi\sigma^2)] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp[-(x^2 + y^2)/(2k^2)] \cdot \exp[-(x^2 + y^2)/(2\sigma^2)]dx\,dy$$

$$= k^2/(k^2 + \sigma^2) = A/(A + 2\pi\sigma^2) \ . \tag{14-23}$$

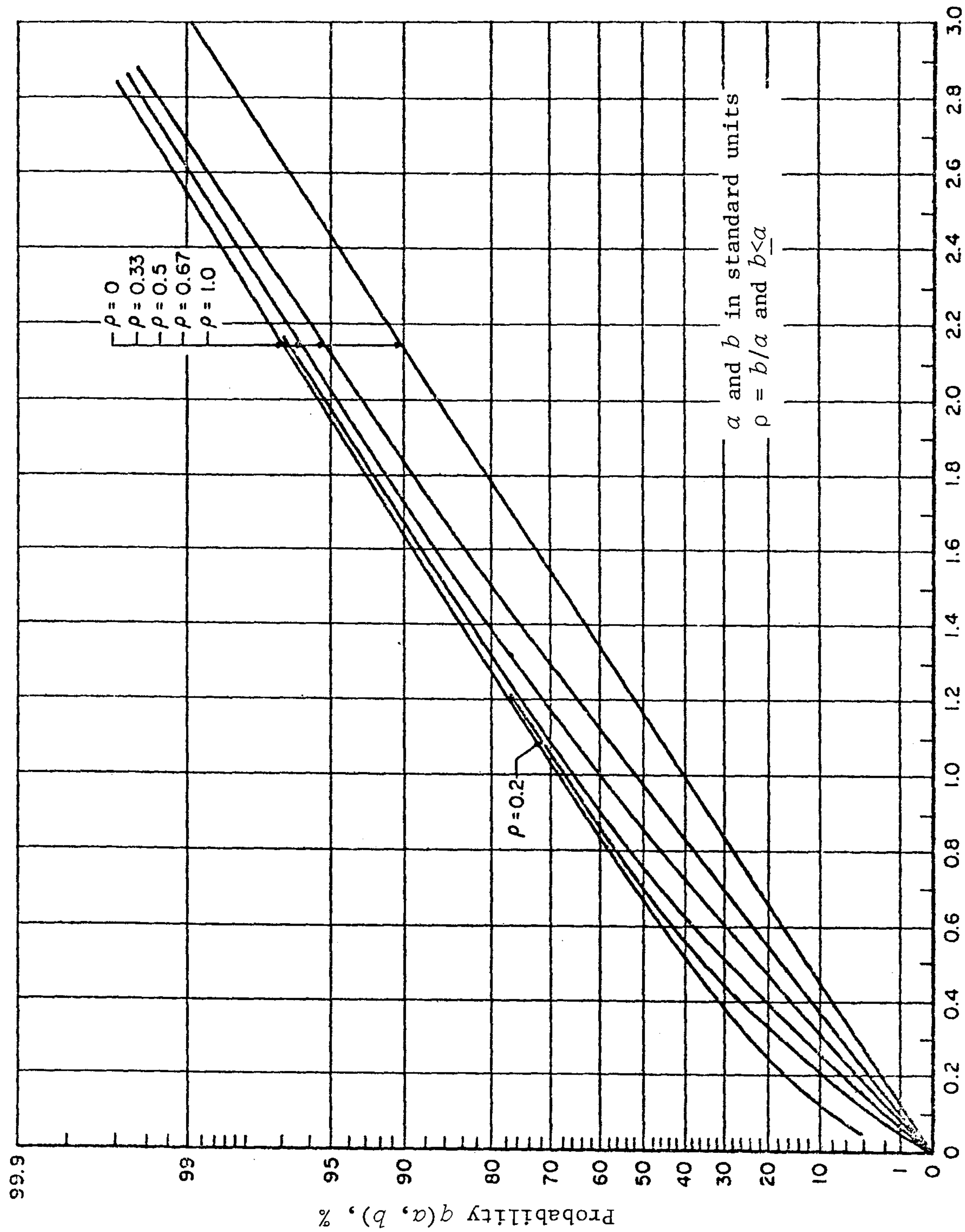

Figure 14-2. Graphs of the Elliptic Normal Coverage Function (Ref. 18)

This is the von Neumann-Carlton diffuse target concept. Note that although we used the term "square target" for illustrative purposes no target dimensions were involved at all, as the diffusion was over an area. This indicates some kind of "independence" from the particular target shape, which could be quite an advantage in weapon systems analyses. Indeed, Eq. 14-23 is merely the ratio of the area A to the total area obtained by adding to A the measure of scatter of rounds about the target, i.e., $2\pi\sigma^2$, which has the proper units of an area!

We know that for a circular target of radius R, $\pi R^2 = A$, and the chance of a hit for the circular normal delivery distribution is

$$p_h = 1 - \exp[-R^2/(2\sigma^2)] = 1 - \exp[-A/(2\pi\sigma^2)] \, . \tag{14-24}$$

In this case, therefore, we can check the accuracy of the von Neumann-Carlton approximation for some particular cases, as follows.

Example 14-6. Let $A = 10$, $\sigma = 5$. Then, by Eq. 14-23, $p(h) = 10/(10 + 50\pi) \approx 0.060$. By Eq. 14-24, $p(h) = 1 - \exp[-10/(50\pi)] \approx 0.062$, which is good agreement.

Example 14-7. Now let $A = 10$, $\sigma = 1$. Then, by Eq. 14-23, $p(h) = 10/(10 + 2\pi) \approx 0.614$ and by Eq. 14-24, $p(h) = 1 - \exp[-10/(2\pi)] \approx 0.796$ and the difference is very significant for this case of $2\pi\sigma^2$ being smaller than the target area A.

In view of Example 14-7, we are led to believe, and it is rather widely accepted, that the von Neumann-Carlton approximation should be used only when $2\pi\sigma^2$ is considerably or many times larger than the target area A. However, as a counter example, let us look at the relatively thin elliptical target of Example 14-5.

Example 14-8. For Example 14-5, we found the exact first-round hit probability was 0.759. Now the area of the elliptical target is $\pi cd = \pi(6)(24) = 452.4$ in^2, and $2\pi\sigma^2 = 2\pi[(1.27)(3.937)]^2 = 157.1$ in^2 Hence, the von Neumann-Carlton approximation gives $452.4/(452.4 + 157.1) = 0.742$, which is good agreement indeed for a relatively high hit probability. On the other hand, if we had used a circular target of the same area as the elliptical silhouette, then the exact hit probability would be $1 - \exp\{-452.4/[(2\pi)(1.27)^2(3.937)^2]\} = 0.944$! Hence, in this case the von Neumann-Carlton approximation is very conservative and very inaccurate since more of the rounds spill off the elliptical target than the circular target of the same area.

In summary we conclude that the von Neumann-Carlton equation is certainly very accurate and useful for small hit probabilities per round, but that a full study of the accuracy of it for various shapes of targets and sizes of the delivery error term would be very worthwhile. We will find in target coverage and target damage studies where hit probability per round is suitably low, and multiple rounds are necessary, that the von Neumann-Carlton approximation becomes of much importance indeed since it simplifies the analysis.

It is informative to observe at this point that for single rounds the σ^2 in the von Neumann-Carlton equation (and the others) may be the total delivery variance, obtained by adding the round-to-round variance, the aiming error variance, and other variance components of delivery error.

Also, for the case of unequal delivery errors, one might consider a slight generalization of von Neumann-Carlton equation, i.e.,

$$p(h) = A/(A + 2\pi\sigma_x\sigma_y) \, . \tag{14-25}$$

Looking ahead, if the vulnerable area of a target is A_v, then the first-round kill probability p_k may be approximated as

$$p_k \approx A_v/(A_v + 2\pi\sigma^2)\,. \tag{14-26}$$

Finally, for an offset aim point such that the C of I is at the point (μ,ν), then the chance of a hit is approximately

$$p(h) \approx [A/(A + 2\pi\sigma^2)]\exp[-(\mu^2 + \nu^2)/(2\sigma^2)]\,. \tag{14-27}$$

14-9 OFFSET PROBABILITIES OF HITTING

The probability of hitting a target (square, rectangular, circular, elliptical, etc.) becomes more complicated when the C of I of the rounds is offset from the target center or desired aim point. In fact, the chance of hitting depends on just where the true C of I is located (which is hardly ever known), and which may be in error due to inaccurate aiming, an ever-present source of error. We discussed the chance of a hit for an offset circular target and circular normal delivery error. It is easy to see that the first-round hit probability for a rectangular target with sides $2a$ and $2b$ parallel to the axes and center located at (μ,ν) for the case of noncircular Gaussian delivery errors will be given by

$$p(\text{hit}) = [1/(2\pi)]\int_{(-a-\mu)/\sigma_x}^{(a-\mu)/\sigma_x}\int_{(-b-\nu)/\sigma_y}^{(b-\nu)/\sigma_y}\exp[-(x^2 + y^2)/2]\,dx\,dy\,. \tag{14-28}$$

This chance of hitting is determined easily by multiplying the probability contents in the x- and y-directions, which are found from a table of the standard normal probability distribution.

As an example, let us consider the case of a rectangular target with offset aim error, so that the center of impact of rounds will not coincide with the target center.

Example 14-9:

An enemy tank is being attacked from the side by an antitank weapon which has horizontal and vertical delivery errors of $\sigma_x = 0.75$ mil and $\sigma_y = 1.0$ mil, respectively. The range to the enemy tank is 500 m, its side dimensions are 9 ft by 6 ft in the x- and y-directions, respectively, and the lead error is such that the C of I or aim point of the rounds is at $\mu = 1.5$ mils to the right of the target center and $\nu = 1$ mil above. What is the chance of hitting for a round fired under these conditions?

A target with dimensions of 9 ft by 6 ft at 500 m subtends angles of $(9)(12)/[(39.37)(0.5)] = 5.49$ mils and 3.66 mils, respectively. Hence, for Eq. 14-28, we have $a = 5.49/2 = 2.75$, $b = 3.66/2 = 1.83$, $\mu = 1.5$, $\nu = 1$, $\sigma_x = 0.75$, and $\sigma_y = 1$. The chance of hitting is therefore

$$p(h) = \left[(1/\sqrt{2\pi})\int_{-5.67}^{1.67}\exp(-x^2/2)dx\right]\left[(1/\sqrt{2\pi})\int_{-2.83}^{0.83}\exp(-y^2/2)dy\right]$$

$$= (0.9525)(0.7944) = 0.76\,.$$

This is an exact computation for the stated conditions.

Example 14-9 requires a relatively simple and straight-forward computation, but it applies only to rectangles and squares. As we will see later for multiple rounds and target damage problems, it is desirable to have a unified approach which applies to either two or three dimensions and which

possesses sufficient accuracy even though it approximates the exact theory which calls for very complex and extensive computations. Moreover, we have seen that we may interchange squares and circles, and rectangles and ellipses of equivalent areas with some success. This helps us to see the effect of target shape on hit probabilities to some extent. As a rather general approach we should have some interest in approximating probabilities of hitting, for elliptic normal distributions, elliptically shaped targets which are offset from the C of I of the impacting rounds. Again, we will consider only elliptical targets which have their axes parallel to the x- and y- (and z-) axes. The general problem here has to do with the probability distribution of quadratic forms in normally distributed variables which we will approximate with a chi-square variate. Consequently, we will call the technique the approximate chi-square method. The procedure is derived from and covered thoroughly in Ref. 15. We will first illustrate the technique for Example 14-5, where a rifleman with circular normal delivery error shoots at an elliptically shaped target. The problem is that of finding the probability that a quadratic form, such as Eq. 14-19, is less than a given value (unity in this case). Ref. 15 points out that if one knows the delivery errors, σ_x and σ_y, the location of the C of I, and the dimensions of the elliptical target, then the mean value and variance of the quadratic form may be determined. Furthermore, if we call $Q = Q(x,y)$ the quadratic form in x and y, and m and v its mean and variance, respectively, then

$$2mQ/v \approx \chi^2(2m^2/v) \tag{14-29}$$

or that the quantity on the left, $2mQ/v$, is approximately distributed in probability as chi-square with $2m^2/v$ degrees of freedom. Then, knowing this, it is easily seen that the chance that Q is less than some constant can be calculated, this being the chance of hitting. As an example, we can identify the conditions of Example 14-5 with those of Example 5, page 60 of Ref. 15. In fact, we put $x = x_1$ (of Ref. 15), $y = x_2$, $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, the semiaxis $c = a_1$, the semiaxis $d = a_2$, $\sigma_x^2/c^2 = c_1$, $\sigma_y^2/d^2 = c_2$ and $N = 2$, the number of dimensions, which relates this Handbook to Ref. 15. Then,

$$m = E(Q) = \sum_{i=1}^{2} c_i \tag{14-30}$$

and

$$v = \text{Var}\,(Q) = 2\sum_{i=1}^{2} c_i^2\,. \tag{14-31}$$

Finally, $2mQ/v \approx \chi^2(2m^2/v)$ and the chance of a hit is the computed probability that Q does not exceed $v/(2m)$ times a chi-square variate with $2m^2/v$ degrees of freedom. By using the Wilson-Hilferty transformation of chi-square (Ref. 22) to an approximate normal variable, the chance of hitting is also the probability that the unit standard normal variable t does not exceed

$$\left.\begin{aligned} t &= \{[\sqrt[3]{1/m} - [1 - v/(9m^2)]\}/\sqrt{v/(9m^2)} \\ &= \{3m^{2/3} - [3m - v/(3m)]\}/\sqrt{v} \end{aligned}\right\} \tag{14-32}$$

which may be read from a table of the standardized normal integral. For the Wilson-Hilferty transformation, the number of degrees of freedom, $2m^2/v$, should be greater than about three for good accuracy, although a smaller value is sometimes good enough. The computation of Eq. 14-32 may be

carried out easily with a scientific pocket calculator. Also, there now exists some tables of chi-square probabilities (especially percentage points) for fractional numbers of degrees of freedom, so that neither interpolation nor the use of Eq. 14-32 is required.

Let us illustrate with Example 14-5. In this case, we have $\sigma_x = \sigma_y = \sigma_1 = \sigma_2 = 1.27$ mil or 5 in., $c = a_1 = 24$ in., $d = a_2 = 6$ in., $c_1 = (5/24)^2 = 0.0434$, $c_2 = (5/6)^2 = 0.6944$, $m = 0.7378$, $v = 0.9683$, and $t = 0.684$. By referring the 0.684 to a table of the standardized normal cumulative integral, we find that the chance of hitting is about 0.753 vs the exact value of 0.759 obtained previously. We note in this connection that the degrees of freedom are only $2m^2/v = 1.12$, but an accurate answer was obtained nevertheless.

If we use chi-square without the Wilson-Hilferty transformation, we note that we want the chance that the quadratic form Q is less than or equal to unity, or that $\chi^2(2m^2/v) = \chi^2(1.12)$ is not greater than $2mQ/v = 2(0.7378)(1)/(0.9683) = 1.52$. Interpolation in a table of the cumulative distribution of chi-square for this value gives a probability of about 0.75 also.

For considerable generality, we now suppose that the C of I of the rounds is offset from the center of the elliptical target, and the delivery errors are unequal. Ref. 15 considers this case in Example 6, page 61. If the center of the target is located at (b_1,b_2) for the two-dimensional case, the C of I of the rounds located at (μ_1,μ_2), the semimajor and semiminor axes have lengths c_1 and c_2, respectively, and $\sigma_x = \sigma_1$ and $\sigma_y = \sigma_2$, then we are seeking the chance that a round will hit on or within the ellipse

$$Q = \sum_{i=1}^{2} (x_i - b_i)^2/c_i^2 \leq 1 . \tag{14-33}$$

In this case, $u_i = (x_i - \mu_i)/\sigma_i$ is normally distributed with zero mean and unit variance, i.e., u_i is a unit standard normal variate, and

$$Q = \sum_{i=1}^{2} V_i(u_i + A_i)^2$$

where

$V_i = \sigma_i^2/c_i^2$

$A_i = (\mu_i - b_i)/\sigma_i$.

Moreover, the mean and variance of the quadratic form Q are

$$m = E(Q) = \sum_{i=1}^{2} V_i(1 + A_i^2) \tag{14-34}$$

and

$$v = \mathrm{Var}(Q) = 2 \sum_{i=1}^{2} V_i^2(1 + 2A_i^2) . \tag{14-35}$$

Then finally, Eq. 14-32 is used to determine the approximate hit probability.

The methodology extends easily to the three-dimensional case, where i now ranges over 1, 2, and 3, so that we may find the chance of a hit within any offset ellipsoidal target for the tri-variate normal distribution with unequal sigmas.

As before, we again have that $2mQ/v$ is distributed approximately as chi-square with $2m^2/v$ degrees of freedom.

As an example and to get some idea of the accuracy in replacing offset rectangles by ellipses, we will approximate the probability of hitting in Example 14-9. In this case, we have

$a = 2.75, c_1 = 2a/\sqrt{\pi} = 3.10$	$b = 1.83, c_2 = 2b/\sqrt{\pi} = 2.06$
$\sigma_x = \sigma_1 = 0.75, V_1 = 0.0585$	$\sigma_y = \sigma_2 = 1, V_2 = 0.235$
$b_1 = 0$	$b_2 = 0$
$\mu_1 = 1.5$	$\mu_2 = 1$
$A_1 = 1.5/0.75 = 2$	$A_2 = 1/1 = 1$

$m = 0.763 \qquad v = 0.393 \qquad 2m^2/v = 2.96$ degrees of freedom.

Then $t = 0.62$, and the probability of hitting by reference to a table of the standard normal integral is 0.73 vs the exact value of 0.76. Again, such differences are rather inconsequential, considering the accuracy of other input data to weapon systems evaluations. We notice that the tank target of Example 14-9 is wider in the x-direction and also that the delivery standard deviation is smaller in that direction, yet the approximation is rather good indeed for representing the rectangle by an ellipse of the same area.

The previous approximate theory—i.e., the use of Eqs. 14-34, 14-35, and 14-32—will cover the case of offset circle probabilities also. In fact, we have checked some 100 representative values in various parts of the Rand tables Ref. 19 and the greatest difference was 0.023 in probability. To obtain such probabilities, we let $P(r,\rho)$ equal the chance that a sample point will fall on or within a circle of radius r, with the aimpoint (C of I) offset by a distance ρ from the origin. We then determine $m = 1 + \rho^2/(2\sigma^2)$ and $v = 1 + \rho^2/\sigma^2$, noting that ρ^2 is the sum of the squares of the x- and y-distances between the target center and C of I of the rounds. Then $P(r,\rho)$ is simply the $Pr[\chi^2 \leq mr^2/(v\sigma^2)]$.

The approximate chi-square method is rather general, easily extending to the three-dimensional problem, and is therefore of considerable use. Moreover, it can be used to find an equivalent CEP for the case of unequal delivery errors, $\sigma_x \neq \sigma_y$ and also for analyses of the general bivariate normal distribution having correlated delivery errors mentioned below.

The CEP for an offset noncircular normal delivery distribution is easily found by the approximate chi-square theory. If x and y follow a bivariate normal delivery distribution with C of I at (μ,ν) and delivery sigmas of σ_x and σ_y, respectively, then the CEP about the origin or aim point may be determined from

$$CEP = R_{0.50} \approx \sigma_T[1 - v/(9m^2)]^{3/2} \tag{14-36}$$

where

$$\sigma_T^2 = \sigma_x^2 + \sigma_y^2$$

$$m = 1 + (\mu^2 + \nu^2)/\sigma_T^2$$

$$v = 2(\sigma_x^4 + \sigma_y^4)/\sigma_T^2 + 4(\sigma_x^2\mu^2 + \sigma_y^2\nu^2)/\sigma_T^4 .$$

14-10 ROTATION OF AXES

So far, we have considered only the case where the delivery error axes are parallel to or coincide with the target axes of symmetry. However, the bivariate normal delivery distribution sometimes can have correlated errors, the delivery distribution density about its own C of I then being

$$f(x,y) = [1/(2\pi\sigma_x\sigma_y\sqrt{1-\rho^2})] \times \exp\{-[x^2/\sigma_x^2 - 2\rho xy/(\sigma_x\sigma_y) + (y^2/\sigma_y^2)]/[2(1-\rho^2)]\} . \tag{14-37}$$

If we make the transformation,

$$\left.\begin{aligned} x &= u\cos\theta - v\sin\theta \\ y &= u\sin\theta + v\cos\theta \end{aligned}\right\} \tag{14-38}$$

where

$$\tan 2\theta = 2\rho\sigma_x\sigma_y/(\sigma_x^2 - \sigma_y^2) \tag{14-39}$$

then u and v are uncorrelated, and have the bivariate normal delivery density given by

$$f(u,v) = [1/(2\pi\sigma_u\sigma_v)]\exp[-(u^2/\sigma_u^2 + v^2/\sigma_v^2)/2] \tag{14-40}$$

where

$$\left.\begin{aligned} \sigma_u^2 &= \{(\sigma_x^2 + \sigma_y^2) + [(\sigma_x^2 - \sigma_y^2)^2 + 4\rho^2\sigma_x^2\sigma_y^2]^{1/2}\}/2 \\ \sigma_v^2 &= \{(\sigma_x^2 + \sigma_y^2) - [(\sigma_x^2 - \sigma_y^2)^2 + 4\rho^2\sigma_x^2\sigma_y^2]^{1/2}\}/2 . \end{aligned}\right\} \tag{14-41}$$

In passing, it is of interest to note that

$$\sigma_x^2 + \sigma_y^2 = \sigma_u^2 + \sigma_v^2 . \tag{14-42}$$

The target axes of symmetry may still not be parallel to or coincide with the new delivery distribution errors in u and v, the result still calling for a complex analysis. Perhaps the approximate chi-square technique might still be of value. We will not discuss such an infrequent problem any further here.

14-11 DAMAGE RADII AND DELIVERY ERRORS

One of the important problems in weapon systems evaluations is to optimize hit or kill probabilities, or specifically to "match" damage radii with delivery CEP's. For example, we see from par. 14-2 that the kill probability p_k of a weapon with "damage" radius R_D against a point target is

$$p_k = 1 - \exp[-R_D^2/(2\sigma^2)] = 1 - (1/2)^{(R_D/CEP)^2} . \tag{14-43}$$

We see easily that when $R_D = 2\ CEP$, then already $p_k = 1 - (1/2)^4 = 15/16 = 0.94$, and for $R_D = 3\ CEP$, $p_k = 0.998$. Hence, halving the CEP has the same effect as doubling the weapon damage radius, etc., but such might amount to very costly trade-offs!

When the kill probability per round is not sufficiently high, then consideration should be given to multiple rounds or higher rates of fire, involving trade-off problems also.

14-12 MULTIPLE ROUNDS

For more than a single round fired at a target, or onto a target area, we are interested in the chance of at least one hit (or a kill) or perhaps the chance of exactly one hit, or exactly two hits, etc. The problem here, as mentioned previously, is that the movement of the C of I becomes of critical importance, and we may have to deal with the case of "correlated rounds", as it is often referred to. In other words, the C of I may move in a pattern such that it is not exactly proper simply to add the round-to-round variance to the aiming error variance to obtain the proper or "total" dispersion. In any event, the chance of at least one hit is often estimated from the simple relation,

$$Pr(\text{at least one hit}) = 1 - (1 - p)^n \tag{14-44}$$

where

p = the chance of a hit for a single round

n = the number of the rounds.

In fact, it is often tempting to use such an equation either for the case of independence, or the case of correlated movements of the C of I. However, considerable errors may result from using Eq. 14-44 and we will discuss this problem more fully in Chapter 20. Nevertheless, multiple rounds are fired when the chance of a hit per round is relatively low or the conditional chance of a kill given a hit is not near unity, or both, and hence it becomes desirable to cover the topic of multiple rounds in terms of overall models for hitting and killing targets, or for expected target damage models for multiple rounds. In view of this, the problem of multiple rounds will be considered in Chapter 20 along with chance of damage problems, which also include terminal effects in addition to hit probabilities.

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CHAPTER 15

VULNERABILITY AND LETHALITY

Vulnerability of targets to attack and the lethality of warheads against personnel or soft targets are presented from the point of view of the weapon systems analyst. In particular, the analyst must deal with the basic concepts of vulnerable areas and lethal areas, or "mean areas of effectiveness", in his evaluations of weapon systems.

15-0 LIST OF SYMBOLS

A_L = lethal area
A_p = presented area of a target
A_v = vulnerable area
$\hat{A}$ = estimate based on a (small) sample
a = parameter of a Weibull distribution
b = parameter of a Weibull distribution
c = parameter of a Weibull distribution
c = constant giving the maximum average kill probability at the center of the lethality pattern (Fig. 15-2)
E = expected fractional coverage
$f(H)$ = probability density function for height of burst
H = height of burst of a warhead
$I(u,p)$ = designation for Karl Pearson's incomplete gamma function
k = a number involved in an average
MAE = mean area of effectiveness = A_L
m = mass of fragment
N = number of rounds
$p(k|h)$ = conditional chance that a hit is a kill
$p(R,r)$ = circular coverage function of Germond = chance that round lands in an offset circle of radius R located r units from the origin or C of I
p_{ij} = probability of damage for position or cell at i, j
p_{ijk} = probability of damage for position or cell at i, j as determined by assessor k
$p_k(x,y)$ = kill probability for a hit at (x,y)
$p_k(x,y|H)$ = conditional probability that a hit on a target element located at the point (x,y) on the ground results in an incapacitation for a given height of burst H of a detonating round
$\hat{p}$ = estimate based on a (small) sample
R = target radius
R_d = damage radius
r = radius of circle dropped randomly on circular target of radius R
r_L = lethal or damage radius
$S(R,r)$ = integral of the circular coverage function $p(R,r)$
$u = x/\sigma$ = transformation or deviation in standard units
$v = y/\sigma$ = transformation or deviation in standard units
v_s = striking velocity

$w = z/\sigma$ = transformation or deviation in standard units
x,y = variables for two dimensions
z = variable for a third dimension
Δx_i = small change in x_i
Δy_i = small change in y_i
σ = standard deviation
σ_{kx}, σ_{ky} = constants or measures of variation in standard deviations for a negative exponential square kill probability fall-off function
$\chi^2(\nu)$ = chi-square for ν degrees of freedom (a statistical variate)

15-1 INTRODUCTION AND SUMMARY

Probabilities of hitting for single rounds having been discussed, and the probability of kill per round as the product of the chance of hitting and the conditional probability of kill given a hit having been noted, it is logical that topics on chance of damage for a target hit, or a round falling in the vicinity of a target should follow Chapter 14. It can be seen that large caliber rounds with relatively slow rates of fire must have sufficiently high overall kill probability per round to be cost-effective, and that some targets may require high rate-of-fire weapons with appropriate terminal effectiveness to obtain a suitable chance of damaging many types of targets, although nothing so universal about such statements of effectiveness can really be made.

Well armored targets such as tanks must be engaged with armor piercing projectiles or shape charge warheads in order to defeat them. On the other hand, fragmenting artillery projectiles which upon detonation scatter many fragments over an area can be very effective against exposed enemy troops. In the first case, it becomes very desirable to have antitank weapons possessing high hit and kill chances per round in order to stop any threatening tank attack. For artillery attacking enemy troops, it also is easily seen that round-to-round delivery errors on the one hand and the fragmenting of the projectile to produce lethal fragments on the other must be optimized in some way for overall effectiveness. Indeed, in this problem it becomes important to guarantee a large number of lethal fragments from the detonating projectile instead of a relatively few large fragments travelling longer distances, but nevertheless resulting in too small a number of fragments to cover areas where enemy troops might be located. Accurately aimed weapons are needed to counter the threat of armored targets like tanks, whereas fragmenting artillery projectiles are needed to attack large area targets of enemy personnel, thinly armored targets, or inaccurately located soft targets. In fact, such evaluations are often carried out by the weapon system analyst.

Vulnerability may be defined as the characteristics of a target which describe its sensitivity to combat damage mechanisms (Ref. 1). Therefore, the vulnerability of a target is a function of the damage producing properties of the attacking weapon and the physical properties of the target, including, for example, geometry, hardness, number and location of critical components, and other properties which determine the overall chance that a random hit on the target will result in a kill or incapacitation. Thus, vulnerability must of necessity involve considerations of a stochastic or probabilistic nature, which in turn will depend markedly on the conditions of the attacking weapons and the target in the combat environment. For example, the striking velocity of an armor piercing round fired against a tank will have a considerable effect on the vulnerability of a tank, as will the striking velocity of a bullet against an infantryman. For particular or specified conditions of the engagement of weapons against targets, it has become conventional to express the vulnerability of a target in terms of an area (or a volume) for a given attack direction. Of course, since combat itself may be largely of a stochastic

nature, then what is really wanted and needed by the systems analyst for his evaluations is the chance of damage on the battlefield, and this represents one of the main problems of the vulnerability analyst who determines target vulnerability. Consequently, it has been found convenient to determine and record vulnerable areas or volumes for all possible targets to various conditions of attack that may occur on the battlefield. Clearly, this involves much experimental work or firings at probable targets or facsimiles in order to determine all of the conditions on the battlefield which may need evaluations for comparing the performance of weapon systems. At the same time, it is desirable to build up a data base and appropriate body of theory which help to summarize results and permit extrapolation to targets and conditions for which experimental firings have not been conducted. (See, for example, Ref. 2).

If we take the vulnerable area of a target for given attack conditions and divide it by the presented area, the result is the conditional probability that a hit is a kill, i.e., $p(k|h)$, which we have referred to before, and this factor is a component of the chance of damage to a target under attack. One should note in this connection that hits on a target may be more or less uniformly distributed over the target presented area or they may follow some probability law. Generally speaking, the ranges of engagement are such that the assumption of a uniform distribution of hits on a target appears quite adequate; although for some conditions such as close ranges, it may be very desirable to compute the vulnerable areas for a peaked (normal) distribution of hits on the presented area of the target. Such problems must be taken into account by the analyst as required in a particular evaluation.

The vulnerability of a target in combat is a prime component of its chance of survival. In fact, survivability is currently a very important area of interest, and it becomes critical in many evaluations to study the survivability of weapon systems in the combat environment. This phase of the evaluation of weapons will be covered in subsequent chapters, as we build up to the more complex situations of conflict of forces. Survivability is defined in MIL-STD-721, *Definitions of Effectiveness Terms for Reliability, Maintainability, Human Factors, and Safety,* as "The measure of the degree to which an item will withstand hostile man-made environment and not suffer abortive impairment of its ability to accomplish its designated mission". In a sense, therefore, survivability of an Army weapon system is a measure of the ability of the weapon system to complete its assigned mission while subject to direct enemy attack, and hence includes the vulnerability of it.

Survivability also depends on angles of attack. Hence, the distribution of attack angles in combat should be included in analyses.

Finally, the determination of vulnerable areas involves much expertise in damage mechanisms, including the effects of armor piercing projectiles, fragments, blast, shaped charge warheads, bullets, etc., against targets. The details cannot be covered fully in this handbook, but the analyst must have a good knowledge of all types of damage mechanisms and their importance to evaluations. The reader is referred to DARCOM-P 706-163 (Ref. 3) which covers the necessary details.

In some cases, a comparison of the vulnerability of competing systems may be all that is needed in an evaluation.

We now describe the concept of vulnerable areas in-so-far as the analyst will use such information in his studies. The bibliography would be of considerable benefit to the systems analyst to further his training.

15-2 ESTIMATION OF VULNERABLE AREAS

The determination of vulnerable areas of targets may be described briefly as follows for the systems analyst. If we observe the presented area of a target, for example a tank, from a single aspect angle for

an attacking weapon, and consider that the target face is divided into and composed of square or rectangular cells, then the vulnerable area A_v for a given attack condition is defined as

$$A_v = \sum_x \sum_y p_k(x,y)\Delta x \Delta y \tag{15-1}$$

for cells of uniform size, where $p_k(x,y)$ is the average chance of kill for the cell located at the equally spaced points (x,y), with $\Delta x \Delta y$ the area of the cell. Thus, the vulnerable area of the target is a computed area obtained by weighting suitable small areas by the conditional chance that a hit is a kill and summing these weighted areas over the whole presented area of the target. If A_p is the presented area of the target, then

$$A_v \le A_p \tag{15-2}$$

and the conditional probability that a hit is a kill is given by

$$p(k|h) = A_v/A_p \tag{15-3}$$

for uniformly distributed hits on the target face.

The quantities A_v and $p(k|h)$ are obviously of fundamental importance to the weapon systems analyst in his evaluations although they are determined by vulnerability personnel.

As we have said, the determination of vulnerable areas of a target can be and often is a long and detailed experimental process, involving expensive firing programs and many technically qualified personnel. The original vulnerability programs were started at the Ballistic Research Laboratories during World War II for aircraft, and since have spread to tanks, personnel (wound ballistics), vehicles of all kinds, missiles and missile sites, and many other possible targets requiring vulnerability and/or survivability analyses. Refs. 1 and 3 through 15 should be studied in this connection.

Vulnerable areas are now calculated on high speed computers using principles of combinatorial geometry (Refs. 16-23), penetration or terminal ballistics data, and the assessment of damage to components and overall targets. These references and the GIFT Code User Manual (Ref. 16) cover a description of the process. A complete account of vulnerability studies is covered in DARCOM-P 706-163, *Basic Target Vulnerability* Handbook.

It is desirable to be able to predict vulnerable areas with no firing or a limited amount if at all possible. As an example of some possible types of models to do this, see the work of Strickland (Ref. 2).

For purposes of evaluation, different types of "kill" or damage have been developed. For tanks, for example, they are:

1. *F-Kill.* A tanks suffers an F (firepower) kill if the main armament is put out of action either because the crew has been rendered incapable of operating it or because the armament or its associated equipment is damaged.

2. *M-Kill.* A tanks suffers an M (mobility) kill if it is incapable of executing controlled movement and is irrepairable by the crew on the battlefield. In actual combat situations this definition would have to be modified to include those tanks that have mobility type damage and are repairable but cannot be repaired due to the battlefield situation.

3. *K-Kill.* A tank suffers a K-Kill if it receives a combined M and F kill or has internal damage severe enough to make economical repair unlikely. In most cases severe damage comprises the results of exploded ammunition and/or fuel fires.

For aircraft, the different types of kill of interest are:

1. *K-Kill.* Damage to the aircraft that causes loss of control immediately—usually taken as 30 s or less.

2. *A-Kill.* Damage that causes the aircraft to be out of control in 5 min or less.

3. *B-Damage.* Damage that requires the mission to be terminated and the aircraft landed as soon as possible.

4. *C-Damage.* Damage that reduces the ability of the aircraft to perform the assigned mission.

Fig. 15-1 presents a typical flow chart of a vulnerability analysis, and Table 15-1 gives some typical categories of damage and defeat criteria for various targets.

15-3 PRECISION OF ESTIMATES OF VULNERABLE AREAS

The precision or standard deviation of the error in estimating vulnerable areas of targets represents a topic which has been under discussion for many years. There are bound to be variations in such estimates since subjective types of judgment by the assessors of damage must be made. Also, in many cases only a single assessor might be used. In any event, if we consider a given cell area of a target and use k assessors to give judgments on the chance of damage, then we might identify the problem by saying that for the cell shot at in the ith and jth position, the subjective estimates (where ^ indicates estimate) of the chances of damage turn out to be

$$\hat{p}_{ij1}, \hat{p}_{ij2}, \ldots, \hat{p}_{ijk}$$

and the average chance of damage is estimated as

$$\hat{p}_{ij} = \left(\sum_{n=1}^{k} \hat{p}_{ijn} \right) / k \tag{15-4}$$

for the cell i,j.

The mean value of this estimate might approach the true value, say p_{ij}, and its variance is

$$\left. \begin{aligned} \text{Var}(\hat{p}_{ij}) &= (1/k^2)(kp_{ij}q_{ij}) \\ &= p_{ij}(1 - p_{ij})/k \,. \end{aligned} \right\} \tag{15-5}$$

For a single assessor the variance is the quantity $p_{ij}(1 - p_{ij})$ on the average. Hence, the variance of the total vulnerable area based on all the cells would therefore be

$$\text{Var}(\hat{A}_v) = \sum_i \sum_j p_{ij}(1 - p_{ij})\Delta x_i \Delta y_j / k \,. \tag{15-6}$$

The square root of Eq. 15-6 would then be the standard error of the estimated vulnerable area and the relative size of the resulting value might show approximately the error to be expected in vulnerability analyses or experimental determinations. Moreover, the standard error of a vulnerable area would help the analyst compare such input data to his analysis with that of other types of inaccuracy such as the chance of hitting, etc.

Example 15-1:

Aircraft vulnerability testing experience would indicate that the warhead of our SA-6 surface-to-air missile has a K-kill blast radius of 55 ft against the enemy ZU-17 tactical bomber, and that the delivery

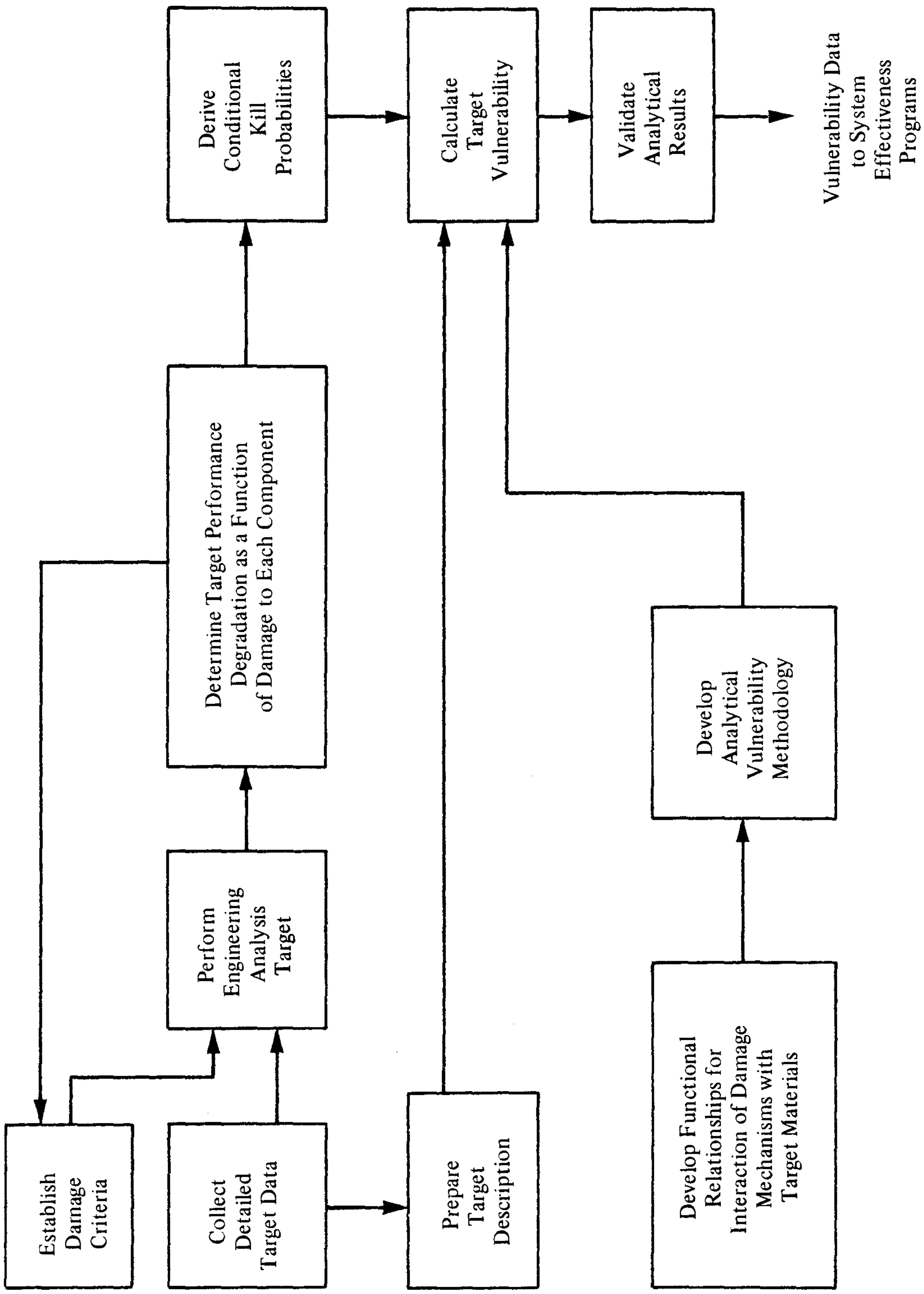

Figure 15-1 Simplified Functional Flow Chart for a Typical Vulnerability Analysis

TABLE 15-1. SOME TYPICAL TARGET DEFEAT CRITERIA

Target	Defeat Criteria	
Personnel	Defense 30 s Assault 30 s Assault 5 min Supply 12 h	The casualty criteria for personnel are dependent upon the tactical situation. A casualty results when an individual's capacity to perform his military duty is reduced by a specified degree in a specified time. For damage fragments, three tactical roles with related time periods have been defined. The times are the periods between injury and incapacitation; if the soldier is wounded by one or more fragments so that he is unable to perform a useful military function within his tactical role and he becomes incapacitated within the time specified, he is considered a casualty.
Tanks	Mobility, M-kill	Damaged so that the tank is uncontrollable and is not reparable by the crew on the battlefield.
	Firepower, F-kill	Defeat of the main armament either because the crew has been rendered incapable of operating it or because the armament or its associated equipment has been rendered inoperative and not reparable by the crew on the battlefield.
	Catastrophic, K-kill	Damaged beyond repair or to the extent that repair is not economically feasible.
Artillery	Firepower, F-kill	Damage that prevents the accurate delivery of munitions on intended targets or damage that cannot be repaired in the field.
Rocket and Launcher	Firepower, F-kill	Sufficient damage to all rockets, launcher tubes, and/or launcher system to make the weapon or system inoperable until it receives major repair.
	Mobility, M-kill	Damage to the carrier vehicle that immobilizes the vehicle within 5 min.
Armored Personnel Carrier	Mobility, M-kill	Damaged to the extent that the vehicle is uncontrollable and is not reparable by the crew on the battlefield.
	Catastrophic, K-kill	Damaged beyond repair or to the extent that repair is not economically feasible.
	Personnel, P-kill	Incapacitation of transported personnel. The P-kill is a calculated ratio of the number of personnel incapacitated to the total number of personnel being transported in the personnel compartment of the APC.
Field Fortifications	Complete K-kill	Breaching of the walls or ceiling causing a complete breakdown of structural integrity and possible filling of the interior by dirt, scabbing, and debris.
Stacked Ammunition		Complete destruction of the stack.
POL Storage Tanks		Destruction of the tank, ignition of contents resulting in sustained fire, or leakage from the tank leaving less than 25 percent of the original contents.
Trucks	Mobility, M-kill	Damage that immobilizes a moving vehicle within a given time. This type of kill is usually conditional, based on provisions for repairing the damage.
	Category A Category B Category C	Vehicle is immobilized within 5 min. Vehicle is immobilized within 20 min. Vehicle is immobilized within 40 min.
	Interdiction Kill	
	Expedient ½, 1-½, and 8 h	Expedient Repair indicates that only those components absolutely necessary to operation are repaired. Repairs may be temporary in nature; e.g., holes in liquid systems may be plugged rather than soldered or welded.
(continued)		

TABLE 15-1. (cont'd)

Target		Defeat Criteria
Trucks (cont'd)	Thorough ½, 1-½, and 8 h	Thorough Repair indicates that all components which contribute significantly to performance, safety, and efficiency of operation are repaired. Repairs are permanent in nature. The repair times indicated mean that at least one component requiring the stated repair time has been damaged. Additional components may have been damaged so that total repair time may be significantly greater than stated. The damage level associated with an 8-h Expedient Repair damage criterion is significantly greater than that associated with an 8-h Thorough Repair.
	Catastrophic, K-kill	Damage that renders the vehicle unfit for any purpose except salvage.
Buildings		Structural damage of a specified level (percent of roof or floor area). Structural damage is damage to principal load-carrying members (trusses, beams, columns, load-bearing walls) requiring replacement or special support during repair. Fifty-percent structural damage implies that half of the total floor space has been rendered unusable.
Parked Aircraft	Prevent Takeoff (PTO)	Damage such that the aircraft is unable to generate sufficient power to takeoff, or the pilot is unable to control the aircraft.
	Catastrophic, K-kill	Irreparable damage to the aircraft that renders it unfit for any purpose except cannibalization and scrap.
Helicopters	K-Damage	Damage from ground fire that causes loss of control immediately (usually taken as 30 s or less).
	A-Damage	Damage from ground fire that causes the helicopter to become uncontrollable in 5 min or less.
	B-Damage	Damage from ground fire that requires the mission to be terminated and the helicopter be landed as soon as possible.
	C-Damage	Damage from ground fire that reduces ability to perform the assigned mission.

errors including fuzing amount to an approximate spherical normal delivery distribution with one-directional sigma of about 20 ft. The SA-6 missile had design requirements for a single shot kill probability of 0.95 against the target. Do these data indicate that such requirement can be met for no aiming bias?

With a linear delivery standard error for the missile of $\sigma = 20$ ft and a K-kill blast radius of $R_d = 55$ ft, the single shot kill chance against the aircraft is

$$\left.\begin{aligned} p_k &= [1/(\sqrt{2\pi}\,\sigma)]^3 \iiint_{x^2+y^2+z^2 \le R_d^2} \exp[-(x^2+y^2+z^2)/(2\sigma^2)]\,dx\,dy\,dz \\ &= (1/\sqrt{2\pi})^3 \iiint_{u^2+v^2+w^2 \le (R_d/\sigma)^2} \exp[-(u^2+v^2+w^2)/2]\,du\,dv\,dw \end{aligned}\right\} \quad (15\text{-}7)$$

where $x/\sigma = u$, $y/\sigma = v$, $z/\sigma = w$; and u, v, and w are unit normal variables. This kill probability is easily evaluated by a transformation of variables or by simply noting that the chance that $u^2 + v^2 + w^2 \le (R_d/\sigma)^2$ is the same as the probability that chi-square with 3 degrees of freedom, i.e., $\chi^2(3)$, does not

exceed $(R_d/\sigma)^2 = (55/20)^2 = 7.56$. Looking up 7.56 in a table of the cumulative integral of $\chi^2(3)$, we see that the kill probability is about 0.944, just slightly less than the requirement (but practically okay).

For the information of the analyst, the kill probability Eq. 15-7 may be read also from *Tables of the Incomplete Γ-Function*, edited by Karl Pearson, Cambridge University Press, Cambridge, England, 1934. The quantity tabled is $I(u,p)$, where

$$u = R_d^2/(\sigma^2\sqrt{2\nu}) \tag{15-8}$$

$$p = \nu/2 - 1 \tag{15-9}$$

and ν is the number of dimensions, i.e., degrees of freedom for chi-square. Thus, for Example 15-1 we have $\nu = 3$, $p = 1/2$, and $u = (55)^2/[(20)^2\sqrt{6}] = 3.09$, so that by looking up $I(3.09,1/2)$ we get the same answer, 0.944.

(Note for the two-dimensional case, $\nu = 2$, then $u = R_d^2/(2\sigma^2)$, and $I(u,0) = 1 - \exp[-R_d^2/(2\sigma^2)]$, which is precisely Eq. 14-2, as it should be.)

Had the blast kill volume been of ellipsoidal shape and the delivery errors unequal, and even with offset center of impact (C of I) or aim, the approximate chi-square technique of par. 14-9 would be quite adequate.

15-4 WOUND BALLISTICS

For personnel, vulnerable areas are not ordinarily considered or used, although they could be. The wound ballistics program at the Ballistic Research Laboratories over the years was developed from the standpoint of providing $p(k|h)$'s, or conditional probabilities that a hit is an incapacitation. Such conditional probabilities, which are of fundamental importance in the evaluation of antipersonnel weapons, are expressed in terms of a law giving chance of incapacitation as a function of the striking velocity and mass of fragments, bullets, or flechettes. For an account of such work, the analyst should study Refs. 24-28. Again, the wound ballistics program involves much effort and consequently has been a special program for many years, providing incapacitation type information to the system analyst and others. Such information also can be used to predict medical loads expected in combat from casualty prediction data.

As a result of wound ballistic studies, the conditional probability that a hit is an incapacitation is expressed in terms of the Weibull law

$$p(k|h) = 1 - \exp[-a(mv_s^{3/2} - b)^c] \tag{15-10}$$

where

m = fragment mass
v_s = striking velocity
a, b, and c = empirically determined constants depending on the type of incapacitation.

The $p(k|h)$ of Eq. 15-10 is for single, not multiple, fragments.

15-5 LETHAL AREAS OR MEAN AREAS OF EFFECTIVENESS

Vulnerability is ordinarily a term used for the case where actual hits are obtained on targets such as tanks and aircraft. Lethality, on the other hand, refers primarily to the case where lethal or incapacitating fragments, for example, are projected over an area on the battlefield to incapacitate personnel. A properly designed artillery projectile usually should be detonated in the air so that it will

project lethal fragments over as large an area as possible and hence incapacitate enemy troops. Thus, in speaking of lethal areas, we must keep in mind that the effected area is often much larger than that of a single target such as a tank or a target element. (In passing, we remark that for Example 15-1 one might speak of a "lethal volume" for the aircraft, since a detonating warhead will cause a kill of the aircraft well outside of the actual volume it occupies.)

Lethal area is defined in a manner similar to that of vulnerable area, except that we keep in mind the projection of fragments over areas that might cover several or many target elements such as personnel. In addition, it should be noted that the height of burst of a fragmenting warhead is of importance, as is also the random variation in the point of burst due to fuzing. To develop the concept, we picture a warhead or artillery projectile detonating on the ground or in the air and the projection of lethal fragments which might strike enemy personnel, who might be standing, kneeling, prone, or even in foxholes. Also, personnel usually are protected by the terrain or ground cover. Such conditions must be taken into account since shielding due to terrain undulations may stop some of the fragments or render them very ineffective. A target under attack by artillery could consist of many personnel performing various functions of combat, and such considerations also must be taken into account in the computation of lethality. Generally speaking, lethal areas are computed for single conditions such as prone personnel. A weighted average for a composite target—consisting of for example, 30% standing troops, 40% kneeling, and 30% prone—could be computed and used in an analysis as required.

The analytical definition of lethal area A_L is

$$A_L = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_k(x,y\,|\,H)dxdy^* \tag{15-11}$$

where $p_k(x,y\,|\,H)$ is the conditional probability that a hit on a target element located at the point (x,y) on the ground results in an incapacitation for given height of burst H.

If a random distribution in height-of-burst is considered, due to fuzing, then the average lethal area becomes

$$A_L = \int\int\int f(H)p_k(x,y\,|\,H)dxdydH \tag{15-12}$$

where $f(H)$ is the probability density function for height of burst, x and y are integrated over infinite limits as before, and the height H is integrated over applicable fuzing limits.

A good way to think of a lethal area determination is to consider an artillery projectile detonating at a height H above the origin and then moving a single target element, such as a standing soldier, all over the ground plane, and computing the chances of incapacitation as the element moves. These chances of incapacitation will depend on the ground cover, the areas or portions of the target exposed as a function of its location (x,y), and the chance of incapacitation based on wound ballistic conditions and criteria. The elementary areas, $dxdy$, are then weighted by kill chances and summed or integrated over the ground plane or terrain to obtain A_L.

The so-called Mean Area of Effectiveness (MAE) is the same as the lethal area, whichever is used being a matter of preference. Mathematically, they are equivalent.

Clearly, lethal areas or MAE's often can be used, or they may be all that is needed to determine the relative effectiveness of munitions. For example, if two artillery projectile designs involve the same

*A_L multiplied by troop density equals expected number of personnel incapacitated.

caliber and have the same delivery errors, but one has a larger lethal area than the other, then it is the more effective. Thus, for example, a suitably large artillery projectile could contain submissiles that would spread over an area and then detonate. We see, therefore, that the lethal area alone may in many cases be a logical measure of effectiveness (MOE) itself.

The lethal area has the dimensions of an area, but it is not a clearly defined area in the physical sense, but rather one for which chances of incapacitation are used as weights. Therefore, it is more or less an "average" area in which incapacitations of target elements would occur if they happen to be at points located in the lethal fragment spray. For the purpose and ease of analysis, lethal areas often are considered to be regular figures such as circles, although detonating artillery projectiles exhibit very different properties, having a predominant side-spray of fragments.

A good useful description of lethal area is given in Ref. 29, and general purpose computer programs may be found in Refs. 30 and 31. Estimates of lethal areas are classified but may be found in appropriate publications, such as Refs. 9, 32, 33, and 34.

Estimates of lethal areas, like those of vulnerable areas, are subject to variation or error, and the variance of the estimates are of interest in connection with sensitivity analyses of the importance of input parameters in an overall weapon systems effectiveness study. The reader might review, for example, Ref. 35. Also, Refs. 36-41 cover details relative to the precision of lethal areas. Ref. 37 indicates standard errors of lethal areas may be of the order of four or five percent.

The so called "cookie-cutter" concept is used along with that of lethal area. If the lethal area happens to cover a target element, then that target element is "killed". A near miss by the boundary of the "cookie-cutter" results in no damage whatever. To illustrate, we give an example.

Example 15-2:

An artillery type rocket has circular normal delivery errors of sigma equal 50 m at 5000 m range and the fuzing is such that the mean lethal area of its warhead against troops on average ground is 2000 m^2. If such a rocket is used to attack an enemy personnel target of 50 m radius, then what is the expected fraction of casualties?

We will assume that the lethal area is circular in shape, and recall from Chapter 14, par. 14-6, that the integral of the circular coverage function $p(R,r)$, which is labeled $S(R,r)$, gives the mean or expected overlap of a circle of radius r dropped randomly on a circular target of radius R with equal normal delivery errors in x and y. As mentioned previously, this is called a "cookie-cutter" type of analysis, where a target element covered by the dropped circular lethal area is killed or incapacitated and those outside its circumference are completely safe. Thus, in accordance with Germond, RM-330, Ref. 19 of Chapter 14, we take the damage radius r_L to be $\sqrt{2000/\pi} = 25.23$ m, the target radius $R = 50$ m, and the delivery sigma as $\sigma = 50$ m. We then look up

$$S(R/\sigma, r_L/\sigma) = S(50/50, 25.23/50) = S(1.0, 0.50)$$

in Table 15-2. We find that $S(1.0,0.50) = 0.0938$, which is the expected fraction of the target covered, hence there would be only about 9.4% casualties per round. For a larger fraction of casualties, multiple rounds must be fired.

The expected fractional coverage for lethal circles dropped on circular targets may also be obtained from M. P. Jarnagin, Jr., "Expected Coverage of a Circular Target by Bombs All Aimed at the Center", Naval Weapons Laboratory Report No. 1941, US Naval Weapons Laboratory (NWL), Dahlgren, VA, 30 June 1965. In using the tables of this report, we put our r_L/σ = Jarnagin's A and our R/σ = Jarnagin's R. For the Example 15-2, we find on page 30 of Jarnagin's table for $N = 1$, (i.e., a single round) his $R = 1$ and $A = 0.5$, the value 0.094.

TABLE 15-2. VALUES OF THE INTEGRAL OF THE CIRCULAR COVERAGE FUNCTION

r \ R	0	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0
0.1	0.0050	0.0047	0.0039	0.0030	0.0022	0.0015	0.0011	0.0006	0.0004	0.0003
0.2	0.0198	0.0186	0.0156	0.0119	0.0086	0.0061	0.0044	0.0025	0.0016	0.0011
0.3	0.0440	0.0414	0.0348	0.0267	0.0193	0.0137	0.0099	0.0056	0.0036	0.0025
0.4	0.0769	0.0724	0.0611	0.0470	0.0342	0.0243	0.0175	0.0100	0.0064	0.0044
0.5	0.1175	0.1109	0.0938	0.0725	0.0530	0.0379	0.0274	0.0156	0.0100	0.0069
0.6	0.1647	0.1557	0.1323	0.1029	0.0756	0.0543	0.0394	0.0225	0.0144	0.0100
0.7	0.2173	0.2057	0.1757	0.1377	0.1019	0.0736	0.0535	0.0306	0.0196	0.0136
0.8	0.2739	0.2598	0.2231	0.1762	0.1314	0.0955	0.0697	0.0400	0.0256	0.0178
0.9	0.3330	0.3167	0.2737	0.2180	0.1640	0.1201	0.0879	0.0506	0.0324	0.0225
1.0	0.3935	0.3751	0.3263	0.2625	0.1994	0.1470	0.1082	0.0624	0.0400	0.0278
1.1	0.4539	0.4339	0.3802	0.3088	0.2370	0.1763	0.1304	0.0755	0.0484	0.0336
1.2	0.5132	0.4919	0.4343	0.3565	0.2766	0.2076	0.1545	0.0898	0.0576	0.0400
1.3	0.5704	0.5483	0.4879	0.4049	0.3178	0.2408	0.1804	0.1053	0.0676	0.0469
1.4	0.6247	0.6022	0.5402	0.4534	0.3601	0.2757	0.2079	0.1221	0.0784	0.0544
1.5	0.6753	0.6529	0.5905	0.5014	0.4031	0.3119	0.2371	0.1400	0.0900	0.0625
1.6	0.7220	0.7001	0.6383	0.5482	0.4464	0.3493	0.2676	0.1592	0.1024	0.0711
1.7	0.7643	0.7432	0.6832	0.5936	0.4896	0.3875	0.2995	0.1795	0.1156	0.0803
1.8	0.8021	0.7822	0.7248	0.6370	0.5322	0.4262	0.3325	0.2009	0.1296	0.0900
1.9	0.8355	0.8171	0.7629	0.6781	0.5739	0.4652	0.3663	0.2235	0.1443	0.1003
2.0	0.8647	0.8478	0.7974	0.7167	0.6142	0.5041	0.4010	0.2471	0.1599	0.1111
2.1	0.8897	0.8745	0.8284	0.7525	0.6531	0.5427	0.4361	0.2718	0.1763	0.1225
2.2	0.9111	0.8975	0.8559	0.7854	0.6900	0.5805	0.4715	0.2974	0.1934	0.1344
2.3	0.9290	0.9171	0.8800	0.8153	0.7249	0.6175	0.5070	0.3239	0.2113	0.1469
2.4	0.9439	0.9336	0.9009	0.8423	0.7575	0.6532	0.5423	0.3513	0.2300	0.1600
2.5	0.9561	0.9473	0.9189	0.8664	0.7877	0.6874	0.5772	0.3794	0.2494	0.1736
2.6	0.9660	0.9586	0.9342	0.8877	0.8155	0.7201	0.6115	0.4081	0.2696	0.1878
2.7	0.9739	0.9677	0.9470	0.9064	0.8408	0.7509	0.6449	0.4374	0.2905	0.2025
2.8	0.9802	0.9751	0.9578	0.9225	0.8636	0.7797	0.6772	0.4671	0.3122	0.2177
2.9	0.9851	0.9810	0.9666	0.9364	0.8841	0.8065	0.7082	0.4970	0.3345	0.2336
3.0	0.9889	0.9856	0.9738	0.9483	0.9022	0.8312	0.7378	0.5272	0.3574	0.2499
3.1	0.9918	0.9892	0.9797	0.9583	0.9181	0.8538	0.7658	0.5573	0.3810	0.2668
3.2	0.9940	0.9920	0.9844	0.9666	0.9320	0.8742	0.7922	0.5872	0.4051	0.2843
3.3	0.9957	0.9941	0.9881	0.9735	0.9439	0.8926	0.8167	0.6168	0.4298	0.3022
3.4	0.9969	0.9957	0.9910	0.9792	0.9541	0.9089	0.8394	0.6459	0.4549	0.3207
3.5	0.9978	0.9969	0.9932	0.9837	0.9628	0.9233	0.8602	0.6744	0.4803	0.3398
3.6	0.9985	0.9978	0.9950	0.9874	0.9701	0.9360	0.8792	0.7021	0.5061	0.3593
3.7	0.9989	0.9984	0.9963	0.9904	0.9761	0.9469	0.8963	0.7288	0.5322	0.3793
3.8	0.9993	0.9989	0.9973	0.9927	0.9811	0.9564	0.9116	0.7545	0.5583	0.3998
3.9	0.9995	0.9992	0.9980	0.9945	0.9852	0.9644	0.9252	0.7789	0.5845	0.4208
4.0	0.9997	0.9995	0.9986	0.9959	0.9884	0.9712	0.9372	0.8021	0.6106	0.4422
4.1	0.9998	0.9996	0.9990	0.9969	0.9911	0.9768	0.9476	0.8239	0.6364	0.4639
4.2	0.9999	0.9998	0.9993	0.9978	0.9932	0.9816	0.9567	0.8442	0.6620	0.4861
4.3	0.9999	0.9998	0.9995	0.9984	0.9948	0.9854	0.9644	0.8631	0.6871	0.5085
4.4	0.9999	0.9999	0.9997	0.9988	0.9961	0.9886	0.9710	0.8804	0.7117	0.5312
4.5	1.0000	0.9999	0.9998	0.9992	0.9971	0.9911	0.9766	0.8962	0.7356	0.5541
4.6	1.0000	0.9999	0.9998	0.9994	0.9979	0.9932	0.9812	0.9106	0.7587	0.5772
4.7	1.0000	0.9999	0.9999	0.9996	0.9984	0.9948	0.9851	0.9234	0.7809	0.6003
4.8	1.0000	1.0000	0.9999	0.9997	0.9987	0.9960	0.9882	0.9349	0.8021	0.6234
4.9	1.0000	1.0000	1.0000	0.9998	0.9992	0.9970	0.9908	0.9451	0.8222	0.6464
5.0	1.0000	1.0000	1.0000	0.9999	0.9994	0.9978	0.9928	0.9540	0.8412	0.6693
5.1	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9945	0.9617	0.8590	0.6919
5.2	1.0000	1.0000	1.0000	0.9999	0.9997	0.9988	0.9958	0.9684	0.8756	0.7141
5.3	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9968	0.9741	0.8908	0.7358
5.4	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9976	0.9790	0.9049	0.7570
5.5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9982	0.9830	0.9176	0.7775
5.6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9864	0.9291	0.7973
5.7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9892	0.9394	0.8163
5.8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9915	0.9486	0.8343
5.9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9934	0.9567	0.8514
6.0	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9949	0.9638	0.8675
6.1	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9961	0.9699	0.8825
6.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9970	0.9752	0.8964
6.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9977	0.9797	0.9093
6.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9983	0.9835	0.9210
6.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987	0.9867	0.9317

The Jarnagin tables may be used for any number of rounds up through $N = 20$. For example, if for the previous data we were to fire a volley of $N = 4$ rounds all aimed at the target center with the same delivery error, then from page 39 of the tables we read that the fractional target coverage or fraction of casualties would be 0.325 or 32.5%.

Appendix B of the Jarnagin NWL Report No. 1941 is also valuable to the systems analyst, since the table therein gives the expected number of rounds, or warheads, of lethal radius $A = r_L/\sigma$ (in standard units) to give expected fractional coverages of $E = 0.05(0.05)0.95$* for a target of radius R/σ.

As we develop the subject of target damage generally, much more will be covered for the case of multiple rounds. We should record in this chapter, however, the idea of damage probability patterns for multiple rounds since this concept will also be used in addition to the cookie-cutter type of analysis.

15-6 DAMAGE OR KILL PROBABILITY PATTERNS

The concept of lethality in the form of lethal areas of warheads along with the use of cookie-cutter type approximations to evaluate the effectiveness of weapons represents one method of approach for systems analyses. In many cases, such an approach is entirely adequate, while for others some questions on accuracy or realism may be raised. As an alternative development, we now cover the concept of using an analytical approximation to the kill or damage probability pattern, which may be integrated along with the delivery error distributions of rounds for overall effectiveness computations. With such an approach, therefore, lethal areas are not employed directly, but in effect are taken care of by use of an analytical function, as discussed in Ref. 42.

To illustrate the process, refer to the pattern of Fig. 15-2, where we consider the problem of volley fire for a battery or battalion of artillery. The table gives values of damage probabilities for a volley of rounds fired and may be somewhat typical for illustrative purposes here. Each two-digit figure in the pattern represents a damage probability within the square of, say, 15 units on a side. For italicized figures, the decimal point should be placed just before the two digits given (for example, *17* = 0.17 for the correct damage probability); for underlined figures, insert a zero to the left of the two digits and then place the decimal just before the zero (for example, 93 = 0.093 in damage probability); and, for the ordinary unmarked figures, add two zeros and then the decimal (for example, 18 is to be taken as 0.0018 in damage probability). The so-called "lethal area" for this pattern (Fig. 15-2) is found from the formula $A_L = \sum\sum p_k(x,y)\Delta x\Delta y$, where $\Delta x\Delta y = 225$, and turns out to be about 14002 square units.

We now consider fitting a kill function of the exponential square fall-off form

$$p_k(x,y) = c\,\exp[-x^2/(2\sigma_{kx}^2) - y^2/(2\sigma_{ky}^2)] \tag{15-13}$$

where x and y are measured from the center of the kill or damage pattern, σ_{kx} and σ_{ky} are measures of the variation (standard deviations) in kill probability weighted distances in the x and y directions, respectively; and c is a constant giving the maximum average kill probability at the center of the lethality pattern. The approximate damage function (Eq. 15-13) may be easily related to the lethal area of the damage pattern. The relation is given by

$$A_L = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_k(x,y)\,dx\,dy = 2\pi c\sigma_{kx}\sigma_{ky} \tag{15-14}$$

*This notation indicates the range is from 0.05 to 0.95 in increments of 0.05.

		1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1							
1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1				
2	2	2	3	3	3	3	3	4	4	4	4	4	4	4	4	3	3	3	3	3	3	3	2	2	2	2	1	1	1	
3	3	4	4	5	5	6	6	5	5	5	5	5	5	5	5	5	5	5	5	4	4	4	4	3	3	2	2	2	2	1
4	5	6	7	8	9	9	9	9	8	8	8	8	8	8	8	8	7	7	7	7	6	6	5	5	4	4	3	3	2	2
6	8	10	12	14	15	15	15	14	13	13	13	14	14	13	13	12	11	11	11	11	10	9	9	7	6	6	5	4	3	3
10	13	17	22	26	28	28	26	23	22	22	22	23	23	22	21	19	18	18	18	17	16	15	14	12	10	8	7	6	5	4
16	22	30	40	52	56	55	47	41	38	36	38	42	42	40	35	32	30	29	29	29	28	26	23	20	16	13	11	9	7	5
30	41	57	79	12	11	12	91	77	70	68	72	83	90	80	64	56	52	51	51	50	49	45	40	33	26	21	17	13	10	8
38	66	12	19	24	19	25	20	15	11	11	14	16	17	16	14	10	83	80	84	90	90	85	71	57	45	35	27	21	16	12
47	64	91	13	19	14	21	16	13	12	12	13	13	13	13	13	12	12	13	15	18	18	17	14	10	80	60	45	33	24	18
68	10	16	24	32	21	34	29	23	20	19	21	25	22	25	22	20	20	24	31	36	30	35	28	19	13	98	72	52	37	27
87	13	20	29	43	38	46	37	32	30	33	40	47	45	48	41	35	33	39	48	59	54	57	43	31	22	16	11	78	55	39
12	18	28	41	56	58	62	54	50	47	53	63	80	72	80	65	56	58	60	75	99	79	96	68	46	32	24	17	12	81	57
18	26	41	61	85	62	95	83	74	65	81	10	13	11	13	11	84	78	86	11	14	97	14	98	72	51	40	27	18	12	82
24	39	64	10	15	11	16	12	10	86	10	12	14	12	14	12	93	83	99	13	17	16	17	13	10	70	60	39	25	16	10
28	45	75	12	20	18	21	16	14	12	14	15	18	15	18	14	12	11	12	14	17	15	16	12	97	74	61	41	27	19	13
28	47	82	14	22	23	25	21	20	16	21	24	29	24	28	22	19	17	17	19	24	20	23	18	15	10	98	68	43	27	18
27	45	78	13	20	18	23	21	22	21	24	28	34	29	35	28	24	19	22	25	30	28	29	23	19	15	14	93	58	35	22
28	44	73	12	19	19	23	21	23	19	24	25	31	30	32	27	26	21	24	23	25	22	25	22	22	15	18	12	72	45	27
27	46	84	14	22	21	26	24	24	22	24	25	29	29	29	24	24	20	23	22	24	18	24	23	26	23	22	14	76	43	26
21	37	64	11	17	16	21	20	23	19	23	20	22	18	22	22	25	24	26	25	26	24	25	23	23	19	19	12	71	42	26
17	28	49	90	15	15	20	22	27	24	27	22	19	18	18	20	24	22	25	22	22	21	22	22	24	19	20	14	78	43	25
12	22	42	71	11	12	16	18	22	17	21	17	13	12	13	16	21	16	21	16	14	14	14	15	19	16	18	11	67	38	23
60	11	20	36	60	79	10	13	18	16	18	13	11	96	13	18	26	23	26	18	13	10	10	13	18	15	17	11	63	38	23
32	52	91	17	27	41	68	11	17	16	17	12	82	68	77	11	17	17	17	11	73	65	87	13	19	17	19	12	68	36	20
18	27	42	70	11	18	34	64	11	10	11	69	45	38	46	67	10	96	10	72	53	44	57	87	14	13	13	76	39	21	12
16	22	36	59	87	15	22	39	55	54	57	44	33	23	34	42	56	56	54	40	29	25	30	48	75	75	73	44	22	12	71
13	23	45	67	95	14	19	25	33	35	32	26	26	24	25	26	35	39	34	24	18	18	22	33	53	37	52	31	17	10	54
3	5	8	12	20	29	31	39	71	11	73	43	41	57	10	19	28	31	28	19	11	85	11	23	42	46	42	22	87	37	17
2	2	2	2	2	2	1	1	2	3	4	5	9	17	35	75	11	11	11	76	37	20	13	10	8	5	6	6	6	5	4
								1	1	2	2	2	2	3	4	4	3	4	4	3	2	2	2	2	1	1				

Figure 15-2. Damage-Probability Pattern for a Volley of Artillery Rounds

which is obtained simply by integrating the right-hand side of Eq. 15-13. Using the marginal totals of damage probability in the x- and y-directions, one may compute in a straightforward manner the standard deviations $\sigma_{kx} = 110$ and $\sigma_{ky} = 61$. Now, since in this case we would put $A_L = 2\pi c\sigma_{kx}\sigma_{ky}$, then by substitution we find $c \approx 0.33$. We also note that the maximum $p_k(x,y)$ within and near the center of the pattern is 0.35; therefore, a good check is provided and the computed or estimated values for σ_{kx}, σ_{ky}, and c may be used with some assurance for the exponential square fall-off lethality law for volley or salvo fire. Thus, these values would be substituted directly into analytical formulas instead of values of A_L for the individual rounds. Of course, this particular treatment of lethality or damage patterns may represent a rather favorable case for the exponential square fall-off assumption, and it may occur sometimes that many other damage patterns could not be fitted so well, or the assumptions here may not apply well. Also, it is to be recognized that since the fragmentation of warheads is random in character, then so are the $p_k(x,y)$ values; hence we assume that the figures given represent average values. We have made some Monte Carlo comparisons with actual data and have found the agreement for the expected fraction of coverage to be quite acceptable, nevertheless, for the cases studied.

The principle of using approximate analytical fits to lethality data may be applied to much advantage in expected target damage problems or models for multiple rounds since most of the computations involve various approximations anyway.

Finally, computer programs are now available and are widely used for lethality computations and for expected target damage or target coverage problems as well. Such computer programs do not necessarily display lethal areas as such but rather often use the original conditional chances of kill for cells. The reader is referred to Ref. 43 for an example of an expected target damage computer program. As the reader will learn, there are a variety of models for estimating expected target damage or coverage, and as a matter of fact not enough has been done to check the models against each other, or otherwise validate them, for the purpose of picking the best or more accurate one as a standard. Different methods of analysis arise at different Army installations and are often programmed for computation; in which case some might think that, since such a computer model is available, all problems are automatically taken care of! However, for some of the computer models it becomes very difficult to know just what particular parameters are included and how random variables are handled in computations. Analysis of the accuracy or variability to be expected in vulnerable areas and lethal areas still represents an area of much concern, for a much improved account may be needed in evaluating subjective judgments relating to the conditional probabilities of kill.

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CHAPTER 16

RATES OF FIRE

In view of the importance of the rate of fire of weapons, this topic is introduced and explored in some preliminary detail for the weapon systems analyst.

16-0 LIST OF SYMBOLS

A = area in which targets may be located
A_T = part of A actually exposing targets
E = expected number of kills
e = natural or Naperian base of logarithms
$f(R)$ = probability density function of R
N_0 = number of rounds initially on hand
n = number of rounds
$P(R \leq R_O)$ = chance that R is less than or equal to R_O
$p(h)$ = hit probability
p_k = kill probability
$p(k|h)$ = chance of kill given a hit
R_I = intrinsic rate of fire (maximum rate)
R_O = operational rate of fire (sometimes designated by R, a random rate of fire)
R_s = sustained rate of fire
r = rate of fire
$S(T_d)$ = number of rounds resupplied during time $T = 0$ to time $T = T_d$
T = time
T_d = firing time duration
t_1 = time required for extraction of a spent casing
t_2 = time required for insertion and arming of a round
t_3 = time required for aiming and firing
ρ = expected or mean rate of fire = $E(R_O)$

16-1 GENERAL

One of the very important parameters by which the performance of a weapon system is characterized is its rate of fire. It is a parameter used to describe the key characteristics of a weapon in addition to weight, caliber, muzzle velocity, range, warhead effectiveness, etc. Increasing rate of fire tends to reduce time to obtain a "kill", and indeed survival in battle often may depend on rate of fire. Rate of fire is defined in Ref. 1 as "The number of rounds fired per weapon per minute." Numerically, this quantity may vary from a small fraction, as in the case of a complex surface-to-surface missile system to several thousand rounds per minute for a VULCAN type weapon.

The push to develop high rate of fire weapons has much historical significance, and all nations have recognized the implication of rapid-fire weapons in combat. The classical example of a rapid-fire weapon is, of course, the "Gatling" gun of Richard Jordan Gatling (1818-1903). At the outbreak of the Civil War, Gatling, an inventor, devoted himself to the perfecting of firearms. In 1861, he conceived the idea of a revolving multibarrel, rapid-fire machine gun that is associated with his name. By 1862, he

succeeded in developing a gun which would discharge 350 rd/min—this representing more or less a "breakthrough" in rate of fire. Although the Civil War was over before Federal authorities consented to official adoption of the Gatling gun, the invention was later adopted by almost every civilized nation because of its substantial improvement in effectiveness. High rate of fire guns have always been mandatory for air-to-air combat and for air defense applications also. The M61A1 six-barrel 20mm VULCAN Gun uses its highest rates of fire (4000 to 6000 rd/min) in aircraft. For the air defense role it is mounted on a ground vehicle and fires 1000-3000 rd/min. Rate of fire is becoming increasingly important in various ground combat operations. The analyst may read AMCP 706-260 (Ref. 2) for general technical information on automatic weapons.

Once a target is engaged, rate of fire, or enough rounds fired in time, may become of great urgency. For example, if the kill probability per round against an enemy target which may return fire is 0.5, then two rounds in time results in an expected kill probability of 0.75, and three rounds in time an expected kill probability of 0.875, etc.

We do not intend to place undue emphasis on rate of fire here, for rounds of ammunition could be wasted or "overkills" result in the indiscriminate use of automatic or high rate of fire weapons against some targets. Nevertheless, since the chance of at least one hit on a target depends on the number of rounds fired from a weapon, then rate of fire becomes important, especially for targets of short duration or "fleeting" targets. Indeed, it easily can be seen that weapon delivery errors, lethality of rounds, and rate of fire must be optimized in some way for the most effective fire.

16-2 COMMENT ON KILL RATES OR ATTRITION COEFFICIENTS

The idea of an expected kill rate for a weapon represents a topic of much interest to the analyst. Recalling that the expected kill probability per round may be described as the chance of a target hit $p(h)$ times the conditional probability $p(k|h)$ that a hit results in a kill, we see that the expected number of kills E for n rounds fired is

$$E = np_k = np(h) \cdot p(k|h) \ . \tag{16-1}$$

Furthermore, if r is the rate of fire, e.g., rd/min of a given weapon, then the expected kill rate or the target kills per unit of time for aimed fire may be described by

$$\text{Kill rate} = rp_k = rp(h) \cdot p(k|h) \ . \tag{16-2}$$

For the case of weapons firing indiscriminately or at random into an area A occupied by "targets", which might cover a part A_T of A, then the kill rate for uniform fire on the average would be given by

$$\text{Kill rate} = rA_T/A \ . \tag{16-3}$$

Thus, kill rates, or attrition coefficients as they are often called, represent the expected number of targets put out of action per unit of time. It becomes important to note that doubling the rate of fire would be expected to double the kill rate, etc. For some weapon-target applications we see that high rates of fire may be essential such as in a "duel", whereas in other cases it may not be required since low kill rates would be quite satisfactory.

16-3 DESCRIPTION OF TYPES OF RATE OF FIRE

In discussing rates of fire, it is useful to define three types:

1. *Intrinsic rate of fire* R_I. This is the maximum rate of fire that can be achieved for a given weapon. This quantity is primarily determined by the physical characteristics of the weapon system and is

given by the reciprocal of the total time required for the extraction of the spent casing, loading of the next round, arming (if appropriate), and firing. Values of R_I for specific weapon systems may be obtained from appropriate FM's and TM's.

2. *Operational rate of fire R_O or simply R.* This is a random variable and is the actual rate of fire achieved by a weapon in a given situation. This quantity includes the effects of the operational and physical environment upon the functioning capability of the weapon, as well as such random factors as operator abilities, training, and ammunition characteristics. Since R_O is a random variable, the operational rate of fire often is described in a probabilistic sense by its corresponding probability density function or distribution.

3. *Mean rate of fire ρ.* This is the expected or mean value of the operational rate of fire. Hence, it too will be a function of factors such as the operational and physical environments, weapon and operator characteristics, and ammunition characteristics. When describing the operational rate of fire of a weapon, it is generally more appropriate to refer to the expected value ρ rather than any instantaneous value R. This method of description is somewhat analogous to that used in describing the reliability of a system in terms of its failure times, i.e., mean time between failures, or other types of statistical parameters.

16-4 APPLICATION

The rate-of-fire parameter may be an important input to any weapon system analysis model. For example, for complex war game models, the rate of fire must be used in conjunction with the scenario employing weapons in a hypothetical or computer played battle to estimate the number of enemy targets destroyed at various times in an engagement. Ref. 3 reviews several Army gaming models for the evaluation of weapon systems (see also Chapter 40).

The nature and detail of the model usually will determine whether to describe the rate of fire in terms of its expected value or as a random variable. If the model is deterministic, then the expected or average value of rate of fire most likely would be used. For example, Ref. 4 describes a deterministic Lanchester combat theory type of modèl for guerrilla warfare in which the use of mean rate of fire is illustrated.

If the model is stochastic such as a Monte Carlo simulation of an engagement, then the rate of fire may be programed by its statistical distribution and hence treated as a random variable. In such a situation, the rate-of-fire inputs are then obtained by sampling the given probability distribution. The exponential distribution with probability density $f(R) = (1/\rho)\exp(-R/\rho)$, where ρ is the mean rate of fire, is often used for this application. With this distribution, the chance of a firing rate less than or equal to an operational rate R_O is

$$P(R \leq R_O) = \int_0^{R_O} f(R)dR = 1 - \exp(-R_O/\rho) \tag{16-4}$$

and the chance of a firing rate greater than or equal to R_O is simply

$$P(R \geq R_O) = \exp(-R_O/\rho). \tag{16-5}$$

(Note that for the exponential distribution, the chance of a firing rate exceeding the expected rate is $e^{-1} = 0.37$.)

Distributions other than the exponential also may be used to represent the random firing rate. Choice of the specific distribution to be used will depend upon the closeness of fit of the assumed distribution function to available observed data for the weapon under study, e.g., distributions such as the normal, lognormal, Rayleigh, or Weibull might be employed. In fact, Ref. 5 indicates for the automatic rifle studied, the average rate of fire is 325.9 rd/min and the standard deviation only 4.27 rd/min, which may be far from exponential! Ref. 6 presents methods for sampling these various types of distributions for a Monte Carlo simulation, and also provides the FORTRAN programs for accomplishing the sampling.

Another method for determining needed random rate-of-fire inputs to a simulation is to sample the distributions of the individual factors which make up the rate-of-fire parameter. To illustrate, assume the firing cycle time—i.e., the reciprocal of the rate of fire for a specific weapon—is given by the sum of the times for extraction of the spent casing t_1, insertion and arming of the next round t_2, aiming and firing at the next target t_3, and beginning the cycle anew. Further, assume that all of these times are random variables with known (or approximately known) statistical distributions. The instantaneous cycle time may then be found by sampling each of the three distributions, e.g., using the sampling procedures described in Ref. 6, and then adding the obtained times to determine the total time for the cycle. The instantaneous rate of fire is then found by taking the reciprocal of the total time for that particular cycle, and the procedure repeated with new random inputs to obtain another rate of fire, etc. Alternatively, the probability distribution of the sum of random variables may be found from proper analytical procedures described in textbooks on statistics.

Rates of fire must be taken into account even for artillery weapons. For example, the so-called "time-on-target" procedure whereby a high volume of artillery fire can be delivered in a short time period has been very effective indeed, as has also the so-called "surge" capability of massing artillery fires. The use of artillery for sustained rates of fire is well known, but there also is a requirement to make studies of burst rates of fire which may be needed from time to time. Some studies (Refs. 7 and 8) covered investigations on the need for burst rates of fire for artillery weapons and especially the 155mm gun-howitzer (G/H) against moving tank and truck targets, and against personnel targets located within an enemy site. Ref. 7 examined the need for increasing the rate of fire over the existing sustained rate of fire for the XM198, 155mm G/H artillery weapon, and attempted to determine what burst rates appeared to be needed. Delivery accuracy, projectile reliability, lethality of projectiles, and several rates of fire were all taken into account. A significant advantage was found for burst rates of fire in some tactical situations, although under other conditions burst rates appeared to have little or no value. Additional field testing was deemed necessary to define better what the expected personnel and vehicular reactions would be for such targets coming under fire from artillery.

Kinematic studies of typical small arms weapons in automatic fire often are made for the purpose of studying rate of fire, muzzle velocity, target dispersion data, trunnion forces, and displacement versus time data. Rate of fire depends on the weight of recoiling mass as indicated, for example, in Fig. 16 of Ref. 9. Some other kinematic type studies are covered in Refs. 5, 10, and 11.

Rate of fire also may have an important effect on the performance of weapons. For example, high rates of fire have a decided effect on the round-to-round dispersion of machine guns as compared to single-shot fire, increasing the dispersion substantially. The increased round-to-round dispersion would, of course, decrease hit probabilities against any single target, although the rapid rate of fire and consequent number of rounds fired in a short period of time make automatic weapons very suitable for defense against multiple targets such as enemy personnel approaching a position, or for air defense in which case rapidly moving targets are involved, as we have indicated already.

Finally, high rates of fire result in heating gun tubes, and some bounds on burst rates may have to be considered, as we next discuss.

16-5 RATE-OF-FIRE BOUNDS

The preceding paragraphs treated rate of fire and indicated it may be a random variable with statistically described characteristics. Further, they covered the concept of an intrinsic rate of fire, the value of which is determined primarily by the physical limitations of the weapon and represents the maximum achievable rate of fire. However, there are additional aspects of the rate-of-fire parameter which often will have to be considered in a weapon systems analysis. Two such aspects are the thermal stress to the weapon and the ammunition supply. These factors depend on the firing duration and will generate bounds or envelopes on the mean rate of fire as a function of firing duration.

16-5.1 THERMAL BOUNDS

A thermal bound will exist for a given rate of fire of a weapon as a result of the increase in barrel temperature with time as successive rounds are fired. A plot of the barrel temperature versus time for the weapon will show an initial steep slope followed by an eventual leveling off at an equilibrium or steady state value. However, if this barrel temperature reaches the self ignition or "cook-off" temperature of the ammunition, the weapon cannot be operated safely. For example, the temperature limit for the 60-mm mortar HE round is 700°F and its maximum firing rate of 30 rd/min is limited to 1 min (Ref. 12). The temperature leveling off and ammunition cook-off bounds have resulted in the concept of "sustained rate of fire" R_s which is defined (Ref. 1) as the "Actual rate of fire that a weapon can continue to deliver for an indefinite length of time without seriously overheating." The sustained rates of fire for existing weapon systems may be found in the appropriate FM's or TM's. The sustained rate of fire for the 60-mm mortar mentioned previously was found experimentally to be 8 rd/min (Ref. 12). The sustained rate of fire thus is the highest rate that falls under the maximum thermal bound that will be reached with time for a given weapon system. For intermediate firing rates, i.e., between the maximum and sustained rates of fire, the bound will be defined by the time the weapon can fire at that rate before reaching the cook-off temperature. Again, referring to the 60-mm mortar example from Ref. 12, the experimental results showed that a firing rate of 18 rd/min could be sustained for 4 min before reaching the thermal bound. Fig. 16-1 illustrates how this thermal envelope would appear for a given weapon system. The region below the curve represents the rates of fire that would be permissible for a given firing period, whereas firing rates in the region above the curve for the same firing period will cause the barrel temperature to exceed its cook-off limit.

Determination of the thermal envelope for the rate of fire of a specific weapon system can be made either by testing the weapon or perhaps also through a theoretical analysis of the thermal effect of the weapon firing process. The Aberdeen Proving Ground and White Sands Missile Range are the primary test facilities used by the Army for experimental investigations of the thermal envelopes of weapons and determination of the maximum and sustained rates of fire. Hence, the weapon systems analyst should consult the published test results of these agencies in determining the thermal bounds for rates of fire in studies of existing weapon systems. Estimated rates of fire for a new weapon for which experimental data do not exist may be obtained either by a prediction model or by extrapolation from tests results obtained for weapons similar to the one being studied. Considerable theoretical work has been accomplished in this area which also can be applied to estimate the thermal bounds of weapon systems. For example, Ref. 13 presents a method for calculating theoretical rates of fire for recoilless rifles.

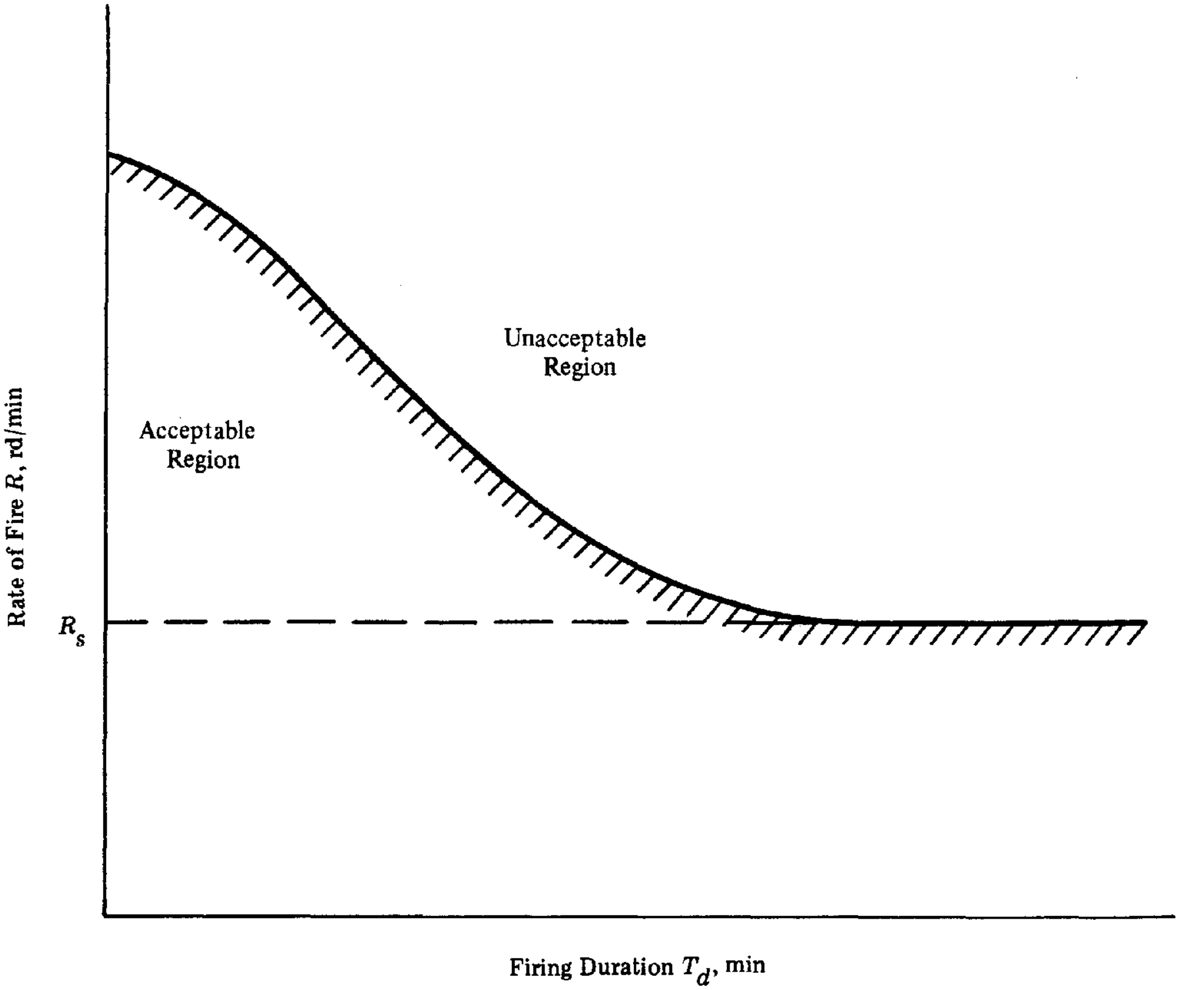

Figure 16-1. Sample Weapon Thermal Stress Envelope

16-5.2 LOGISTIC BOUNDS

The rates of ammunition expenditure and replenishment will produce another time-dependent bound on the rate of fire of a weapon system. Whereas the sustained rate of fire developed for the thermal bound is related directly to the physical characteristics of the weapon, the logistic bounds are related instead to the operational policy established for support of the weapon, e.g., as specified by the Ammunition Day of Supply Bulletin. Therefore, while there is only one thermal bound for a given weapon, there can be as many logistic bounds as there are support policies. In general, these logistic bounds can be determined by simply finding the rate of fire R in time interval $T = 0$ to $T = T_d$ which satisfies the equation

$$N_0 - RT_d + S(T_d) \geq 0 \qquad (16\text{-}6)$$

where

N_0 = number of rounds initially on hand
R = rate of fire, rd/min
T_d = firing duration, min
$S(T_d)$ = number of rounds resupplied during the time interval $T = 0$ to $T = T_d$

$S(T_d)$ in Eq. 16-6 is subject to variation, of course.

16-6 TYPICAL RATES OF FIRE

Table 16-1 presents a summary of the maximum and sustained rates of fire for some typical weapon systems within the Army inventory.

TABLE 16-1. RATES OF FIRE SUMMARY OF REPRESENTATIVE ARMY WEAPON SYSTEMS (Ref. 14)

Weapon System	Maximum Rate of Fire, rd/min	Sustained Rate of Fire, rd/min
75-mm Howitzer	8	2.5
105-mm Howitzer	10	3
155-mm Howitzer	4	1
8-in. Howitzer	1.5	0.5
175-mm Gun	1.5	0.5
4.2-in. Mortar	15-20	15-20
.45-cal Pistol	21-28	10
.45-cal Submachine Gun	450	40-60
.30-cal Carbine	750-775	40-60
.30-cal Rifle	16-24	16
5.56-mm Rifle	700-800	45-65 (semi), 150-200 (auto)
7.62-mm Rifle	750 (auto)	15 (semi), 20 (auto)
.30-cal Heavy Machine Gun	450-600	125
.30-cal Light Machine Gun	600-650	60
7.62-mm Machine Gun	550	100
.50-cal Heavy Machine Gun	400-600	40
2.36-in. Rocket Launcher	8	4
3.5-in. Rocket Launcher	8	4
.30-cal Browning Automatic Rifle	350-550	40-60

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CHAPTER 17

INTRODUCTION TO STOCHASTIC AND OTHER DUELS

Hit probabilities, conditional chances that hits are kills, and rates of fire are the basic parameters in the analysis of duels. These quantities are combined in models for stochastic duels, and the chances of winning can be determined for various firing strategies, thereby predicting weapon performance.

17-0 LIST OF SYMBOLS

a,b,c,d = integers in Eq. 14-27

$[a_{ij}]$ = chance the duel undergoes transition from state i to state j when Blue alone fires a shot

$[b_{ij}]$ = chance the duel undergoes transition from state i to state j when Red alone fires a shot

$[c_{ij}]$ = chance the duel undergoes transition from state i to state j when Blue and Red fire simultaneous shots

f_t = probability density function for the time variable t

g = number of hits to kill Red

$g_s(t)$ = normal probability density function for a sighting-time variable

$H(x)$ = $M(x)/\sqrt{2\pi}$

h = number of hits to kill Blue

$I_x(g,h)$ = Karl Pearson's Incomplete Beta Function Ratio (Ref. 12)

k = number of shots Blue gets off before Red returns fire

M = Red's limited number of shots

$M(x)$ = Mill's Ratio = upper tail area of the standard normal distribution divided by the ordinate at that point

m = an exponent

N = Blue's limited number of shots or weapons

n = number of rounds or shots

n = number of combatants on a side

n = number of duels

$P(B)$ = chance that Blue wins the duel

$P(BR)$ = chance of draw (B and R kill each other)

$P(R)$ = chance that Red wins the duel

$P(r)$ = lower limit of binomial probability summation in Eq. 17-17 expressing the chance of winning at least r of n duels

p = chance of winning a single duel

p = pth shot of Blue

p_B = $p_B(h) \cdot p_B(k|h)$ = single shot kill probability of Blue against Red

p_b = $p_B(h)$ = single shot hit probability of Blue against Red

p_R = $p_R(h) \cdot p_R(k|h)$ = single shot kill probability of Red against Blue

p_r = $p_R(h)$ = single shot hit probability of Red against Blue

$p_B(h)$ = single shot hit probability of Blue against Red

$p_B(k|h)$ = conditional chance that a hit is a Red kill for Blue

$p_R(h)$ = single shot hit probability of Red against Blue
$p_R(k|h)$ = conditional chance that a hit is a Blue kill for Red
q = qth shot of Red
$_Bq_R$ = $1 - p_R$ = single shot survival probability for Blue when fired on by Red
$_Rq_B$ = $1 - p_B$ = single shot survival probability for Red when fired on by Blue
r = lower limit of binomial probability summation in Eq. 17-17 expressing the chance of winning at least r of n duels
S_n = $(S_{1n}, S_{2n}, S_{3n}, S_{4n})$ = state vector for each of four states (1,2,3,4) after n rounds or shots

$$[T] = \text{matrix defined as} \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$[T_B]$ = $[a_{ij}]$ = transition matrix for Blue firing a shot
$[T_{BR}]$ = $[c_{ij}]$ = transition matrix for Blue and Red firing simultaneous shots
$[T_R]$ = $[b_{ij}]$ = transition matrix for Red firing a shot
t = time
t_n = time at which nth round is fired
$t_{p,B}$ = time at which Blue fires his pth shot, s
$t_{q,R}$ = time at which Red fires his qth shot, s
t_s = sighting time
t_1,t_2,t_3,t_4 = elements of top row of patterned transition matrix
v_1 = $(1 - p_B)^k$
v_2 = $p_B \sum_{i=1}^{k-1} (1 - p_B)^i$
z = least common multiple (LCM) of b and d
Δ = least common multiple (LCM) of $z\rho_B$ and $z\rho_R$
δ = time Blue fires before Red at start of duel, s
λ = period of patterned transition matrix cycle = $60\Delta/(z\rho_B\rho_R)$, s
μ_B = mean failure rate of Blue's weapon
μ_R = mean failure rate of Red's weapon
ρ_B = mean rate of fire for Blue (B)
$1/\rho_B$ = mean time between shots for Blue
ρ_R = mean rate of fire for Red (R)
$1/\rho_R$ = mean time between shots for Red
σ = standard deviation of normal sighting time distribution
τ_B = time of flight of Blue's shot
τ_R = time of flight of Red's shot

17-1 INTRODUCTION

Battles generally represent very complex fighting conditions involving chance occurrences. Nevertheless, it is instructive to study engagements between individual combatants or weapons in order to

obtain some idea of the relative effectiveness of weapons and the way they should be employed. Moreover, there are many battles which may be broken down into a series of engagements among individual contestants, i.e., one versus one type duels. We believe it represents a desirable effort to describe, for example, a tank versus tank engagement in stochastic terms and then try to extrapolate such outcomes to the more complex battles. Also, it is always of interest to keep in mind some measures of effectiveness or performance of individual combatants or systems in an engagement. Hence, the need to model and study "duels". It is widely realized that chance plays a very important role in duels, and that the one who fires first has a much improved chance of winning, other conditions being equal. In this chapter we will study the important and very useful class of duels known as "stochastic duels" which we define as duels in which at least one of the contestant's firing times between rounds is randomly distributed in accordance with a reasonable probability distribution. The work we will cover here is primarily that due to Clinton J. Ancker, Jr., and G. Trevor Williams (Refs. 1-5) because their contributions to the subject are excellent and their models of duels are believed to be of sufficient accuracy to cover many important practical cases occurring in combat.

Our studies of hit probabilities (Chapter 14), vulnerability and lethality (Chapter 15), and rates of fire (Chapter 16) lead us directly into the subject of duels.

17-2 THE FUNDAMENTAL DUEL

In a "fundamental" duel, it is hypothesized that two duelists, B and R, or Blue and Red, fire at each other until one is put out of action. The firing times, or time between rounds, for each duelist is considered to be of a random character with known probability density functions, the parameters for which may be different for Blue and Red. At the start of the engagement, each contestant loads, aims, and fires the first round at his opponent. Thus, in the "fundamental" duel, both start with unloaded weapons. It is also assumed here that each time Blue and Red fire at each other they have constant single shot kill probabilities, although such kill probabilities of Blue and Red may be different. (The case of changing or dependent kill probabilities from round to round is mentioned later.) Both B and R have unlimited ammunition supplies, so that a kill is certain. To model duels, we define the following:

ρ_B = mean rate of fire of Blue (B)

ρ_R = mean rate of fire of Red (R)

p_B = single shot kill probability of Blue against Red

p_R = single shot kill probability of Red against Blue

$P(B)$ = chance that B wins the duel

$P(R)$ = chance that R wins the duel $= 1 - P(B)$.

The mean rates of fire, ρ_B and ρ_R, are, respectively, the reciprocals of the mean times between rounds fired by Blue and Red.

The single shot chances of kill, p_B and p_R, may be built up or determined by taking the product of the chance of a hit and the conditional probability that a hit is a kill; i.e., $p_B = p(h) \cdot p_B(k|h)$ and $p_R = p_R(h) \cdot p_R(k|h)$.

Finally, we make an assumption that appears of practical value; namely, that the time to fire the first round and the times between rounds fired for B and R follow single parameter negative exponential distributions. Thus, for random times t

$$f(t) = \rho \exp(-\rho t) \tag{17-1}$$

where $\rho = \rho_B$ or ρ_R, as needed. Mean time between rounds $= 1/\rho$.

Since the exponential distribution is equivalent to the chi-square distribution with two degrees of freedom, this means that the time at which the nth round is fired is the sum of n independent selections from Eq. 17-1, or the chi-square distribution with $2n$ degrees-of-freedom (or the gamma distribution) given by

$$f(t_n) = \rho^n (t_n)^{n-1} \exp(-\rho t_n)/(n-1)! \,. \tag{17-2}$$

Then, with these formulations and assumptions, it is well known (Ref. 1) that the chance that Blue wins is

$$P(B) = \frac{p_B \rho_B}{p_B \rho_B + p_R \rho_R} \tag{17-3}$$

and the chance that Red wins is

$$P(R) = 1 - P(B) = \frac{p_R \rho_R}{p_B \rho_B + p_R \rho_R} \,. \tag{17-4}$$

Thus, for exponentially distributed firing times between rounds, the chance that a side wins is the kill rate for that side divided by the sum of the kill rates for both sides, which is a rather simple outcome. Hence, the value of kill rate as a key measure of effectiveness is evident. Note that if the single shot kill probabilities of B and R are equal, then their rates of fire take over; and if their rate of fire also are equal, each B and R have a 50% chance of winning. The chance of a draw, or both being killed, here is zero.

Example 17-1:

A Blue tank and a Red tank meet at an engagement range such that average conditions of firing are:

$p_B(h)$ = single shot hit probability of B = 0.6
$p_R(h)$ = single shot hit probability of R = 0.8
$p_B(k|h)$ = conditional chance that a hit is a Red kill for Blue = 0.7
$p_R(k|h)$ = conditional chance that a hit is a Blue kill for Red = 0.8
ρ_B = 2 rd/min for Blue
ρ_R = 1 rd/min for Red.

What are the chances of winning?

We see that by Eq. 17-3

$$P(B) = \frac{(0.6)(0.7)(2)}{(0.6)(0.7)(2) + (0.8)(0.8)(1)} = 0.57 = \text{Blue's chance of winning the duel}$$

$$P(R) = 1 - 0.57 = 0.43 = \text{Red's chance of winning the duel.}$$

Note that for this example the single shot kill probabilities against opponents for Blue and Red are:

$$p_B = p_{ssk}(B) = (0.6)(0.7) = 0.42$$
$$p_R = p_{ssk}(R) = (0.8)(0.8) = 0.64$$

The kill rates for Blue and Red are:

$$p_B\rho_B = 0.84 \text{ kill per min}$$
$$p_R\rho_R = 0.64 \text{ kill per min .}$$

As an extension of the example, if two Blue tanks simultaneously take on a Red tank, then other things being equal, the initial kill rate of Blue would effectively be doubled, etc.

It is important for the systems analyst to keep these simple concepts in mind since they are very informative for quick mental evaluations of weapons and are basic building blocks for more complex studies.

17-3 FIXED RATE OF FIRE FOR BLUE

For contrast, it would be well here to compare the previous case where both Blue and Red fire their rounds so that the times between rounds follow exponential laws, as contrasted to the case where Red fires this same way but Blue is assumed to have a fixed rate of fire, i.e., the time between Blue's rounds fired is a constant. Ancker (Ref. 2) has shown for this case that the chance of Blue winning the duel is given by

$$P(B) = \frac{p_B}{\exp(p_R\rho_R/\rho_B) - {}_Rq_B} \tag{17-5}$$

where

${}_Rq_B = 1 - p_B$ = single shot survival probability for Red when fired on by Blue .

Example 17-2. Using the data of Example 17-1, we calculate from Eq. 17-5 the chance of Blue winning this kind of duel to be

$$p(B) = \frac{0.42}{\exp(0.64/2) - 0.58} = 0.53$$

where we used the constant time between B's rounds fired as 0.5 min, so that his rate of fire averages 2 rd/min.

We note that this is a somewhat lower chance of Blue winning when his time between shots is absolutely constant and not random. In fact, if we compare Eq. 17-5 with Eq. 17-3, we see that B's chance of winning under Eq. 17-5 is less than his chance of winning under Eq. 17-3, provided

$$1 + p_R\rho_R/\rho_B < \exp(p_R\rho_R/\rho_B) \tag{17-6}$$

which clearly is always true as seen by expanding the exponential term in a power series. Thus, random firing according to exponential times to the first shot and between shots has the advantage. A little reflection will show that this makes practical sense, for one tends to fire his shots as quickly as possible. This results in peaking the frequency of shots to values less than the mean times between rounds.

17-4 THE CLASSICAL DUEL

In the classical duel, which is a very dangerous one, Blue and Red start with loaded weapons and fire their first rounds simultaneously, and proceed as in the fundamental duel. Thus, we can illustrate

the sensitivity of results to initial conditions of firing. Again, we assume that Blue's and Red's times between shots are randomly distributed according to exponential laws.

First, we note that both Blue and Red could be killed on their first shots, and the chance of this (a draw) is

$$P(BR) = p_k(BR) = p_B p_R \, . \tag{17-7}$$

For Example 17-1, this is $p_k(BR) = (0.42)(0.64) = 0.27$ which is significant. For the classical duel, Blue's chance of winning can be shown to be (Ref. 1)

$$p(B) = \frac{(p_B)({}_Bq_R)(p_R\rho_R + \rho_B)^*}{p_B\,\rho_B + p_R\,\rho_R} \, . \tag{17-8}$$

For this duel

$$P(B) + P(R) + P(BR) = 1 \, . \tag{17-9}$$

Example 17-3. If we use the data of Example 17-1 and Eq. 17-8, we find that for the classical duel the chance of Blue winning is

$$P(B) = \frac{(0.42)(0.36)[(0.64)(1) + 2]}{(0.42)(2) + (0.64)(1)} = 0.27$$

and

$$P(R) = \frac{p_{RR}q_B(p_B\rho_B + \rho_R)}{p_B\rho_B + p_R\rho_R} = 1 - P(B) - P(BR) = 1 - 0.27 - 0.27 = 0.46 \, .$$

Hence, Blue's chance of winning is drastically reduced because for Red's first shot he has much higher kill probability, and the chance of both being killed (a draw) is very significant.

By comparing Eq. 17-3 with Eq. 17-8 we see that Blue's chances of winning a classical duel are greater than winning a fundamental duel only if

$$({}_Bq_R/\rho_B)(p_R\rho_R + \rho_B) > 1$$

or that is

$${}_Bq_R\rho_R > \rho_B, \quad {}_Bq_R \neq 1 \, . \tag{17-10}$$

As the single shot kill probability of Blue approaches unity, the chance of Blue winning the duel approaches $1 - p_R$, the single shot survival probability of Blue, i.e.,

$$P(B) \approx 1 - p_R \, , \; p_B \rightarrow 1 \, . \tag{17-11}$$

*Blue's chance of winning is the probability that Blue kills Red on the first shot, and survives the first shot, plus the probability both survive the first shot times the chance that Blue wins the ensuing fundamental duel.

Ref. 1 displays some graphs (Fig. 6) of Blue's chances of winning for the classical duel, and for the relative rates of fire of $\rho_B = \rho_R/2$, $\rho_B = \rho_R$, and $\rho_B = 2\rho_R$. The results are relatively insensitive to rates of fire, due to both firing simultaneously at each other at the start of the duel, as can be seen from Eqs. 17-8 and 17-11. For very low single shot kill probabilities, the chance-of-winning contours become identical with those of the fundamental duel.

17-5 THE TACTICAL EQUITY DUEL

A very plausible hypothesis concerning the start of a duel is for Blue and Red each to shoot first 50% of the time. This is called the tactical equity duel or the duel with equal initial surprise. Here, Blue sights Red first one-half of the time, and fires one round at Red who is alerted and immediately returns fire with a shot. The duel then goes on as in the fundamental case. The other half of the time Red sights Blue first and fires a shot.

The chance that Blue wins is the probability that he fires first (1/2) multiplied by the sum of the chance that he wins on the first round and the chance that he fails on the first round but wins on any subsequent round, plus the chance that Red fires first (1/2) multiplied by the chance that Red fails to win on the first round and the chance that Blue wins the subsequent duel. For the assumption of negative exponential firing times between rounds for Blue and Red as before, the chance that Blue wins is (Ref. 4)

$$P(B) = (p_B/2)\,\frac{(2 - p_R)\rho_B + p_R\rho_R}{p_B\rho_B + p_R\rho_R}\,. \tag{17-12}$$

Example 17-4. If we substitute into Eq. 17-12 the conditions of Example 17-1, we get for the tactical equity duel

$$P(B) = (0.42/2)\frac{(2 - 0.64)2 + 0.64(1)}{0.42(2) + 0.64(1)} = 0.48\,.$$

Thus, Blue now has just slightly less than a 50% chance of winning, significantly less than for the fundamental duel with his much higher rate of fire than Red. In this case

$$P(R) = 1 - P(B) = 0.52$$

giving Red the slight advantage.

17-6 THE RANDOM INITIAL SURPRISE DUEL

Another reasonable assumption in combat has to do with target sighting times of Blue and Red. In fact, we might say that depending on battlefield surveillance capabilities, one side may have a better capability to detect targets long before his enemy on the average. Hence, the concept of a random sighting time distribution $g_s(t)$ for Blue and Red which, for example, might be considered to be normally distributed

$$g_s(t) = [1/(\sqrt{2\pi}\,\sigma)]\exp[-t^2/(2\sigma^2)] \tag{17-13}$$

where σ is the standard deviation of the Gaussian sighting time distribution. Here, a positive sighting time t_s could represent an advantage for Blue and a negative t_s an advantage for Red. The sighting time

is a period during which one combatant may fire with impunity at his opponent. At the end of the sighting time period, if the duelist who was firing has not killed his opponent, then the fundamental duel goes into effect. The probability that Blue wins is equal to the chance that the time for Blue to make a kill is less than that for Red to make a kill plus Blue's time advantage.

For exponential firing times between rounds for Blue and Red, which have mean firing rates ρ_B and ρ_R, respectively, as before then the chance that Blue wins is (Ref. 1)

$$P(B) = \frac{1}{2} + \left(\frac{p_B\rho_B}{p_B\rho_B + p_R\rho_R}\right) H(\sigma p_R \rho_R) - \left(\frac{p_R\rho_R}{p_B\rho_B + p_R\rho_R}\right) H(\sigma p_B \sigma_B) \quad (17\text{-}14)$$

where

$$\sqrt{2\pi}\, H(x) = M(x) = \int_x^\infty \exp(-t^2/2)dt/\exp(-x^2/2) \quad (17\text{-}15)$$

and $M(x)$ is known as Mill's Ratio, the upper tail area of the standard normal distribution divided by the ordinate at that point. $M(x)$ therefore is easily found from a suitable table of the ordinates and integral of the normal probability function. (Mill's Ratio, incidentally, is also equal to the reciprocal of the "intensity" function in reliability theory for a normal failure time distribution.)

Thus, in addition to the kill rates of Blue and Red, one needs only the standard deviation of the sighting times, which are assumed to be normally distributed, in order to find Blue's and Red's chances of winning.

Example 17-5:

Suppose we continue to use the same basic data of Example 17-1 and the assumption of a normal sighting time distribution with a standard deviation of σ = 2 min. What would be Blue's and Red's chances of winning now?

For this duel, we note the fractions in Eq. 17-14 are the chances that Blue and Red win the fundamental duel, i.e., 0.57 and 0.43, respectively, as determined from Example 17-1.

Also, we have

$$\sigma p_R \rho_R = (2)(0.64) = 1.28\,, \quad \text{and} \quad \sigma p_B \rho_B = (2)(0.84) = 1.68\,.$$

Thus

$$H(1.28) = 0.1003/(0.1758\sqrt{2\pi}) = 0.228$$

and

$$H(1.68) = 0.0465/(0.0973\sqrt{2\pi}) = 0.191\,.$$

Therefore, the chance of Blue winning the duel is

$$P(B) = 0.50 + (0.57)(0.228) - (0.43)(0.191) = 0.55\,.$$

Thus, the sighting time distribution with a sigma of 2 min degrades Blue's chance of winning only slightly. If $\sigma = 5$ min, then $P(B) = 0.53$. Hence, after the initial random sighting problem, then the fundamental duel takes over with original parameters. We comment that as sigma approaches zero then Blue's chance of winning becomes that of the fundamental duel, as would be expected.

17-7 COMMENT ON SEVERAL COMBATANTS OR WEAPONS ON A SIDE

It is of some interest to make a comment here concerning a simple engagement in which several Blue tanks (combatants) attack a single Red tank (combatant), or vice versa. Ordinarily, such an occurrence may not be expected to happen very frequently in an actual battle unless terrain conditions are favorable, clever tactics on the part of one side are employed, etc. Moreover, both sides or forces often would be better trained. However, the effect of such concentration of Blue tanks would tend to overwhelm Red. In this connection, we could argue that single shot hit and kill probabilities are the same as before, but with Blue employing two tanks against Red's single tank the rate of fire of Blue has doubled initially, and for three Blue tanks it has tripled for the beginning of the battle, etc. Thus, for n-Blue tanks attacking a single Red tank, one might be tempted to use as a quick, "back of the envelope" type calculation (but obviously inaccurate one) that says the chance of Blue winning now becomes

$$P(B) = \frac{np_B\rho_B}{np_B\rho_B + p_R\rho_R} . \tag{17-16}$$

In other words the kill rate of Blue against Red on the face of it is effectively multiplied by the number of weapons employed, at least initially.

For $n = 2$ Blue tanks, and the data of Example 17-1 for the fundamental duel, then such calculation gives $P(B) = 0.72$, instead of 0.57, and for $n = 3$ Blue tanks, $P(B) = 0.80$, etc. However, we desire to point out here that such a naive analysis may be subject to considerable error, for the conditions of battle, or the "states" as they are called, change on a probabilistic basis after each round is fired. For Blue's first firing of, for example, three rounds simultaneously from his three tanks, we note that the chance of killing the single Red tank is $1 - (1 - p_B)^3 = 1 - (1 - 0.42)^3 = 0.80$. That is, Red's survival probability after the first exchange is 0.20. Then again, Red might kill one of the Blue tanks, with chance equal to 0.64, if he gets a shot off, in which case Blue arrives at the "state" where he has two tanks left now with only double his original kill rate per tank. Hence, for several tanks (weapons) on a side the calculations can become a bit complex, and elegant mathematical methods are required for such analyses (see references and bibliography). Nevertheless, the idea of dealing with simple measures of effectiveness such as the total or initial potential kill rate on each side contributes some, but not a completing understanding of just what will occur stochastically in a complex battle situation. It can be said that the initial conditions on numbers of weapons on the two sides, the kill rate potential, and other such characteristics often will go a long way toward determining battle outcome, in spite of many chance occurrences otherwise during the battle. The number of weapons on a side multiplied by the kill rate has been often referred to as the "fighting power" of that side. The papers of Ancker and Williams (Ref. 5), Robertson (Ref. 6), and Helmbold (Ref. 7) throw much light on this broad and complex subject. Also, suitable models of two-sided conflict for many battles are available from the considerations of (Lanchester type) combat theory (Chapters 28 and 29). Kill rates, as we describe them here, are widely referred to as attrition coefficients in Lanchester type laws of combat.

The principle of using the "states" of a duel after each shot, along with associated probabilities of the various "states", is discussed in par. 17-9.

17-8 COMMENT ON BATTLES OF SEVERAL OR MANY DUELS

As we have indicated, it might be argued that some battles can be described perhaps as a series of independent, individual duels—i.e., one Blue versus one Red at a time—and such a hypothesis could be verified or refuted with an analysis of appropriate historical data. For such a hypothesis, it is easy to see that for an equal number of Blues and Reds, say n, and their engagement in a battle involving n-simultaneous duels, we may use the simple binomial probability distribution to calculate the chance that Blue (or Red) will win at least r of n such engagements. This is given by

$$P(r) = \sum_{x=r}^{n} \binom{n}{x} p^x (1-p)^{n-x} \tag{17-17}$$

where p is the chance of a win in a single duel. Thus, suppose as an example we assume that 20 Blue and 20 Red tanks meet in an engagement involving as many duels, and that either side would withdraw as soon as he lost as many as 40% (a total of 8) of his tanks. Then, for the result of the fundamental duel in Example 17-1, where Blue's chance of winning a single duel is 0.57, we see that the probability Red loses 8 or more tanks in the engagement of 20 B-R duels will be

$$P_{20}(B) = \sum_{x=8}^{20} \binom{20}{x} (0.57)^x (0.43)^{20-x} = 0.96$$

which is easily found from a table of binomial probabilities, such as AMCP 706-192, *Tables of the Cumulative Binomial Probabilities* (Ref. 8).

As a further calculation, the chance that in 20 B-R tank duels Blue will win at least 50% of the duels, or kill at least 10 Red tanks, is about 0.81.

Alternatively, and as another type of analysis, Blue might ask his weapon systems analyst how many tanks should he commit, or how many duels would be involved, in order for him to kill 8 Red tanks; which would result in Red's withdrawal? Here, the negative binomial distribution of par. 10-6, Chapter 10, would be useful. That is, the expected number of B-R fundamental duels to kill 8 Red tanks would be $8/P_1(B) = 8/0.57 = 14$ duels. Therefore, for average conditions or expectations, commit 14 Blue tanks, each of which will take on a Red tank as it appears.

We remark that many questions could be raised about these two very superficial analyses, although they are at least provocative enough to get the young weapon systems analyst thinking about the complexities of two-sided engagements or battles involving many weapons. We will not go beyond these considerations here because combat theory, war games, and simulations will be covered in later chapters as we develop the subject more fully.

17-9 FIXED OR NONRANDOM RATE-OF-FIRE DUELS

So far we have dealt only with stochastic type duels. However, there exists also a wide class of dueling procedures for which the firing times between rounds are not randomly distributed, but rather are more or less fixed, or at least the duel is broken down into consecutive shots by Blue or Red, and it becomes of interest to evaluate possible occurrences after each shot. Groves (Ref. 9) has developed a clever analytical procedure to handle such cases. For the simple duel, we observe that at any time just before or after a shot by either Blue or Red, then the duel can be said to be in one of precisely four *states* or conditions. These states are:

1. State 1: B and R are both alive (neither has won the duel).

2. State 2: R is dead, B is alive (B has won the duel).
3. State 3: B is dead, R is alive (R has won the duel).
4. State 4: B and R are both dead (a draw).

As before, a duelist must kill his opponent while remaining alive in order to win the duel. State 4 may result when B and R exchange shots simultaneously, as will sometimes occur. Once the duel starts, we note that any time a shot is fired by either B or R, then a *transition* from one state to the same or a different state takes place. Such transitions are stochastic in nature; hence, the term "transition probabilities" for a change from one state to another. Note here that the dueling process may be indexed by a discrete variable, the number of shots fired, rather than the continuous variable, time, as before.

Now the associated probabilities for the given four possible states may be described analytically by a row vector we define as

$$S_n = (s_{1n}, s_{2n}, s_{3n}, s_{4n}), \quad \sum_{i=1}^{4} s_{in} = 1 \tag{17-18}$$

where the ith component is the chance that the duel is in the ith state (1, 2, 3, or 4 as given) after n-"transitions" brought on by shots from Blue and Red. At the beginning of the duel both B and R are alive, and the state vector Eq. 17-18 is $S_0 = (1, 0, 0, 0)$.

Next, we define three transition matrices, which depend respectively on Blue firing alone, Red firing alone, or Blue and Red firing simultaneously, namely:

$$[T_B] = [a_{ij}] , \ [T_R] = [b_{ij}] , \text{ and } [T_{BR}] = [c_{ij}]$$

where

$[a_{ij}]$ = chance the duel undergoes transition from state i to state j when Blue alone fires a shot
$[b_{ij}]$ = chance the duel undergoes transition from state i to state j when Red alone fires a shot
$[c_{ij}]$ = chance the duel undergoes transition from state i to state j when Blue and Red fire simultaneous shots.

As before, the single shot kill probability of Blue against Red is p_B, and the single shot kill probability of Red against Blue is p_R. Thus, consider the following transition matrix based on Blue having fired a shot:

$$T_B = \begin{bmatrix} 1 - p_B & p_B & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{17-19}$$

Now note that if we take the state vector at the beginning of the duel, i.e., $S_0 = (1, 0, 0, 0)$, and multiply it on the right by the transition matrix Eq. 17-19 for a shot from Blue, the result is

$$S_0[T_B] = (1 - p_B, p_B, 0, 0) . \tag{17-20}$$

That is to say, the chance that state 1 now exists (i.e., Blue and Red both alive) is $1 - p_B$, the chance the state 2 exists (i.e., Red is dead, Blue is alive—Blue won) is p_B, and states 3 and 4 at this instant have zero probabilities of occurrence.

In a like manner, if we multiply the starting state vector $S_0(1, 0, 0, 0)$ on the right side by the following transition matrix, depending on a shot fired by Red, i.e.,

$$T_R = \begin{bmatrix} 1-p_R & 0 & p_R & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (17\text{-}21)$$

then we obtain

$$S_0[T_R] = (1 - p_R, 0, p_R, 0) \,. \qquad (17\text{-}22)$$

That is to say, if Red fires first at Blue, the chance that Blue and Red both are alive (state 1) is $1 - p_R$; the chance that Red is dead and Blue alive (state 2) equals zero (since only Red fired); the chance that Red is alive and Blue dead (state 3) equals p_R (the chance that Red kills Blue on a shot); and the chance both are dead (state 4) is zero.

Finally, the transition matrix which transforms an existing state to one where Blue and Red shoot simultaneously and kill each other can be seen in probability terms to be

$$T_{BR} = \begin{bmatrix} (1-p_B)(1-p_R) & p_B(1-p_R) & p_R(1-p_B) & p_Bp_R \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \qquad (17\text{-}23)$$

Furthermore, it is clear that no matter what state exists, for example, the general state Eq. 17-18, then it may be operated on by $[T_B]$, $[T_R]$, or $[T_{BR}]$—depending on who then fires—to give the occurrence chances for the next state.

Therefore, the three transition matrices—$[T_B]$, $[T_R]$, and $[T_{BR}]$—give all that is necessary in order to find the probabilities of occurrence of each of the four states, once the order of shots by Blue and Red are known or given. The duel may be stopped at any desired state (shot) to get an idea of chances of each of the four states at that point.

The analysis referred to here is widely recognized as a "Markov" process to honor the mathematician who made some very profound and early contributions to modeling this area of investigation. In a Markov process, any state depends on the immediately preceding state, as we saw by the previous discussion.

For this duel, it is assumed that Blue (or hence Red by interchanging letters) fires his first shot at time $t = 0$ and Red fires his first shot at time $t = \delta$. Hence, the time at which Blue fires his pth shot is

$$t_{p,B} = 60(p - 1)/\rho_B, \text{ s} \qquad (17\text{-}24)$$

where ρ_B is, as before, Blue's rate of fire in rounds per minute.

The time at which Red fires his qth shot is

$$t_{q,R} = \delta + 60(q - 1)/\rho_R, \text{ s} \qquad (17\text{-}25)$$

ρ_R being Red's rate of fire in rounds per minute.

The order or position of shots by Blue and Red, and the points of occurrence of their simultaneous shots (if any), therefore can be found. It can be seen that the details of the evaluation process are somewhat tedious since considerable matrix multiplication may be required. Groves (Ref. 9) shows that, when Red starts firing, the sequence of multiplication of transition matrices based on shots fired by Blue and Red becomes periodic; and this can be an aid in evaluating the iterations required. The number of shots k which Blue fires initially before Red returns fire is the greatest integer in

$$k = [(\delta p_B/60) + 1] \,. \tag{17-26}$$

Other parameters needed for the evaluation are the following (Ref. 9):

1. The rates of fire, which are assumed to be rational numbers, are expressed as fractions:

$$\rho_B = a/b \quad , \qquad \rho_R = c/d \tag{17-27}$$

where a, b, c, and d are all integers.

2. A quantity z is taken as the least common multiple (LCM) of the integer denominators b and d, i.e.,

$$z = \text{LCM}\ (b,d) \,. \tag{17-28}$$

3. A quantity Δ is defined similarly, i.e.,

$$\Delta = \text{LCM}\ (z\rho_B, z\rho_R) \,. \tag{17-29}$$

4. The period of the patterned cycle is given by

$$\lambda = 60\Delta/(z\rho_B\rho_R),\ \text{s} \,. \tag{17-30}$$

5. The shots per cycle by B and R are:

$$\text{shots per cycle by } B = \Delta/(z\rho_R) \tag{17-31}$$

$$\text{shots per cycle by } R = \Delta/(z\rho_B) \,. \tag{17-32}$$

Then with Blue's and Red's firing times and the length of the patterned cycle, the firing order and transition matrices involved in the particular patterned sequence may be determined. The product of the transition matrices in the cycle, and in proper order, gives a matrix $[T]$ defined as

$$T = \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \,. \tag{17-33}$$

With the quantities

$$v_1 = (1 - p_B)^k \tag{17-34}$$

and

$$v_2 = p_B \sum_{i=1}^{k-1} (1 - p_B)^i \tag{17-35}$$

the states and associated chances of each state after m-cycles and, at the end of the duel, are as given in Table 17-1.

Example 17-6: A Blue tank sights a Red tank at long range and fires on it. Both tanks are armed with antitank guns. Each tank can fire two rounds per minute and Red returns fire after 15 s. The calculated single shot kill probabilities against opponents for Blue and Red are 0.3 and 0.4, respectively. What are the duel chances of winning?

From the given data, note that B fires 15 s before R, and has this advantage, at least. Also we have

$p_B = 0.3$, $p_R = 0.4$, $\delta = 15$ s, $\rho_B = \rho_R = 2$ shots per min. Moreover, since $\rho_B = \rho_R = 2/1$, then

$$\begin{aligned}
z &= \text{LCM }(1,1) = 1\\
\Delta &= \text{LCM }(z\rho_B, z\rho_R) = \text{LCM }(2,2) = 2\\
\lambda &= 60\Delta/(z\rho_B\rho_R) = 30 \text{ s per cycle}\\
\Delta/(z\rho_B) &= 1 \text{ shot per cycle by } B\\
\Delta/(z\rho_R) &= 1 \text{ shot per cycle by } R\\
k &= [\delta\rho_B/60 + 1] = 1 \text{ Blue shot before the cycle starts.}
\end{aligned}$$

By Eq. 17-24, Blue fires at 0, 30, 60, 90, 120, etc., seconds, and by Eq. 17-25, Red fires at 15, 45, 75, 105, 135, etc., seconds.

The transition matrices are by Eqs. 17-19, 17-21, and 17-23

$$T_B = \begin{bmatrix} 0.7 & 0.3 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_R = \begin{bmatrix} 0.6 & 0 & 0.4 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

TABLE 17-1. TABLE OF STATES AND ASSOCIATED PROBABILITIES

State	Description	Probability	
		After m-Cycles	At End of Duel
1	Neither B nor R wins	$v_1 t_1^m$	0
2	B wins	$v_2 + v_1 t_2(1 - t_1^m)/(1 - t_1)$	$v_2 + v_1 t_2/(1 - t_1)$
3	R wins	$v_1 t_3(1 - t_1^m)/(1 - t_1)$	$v_1 t_3/(1 - t_1)$
4	B and R kill each other	$v_1 t_4(1 - t_1^m)/(1 - t_1)$	$v_1 t_4/(1 - t_1)$

and

$$T_{BR} = \begin{bmatrix} 0.42 & 0.18 & 0.28 & 0.12 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Now we see that B fires one shot before the beginning of the first cycle, and the cycle consists of one shot by R and then one by B. Hence, the matrix

$$T = T_R T_B = \begin{bmatrix} 0.6 & 0 & 0.4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.42 & 0.18 & 0.40 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Hence, $t_1 = 0.42$, $t_2 = 0.18$, $t_3 = 0.40$, and $t_4 = 0$
Further, from Eqs. 17-34 and 17-35

$$v_1 = (1 - p_B)^k = (0.7)^1 = 0.7$$

$$v_2 = p_B \sum_{i=0}^{k-1} (1 - p_B)^i = 0.3(1 - 0.3)^{0.42} = 0.3.$$

The results for a general number of cycles m and kill chances at the end of the duel are given in Table 17-2.

After 5 cycles, for example, the state vector is $S_m = (0.01, 0.51, 0.48, 0)$.

Hence, Blue's 15-s advanced firing advantage is enough to overcome his lower single shot kill probability of 0.3 versus Red's 0.4.

We see that setting up and evaluating this type of duel may become somewhat tedious. Groves (Ref. 9) gives extensive details and more complex examples in addition to Example 17-6.

17-10 STOCHASTIC DUELS WITH LIMITED AMMUNITION SUPPLY

Ancker (Ref. 10) also has made a study of stochastic duels with limited ammunition supply. In other words, we have the same parameters—p_B, p_R, ρ_B, and ρ_R—as in the stochastic duels discussed

TABLE 17-2. RESULTS FOR EXAMPLE 17-6

State	Description	Probability After m-Cycles	Probability At End of Duel
1	Neither wins	$(0.7)(0.42)^m$	0
2	B wins	$0.3 + 0.21724(1 - 0.42^m)$	0.51724
3	R wins	$0.48276(1 - 0.42^m)$	0.48276
4	Draw	0	0

previously, except now Blue is limited by N-rounds and Red by M-rounds. The equation for the chance that Blue (or Red) wins is complicated, and given in Ancker's Equations (35) or (39) of his paper (Ref. 10).

In case Blue has a fixed number of rounds equal to N, and Red has an unlimited supply of ammunition, then for the assumption of exponential firing times between rounds as before, the chance that Blue wins is nevertheless rather simple and given by

$$P(B) = [p_B\rho_B/(p_B\rho_B + p_R\rho_R)]\{1 - [{}_Rq_B\rho_B/(p_R\rho_R + \rho_B)]^N\} \tag{17-36}$$

and

$$P(BR) = 0\,. \tag{17-37}$$

For the data of Example 17-1, we found $P(B) = 0.57$. If we suppose that B is limited to $N = 5$ rounds, then Eq. 17-36 gives

$$P(B) = (0.57)[1 - (0.439)^5] = (0.57)(0.984) = 0.56$$

which indicates very slight limitation on Blue in this particular case. However, for small p_B, and hence large ${}_Rq_B = 1 - p_B$, approaching unity, the effect could be drastic.

If Blue has an infinite number of rounds available to fire and Red is limited to M-rounds, then

$$P(B) = \frac{p_B\rho_B}{p_B\rho_B + p_R\rho_R} + \left[\frac{p_R\rho_R}{p_B\rho_B + p_R\rho_R}\right]\left[\frac{{}_Bq_R\rho_R}{p_B\rho_B + \rho_B}\right]^M \tag{17-38}$$

and

$$P(BR) = 0\,. \tag{17-39}$$

Again, this chance of Blue winning may approach rapidly the same value as in the case of the fundamental duel, unless Red's single shot miss probability ${}_Bq_R$ is large, approaching unity.

17-11 STOCHASTIC DUELS WITH "LETHAL DOSE"

In an interesting paper entitled "Stochastic Duels with Lethal Dose", N. Bhashyam (Ref. 11) has studied the case where Blue and Red fight a stochastic duel with unlimited supply of ammunition, but Blue kills Red—i.e., wins the duel—when Blue obtains $g \geq 1$ hits on Red. On the other hand, Red wins when he obtains $h \geq 1$ hits on Blue. Thus, the number of hits g, required to kill Red, and the number of hits h required to kill Blue are called "lethal doses". In this case, we do not use or need the single shot kill probabilities p_B and p_R used before, nor do we use conditional probabilities of a kill, given a hit. In fact, the quanitities g and h take care of the latter. We do, nevertheless, need the single shot hit probabilities for Blue and Red. To shorten the notation, let

$p_b = p_B(h)$ = single shot hit probability of Blue against Red
$p_r = p_R(h)$ = single shot hit probability of Red against Blue

where we have simply lower case letters for hit probabilities.

Then, the hit rates are

$\rho_B p_b$ = rate at which Blue hits Red
$\rho_R p_r$ = rate at which Red hits Blue .

With these parameters, and the assumption of negative exponential interfiring times, Bhashyam shows that the chance Blue gets g-hits on Red before Red gets h-hits on Blue, i.e., the chance that Blue wins is,

$$P(B) = I_x(g,h) \quad (17\text{-}40)$$

where

$$x = \rho_B p_b / (\rho_B p_b + \rho_R p_r) \quad (17\text{-}41)$$

and $I_x(g,h)$ is Karl Pearson's Incomplete Beta Function Ratio (Ref. 12). The quantity $I_x(g,h)$ also may be expressed in terms of the binomial probability. In fact, it is the chance of obtaining at least g successes in $(h + g - 1)$ trials when the chance of success in a single trial is x as in Eq. 17-41. Thus,

$$I_x(g,h) = P(g, g + h - 1, x) \quad (17\text{-}42)$$

may be found from AMCP 706-109, *Tables of the Cumulative Binomial Probabilities* (Ref. 8), by entering the tables with $c = g$, $n = g + h - 1$, and $p = x$.

Example 17-7:

Blue and Red over many years of developing armor protection have arrived at the point where on the average two hits are required to penetrate their armor sufficiently to kill the tank. Blue's intelligence information indicates that Red's new tank can fire 2 rounds per min in a combat duel and that his single shot hit probability is 0.8 for the expected range of engagement. What would be required for Blue's single shot hit probability and rate of fire capability to guarantee that Blue would have a 90% chance of killing Red in a tank duel?

We have, $g = 2$, $h = 2$, $\rho_R = 2$, $p_r = 0.8$, and $I_x(2,2) = 0.90$, and we need to solve for x. We note using AMCP 706-109 that

$$I_x(2,2) = P(2,3,x) = 1 - P(2,3,1 - x) = 0.90$$

and from page 3 for $n = 3$, $c = 2$, we find that for $1 - x = 0.20$, then $1 - P(2,3,1 - x) = 1 - 0.104 = 0.896$, which is practically the 0.90 chance we seek. Hence take $1 - x = 0.2$, or $x = 0.8$. But we know that

$$x = \frac{\rho_B p_b}{\rho_B p_b + \rho_R p_r}$$

so that solving for $\rho_B p_b$, we get

$$\rho_B p_b = \rho_R p_r x / (1 - x) \quad (17\text{-}43)$$

$$= (1.6)(0.8)/(1 - 0.8) = 6.4 .$$

Thus, Blue's task is almost an unsurmountable one, for his single shot hit probability must approach one for any reasonable rate of fire. If Blue's single shot hit probability is 0.8, i.e., equal to Red's, then Blue needs an "antiarmor automatic cannon" to fire at least 8 rounds per min.

This example should give some appreciation of the very difficult problem of armament, and especially trying to guarantee high kill probabilities on a weapon versus weapon basis. Such one-sided chances of a win as 0.90 cannot be expected against any alert enemy. In fact, each nation needs to establish and maintain a significant "edge" over his possible enemy, by at least attempting to field weapons which will guarantee chances of win against similar foreign weapons as much above 0.5 as economically possible. In this way, some duels will be lost, but battles will be won!

17-12 EFFECT OF PROJECTILE TIME OF FLIGHT

Ancker (Ref. 13) has also considered the effect of time of flight of projectiles on chances of winning for the fundamental duel. If the times of flight of Blue's and Red's shots are fixed at values τ_B and τ_R, respectively, and if the duelists fire as rapidly as possible (without delay between rounds), then again for exponential firing times

$$P(B) = [p_B\rho_B/(p_B\rho_B + p_R\rho_R)]\exp(-p_R\rho_R\tau_B) \; . \qquad (17\text{-}44)$$

Thus, Blue's chance of winning in the fundamental duel goes down exponentially in terms of the product of Red's kill rate and time of flight of Blue's rounds. The degradation could be serious, and waiting to sense each round is not advisable!

17-13 OTHER CONSIDERATIONS

Bhashyam (Ref. 14) has studied stochastic duels with "nonrepairable" weapons. Here, he assumes the same parameters used in the fundamental duel, but includes in his analysis the idea of weapon failure times. In other words, Blue's and Red's weapon failure times are assumed to be exponentially distributed with mean failure times $1/\mu_B$ and $1/\mu_R$, respectively, or mean failure rates of μ_B and μ_R. If we further assume that Blue and Red have unlimited ammunition supplies, Blue has a limited number of weapons N, and Red has a failure-free weapon ($\mu_R = 0$), then the chances that Blue and Red win are

$$P(B) = \frac{\rho_B p_B}{\rho_B p_B + \rho_R p_R}\left[1 - \left(\frac{\mu_B}{\mu_B + \rho_B p_B + \rho_R p_R}\right)^N\right] \qquad (17\text{-}45)$$

$$P(R) = 1 - P(B), \quad P(BR) = P(\text{Draw}) = 0 \; .$$

Ancker (Ref. 15) considers stochastic duels of limited time duration in an interesting paper.

Duels with round-dependent hit probabilities are also covered in the research of Ancker (Ref. 16).

Fox and Kimeldorf discuss some strategies and values in "noisy" duels (Ref. 17). A duel is said to be noisy if each combatant hears the other's shots.

Strickland (Ref. 18) gives a compilation of single tank versus single tank duel results.

Ancker (Ref. 3) gives a highly informative and very useful annotated bibliography on stochastic duels in his 1967 survey paper. Further references to papers of interest are given in the short bibliography included herewith.

17-14 SUMMARY

Duels depend on kill rates (the product of rate of fire, single shot hit probability, and the conditional chance that a hit is a kill), which as a single measure of effectiveness summarizes the expected performance of a weapon. The analysis of duels contributes much toward evaluating the parameters which affect weapon performance. Also, chances of winning a duel in which a single weapon or weapon system is engaged against another throw much light on the performance of potential weapons in combat. Then again, duels may be used to study various strategies in the employment of weapons, and hence give some insight in this area. There are many different types of duels, and the volume of literature covering investigations of duels and their outcomes is huge indeed, though worthy of review by the analyst.

It should not be expected that one side, or nation, can in all cases afford to field weapons which possess high duel chances of winning, but rather each side must field weapons which have an edge over similar weapons of a potential enemy at the time, thereby guaranteeing that battles will be won nevertheless.

The concept of "fighting power" which involves the number of weapons and warheads available multiplied by the kill rate of that type of weapon may be used to build up the potential combat effectiveness of a side. Hence, we may now return to par. 8-7 and the question raised there about the problem of quantifying the overall effectiveness or firepower of an infantry division, or armored division, etc. Indeed, we begin to see that the concept of "total fighting power" of a side may begin to throw some light on the organization's combat capability or potential.

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CHAPTER 18

RESPONSE TIME

Some implications of response times for weapons are discussed.

18-0 LIST OF SYMBOLS

$Pr(T_R < T_T)$ = probability of weapon response time being less than target exposure time
$Pr(t_B < t_R)$ = probability that the response time of B is less than that of R
$P_K(B)_1$ = chance weapon B kills weapon R with the first (a single) shot
$p_R(T)$ = probability density function of weapon response time
$P_{T,R}$ = $Pr(T_R < T_T)$ = chance weapon response time is less than target exposure time
$p(t\,|\,i)$ = probability density of time t, given i rounds
p_B = $p_k(B)$ = single shot kill probability of weapon B
p_R = $p_k(R)$ = single shot kill probability of weapon R
$p_T(T)$ = probability density function of target exposure time
T, t = time
T_C = random value of weapon cycle time
T_R = random value of weapon response time
T_T = random value of target exposure time
t_B = random value of the response time of weapon B
t_R = random value of the response time of weapon R
$t_R^{(i)}$ = ith cycle time of weapon R
α = τ/τ_T
λ_B = expected (population) rate of fire for weapon B
λ_R = expected (population) rate of fire for weapon R
τ = expected value of weapon response time (a population parameter)
τ_B, τ_R = mean (population) response times of weapons B and R, respectively
τ_T = expected value of target exposure time (population parameter)

(Other symbols are defined in Table 18-1 or made clear on the figures.)

18-1 GENERAL

Every weapon system will experience a delay between the time when a firing demand is imposed upon it and the time that the weapon is actually fired. This delay is defined as the response time of the weapon or weapon system. It is a complex function of the characteristics of the weapon, the personnel operating it, and the tactical and physical environments in which it is employed. Since its constituents are random variables, then it follows that the response time itself is a random variable. The response time often is represented by its mean or expected value τ. (Greek letters are used to indicate population parameters.)

The response time T_R and cycle time T_C will have a complementary relationship, i.e., if the firing requirement is generated at time $T = 0$, the first round will be fired at $T = T_R$ and the next round at time $T = T_R + T_C$. Usually, the response time will be greater than the cycle time since the response time usually encompasses more elemental actions than the cycle time, e.g., bringing the system up to a state of combat readiness. In addition, response time may be dependent upon elements external to the

system, e.g., the request to fire an artillery battery could originate at a forward unit and hence the message would experience a delay in transmission through the fire control net.

We already have seen the importance of firing first in the duel situations of Chapter 17—hence, the need for quick response times. Rate of fire is also of importance in duels, but our primary emphasis in this chapter is to cover only the response time of weapons.

18-2 SIGNIFICANCE OF RESPONSE TIME

Response time can affect significantly weapon system performance. For example, it may limit the ability of the weapon to respond to a target of opportunity or lose the kill advantage that may be realized in a dueling situation by the system which otherwise would be firing first.

To illustrate further the significance of the response time parameter, consider the following situation. Assume the time that a target is exposed is a random variable whose distribution is described by the exponential probability density function

$$p_T(T) = (1/\tau_T)\exp(-T_T/\tau_T), \quad T_T \geq 0 \tag{18-1}$$

where

τ_T = expected value of target exposure time (population parameter)

T_T = random value of target exposure time .

The exponential density function is used frequently to represent this exposure time because of its computational simplicity and its usual representativeness of the situation. However, other densities could readily be employed where the situation or the observed data indicate them to be more representative. The exponential density is usually a good fit. In fact, for "time to perform" type activities, the random variable must be positive and natural distributions to fit such data are usually exponential, lognormal, or Weibull.

The response time of the weapon is also a random variable whose probability density function is also assumed to be exponential

$$P_R(T) = (1/\tau)\exp(-T_R/\tau), \; T_R \geq 0 \tag{18-2}$$

where

τ = expected response time of the weapon system

T_R = random value of weapon response time .

Again, the response time represents a "time to perform" type activity, which often follows an exponential distribution. Then, the chance that the weapon response time is less than target exposure time is

$$P_{T,R} = Pr(T_R < T_T) = \tau_T/(\tau_T + \tau) = 1/(1 + \alpha) \tag{18-3}$$

where

$\alpha = \tau/\tau_T$.

$Pr(T_R < T_T)$ = probability of weapon response time being less than target exposure time

It is relatively easy to establish this result for two exponential distributions, and it will often be used for other applications in this handbook, for example, for "availability" of a weapon system. To obtain Eq. 18-3, one merely multiplies the two probability density functions, Eqs. 18-1 and 18-2; with this product multiplied by $dT_R dT_T$, then T_R is integrated from zero to T_T; and finally T_T is integrated over its range from zero to infinity to get the result.

The effects of the response time parameter may be observed from the plot of Eq. 18-3 given in Fig. 18-1. For example, when the expected response time equals the expected target exposure time, i.e., $\alpha = 1$, the probability of the weapon system being able to fire (assuming it is available) before the target leaves the weapon zone of coverage is 0.5. The weapon effectiveness may thus be reduced by 50% since all of the munitions effectiveness measures are dependent upon the firing of the weapon. (It should be noted that the time of flight of projectiles also must be included in the response time of the weapon.)

18-3 APPLICATION OF RESPONSE TIME

The primary application of the response time parameter in a weapon systems analysis, as indicated by the preceding discussion, will be as an input quantity to the system model. Such a model would describe and include the elapsed time between the origination of a firing requirement and the actual firing of the first round to the target. This elapsed time will be caused by both hardware- and operator-induced delays, and includes such elemental tasks as:

1. Transmitting target detection information to the weapon system (if the detection means or firing demand is external to the weapon system itself)
2. Bringing the operator (crew) and the weapon system to a state of combat readiness (i.e., availability)
3. Determining target coordinates, and target acquisition and target information tracking
4. Preparing firing data
5. Arming and aiming the weapon
6. Firing (including time of flight, if applicable).

The specific components of the response time would be determined by the type of system being considered. For example, the delay elements for an air defense system would differ from those of an antitank weapon. The response time of a system is affected also by the tactical role of the system, i.e., whether it is in an offensive or defensive role. Similarly, whether the weapon is fired from an emplaced position or must be relocated before firing will affect its response time. Thus, in developing estimates of a weapon systems response time, care must be taken to ensure that the derivation corresponds to the manner in which the weapon is being employed. Modification of existing or experimentally derived estimates of the expected response time may be necessary to incorporate any added or deleted delay elements. For example, if experimental data are obtained for the expected response time of an antitank weapon in an emplaced position and the scenario being considered entails movement and set up of the weapon, the expected response time estimate should be modified to include the estimated additional times required for the relocation of the weapon to the firing position.

18-4 TECHNIQUES FOR MEASURING RESPONSE TIME EFFECTS ON WEAPON SYSTEM PERFORMANCE

As previously noted, response time is used by the weapon systems analyst as an input to the system evaluation model. However, a detailed description of the specific use of this parameter within the model is inappropriate for this chapter, being reserved for applications of models and measures of effectiveness. Nevertheless, some discussion concerning the techniques for determining the impact of response time on weapon performance is warranted in this chapter. Earlier (par. 18-2), the effect of response time on the weapon system performance using a probabilistic approach was discussed. Within this paragraph, three additional approaches will be briefly presented, namely:

1. A simple graphical approach

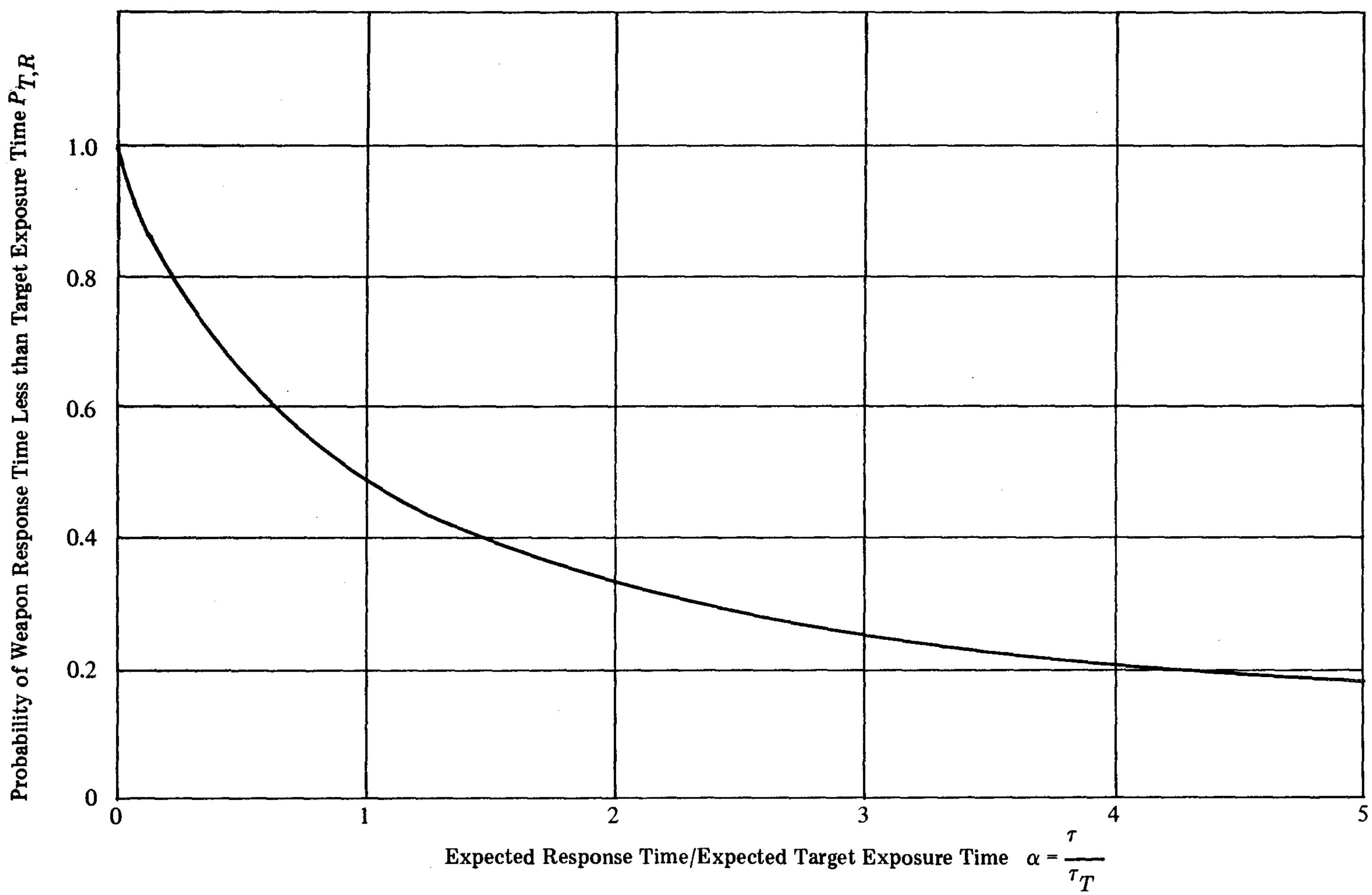


Figure 18-1. Probability of Weapon Response Time Being Less Than Target Exposure Time $P_{T,R}$ vs Ratio of Expected Response Time τ to Expected Target Exposure Time τ_T

2. A probabilistic approach applied to a dueling situation
3. A simulation.

18-4.1 GRAPHICAL APPROACH

A graphical analysis provides a simple means for investigating the impact of system response times in situations wherein an intercept capability is being investigated, e.g., an air defense system. In order to illustrate this approach, consider the example of a missile defense system being investigated for its capability to defend a point target to a specified level of overpressure, which might occur from the nuclear warhead explosion of a ballistic type missile launched by the enemy. The following assumptions are made for the purpose of this illustration:

1. Representative ballistic missile trajectories will have been determined from the available threat information.

2. The minimum permissible burst altitude has been determined from the estimated warhead yield and the specified overpressure level (using, for example, Ref. 1).

3. Interceptor missile flight time contours have been determined. (The flight time contours are a series of isochronal contours for the interceptor plotted for altitude versus ground range in the plane of the incoming warhead trajectory and referred to the coordinates of the defended target.)

4. The air defense system, whose parameters are summarized in Table 18-1, consists of a detection radar, an acquisition and tracking radar, the interceptor missile, and the associated computer and launch equipment.

In order to conduct the analysis, the warhead trajectory data are first plotted as a function of ground range versus altitude in the plane of the trajectory with the origin at the defended target coordinates and with the trajectory passing through the defended altitude or minimum intercept point. Time is then labeled on the trajectory with $t = 0$ taken as at the minimum intercept point. The interceptor flight contours are then overlaid on the trajectory and the maximum radar ranges and their corresponding times marked on the trajectory. Fig. 18-2 depicts the typical plot which results. The intercept zone is then determined by taking into account on the plot the identified delay times. The lower bound of the intercept zone is established by beginning with the minimum intercept point and moving back up the trajectory, taking into account each of the delays and ascertaining whether the maximum range constraints are satisfied. If any of the constraints is exceeded, then interception cannot be accomplished and the target cannot be defended to its required altitude. The upper bound of the intercept zone is determined by beginning with the maximum detection range and moving down the trajectory, again taking into account each of the delays and range constraints. If the intercept point moving in this direction is below the altitude being defended, the interception again is not accomplished and the target is not defended to its required altitude. Once the upper and lower intercept bounds are established and determined to satisfy the defense requirements, the corresponding zones for detection,

TABLE 18-1. SAMPLE ABM DEFENSE SYSTEM PARAMETERS

Maximum Detection Range R_D
Minimum Handoff Time to Acquisition Radar T_{DA}
Maximum Acquisition Range R_A
Minimum Handoff Time to Tracking Radar T_{AT}
Maximum Tracking Range R_T
Minimum Time to Launch T_L
Minimum Interceptor Flight Time (to Defended Point) $T_{I_{min}}$
Maximum Interceptor Flight Time (to Interceptor Maximum Range) $T_{I_{max}}$

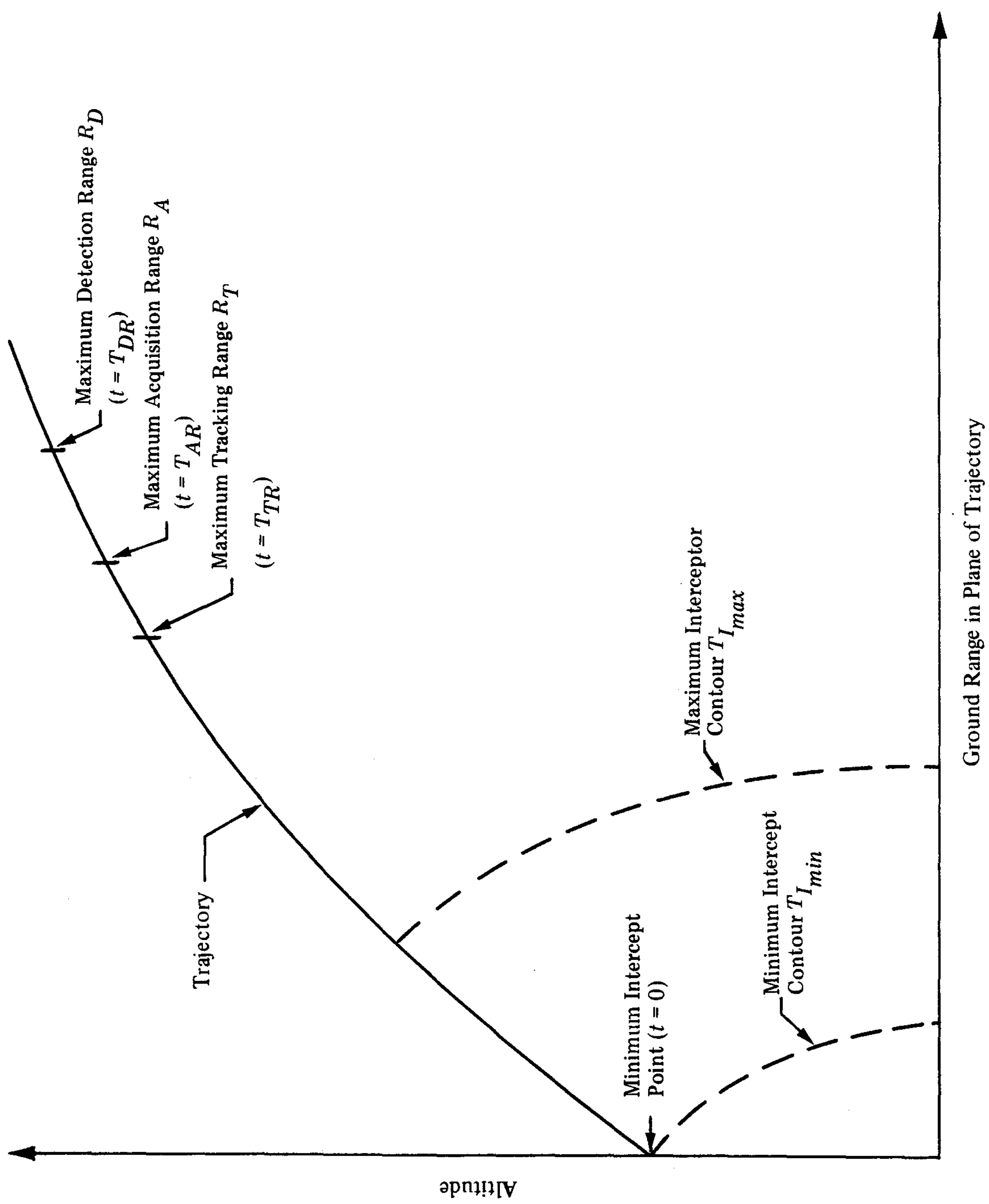


Figure 18-2. Intercept Engagement Representation

acquisition, tracking, and launch may also then be identified. In the event the defense requirements cannot be met, the weak link readily is identified and requirements for reducing a specific delay time or increasing a maximum range are established. Figs. 18-3(A) and 18-3(B) present simple flow diagrams of the processes for establishing the two intercept bounds. The graphical approach provides a simple but useful means for assessing the response time effects in an intercept engagement and identifying any potential weak links in the system. Thus, a plot of this nature helps enormously in any preliminary evaluation of system response time problems.

18-4.2 PROBABILISTIC APPROACH

Another approach for assessing the impact of response time arises in a weapon dueling situation wherein a probabilistic approach may be employed. In order to illustrate this approach, assume the following set of conditions for any general type of weapon duel:

1. The single shot kill probability of weapon B is

$$p_B = p_k(B)\,.$$

2. The single shot kill probability of weapon R is

$$p_R = p_k(R)\,.$$

The single shot kill probabilities as used in this example are conditional kill probabilities; i.e., the probability the weapon will achieve a kill, given the weapon is fired.

3. The response time distribution of each weapon is exponential with means τ_B and τ_R, respectively.
4. The cycle time of weapon R is exponential with expected rate of fire λ_R.
5. Both weapons are available with perfect reliability.
6. Both weapons detect each other simultaneously.

The measure of effectiveness, we use for this example, is the probability $P_K(B)_1$ that weapon B kills weapon R with the first (a single) shot.

The purpose in using this measure of effectiveness is only to present the impact of response time. The example is not intended to present the modeling of duels *per se*, as this was covered in Chapter 17.

Let:

t_B = random value of the response time of weapon B

t_R = random value of the response time of weapon R

$t_R^{(i)}$ = ith cycle time for weapon R.

The measure of effectiveness of weapon B is then given by:

$$\left.\begin{aligned} P_K(B)_1 &= Pr(t_B<t_R)\cdot p_B \\ &\quad + Pr(t_R<t_B<t_R + t_R^{(1)})\cdot(1 - p_R)\cdot p_B \\ &\quad + Pr(t_R + t_R^{(1)}<t_B<t_R + t_R^{(1)} + t_R^{(2)})(1 - p_R)^2\cdot p_B \\ &\quad + \cdots \\ &= p_B\{Pr(t_B<t_R) + \sum_{i=1}^{\infty}[Pr(t_R + \sum_{j=1}^{i-1} t_R^{(j)}<t_B<t_R + \sum_{j=1}^{i} t_R^{(j)}(1 - p_B)^i]\} \end{aligned}\right\} \quad (18\text{-}4)$$

$T_{I_{min}}$ = minimum intercept time
T_L = launch initiation time
T_T = tracking initiation time
T_{TR} = time to tracking range
T_{DR} = time to detection range
T_{DA} = minimum handoff time to acquisition radar
T_{AR} = time to acquisition range
T_{AT} = minimum handoff time to tracking radar
$T_{I_{max}}$ = maximum intercept time
T_I = intercept time

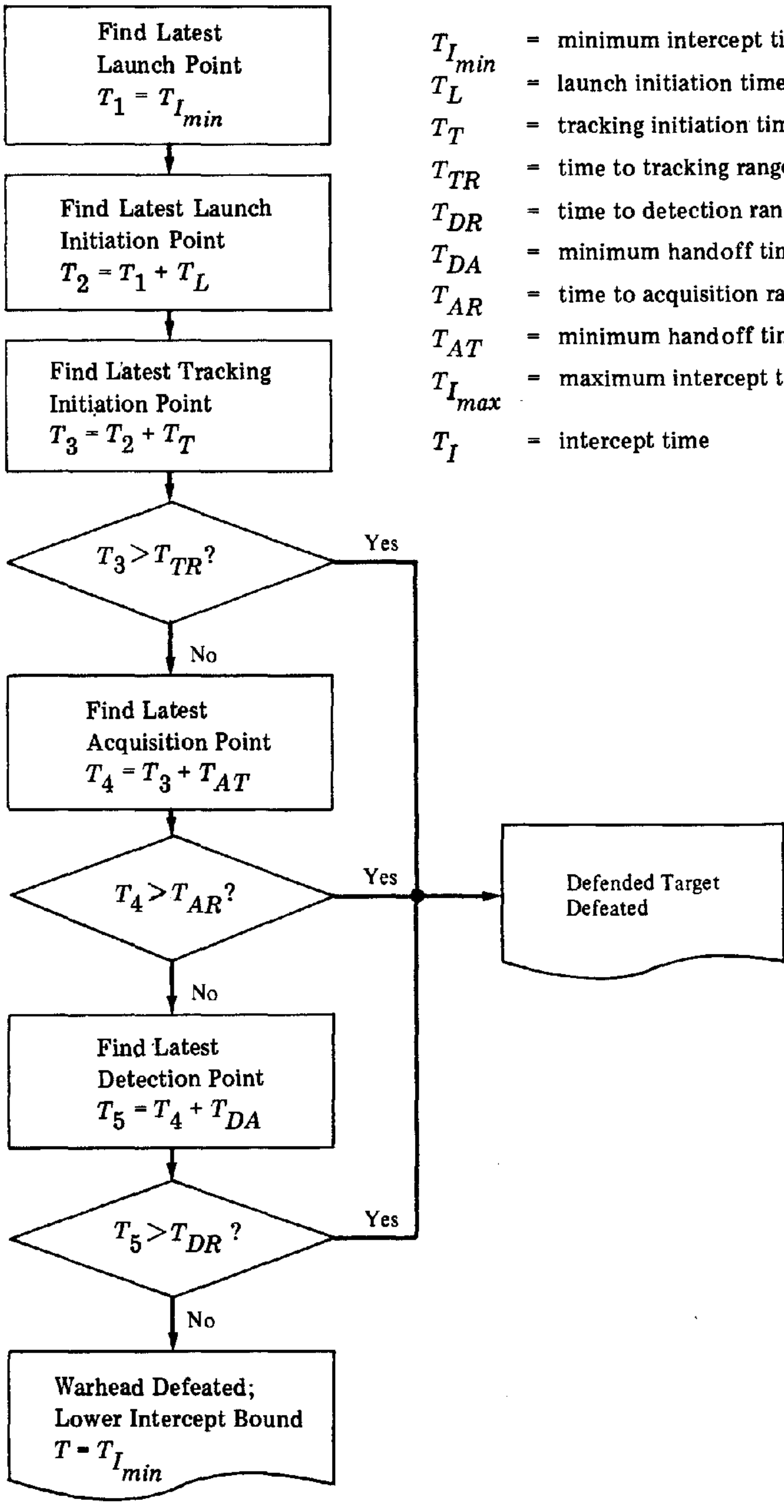


Figure 18-3(A). Determination of Lower Intercept Bound

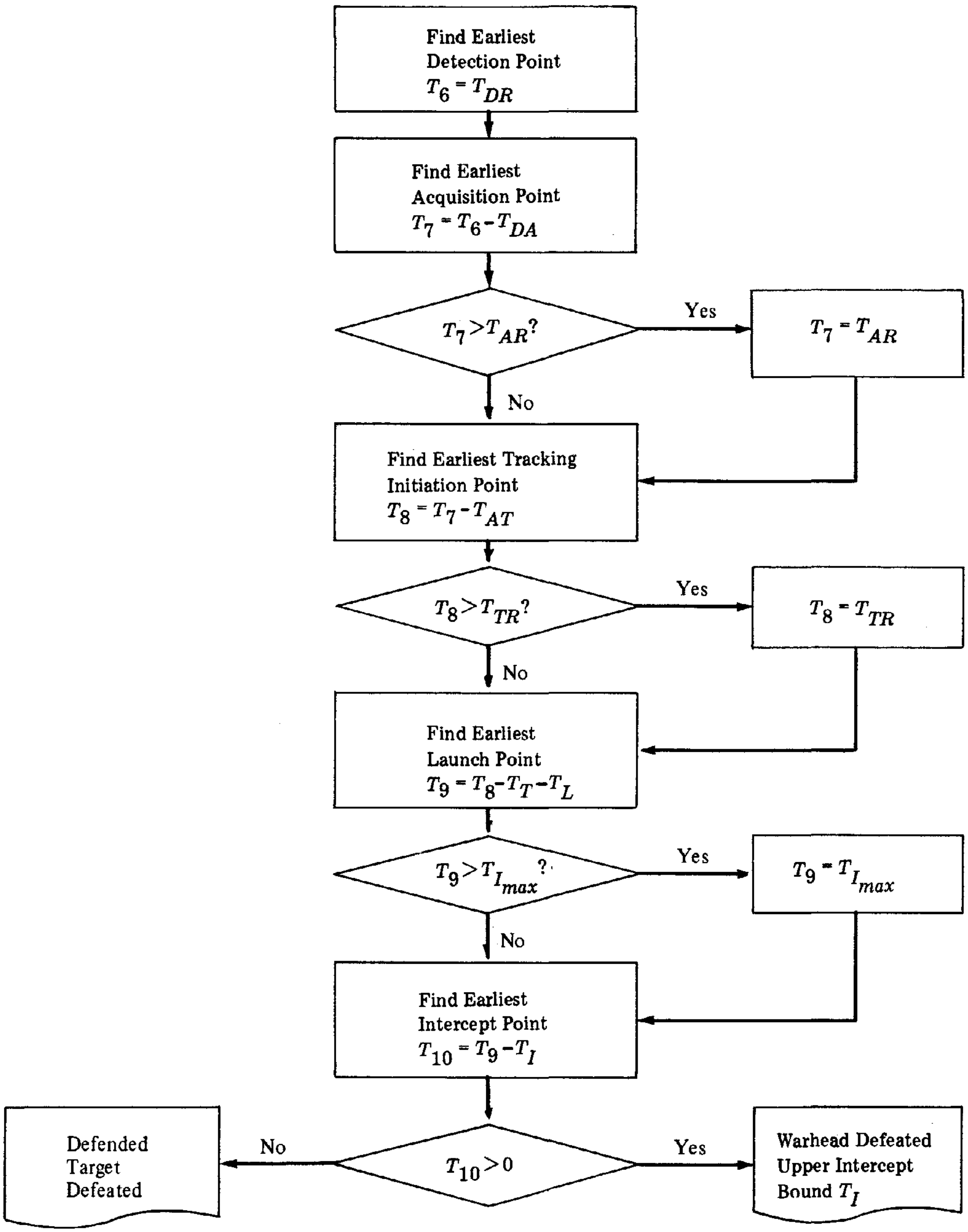


Figure 18-3(B). Determination of Upper Intecept Bound

The first term in Eq. 18-4 represents the probability of kill when B fires first; the second when R fires first but does not kill and B fires and kills before R can refire; and so on. The probability of B firing first when both have exponential response time probability distributions is similar to the case presented in par. 18-2, Eq. 18-3, and is given by

$$Pr(t_B<t_R) = \tau_R/(\tau_B + \tau_R)\,. \tag{18-5}$$

Evaluation of the subsequent terms is considerably more difficult. However, for most practical problems only the first few terms of the summation need to be evaluated. In fact, evaluation of the summation in Eq. 18-4 may be accomplished by recognizing that the time to fire i additional rounds will obey the gamma distribution

$$p(t\,|\,i) = \frac{\lambda}{(i-1)!}\,(\lambda t)^{i-1}\exp(-\lambda t),\ t \geq 0\,. \tag{18-6}$$

The use of this distribution then leads to the result

$$Pr\left[t_R + \sum_{j=1}^{i-1} t_R^{(j)} < t_B < t_R + \sum_{j=1}^{i} t_R^{(j)}\right] = \frac{(\tau_B)^i(\lambda_R)^{i-1}}{(\tau_B+\tau_R)(\lambda_R\tau_B+1)^i}\,. \tag{18-7}$$

Finally, the probability that B eliminates R in one round, using especially R's firing rate λ_R is the manageable equation

$$P_K(B)_1 = p_B\left[\frac{\tau_R}{\tau_B+\tau_R} + \frac{\tau_B}{\tau_B+\tau_R}\sum_{i=1}^{\infty}\frac{(\lambda_R\lambda_B)^{i-1}(1-p_R)^i}{(\lambda_R\lambda_B+1)^i}\right]. \tag{18-8}$$

It can be seen that usually only a few terms, perhaps no more than three or four, will be required to give a sufficiently accurate answer.

Fig. 18-4 presents the evaluation of Eq. 18-8 for several values of p_B and λ_B = 4 rounds per min. The sensitivity of the measure of effectiveness to the expected response time of weapon B is readily apparent from the figure, and the effects of response times therefore can be evaluated in detail.

18-4.3 SIMULATION APPROACH

The final approach for assessing the impact of response time on weapon performance to be discussed here is the employment of a simulation study that represents the engagement as realistically as possible. Estimates of the impact of response time on the weapon performance are developed through repeated replications of the simulation and analyses of outcomes. The response times are simulated using Monte Carlo techniques applied to specific distributions. (This approach is particularly useful for situations in which the distributions are difficult to manipulate analytically, e.g., the lognormal probability density function or a more complex one.) For example, the dueling situation described in the preceding paragraph could be approached using a simulation rather than a probabilistic approach. Fig. 18-5 presents a simple flow diagram of a simulation that could be developed to examine this particular situation. The model described by Fig. 18-5 may be easily programmed in either BASIC or FORTRAN language. Specific techniques for implementing the Monte Carlo processes shown within

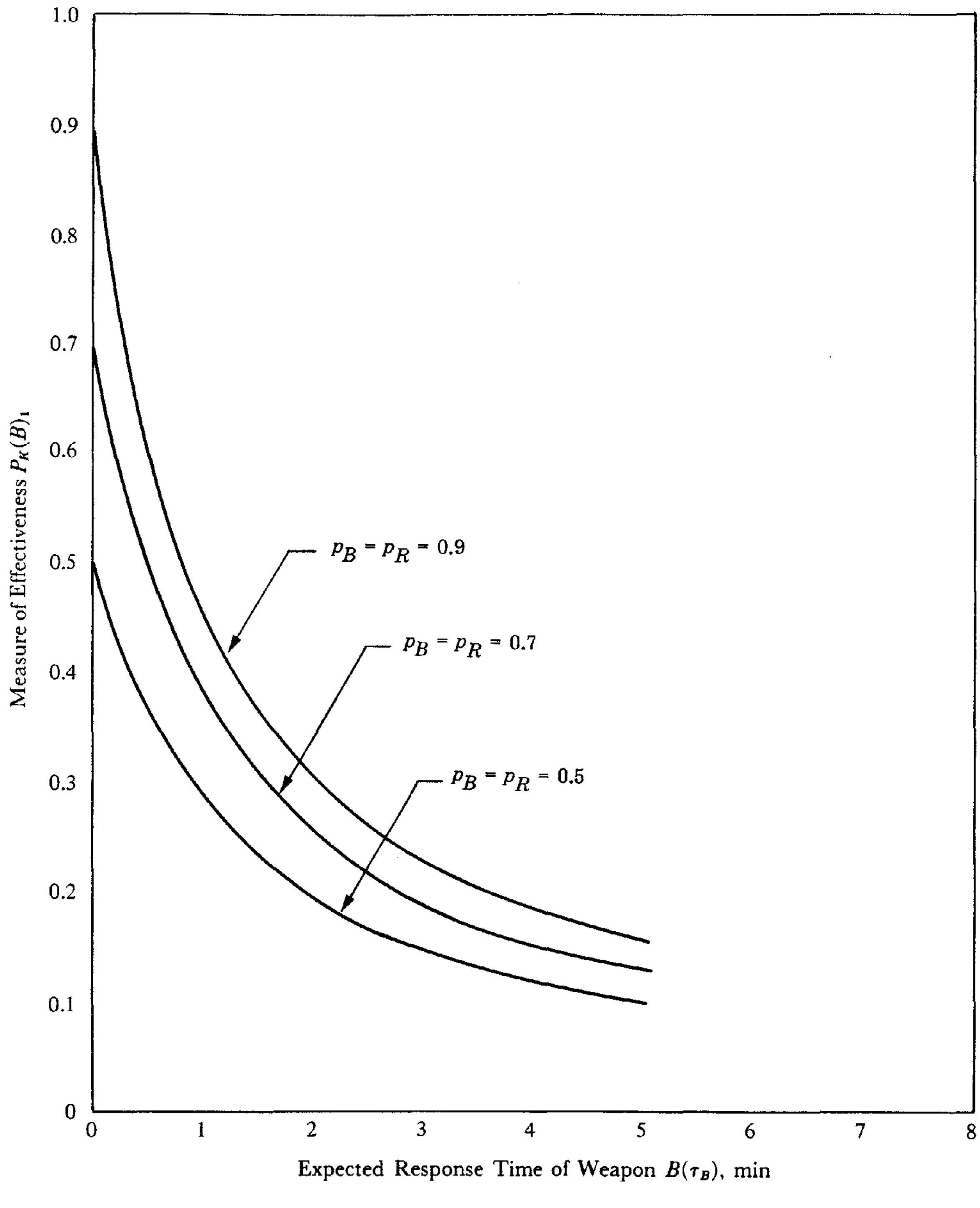
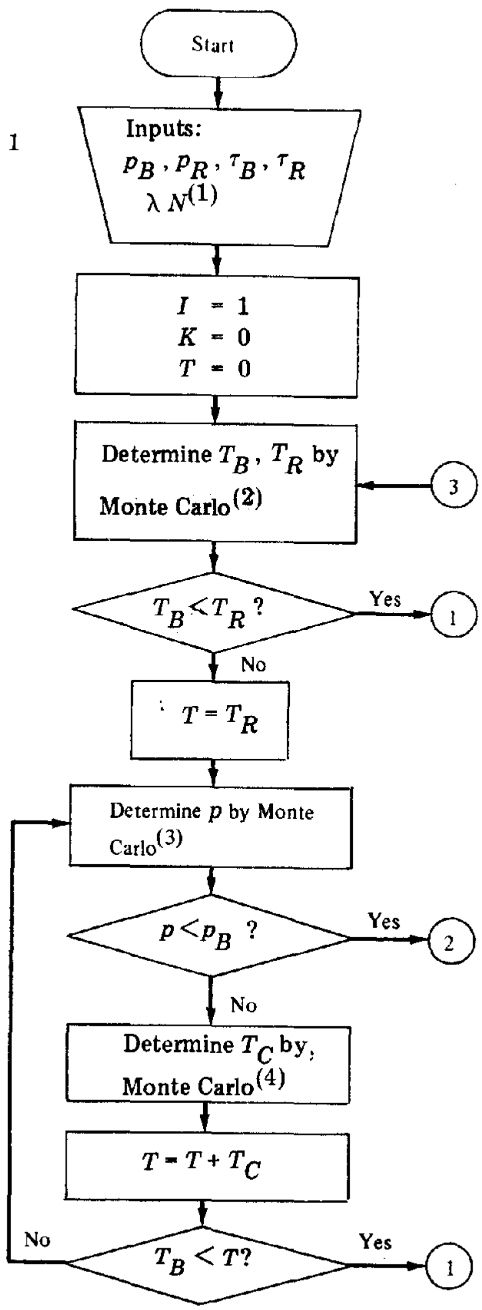
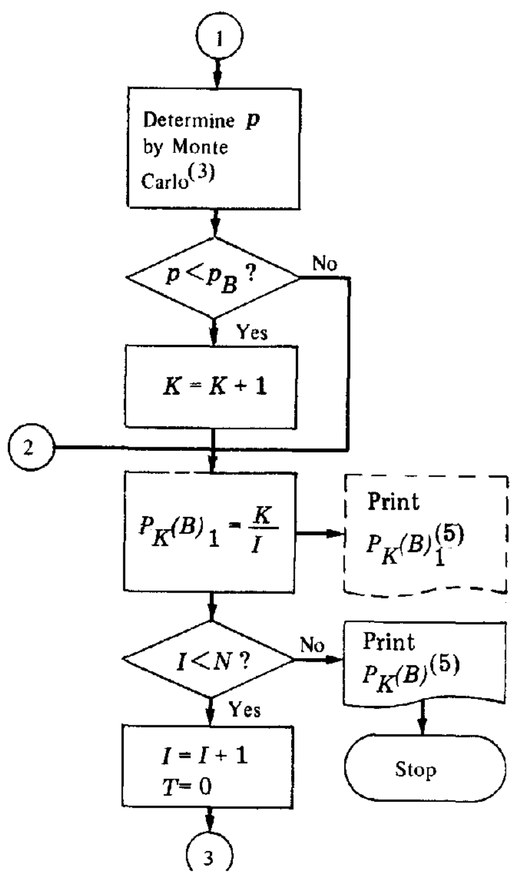


Figure 18-4. Weapon *B* Effectiveness $P_K(B)_1$ vs Expected Response Time τ_B When Weapon *R* Has an Expected Response Time τ_R of 1 min and an Expected Rate of Fire λ of 4 Rounds/Minute





FOOTNOTES

(1) p_B = single shot kill probability of Weapon B
p_R = single shot kill probability of Weapon R
τ_B = expected response time of Weapon B
τ_R = expected response time of Weapon R
λ_R = expected firing rate of Weapon R
N = number of simulation replications

(2) T_B = response time of Weapon B
T_R = response time of Weapon R
(Determined by Monte Carlo against respective exponential distributions)

(3) Determined by Monte Carlo against uniform distribution (p = a probability)

(4) T_C = cycle time of weapon R (determined by Monte Carlo against exponential distribution)

(5) $P_K(B)_1$ = probability that weapon B kills R with the first shot (intermediate values of $P_K(B)_1$ may be printed to determine rate of convergence of $P_K(B)_1$ with increasing replications).

Figure 18-5. Flow Diagram for Sample Problem Simulation

the flow diagram are given in Ref. 2. One should note in particular the use of population parameters as contrasted to the generation of random variables as required. The reader may follow the step-by-step procedure to estimate $P_K(B)_1$ by Monte Carlo, and hence see the enormous potential of the process.

The reader can easily see that in a simulation the input parameters may be varied over any ranges thought to be of practical interest and thereby study the implications of any broad conditions likely to be encountered. Simulations are also useful to study the robustness of various analytical techniques or models.

18-5 FURTHER DISCUSSION

The response times of weapons or weapon systems cover a topic which in a single chapter must be of a somewhat broad nature, for the particular times to respond may vary greatly with the application, i.e., the system studied. Hence, the analyst must become familiar with the area of application and select the best approach to analyze or model the response times involved. Indeed, each weapon systems analysis may call for something new beyond what we are able to cover in such a limited chapter as this one. Also, response times usually are analyzed along with rate-of-fire problems. Therefore, it becomes of some interest to mention the two applications that follow.

Benton (Ref. 3) gives a discussion of response times for the PERSHING missile system. In this connection, the times required to carry out tactical fire missions, before and after launch, were analyzed from data based on the annual service-practice firings of the PERSHING missile. The tactical operation response times included length of times spent by the unit for assembly, occupation, firing, and evacuation of the missile or missile system to accomplish the assigned fire mission. Table 4 of Ref. 3 gives a summary of the times required for this analysis based on 34 missions. Tactical count times and total tactical times also were analyzed. The chance of firing the missile within the prescribed time was found to be high—about 0.90, so that the system response time seemed to be satisfactory. An analysis similar to this might well be carried out for other systems. In particular, one might well try also to fit some reasonable probability distribution to the data analyzed on response times and its components so that future inferences could be made as required.

In another overall type study, Pitts and Brewer (Ref. 4), report on an evaluation of some armor and antiarmor weapon systems. The study covers delivery accuracy, rate of fire, and terminal effects, but also involves response times which have an effect on the expected overall performance of such systems. In fact, this and other such analyses might give the reader a general idea of what may be expected in connection with the weapon system response time problem.

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CHAPTER 19

FUZING

Fuze action generally has some effect on both the delivery accuracy of projectiles or missiles along their trajectories and also the terminal effectiveness of the warhead. Thus, the analyst must be familiar with the principles of fuze operation and evaluate fuze performance, including in particular random variations which must be taken into proper account in the analysis of weapon systems. The reliability and safety of fuzing systems represent major considerations to be reckoned with.

19-0 LIST OF SYMBOLS

A_i = ith possible path of a system
E_j = measure of weapon effectiveness for warhead detonation in jth trajectory segment
e = voltage
$F_i(s) = \sum_{j=1}^{s} p_{ij}$
I = antenna current
I_i = segment of a trajectory
I_j = jth trajectory segment
I_s = most desirable trajectory segment for fuze functioning
i = index for the number of fuzes
j = index for the segments of a trajectory
k = kth fuze to function for a k-out-of-n fuze design
ℓ = running variable (integer)
m = number of segments a trajectory is divided into
n = number of fuzes or fuze elements
P_j = chance the fuze system detonates the warhead in the jth trajectory segment
P_m = chance of a dud for a multifuze system
P_s = chance that the fuzing system functions the warhead in the most desirable trajectory segment
P_1 = chance of a premature for a multifuze system
$P(A)$ = probability of A occurring
p_1 = chance of premature for a single fuze of a multifuze system
p_{ij} = probability that the ith fuze or element functions in I_j, the jth trajectory segment ($\sum_{j=1}^{m} p_{ij} = 1$)
p_m = chance of a dud for a single fuze of a multifuze system
r = integer for highest i
s = integer, denoting optimum j
$t_1, t_2, t_3, \ldots$ = fuze running times
Z = antenna impedence
σ = standard deviation of an individual fuze

(Other symbols are defined on Fig. 19-14, but are restricted to that figure only.)

19-1 INTRODUCTION

Normally, a fuze is a device which is used to initiate the high explosive charge of a projectile or warhead at the desired time and under the circumstances required. Fuzes are used also as ignition devices or to initiate action for opening a warhead to release submissiles, for example. Formerly, fuzes were classified generally according to their position on the projectile—e.g., "point-detonating", "base-detonating", or "concrete piercing". More recently, however, they may be classified according to type of physical phenomena involved—such as mechanical, electrical, barometric, radio proximity, optical, computer, or command guidance. Fuzes must possess very high reliability—i.e., should produce a very small dud rate, should be safe, should not premature or function before the desired instant, and should self-destruct as required. Finally, a point of much interest to the analyst also is that fuzes should be precise—i.e., have small standard deviation in functioning time, functioning height above ground, or in delay action, etc.

Fuzes perform the four major functions of safing, arming, sensing, and firing as follows:

1. *Safing.* The fuze must be designed so that it will not function prior to the desired time. The missile or projectile must be safe to store, transport, handle, and launch. The safing elements must not permit premature functioning due to rough handling, shipment, or the forces of launch (up to 30,000 g's acceleration in some projectiles.)

2. *Arming.* At the proper time after launch the fuze must arm and be ready for detonation at the desired time. Arming can be accomplished by proper utilization of the natural phenomena which occur during projectile or missile flight (such as set-back or centrifugal force), by a timing device, or by external signal (usually a radio signal).

3. *Sensing.* The fuze must sense the presence of the target. It may do this by (a) contact with the target, (b) sensing proximity of the target, (c) measuring a predetermined time from launch, or (d) measuring characteristics of the trajectory (e.g., deceleration) which might indicate presence of the target.

4. *Firing.* Under circumstances determined to produce optimum effects, the fuze must detonate the projectile or missile. Again, this may be on contact with the target, after a short delay, or in proximity to the target. The fuze also may be fired by an external signal.

All fuzes, regardless of the projectile or missile warhead which they must detonate, perform these four functions. In guided missile fuzes, however, the degree of reliability and safety required is usually greater than in smaller projectiles. Nuclear or other special type warheads and complicated guidance circuitry make the guided missile a very expensive item; to waste one because of a fuze malfunction, is, to say the least, quite uneconomical, and in the case of a nuclear warhead could determine the outcome of a battle. On the other hand, a premature nuclear, or even HE explosion, may cause deaths of friendly troops or local population and the loss of valuable facilities. It should be noted that with more sophisticated weapons it is often difficult to separate the fuze and guidance functions. Hence, special fuzing systems or adaption kits are required for such warheads.

Space and weight limitations, while still critical, are not nearly so restrictive in a guided missile as in an artillery projectile. Therefore, missile fuzes usually can be built to contain a great deal more intelligence than smaller, more compact and lighter fuzes for grenades, projectiles, rockets, and bombs.

This chapter provides an overview of the fuzing problem primarily from the standpoint of the analyst, who must take into consideration the functioning characteristics of fuzing systems in so far as they affect the performance of weapons. In order to suit the needs of the analyst, this chapter is partly descriptive and partly analytical. No attempt is made in this account to cover all details of the physical phenomena involved in the performance of the various types of fuzes. Rather, for the analyst, we present outlines of fuze types with some physical characteristics, and attempt to cover some of the more

important analytical considerations the systems analyst might be confronted with. Much of the descriptive material here is taken from a good introduction to the subject, i.e., Chapter 11, Fuzes, of Ref. 10. For a good background the analyst should read Refs. 1-9.

We comment here that terminal ballistics is the study of the effect and action of projectiles and warheads of all types against targets that might be encountered on the battlefield. The ability of a projectile or warhead to bring about the damage desired against a target depends on many factors (if applicable), including, for example, the amount of explosive in the warhead, the density and thickness of case metal, striking velocity, shape and weight of projectile, angle of impact, rotation of projectile, vulnerability characteristics of the target attacked, and especially the performance of the fuze or fuzing system. The fuze is of considerable interest, for it usually affects both the terminal ballistic performance of the warhead or projectile and also the delivery accuracy of the warhead along the trajectory. In this connection, the analyst must realize and take into account the statistical variations brought about due to fuze performance, for this also brings about variation in weapon effectiveness.

The fuzing function should be such as to cause a warhead to detonate at a position that results in maximum damage to the target. The optimal burst point will depend on considerations such as those previously given. Inasmuch as prior knowledge of the optimal point may or may not always be known, the effect of fuze function on weapon effectiveness is influenced by two considerations. The first is the ability of the delivery system to place the warhead on a path which will pass through the optimal burst point, and secondly the capacity of the fuze to function, i.e., to produce detonation at that point. If the delivery system fails in the achievement of its objective, then the fuze should operate in a way which is nearly optimal under the degraded circumstances. Such optimal behavior may, under certain circumstances, be fuze deactivation, or warhead detonation after some arbitrary passage of time, i.e., self detonation.

For all fuzes, high reliability in safing and arming is a necessity. The fuzes must have highly dependable safety measures to prevent functioning at a time other than that desired. Arming features must guarantee the sequence of events which transforms the fuze from a safe condition to a state where proper functioning results in defeating the target.

We go into some detail here on fuzes, for they represent the "brains" of many weapons, and the analyst must evaluate their performance.

19-2 SOME GENERAL TYPES OF FUZES

General and detailed information on the design, construction, operation, and limitations of the various types of fuzes may be found in appropriate documents such as technical manuals covering ammunition types, the references listed herewith, and other publications on the subject. The particular type of fuze used in a given application will depend, of course, on the weapon, the target and its vulnerability, the terminal performance required for target defeat, fuze space available, and cost—a few of the considerations. For the weapon systems analyst, we give here only a brief and broad description of fuze types and of some general interest, namely:

1. Contact or impact fuzes
2. Time fuzes
3. Proximity or influence fuzes
4. Optical fuzes
5. Ambient fuzes
6. Command fuzes
7. Computer fuzes
8. Guidance fuzes.

The analyst must keep in mind that his job is that of converting the performance characteristics of the various fuzes under study to functioning descriptions as they affect the effectiveness and potential combat capability of weapons and warheads. In particular, fuze functioning characteristics may have a decided effect in terminating trajectories or the end flight conditions of the warhead, or the detonating position of the warhead, and terminal ballistic effects.

Some typical ground targets and the recommended fuzes for attack of each type of target are shown in Table 19-1.

19-2.1 CONTACT OR IMPACT FUZES

These fuzes often have been classified according to position on the projectile and method of functioning. Examples include point detonating (PD) fuzes, base-detonating (BD) fuzes, and point-initiating base-detonating (PIBD) fuzes.

The impact fuzes are designed to function on projectile impact with the target, or at some short time thereafter, measured from instant of impact. They are classified also by type of action as superquick, supersensitive, delay, or nondelay.

1. *Superquick.* This fuze is designed to detonate the projectile or missile warhead immediately upon impact, resulting in very little cratering and a maximum effect above ground. The period of time from impact until detonation of the bursting charge for superquick fuzes is on the order of 100 μs. The firing pin is driven directly into the primer by the force of impact. This fuze is used on chemical projectiles and on HE projectiles where the target is above ground.

2. *Supersensitive.* This type fuze is the same as superquick, except that it is designed to function upon impact with a very light target, such as an aircraft wing. These fuzes are always point detonating fuzes.

3. *Delay.* This type is designed to burst the projectile in 0.50-0.25 s after impact. It includes a primer assembly of the nondelay type together with a delay powder pellet between the primer and detonator.

TABLE 19-1. RECOMMENDED FUZE APPLICATIONS

Type of target	Projectile	Fuze	Type of fire	Remarks
Armored vehicles (rendezvous)	HE, HEAT	VT, ti, Q[6]	Neutralization, Destruction, Assault	(1), (2), (3).*
Armored vehicles (moving)	HE, WP, HEAT	VT, ti, Q	Neutralization, Destruction, Assault	HE projectiles force tanks to "button up". Fire of sufficient intensity may demoralize and break up attack. WP may blind vehicle drivers, but it may also obscure adjustment (2).
Boats	HE	VT, ti, Q	Neutralization, Direct	Air bursts against personnel manning boats. Destruction by direct fire.
Bridges	HE	Q, CP, delay	Destruction, Harassing	Direction of fire preferably with long axis of bridge. Destruction of permanent bridges is accomplished best by knocking out bridge support. Fuze quick for wooden or pontoon bridges.
Buildings (frame)	HE, WP	Q	Neutralization	(4).
Buildings (masonry)	HE	CP, delay, Q	Destruction, Neutralization of large areas	Several weapons can be converged on one building. In destroying masonry buildings, the fact that rubble aids defensive fighting and delays friendly mobile elements must be considered. (5).

(continued)

TABLE 19-1. (cont'd)

Type of target	Projectile	Fuze	Type of fire	Remarks
Fortifications (armor)	HEAT, HE (large calibers)	Q	Destruction, Assault, Direct	Fire should be adjusted at apertures of steel turrets and pillboxes (5). Use highest practicable charge.
Fortifications (concrete)	HE	CP, delay, Q	Destruction, Assault, Direct	Use highest practical charge (5).
Fortifications (earth, logs, etc)	HE	Delay, Q	Destruction, Assault, Direct	Use highest practical charge (5).
Personnel (in open)	HE	VT, ti, Q	Neutralization, Harassing	TOT missions are most effective. Fuze quick should be fired at lowest practical charge (steep angle of fall gives better fragmentation). Intermittent fire is better than continuous fire (1).
Personnel (dug in)	HE, WP	VT, ti, delay (ricochet)	Neutralization, Harassing, Destruction	Air bursts are necessary. Surprise not necessary. WP is useful in driving personnel out of holes and into open.
Personnel (in dugouts or caves)	HE	Delay, Q	Destruction, Assault, Direct	(5).
Personnel (under light cover)	HE	Q, VT, ti, delay (ricochet)	Neutralization	(4).
Roads and railroads	HE	Delay, CP	Destruction	Attack critical points: defiles, fills, crossings, culverts, bridges, and narrow portions. Direction of fire should coincide with direction of road.
	HE	VT, ti, Q	Harassing, Interdiction	
Supply installations	HE, WP	Q, VT, ti	Neutralization, Destruction	(1), (4).
Vehicles (rendezvous)	HW, WP	Q, VT, ti	Neutralization, Destruction	(1), (2), (4).
Vehicles (moving)	HE, WP	Q, VT, ti	Neutralization, Destruction	(2), (4).
Weapons (fortified)	HE	Q, CP, delay	Destruction, Neutralization	Air bursts are desirable if weapon is firing. After weapon is silenced, it is attacked for destruction. Choice of fuze is determined by type of fortification. See fortifications
Weapons (in open)	HE, WP	VT, ti	Neutralization, Destruction	(1), (2), (4).

The following notes apply to remarks column:

(1) Area is neutralized with HE projectiles (air bursts if practical). Surprize is essential to produce casualties.

(2) Materiel remaining in area should be attacked for destruction by using appropriate projectile and fuze.

(3) HEAT projectiles may be used in fire-for-effect provided that ranges and observing distances are short enough to permit sensing rounds.

(4) WP projectiles should be combined with HE when the target contains inflammable materiel and the smoke will not obscure adjustment.

(5) HE projectiles with fuze quick are fired at intervals to clear away camouflage, earth, cover, and rubble.

(6) HE: High Explosive
HEAT: High Explosive Antitank
WP: White Phosphorus
VT: Variable Time (proximity)
ti: Time
Q: Quick
CP: Concrete Piercing
delay: delay

These fuzes are used when some penetration of the target is desired; for example, against earth or lightly constructed emplacements. They are also used to obtain ricochet action when firing against personnel. On armor piercing (AP) projectiles the fuze is always in the base, but for other projectiles it may be either a base or a point detonating fuze, or both.

4. *Nondelay.* A nondelay fuze is designed to detonate the projectile or missile before complete penetration of the target occurs. A small crater is obtained, most of the effect being above ground. Nondelay fuzes do not act as quickly as superquick fuzes. This is because the firing pin is of the inertia type and is carried forward against the primer by the force of inertia (setforward force) on impact. The functioning delay is on the order of 500 μs. These fuzes were used in World War II antitank HEAT (shaped charge) projectiles, but have been replaced by piezoelectric ceramic fuzes for this application to obtain faster action.

5. *Selective superquick or delay.* This fuze may be set for either superquick or delay action, thus giving versatility of performance.

6. *Piezoelectric fuzes.* For applications where extremely rapid action is required (e.g., high velocity HEAT rounds), a fuze depending upon a piezoelectric ceramic for its initiation is used. Contact with the target stresses the ceramic delivering energy to an electric detonator and starting the fuze action. This type fuze often is used as a "back up" device on nuclear weapons when it is desired to obtain very rapid action upon contact with a surface so as to avoid deformation of the warhead before burst.

19-2.2. TIME FUZES

There are three types of time fuzes: powder train, mechanical, and proximity (sometimes called VT or variable time fuze and discussed separately in par. 19-2.3.) Refer to Refs. 2 and 11 for a thorough treatment of time fuzes. Powder train fuzes, which are now obsolescent, make use of compressed black powder rings that burn for a predetermined length of time and then initiate the high-explosive element in the fuze. Mechanical time fuzes incorporate a clocklike mechanism. Through a gear train and escapement, this mechanism trips a firing pin at a predetermined time, causing the fuze to function.

In time fuzes, the time delay is initiated upon launch of the missile or projectile. The time element is obtained by the use of a train of pyrotechnic powder, an electronic oscillator and cycle counter, or a mechanical device similar to a watch mechanism. Since the burning of a pyrotechnic powder is affected by moisture, compression of the powder, and air density, mechanical time and electronic time fuzes are replacing pyrotechnic time fuzes in many applications.

Time fuzes have been used in air defense artillery projectiles where time of flight to burst could be predicted. They are not used, however, in air defense guided missiles which follow a changing trajectory. The chief uses of artillery time fuzes today are for illumination and high burst registration.

19-2.3 PROXIMITY OR INFLUENCE FUZES

These fuzes usually are divided into two classes:

1. Those depending on electric or magnetic phenomena, including electromagnetic waves
2. Pressure-sensitive fuzes, e.g., barometric.

These two classes cover almost the entire spectrum of application to weapons since missiles, bombs, artillery projectiles, and mortar projectiles all use proximity fuzes. The radio proximity fuzes initially were based on the cw Doppler principle, although microwave systems using both fm and cw modulation have been developed. The barometric fuze senses the static pressure outside the warhead in which it is mounted and initiates detonation when the pressure reaches a predetermined level. The barometric fuze thus measures indirectly proximity to the earth. They are relatively simple and cheap, but also may be relatively inaccurate and or imprecise.

The so-called VT (variable time) proximity fuze was one of the spectacular developments of World War II. This fuze is actually a combination radio or radar broadcasting and receiving set. The energy waves broadcast by the fuze are reflected from the target and picked up by the receiver in the fuze. The fuze functions when it comes within proper proximity of any target capable of reflecting sufficient energy from the radiated waves. Proximity fuzes employ many radio and radar principles of operation, including continuous wave Doppler, frequency modulation, pulse radar, and pulsed Doppler. Various other proximity devices—such as photoelectric sensers, sound detectors, magnetometers, and capacitance measuring circuits—have been explored without notable success to date in this country. With a proximity fuze, the point of detonation is determined automatically and usually more accurately than with a time fuze, especially when the distance between the firing point and the target is changing rapidly.

Most common proximity fuzes depend upon reflected electromagnetic energy for target intelligence and are, therefore, susceptible to electronic countermeasures. Subject to cost and space limitations, circuitry refinements are employed to reduce this susceptibility. Ref. 22 treats the electronic warfare vulnerability of fuzing systems.

19-2.4 OPTICAL FUZES

These fuzes depend for their action on the reception of light from the target. They operate in four conventional modes: active, semiactive, semipassive, and passive. The literature is largely classified. Optical detectors are characterized by their response time, frequency response, sensitivity, and stability. The three main phenomena by means of which detection takes place are thermal, photovoltaic, and photoconductive. Thermal detectors depend on the heating of a heat-sensitive element by incident radiation. Photovoltaic cells produce an electric potential when excited by incident radiation. Germanium cells have maximum sensitivity to radiation at 1.4 μ. Cuprous oxide and selenium have maximum sensitivity in the middle part of the visible spectrum with little sensitivity in the ultraviolet and infrared. For the photoconductive process, electrical resistance varies with incident illumination. Typical materials possessing this property are thallous sulfide, lead sulfide, and selenium that is both photovoltaic and photoconductive.

When used against airborne targets, the narrow, sharply defined radiation pattern that is possible to achieve with an optical fuze makes precise location of the target relatively simple. Furthermore, the narrow angular detection window makes active countermeasures virtually impossible because the fuze does not see the target until it reaches the optimum position for detonation.

On the other hand, the proper functioning is critically dependent on the type of missile and target. Semipassive systems, which depend on an external source, not under the control of the missile operator to illuminate the target, can only be used when such a source is available, e.g., sunlight.

19-2.5 AMBIENT FUZES

These fuzes make use of conditions existing in the vicinity of the target to estimate distance from the target. One type of ambient fuze might measure air pressure above a target to indicate altitude. Since these ambient conditions change day by day, an accurate height of burst requires accurate target meteorological intelligence, which is not always available. In addition, at high velocities it is difficult to measure accurately the air density outside the missile. These two factors combine to reduce the effectiveness of the barometric fuze when extremely accurate height of burst is desired. However, this type of fuze has the advantage of being immune to electronic countermeasures. Another type of ambient fuze has been used on depth charges where depth is determined as a function of water pressure as the charge descends toward the target.

19-2.6 COMMAND FUZES

Some missiles are detonated upon command from the firing station (which can be stationary or moving). The fire command can be sent over wires trailing from the missile, by radio, or optically.

Command fuzes dependent on radio fire commands are naturally susceptible to electronic countermeasures. However, this disadvantage can be greatly reduced by the use of coded signals, frequency shifts, directional antenna, etc. See Ref. 22 for a treatment of the subject.

19-2.7 COMPUTER AND GUIDANCE FUZES

These fuzes are often considered in the same classification.

A *computer* fuze computes the optimum time for warhead detonation from information provided by a missile target seeker. Such fuzes have been designed in a number of guided missile applications where omnidirectional warheads are used.

A *guidance* fuze derives all information from the missile guidance system. The guidance fuze thus is essentially a computer that uses guidance signals to determine the best time of detonation.

19-2.8 ILLUSTRATIONS OF APPLICATION OF VARIOUS FUZE TYPES

Fig. 19-1 shows an artillery application of a superquick fuze. Fig. 19-2 indicates an antitank application of a nondelay type fuze, which may involve a nose sensing (piezoelectric) device to initiate instantly a base detonating fuze for a shaped charge warhead. Fig. 19-3 shows an application of a delay type of fuze, and Figs. 19-4 and 19-5 indicate two applications of time or proximity fuzes.

Fig. 19-6 shows a view of a combination superquick and delay fuze ordinarily used on many artillery projectiles. The major elements of the fuze are listed on the figure.

Fig. 19-7 indicates a view of and some of the features of a typical artillery proximity fuze.

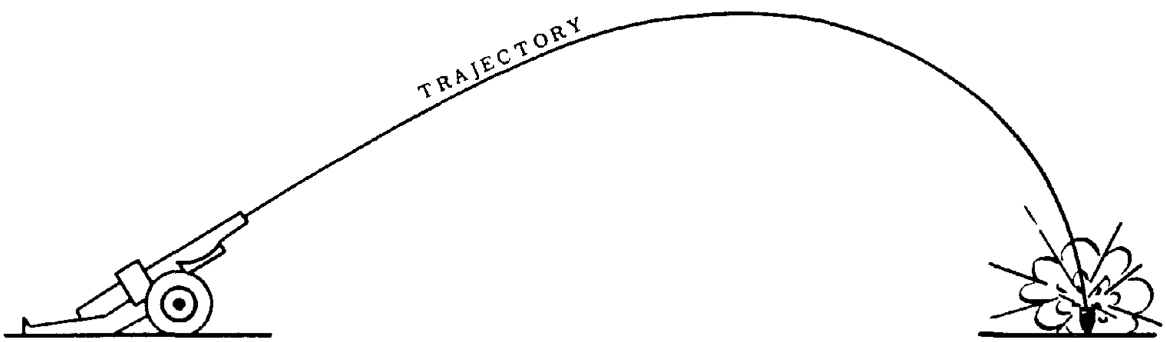


Figure 19-1. Application of Superquick Fuze

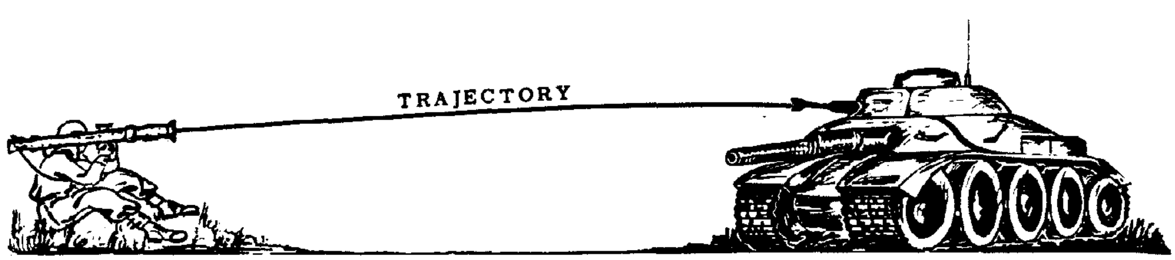


Figure 19-2. Application of Nondelay Fuze

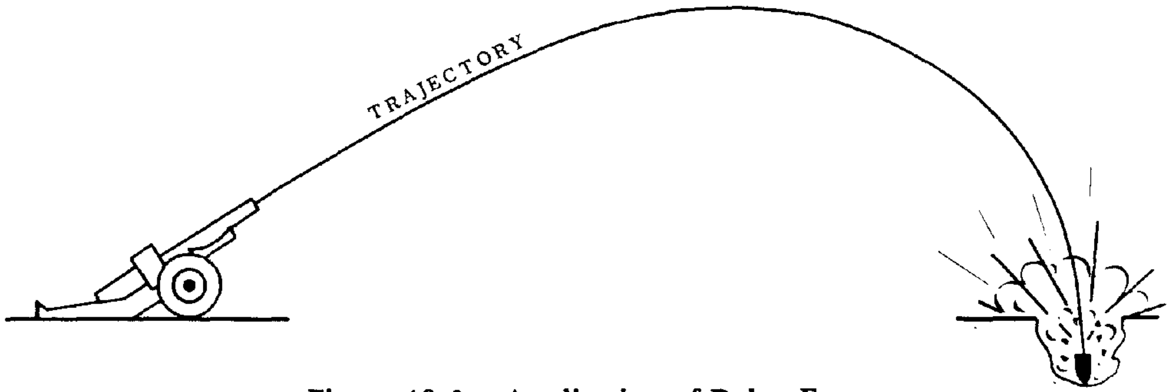


Figure 19-3. Application of Delay Fuze

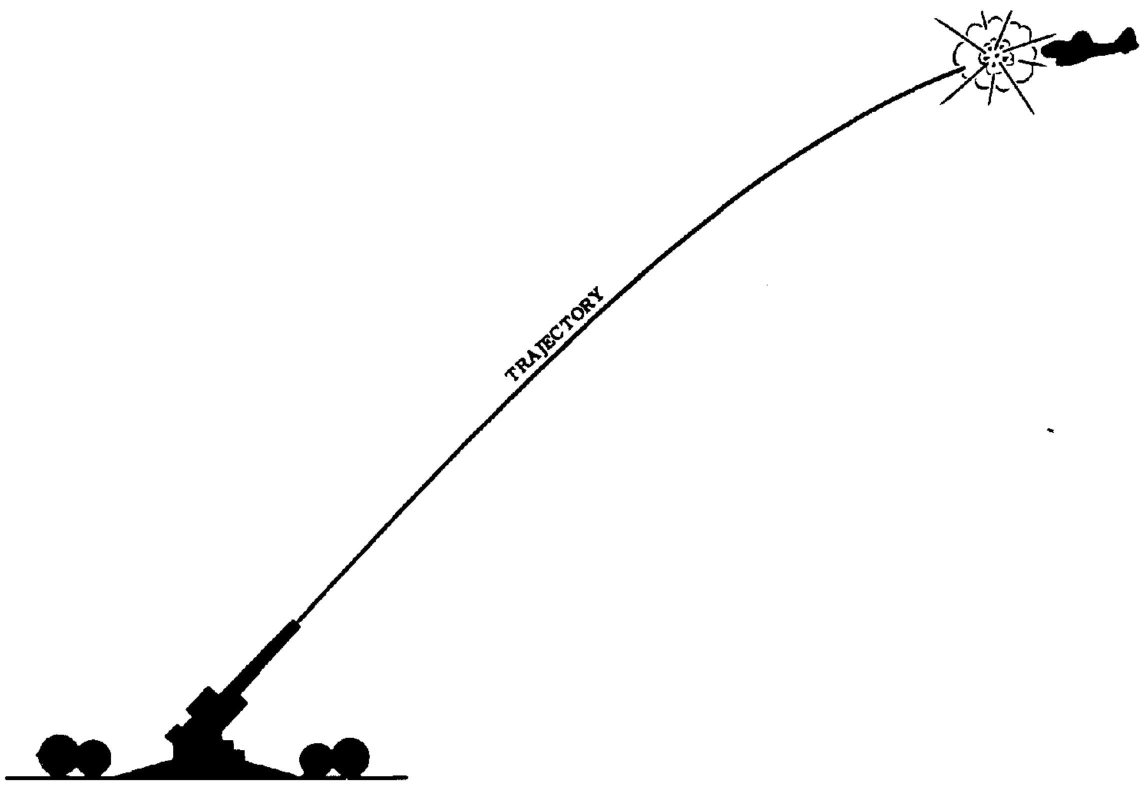


Figure 19-4. Application of Time and Proximity Fuzes to an Aerial Target

19-3 FORCES ACTING ON A FUZE

In performing the four basic fuze functions—safing, arming, sensing, and firing—advantage is taken of the natural environment of the fuze in flight and the forces resulting from this environment. These forces illustrated in Fig. 19-8 are:

1. *Setback.* Setback is the force of inertia or reaction to the extreme acceleration of the projectile in the bore of a gun, or during launch until burnout in the case of a rocket. It is the same force which throws an automobile driver back against the seat when he suddenly accelerates the vehicle. Setback is available from the instant the projectile starts to move until it stops accelerating. It is maximum for tube-launched projectiles where acceleration, hence pressure, is maximum.

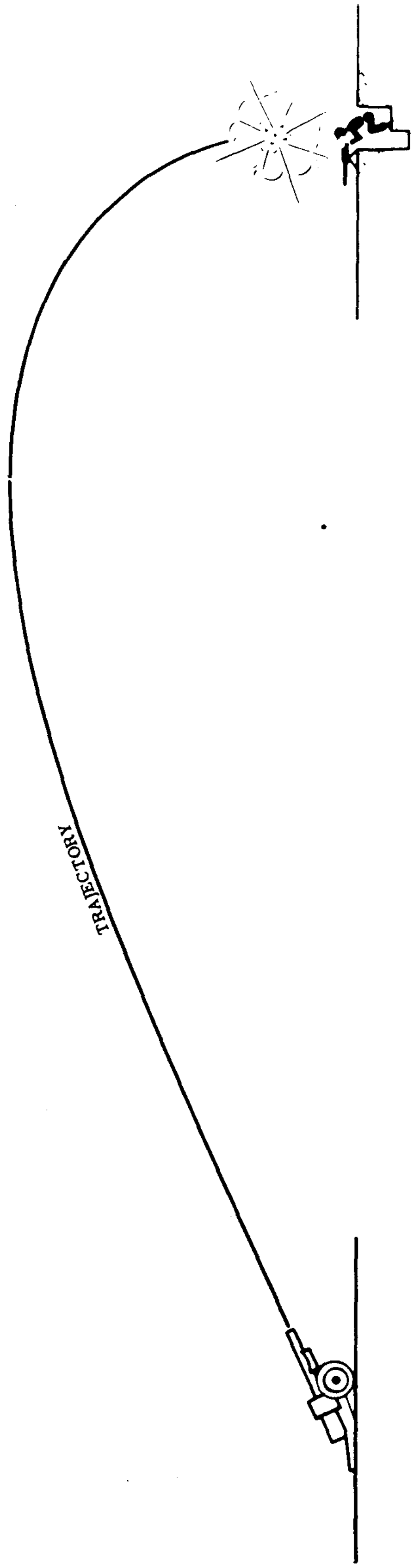
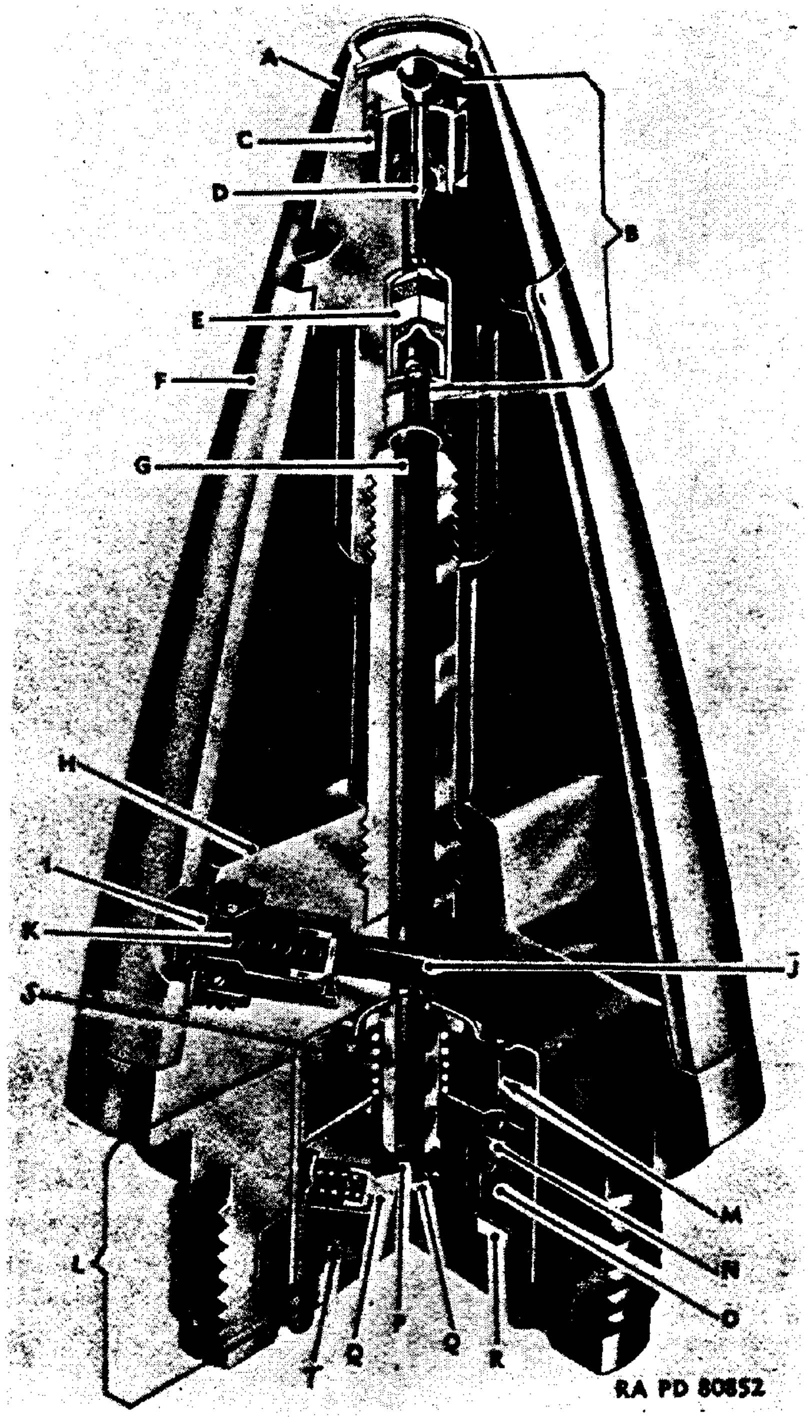
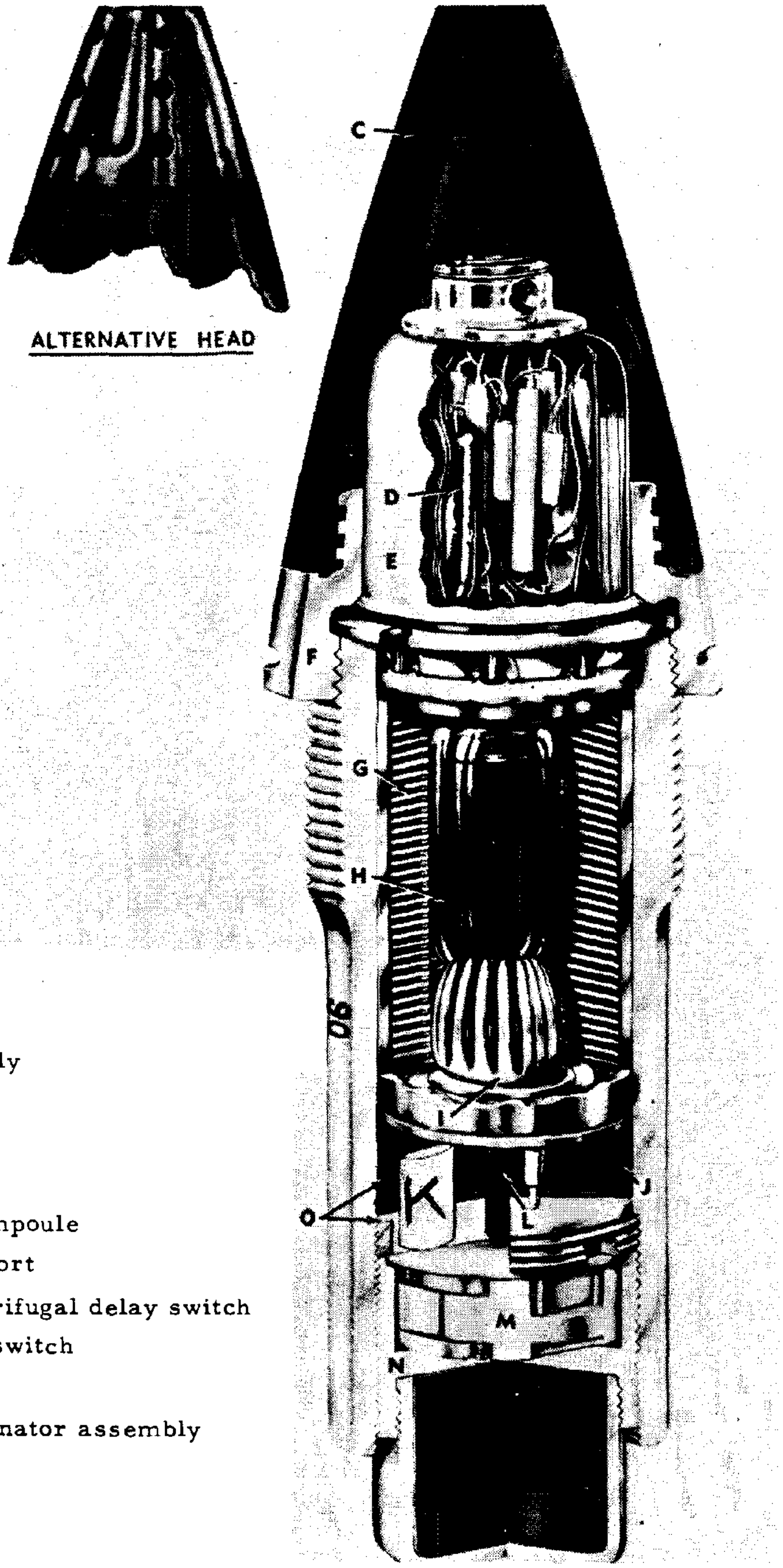


Figure 19-5. Application of Time and Proximity Fuzes to a Ground Target



A - Head
B - Superquick action
C - Gilding-metal cup
D - Firing pin for SQ action
E - Detonator
F - Ogive
G - Flash tube
H - Body
I - Setting sleeve
J - Interrupter
K - Interrupter spring
L - Delay-plunger assembly
M - Firing pin for delay action
N - Primer
O - Black powder delay pellet
P - Delay-plunger pin lock
Q - Plunger pins
R - Detonating relay charge
S - Plunger support
T - Plunger body

Figure 19-6. Selective Superquick Delay Fuze



C - Plastic ogive
D - Radio assembly
E - Shield can
F - Fuze base
G - Battery plates
H - Electrolyte ampoule
I - Ampoule support
J - Mercury centrifugal delay switch
K - Self destruct switch
L - Electric squib
M - Auxiliary detonator assembly
N - Booster
O - Rear fitting

Figure 19-7. Artillery Proximity Fuze

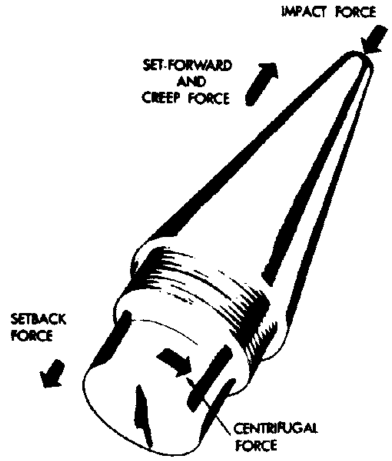


Figure 19-8. Principal Forces Used in Artillery Fuze Design

2. *Setforward.* The force of inertia or reaction to the extreme deceleration of the projectile upon impact with the target is setforward. This force is comparable to that which throws an automobile passenger foward when the brakes are applied suddenly.

3. *Creep.* The tendency of movable parts of a fuze to move forward in a projectile or missile experiencing air drag in otherwise free flight is called creep. It is actually a form of weak "setforward". In long range, high speed ballistic missiles, this force reaches large magnitudes during re-entry.

4. *Centrifugal.* This force is caused by the rotation of spinning projectiles. Whereas the force of setback exists only during the time the projectile is experiencing positive acceleration, centrifugal force exists from the time the projectile starts to spin until impact or detonation. Centrifugal force is greatest where linear velocity is greatest, assuming the projectile is fired from a rifled tube.

In addition to these forces, some of the characteristics of the trajectory which may be employed are:

1. Time of flight
2. Barometric pressure.

Some additional forces which might be employed are electromagnetic force, spring force, friction force, and magnetic force.

In general, the same forces—to a greater or lesser degree—are available in both guided missiles and in artillery projectiles. Missile acceleration (setback) is usually lower (2 to 100 g's) than artillery projectile acceleration (up to 30,000 g's). Centrifugal force, if present at all in missiles, is usually so low that it is not utilized. On the other hand, creep (deceleration) is much higher for ballistic missiles (in the order of 100 g's maximum during re-entry) than for artillery projectiles and can be used in either an arming or a firing device.

19-4 SAFING AND ARMING

Mechanical and electrical devices are used to insure the safety during storage, handling, shipment and firing; and to perform arming after launch so that the fuze will function when proper target intelligence is received. Arming also can be accomplished by an external signal, although this is not too common.

19-4.1 SAFING

Safing can be accomplished by the following means:

1. *Mechanical Safing.* This can be obtained by many means, such as (a) mechanical interruption between two elements of the explosive train, (b) physically restraining the firing pin from striking the primer, (c) holding one element of the explosive train out of line of the other elements, or (d) any combination of the above. Under the influence of forces arising after launch, the restraints are removed to permit arming and firing of the fuze.

2. *Electrical Safing.* In fuzes employing electric circuits, safety can be obtained by (a) interrupting the circuits, (b) short circuiting certain elements such as electric squibs, (c) providing battery power only after launch, or (d) charging firing capacitors through resistances, thus giving a short time delay.

19-4.2 ARMING

After launch, the fuze must prepare the missile to be detonated. Power must be supplied to electrical circuits, opens and shorts must be removed, out-of-line explosive elements must be alined, firing pin restraints removed, interrupters moved, capacitors charged, and all other devices providing safety during launch eliminated. To do this, advantage is taken of the forces previously discussed. The application and removal of required safety devices may be obtained by applying the available forces through various mechanical devices. Many of these devices are quite intricate. A few are shown in Fig. 19-9. A brief description of some of the more common devices follows:

1. *Accelerometers*:

These are devices which are activated under the force of setback. The simplest form of accelerometer is the setback pin (Fig. 19-9). In modern missiles, accelerometers usually take a more sophisticated form so that they will not be actuated by accelerations resulting from rough handling. These devices, called integrating accelerometers, are actuated only if acceleration in excess of some definite level exists for a specific time interval. A simple mechanical integrating accelerometer consists of several setback leaves which must operate in sequence under sustained acceleration.

Another simple integrating accelerometer consists of a "g-weight" held in position by a spring. Under the force of setback, the weight moves to the rear, the speed of its movement controlled by an escapement mechanism such as silicon flowing through a small orifice. Thus, the acceleration must persist for a definite period of time before the "g-weight" is fully moved to the rear.

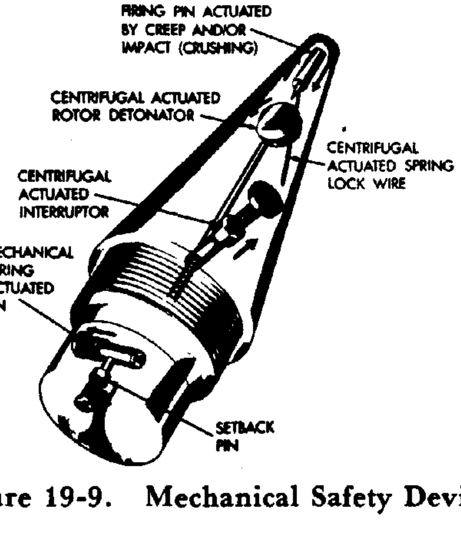


Figure 19-9. Mechanical Safety Devices

Accelerometers can be used to unlock timers or centrifugal devices, to close switches, and to perform other operations included in the arming sequence.

2. *Rotors*. These are devices which, through rotational motion, close switches, unlock mechanical devices, or line up elements of the explosive train. Some rotors respond to acceleration, while others depend on springs to drive them into the armed position.

3. *Centrifugal Actuated Devices*. Various devices are described:

a. *Centrifugal Pins*. Spring loaded centrifugal pins are mounted perpendicularly to the axis of the missile. The pin protrudes into, and prevents movement of other devices until centrifugal force moves the pin outward by compressing the spring which has been holding it in place.

b. *Centrifugal Interrupter*. The centrifugal interrupter is similar in operation to a centrifugal pin. It is used to block off the flash hole between the detonator and booster, interrupting the explosive train.

c. *Semple Centrifugal Plunger*. The Semple centrifugal plunger is a typical example of an arming mechanism used in many base detonating fuzes. Although this plunger is operated primarily by centrifugal force, it also makes use of the force of setback to prevent arming until the missile has left the launcher, and of setforward to drive the firing pin into the primer.

d. *Slider*. The slider is a device similar to an interrupter; however, it contains the primer or primer-detonator, which is held out of line with a firing pin and the rest of the explosive train until centrifugal force moves the slider. In the case of smooth bore mortar projectile the slider is moved into the armed position by spring force.

4. *Timers*. Timers of all varieties can be used to perform arming as well as firing functions. They can release rotating or spring loaded devices such as interrupters, sliders, and firing pins, or can open or close electrical switches. With a timer, arming of a warhead can be delayed until a very few seconds prior to the time that burst action is desired.

5. *Barometric Switches*. The same type switch used to obtain firing signals at a height determined by ambient air pressure can be used to close or open electrical switches, thereby contributing to the arming of the fuze.

6. *Miscellaneous Devices*.

In addition to the principal safety devices discussed, the following miscellaneous means are also used:

a. Springs to overcome creep

b. Crush collars to support the firing pin and hold it away from the primer until impact

c. Shear wires and shear pins which restrain firing pins and plungers and which are sheared off on impact

d. Resistance rings and spiral wrappings which support the firing pin and which are moved out of place by centrifugal force after the projectile leaves the rifled bore

e. Cotter pins which restrain the movement of some mechanism and which must be withdrawn by hand before loading the projectile into the weapon.

19-5 RADIO PROXIMITY FUZE CIRCUITRY

As previously indicated, a radio proximity fuze radiates electromagnetic waves and senses the proximity of a target by the reception of waves reflected from the target. A typical proximity fuze has the following parts:

1. Antenna

2. Radio transmitter and receiver composed of miniature tubes or solid state components which in the case of artillery projectiles must be rugged enough to withstand accelerations on the order of 30,000 g's. (Setback affects reliability.)

3. Selective amplifier
4. Electronic switch
5. Power supply.

A radio proximity fuze circuit is indicated schematically by the block diagram in Fig. 19-10.

The same antenna usually is used for transmission and reception. Dipole or loop antennas, either longitudinal or transverse with respect to the projectile axis, are used. This provides directional sensitivity patterns that are required for particular fuze applications. A typical radiation pattern for the proximity fuze is shown in Fig. 19-11.

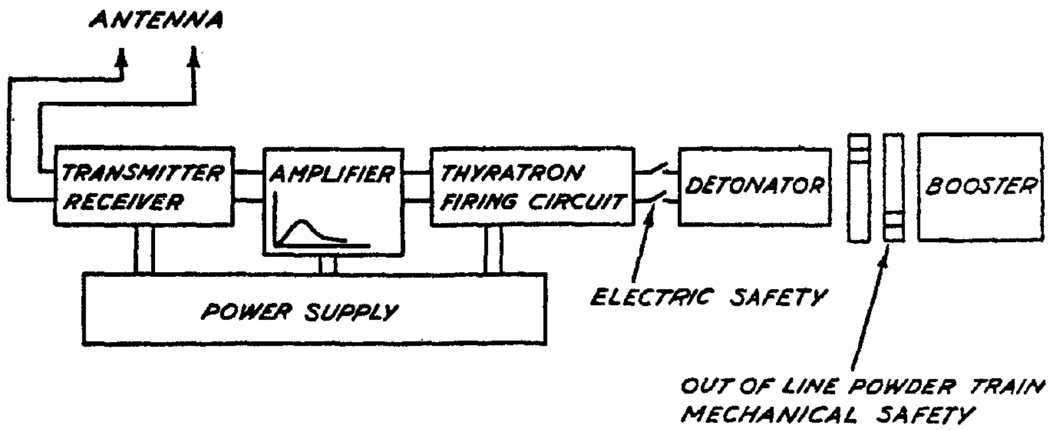


Figure 19-10. Block Diagram of Typical Proximity Fuze System

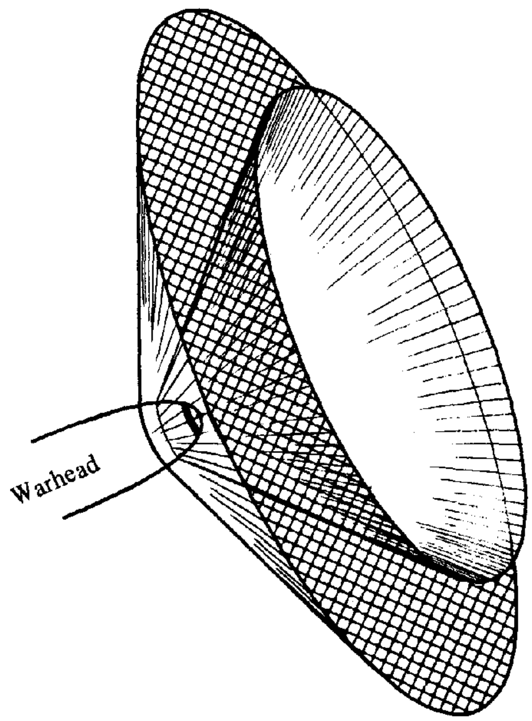


Figure 19-11. Firing Region for Proximity Fuze

Most modern proximity fuzes have an arming delay which consists of a (mechanical) timer set for less than the expected time of flight. This timer is used to prevent the fuze from transmitting until a few seconds prior to arrival of the projectile in the target area. This reduces the effectiveness of enemy countermeasures and prevents prematures or early bursts from low, dense cloud formations; hill masses over which the projectile must pass en route to the target; or other objects not targets en route to the target.

In general, three types of power supplies are employed in radio proximity fuzes:

1. *Wet Cell Batteries.* Artillery fuzes often use a wet cell electrolytic battery. The electrolyte is contained in a glass ampoule which is shattered on setback. Centrifugal force causes the electrolyte to be distributed among the plates of the battery.

2. *Generators.* In older fuzes for bombs and some nonrotating projectiles, wind driven generators were used.

3. *Thermal batteries.* Thermal batteries are now employed in many proximity fuzes of more recent design, particularly in nonrotating missiles. Thermal batteries are electrochemical power sources employing electrolytes which remain solid and nonconducting at all storage temperatures. The electrolyte melts and becomes conducting at elevated temperatures; electrical energy is then available from the battery.

Within a fraction of a second after the power supply has been activated, the fuze circuit warms up and all transients die out so that the fuze is ready for operation upon completion of the arming sequence.

For the principle of operation, suppose that the proximity fuze has come close enough to the target to receive appreciable energy reflection. The fuze antenna sends out a continuous energy wave. Part of the energy is reflected by the target and received by the antenna. This sets up a small voltage in the antenna which is proportional to the sending current but not necessarily in phase with sending current. Thus, the presence of the reflecting target has the effect of changing the effective antenna impedance vectorially by an amount $\mathcal{Z}$ such that

$$\mathcal{Z} = e/I \tag{19-1}$$

where I is the antenna sending current and e is the voltage component due to the reflected energy. As the distance to the target decreases the size of e increases and the phase angle between e and I changes by 180 deg each time the distance is reduced by a quarter wavelength. The fuze operates on the principle of the Doppler effect. An amplifier is used to magnify the returning signal until it can operate a thyratron firing circuit. As the fuze approaches the target, the output of the amplifier increases. The fuze functions when the output of the amplifier increases beyond a certain preset critical value.

19-5.1 ELECTRICAL SAFETY DEVICES

All proximity fuzes have several electrical safing devices. Some of these, which vary with the type of fuze, are:

1. *Thermal or liquid battery activation for battery powered fuzes.* This element of safety is provided by the time and the means required to activate the battery.

2. *Resistor-capacitor time.* The firing circuit contains a resistor and capacitor combination. The time required for the capacitor to reach the firing potential provides a minimum arming time for the fuze after battery activation.

3. *Self-destruction switch.* Centrifugal forces causes a normally closed switch to remain open as long as rotational velocity is sufficiently high. When the rotational velocity drops below a preset value, the switch closes and activates the firing circuit.

4. *Rear fitting (arming mechanism)*. Part of the arming mechanism short circuits the electric detonator and removes it from the firing circuit. Centrifugal force removes the short circuit and thereby electrically places the detonator in the firing circuit.

In addition to special electrical safety devices, proximity fuzes include mechanical safety features comparable to those found in mechanical (impact) fuzes. An impact element also may be included to function the fuze should the radio section fail to function.

19-5.2 GENERAL CHARACTERISTICS

A warhead or projectile must be detonated at a certain position relative to the target for the ensuing blast and fragmentation effects to have maximum effect. The burst position will vary as a function of the angle of approach to the target, and the shape and location of the field of sensitivity of the fuze. Thus, the burst position may be different according to the antenna type used and/or the angle of approach of the fuze for the same target.

For artillery fire, burst height is controlled to a certain extent by angle of fall of the projectile. The burst height may also be influenced by characteristics of the target itself, such as the degree of moisture contained in the target area, ranging from low burst height for dry soil targets to maximum burst height over a watery target. If the target area is wooded, especially with deciduous trees (trees which have broad leaves), the burst height may be increased by as much as the height of trees themselves, depending on the density of the woods.

Proximity fuzes are useful wherever air bursts are desired, but cannot be used against some types of targets. They are extremely effective against personnel in uncovered positions since bursts occurring overhead will reach even into foxholes. They are also extremely useful against light materiel, such as aircraft or trucks. However, they are not effective against armored vehicles, or against any target where penetration before detonation is desired.

We have given this description to indicate that there exists many sources of VT fuze operation which could lead to variation in sensitivity. Such variation in sensitivity results in a random variation in height of burst of the projectile or warhead. Therefore, it is this characteristic that the analyst must take into account since it affects the analysis of weapon performance.

19-6 GUIDED MISSILE FUZES

Although guided missile fuzes must perform all the same functions as fuzes in artillery projectiles, they usually differ greatly in appearance from their smaller counterparts. The requirement for reliabilities on the order of 99.9% means generally that these fuzes must be comparatively large, complex, rugged, and therefore expensive. These fuzes usually contain two separate devices, known as the Safety and Arming (S and A) device, and the Target Detection Device (TDD). The S and A device mechanically or electrically performs the necessary functions to render the fuze as safe as the military characteristics demand and arm it at the proper time. The TDD is the sensor—the device which detects the presence of the target and sends the proper detonation signal to the warhead at the proper time.

In order to obtain maximum warhead effectiveness when large yield weapons are used, the great majority of modern missiles employ some type of proximity element as the target sensor. Ambient fuzes are used to some limited extent, but radio is the most accurate sensor available today and is therefore in very common use. Electrical energy must be made available to the TDD in these fuzes; if this energy is not available, the fuze is safe; when the energy is made available, the fuze becomes armed. So the S and A consists to a great extent of switches and relays which successively supply voltage to elements of the target detector.

The operation of the fuzing systems in most of today's missiles is classified information, so for our purposes here a hypothetical missile will be postulated and an equally hypothetical fuze described. The principles embodied in this fuze, however, are those which are actually used in guided missiles; actual fuzes will differ only in the methods used to apply these principles.

The hypothetical missile for which the fuze is to be used is an inertially guided ballistic missile with a maximum range of 100 mi. It employs a nuclear warhead which will have its maximum effectiveness with a burst height of 2000 ft. The fuzing problem is to supply current safely and reliably to an electrical primer when the missile descends to a height of 2000 ft above the target. A possible arrangement for fuzing this warhead is shown schematically in Fig. 19-12.

The sequence of events along the missile trajectory is as follows: (The paragraph numbers refer to the circled numbers in Figs. 19-12 and 19-13.)

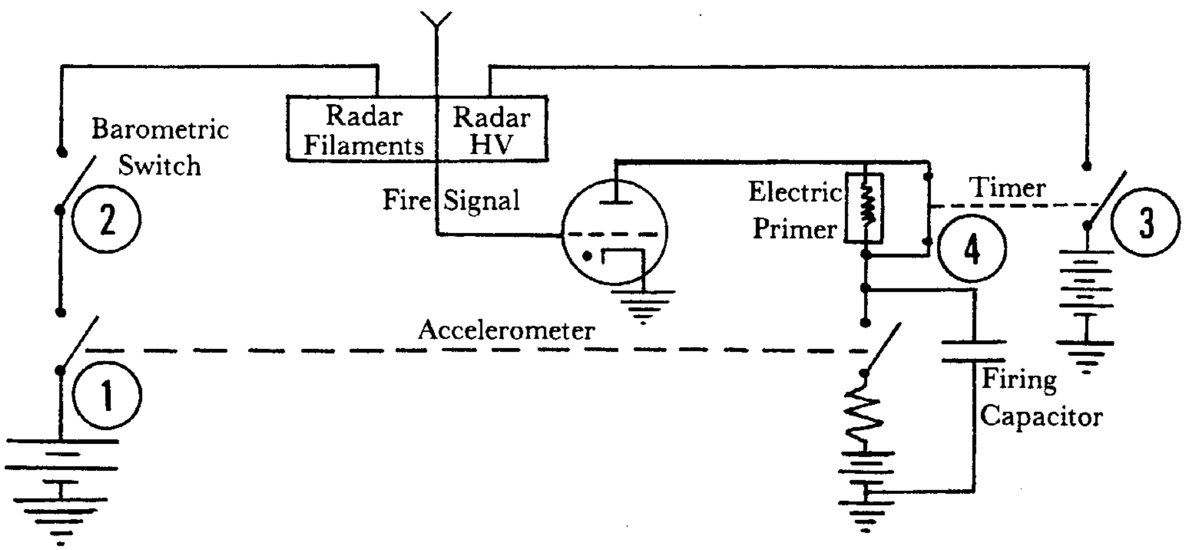


Figure 19-12. Schematic of a VT Fuze for a Guided Missile

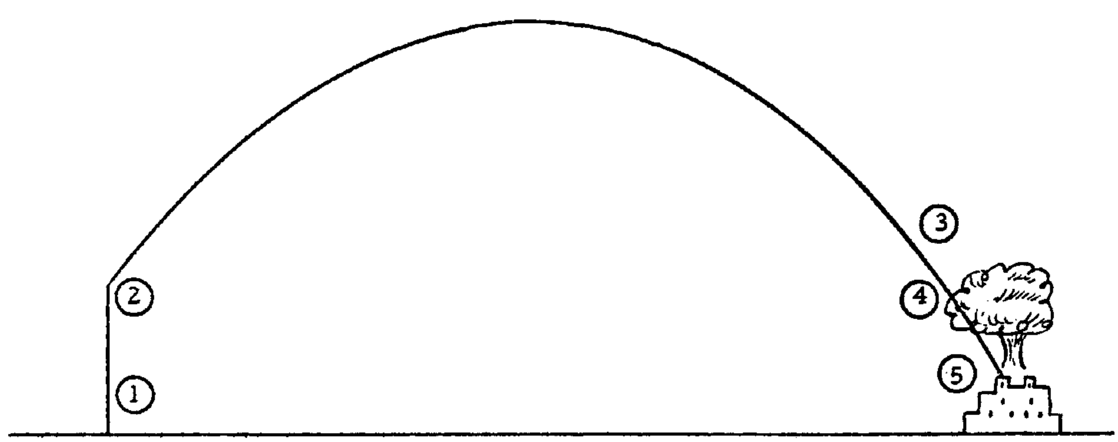


Figure 19-13. Missile Trajectory

1. The missile is subjected to acceleration at launch. An integrating accelerometer closes switch 1 provided the acceleration retains proper magnitude over the required length of time. This begins the charging of the firing capacitor and applies voltage to one side of the barometric switch.
2. The barometric switch closes when the missile reaches minimum safe altitude. This provides energy for the radar filaments.
3. Missile deceleration upon re-entry into the atmosphere is sensed by another integrating accelerometer. If deceleration is of the proper magnitude and duration, a mechanical timer begins operations.
4. After proper time delay (about 5 s), switch 4 opens, removing the short from the electric primer; and switch 3 closes, applying high voltage to the radar which then begins to transmit.
5. At the proper burst altitude, the radar sends a fire signal to the grid of the thyratron which begins to conduct. The flow of current from the firing capacitor through the electric primers initiates the explosion of the warhead.

19-7 COMMENT ON VT AND MECHANICAL TIME FUZES

A few historical remarks concerning some problems of fuzing is of interest here. Prior to and during much of World War II, artillery projectiles were fuzed with either point detonating or mechanical time fuzes. For purposes of improved lethality against enemy troops, it is desirable to obtain as large a number of fragments as possible from the steel projectile casing and with the proper velocity of fragments to insure incapacitations—as we have seen in Chapter 15, the wounding power of fragments depends on striking velocity and mass of fragments. (The greater the number of fragments, the greater is the chance of at least one hit.) In other words, it becomes important to obtain as large a lethal area as possible for standing, kneeling, or prone troops, or troops in fox holes—this latter condition representing a very difficult target since the chance of hitting a fox hole is often very small indeed. Artillery projectiles fuzed with point detonating or delay fuzes would result only in too large a percentage of lethal fragments going into the ground with little or no effect otherwise. Therefore, air burst of projectiles at the best height becomes of considerable importance. Hence, the powder train or mechanical type time fuzes were formerly used to obtain air burst. However, the dispersion (standard deviation) in running time of even the mechanical fuze was so large that consistent lethality was not obtained on the ground. In the adjustment of fire process, there was some effort to get a mix or effective proportion of air and ground bursts, so that lethality would be improved. However, this procedure did not result in optimality either for the fuze dispersion characteristics and its match to the pattern of projectile fragments. It is for these reasons that the VT or variable time fuze was developed during World War II. As already indicated, the VT fuze operates on the principle of propagating radio waves to the ground on the descending branch of the trajectory and finally detonation of the projectile occurring when the returning signal was sufficiently strong enough, depending on VT fuze design considerations. The VT fuzes did indeed result in sufficiently small dispersion in height of burst so that the lethality of artillery projectiles could almost be optimized. However, a rather unacceptable feature of VT fuzes was the problem of "early functions" or "prematures" along the trajectory, long before detonation of the projectile was desired. This was cured by the use of a mechanical time arming feature incorporated into the VT fuze which prevented early function, so that the radio proximity feature was allowed to function properly at the terminal end of the trajectory. The problem of obtaining optimum average height of burst over all conditions of the terrain has been the subject of much study and redesign of VT fuzes over the years. The cost of the VT fuze ordinarily runs several times that of the mechanical time fuze, however, its average height of burst and standard deviation of burst height proved to be vastly superior.

19-8 SOME STATISTICAL CONSIDERATIONS FOR HIGH RELIABILITY AND PRECISION OF FUZE SYSTEMS

Since we have mentioned the importance of suitably small dispersion for a fuze or fuzing systems, it is of interest to indicate some statistical properties and their effect for fuzing systems for early nuclear missiles or warheads for artillery. At first, it was necessary to use mechanical time fuzes which would have suitably low dud rates as compared to the early variety of VT fuze. Of course, small dispersion for a nuclear burst in air was also considered necessary, but the dud and premature rates were critical indeed. Initially, the concept of using a number of mechanical time fuzes in parallel was considered. If each fuze of a parallel system was connected so that the nuclear warhead would be detonated when any fuze operated, then it is obvious that—since there are statistical or random variations in running times of the individual fuzes—the fuze which functioned first (shortest time) would be the one that actually detonated the warhead. Thus, we are dealing with the principle of order statistics, where random functioning times would be generated by individual fuzes and the smallest or lowest running time would determine the warhead detonation point. Thus, suppose we consider three mechanical time fuzes in parallel, all with the same setting. Then due to random dispersion among individual fuzes, the actual running times would be the distinct values t_1, t_2, and t_3 determined by the size of the standard deviation σ. One of the three times would be shorter than the other two, i.e., if the times were ordered, say,

$$t_1 \leq t_2 \leq t_3$$

then the fuze with the shortest running time t_1 would initiate warhead detonation. (Of course, there is no way to know in advance which fuze would produce the shortest running time).

This arrangement also had the desirable property of decreasing the dud rate, for if the fuze which happened to have the smallest running time were a dud, then a detonation was not attained until the next or subsequent operable fuze initiated. Moreover, there was also a slight gain in precision since the smallest statistical order statistic in a sample had a smaller standard deviation than that of an individual on the average. However, a most undesirable defect of such a parallel system was that the nuclear warhead would detonate as soon as any fuze would premature! Nevertheless, by designing the circuitry so that, for example, if the fuzing system consisted of five mechanical time fuzes, and a detonation signal was not given until three fuzes functioned (a "three-out-of-five system"), then the premature and dud rate both could be controlled satisfactorily. Moreover, with the third fuze in order actually functioning the system, the dispersion in running time for the system as measured by the standard deviation is about $\sqrt{\pi}\sigma/\sqrt{10}$, whereas that for the sample mean is $\sigma/\sqrt{5}$, where σ is the standard deviation of an individual fuze. Ref. 12 gives an early account of multiple fuzing studies of this kind.

We have given this partial background information to indicate that statistical considerations are required for the evaluation of fuze function just as was the case for delivery accuracy studies in Chapter 13. Also, it is a good idea for the analyst to keep these principles in mind for his evaluations in general.

19-9 FUZING PRINCIPLES AND STRATEGIES

The subject of fuzing principles or fuze effectiveness on one hand and the subject of reliability on the other turn out to be quite inseparable. Thus, the principles of reliability are directly applicable to the problems of arming, safing, precision, and overall reliability of fuze systems—especially the use of multiple fuze elements. The use of multiple fuzes, including redundant elements, is necessary to guarantee high reliability. Therefore, the design and operation of fuze systems, which are often the "brains" of

the weapon, must take into proper account any principles or analyses that guarantee optimum warhead functioning. Therefore, we will illustrate or highlight some of the fundamental principles here.

Moore (Ref. 13) has suggested an analysis based on the division of projectile or missile trajectories into seven segments or intervals of primary interest as in Table 19-2. The reader can see easily that such categorization includes the more important functioning possibilities of a fuzing system which can occur with significant chances. Clearly, it is of the utmost importance that the fuze system function properly in segment I_3 (or I_s as we later generalize it) and with a probability of occurrence as near 100% as possible. Premature or early detonations (I_1 and I_2) could be catastrophic for some weapons, and fuze functioning in intervals I_4, I_5, I_6, or I_7 would bring about reduced or no effectiveness—a costly occurrence or even a hazard for many weapon uses.

Depending on the particular application, it may be necessary to consider fuzing systems which involve multiple fuzes or elements in series, parallel, series-parallel, or other arrangements such as the so-called "*k*-out-of-*n*" structure, for which the system operates when a total of k elements of n elements ($1<k<n$) functions, as mentioned in par. 19-8. We will therefore give a brief account of multiple fuzing strategies (Ref. 13) in terms of the trajectory segments of Table 19-2.

Let there be n fuzes or fuze elements in some arrangement indexed by $i = 1, 2, \ldots, n$ and let the segments of the trajectory be represented by $j = 1, 2, \ldots, s, \ldots, m$. (For Table 19-2, $m = 7$ and s the optimal detonation segment is 3). If

P_j = chance the warhead will detonate in the jth segment of the trajectory

and

E_j = measure of effectiveness for warhead detonation in jth segment

then the expected value of the warhead effectiveness *Eff* based on fuze system operation is easily seen to be

$$Eff = \sum_{j=1}^{m} P_j E_j \,. \tag{19-2}$$

Now let

p_{ij} = probability the ith fuze functions in I_j, the jth trajectory segment.

With this terminology, we can now discuss reliability based on the assumption of series, parallel, or other type circuits.

19-9.1 PARALLEL FUZING

If the fuzes or fuze elements are in parallel, then the fuzing system will detonate the warhead when any element (the first) functions. For this case, the chance P_s that the fuzing system functions the warhead in the desirable segment I_s is given by

$$P_s = \prod_{i=1}^{n} \left(\sum_{j=s}^{m} p_{ij} \right) - \prod_{i=1}^{n} \left(\sum_{j=s+1}^{m} p_{ij} \right) \,, \quad s < m \,. \tag{19-3}$$

TABLE 19-2. TRAJECTORY SEGMENTS OF PRIMARY INTEREST

Trajectory Segment	Characterization of Segment
I_1	Premature detonation
I_2	Early detonation
I_3	Optimal detonation
I_4	Late detonation
I_5	Impact detonation
I_6	Delayed detonation
I_7	Dud (failure to detonate)

A dud will occur only if all fuze elements are duds, the chance P_m of which is

$$\left.\begin{aligned} P_m &= \prod_{i=1}^{m} p_{im} \\ &= (p_m)^n\,,\ \text{if}\quad p_{im} = p_m\,. \end{aligned}\right\} \tag{19-4}$$

We see from Eq. 19-4 that a parallel system decreases the chance of a dud very rapidly as n, the number of elements, increases.

The chance P_1 of a premature warhead detonation for the parallel fuze arrangement is

$$P_1 = 1 - \prod_{i=1}^{n} \left(\sum_{j=2}^{m} p_{ij} \right). \tag{19-5}$$

Unfortunately, therefore, it is seen from Eq. 19-5 that the chance of a premature increases rapidly with the number n of fuzes for a parallel system, this being a major or even perhaps catastrophic drawback. Hence, we must consider other arrangements.

19-9.2 SERIES FUZING

For fuze elements in series, each and every element must operate properly, or the "last" fuze in the series chain must function to detonate the warhead. Thus, the warhead will detonate in the desired trajectory segment I_s if one or more fuzes function in I_s, given that all needed serial fuze elements functioned properly in prior trajectory segments. For the case of series fuzes, the chance of optimum functioning P_s is seen to be

$$\begin{aligned} P_s &= \prod_{i=1}^{n} \left(\sum_{j=1}^{s} p_{ij} \right) - \prod_{i=1}^{n} \left(\sum_{j=1}^{s-1} p_{ij} \right), \quad s > 1 \\ &= \left(\sum_{j=1}^{s} p_j \right)^n - \left(\sum_{j=1}^{s-1} p_j \right)^n, \quad \text{if } p_{ij} = p_j\,. \end{aligned} \tag{19-6}$$

The chance of warhead premature is

$$P_1 = \prod_{i=1}^{n} p_{i1} \tag{19-7}$$

and the chance of a dud is

$$P_m = 1 - \prod_{i=1}^{m} (1 - p_{im})\,. \tag{19-8}$$

The series fuzing strategy for an increasing number of fuzes results in decreasing the chance of a premature, (a desirable feature) but also increases the probability that a warhead will fail to function. Hence, series element systems are not generally useful for many fuzing principles, but have their place in safing and arming features of fuze systems.

Having seen some disadvantages for series or parallel fuze arrangements or designs, we turn to another design.

19-9.3 THE *k*-OUT-OF-*n* STRUCTURE

If the fuzes or elements are "wired" and "cross-wired" such that the warhead detonates when the kth number of fuzes (of n fuzes) functions $(1<k<n)$, the fuzing principle or strategy is called a "k-out-of-n" system. A one-out-of-n structure is the parallel system, and an n-out-of-n structure is the series system, indicating the generality of the k-out-of-n structure.

If for simplicity we let $p_{ij} = p_j$, giving each fuze the same chance of operating in trajectory segment I_j, then the chance of warhead premature is clearly

$$P_1 = \sum_{i=k}^{n} \binom{n}{i} (p_1)^i (1 - p_1)^{n-i} \tag{19-9}$$

where p_1 is the probability of a single fuze premature. The chance of a dud is

$$P_m = \sum_{i=0}^{k-1} \binom{n}{i} (1 - p_m)^i (p_m)^{n-i} \tag{19-10}$$

and the chance of desired functioning in I_s is

$$P_s = \sum_{i=k}^{n} \binom{n}{i} \left(\sum_{j=1}^{s} p_j\right)^i \left(1 - \sum_{j=1}^{s} p_j\right)^{n-i} - \sum_{i=k}^{n} \binom{n}{i} \left(\sum_{j=1}^{s-1} p_j\right)^i \left(1 - \sum_{j=1}^{s-1} p_j\right)^{n-i} \tag{19-11}$$

where $s = 2, 3, \ldots, m - 1$.

For the general case of distinct p_{ij}, then it can be shown that

$$P_s = F(s) - F(s - 1) \tag{19-12}$$

where the form of $F(s)$ can easily be seen for three fuzes, and hence generalized, i.e., for three fuzes,

$$F(s) = F_1(s)F_2(s)[1 - F_3(s)] + F_1(s)[1 - F_2(s)]F_3(s) + [1 - F_1(s)]F_2(s)F_3(s) + F_1(s)F_2(s)F_3(s) \tag{19-13}$$

and

$$F_i(s) = \sum_{j=1}^{s} p_{ij} . \tag{19-14}$$

An alternative method for computing the reliability of a k-out-of-n structure is to note that there are $\binom{n}{k}$ parallel paths for which the system functions. Let us suppose that the combination of n things taken k at the time is equal to r, say, and A_i represents the ith possible (parallel) path. Then the chance of fuze system success is

$$Pr(A_1 + A_2 + \cdots + A_r) = \sum_{i=1} [P(A_i)] - \sum_{i}\sum_{j} {}_{i \neq j} P(A_iA_j) + \sum_{i}\sum_{j}\sum_{\ell} {}_{i \neq j \neq \ell} P(A_iA_jA_\ell) - \cdots \pm P(A_1A_2 \cdots A_r) \tag{19-15}$$

It is noted that this is the basic law of calculating the probability of a sum of possible events, which is given in textbooks on statistics.

Summarizing, the parallel design functions as soon as any (the first) fuze operates, and the series design does not function if any fuze in the chain fails. The *k*-out-of-*n* design, however, must perform a counting and switching process until at least *k* number of fuzes get signals to operate.

The following example of Moore (Ref. 13) will show the advantages of the *k*-out-of-*n* design. We assume three fuzes with characteristics as in Table 19-3 are arranged in parallel, series, or *k* out of *n*. (The *j*th subscript of p_{ij} refers to the trajectory segment of Table 19-2)

With the data of Table 19-3, the warhead functioning probabilities or reliabilities for the parallel, series, and 2-out-of-3 system are given in Table 19-4.

The relative superiority of the 2-out-of-3 system is seen in giving a low premature rate, a low dud rate, and improving the chance of optimum detonation considerably.

There is another advantage of the *k*-out-of-*n* structure, it being that for fuzes operating on a continuous scale—i.e., time, altitude, etc.—instead of the "success" or "fail" dicotomy the fuze that actually detonates the warhead tends to be the sample median, which improves on precision (see par. 19-8).

19-10 ESTIMATION PROBLEMS INVOLVING TERMINATION OF TRAJECTORIES

Fuze action results in termination of the trajectories of projectiles, missiles, and hence warheads, except in the case of a fuze dud. Therefore, in addition to range and deflection dispersion, the analyst

TABLE 19-3. FUZE FUNCTIONING PROBABILITIES

Fuze No. 1	Fuze No. 2	Fuze No. 3
$p_{11} = 0.001$	$p_{21} = 0.002$	$p_{31} = 0.001$
$p_{12} = 0.015$	$p_{22} = 0.028$	$p_{32} = 0.060$
$p_{13} = 0.930$	$p_{23} = 0.900$	$p_{33} = 0.850$
$p_{14} = 0.030$	$p_{24} = 0.036$	$p_{34} = 0.080$
$p_{15} = 0.009$	$p_{25} = 0.003$	$p_{35} = 0.003$
$p_{16} = 0.005$	$p_{26} = 0.001$	$p_{36} = 0.001$
$p_{17} = 0.010$	$p_{27} = 0.030$	$p_{37} = 0.005$

TABLE 19-4. WARHEAD FUNCTIONING PROBABILITIES

	Parallel System	Series System	2-out-of-3 System
P_1	0.00400	0.00000	0.00003
P_2	0.09970	0.00003	0.00319
P_3	0.89600	0.80100	0.98300
P_4	0.00033	0.13300	0.01280
P_5	0.00000	0.01440	0.00059
P_6	0.00000	0.00676	0.00024
P_7	0.00000	0.04450	0.00050

must be sure to take into account the amount of scatter which is brought about by fuze action. In fact, one of his important problems is to estimate the distribution of burst positions and to recommend techniques for controlling detonation points to acceptable regions. The amount of scatter or dispersion will depend on the normal dispersion of trajectories not including fuze action and the precision in time, altitude, height of burst, etc., of the particular type of fuze or fuze system used. We therefore comment briefly on this problem.

For air defense studies and artillery evaluations, the analyst often has to estimate the dispersion in air-burst position for three independent directions. In air defense, some special training may be required for the analyst to become conversant in the delivery errors which result from the engagement geometry between a missile and an aerial target. If accurate evaluations are to be made, then the variation in guidance errors and fuzing errors must be estimated properly since these may have a very significant effect on the suitability of the analytical evaluation. Guidance and fuzing dispersion errors as measured by the standard deviation, for example, are not measured directly by observation and often must be estimated from ballistic considerations and fuze performance data. Fuze dispersion often may be more important than guidance errors or normal dispersion among trajectories.

For cases where data are not otherwise available, the analyst must resort to an engineering error analysis to describe system precision and accuracy.

Even for artillery firing and for fuzes which have randomly varying running times, there may be a problem in estimation of air-burst geometry. A method is given by Grubbs and Prevas in Ref. 14 to predict dispersion in burst positions for artillery projectiles, the trajectories of which are terminated by fuze action. The method utilizes probable or standard errors in range and deflection, which are given in firing tables, and fuze dispersion times which may be obtained from other tests such as acceptance or surveillance firings. With such basic input data and the cutoff points of trajectories for constant time of flight, the terminated air-burst position for dispersion in range, deflection, and height of burst can be expressed in terms of the basic standard errors involved. An example is given in Ref. 14 for the now obsolete T123 HE Projectile for the 280mm gun, although the same principles may be used for any current artillery projectiles as required. To illustrate for the T123 Projectile, the elevation for the range of 26,300 yd was used, with probable error (PE) in running time of a mechanical time fuze giving a PE of 0.2 s. The estimated PE in height of burst then turned out to be 69 yd and was as large as the PE in range of 67 yd, showing the significance of fuzing action.

Groves (Ref. 15) estimates the distributional properties and dispersion patterns for air-burst positions of large free-flight rockets with a parallel combination of a time fuze and an altitude fuze. The altitude fuze terminates trajectories in case the time fuze would run too long for suitable mean fuze time. Hence, the time fuze can be set to run longer (lower altitude of burst) while still maintaining the needed percentage of air burst instead of ground bursts or duds. All functions are expressed in terms of widely tabulated probability distributions. Ref. 16 extends this study to include individual reliabilities of the timer and altitude fuzes.

Some readers might find the study of impact versus powder train time fuzes for hand grenades by Rotkin (Ref. 17) of interest, to mention a comparison of different type fuzes for a simple ammunition system.

19-11 FUZE FUNCTIONING AND GROUND BURST PROBLEMS

Fuze action often has an effect on the percentages or chances of ground bursts for some surface-to-surface weapons, as will also a change in warhead weight or type. Consequently, the analyst often may have to calculate the chance of ground impact. This problem arises also as indicated in Refs. 14, 15,

and 16, and calculations therefore may be based on methods for predicting fuze dispersion characteristics in these references. In troop training programs for the HONEST JOHN Rockets M31 and M50, for example, it was noticed that some of the firings did not result in air-burst. Mioduski and Bell (Ref. 18) made a study of the problem for the HONEST JOHN Rocket when firing a heavy HE or practice warhead, and in particular they provide tables which give the probability of a ground impact for various intended height-of-burst (HOB) settings. As an example in this reference, it was assumed that an M50 Rocket with M38 Practice Warhead was fired from a M386 Launcher at a range of 20,000 m and for an intended HOB of 200 m. Firing Table FTR 762-H-1 indicated that under these conditions the PE in HOB is 90 m. Thus, for an intended height of burst of 200 m, and PE of 90 m, the deviation from the mean is 200/90 = 2.2 PE's or 1.495 sigmas. Using tables of the standardized normal distribution, one finds that the chance of ground impact under these conditions is about 0.067. Ref. 18 gives a rather extensive table of chances of ground burst for HONEST JOHN Battalion Commanders or others concerned with the problem of firing such rockets close to the ground.

19-12 OPTIMUM BURST HEIGHT

Clearly, the effectiveness of surface to surface weapons is highly dependent on the best or optimum mean height of burst brought about due to fuze system operation. Blomquist and Young (Ref. 19) study this problem for single proximity (VT) fuzes for the 105mm M1 Howitzer, the 105mm M548 Howitzer, the 4.2-in. M329A1 Mortar, the 155mm M107 Howitzer, the 155mm M549 Howitzer, the 175mm M437 Gun, and the 203mm M106 Howitzer. The optimum burst height distribution for this study is characterized by that combination of mean or average burst height and standard deviation in burst height for the fuze which maximizes the composite lethal area of the subject projectiles against standing, prone, and entrenched (foxhole) personnel. Open terrain and marsh grass type environments are the only ones considered in the study, not tree canopies or forests which have a decided effect on fuze functioning. Posture frequencies are assumed to be 17% standing, 33% prone, and 50% of the troops in foxholes. Curves are given relating average lethal areas versus average height of burst and different standard deviations in height of burst. The effect of a single type of VT fuze for all projectiles is studied, and the great improvement in effectiveness of VT over point detonating fuzes is indicated on the graphs. Fig. 19-14 indicates a typical probability density function of heights of burst above the ground.

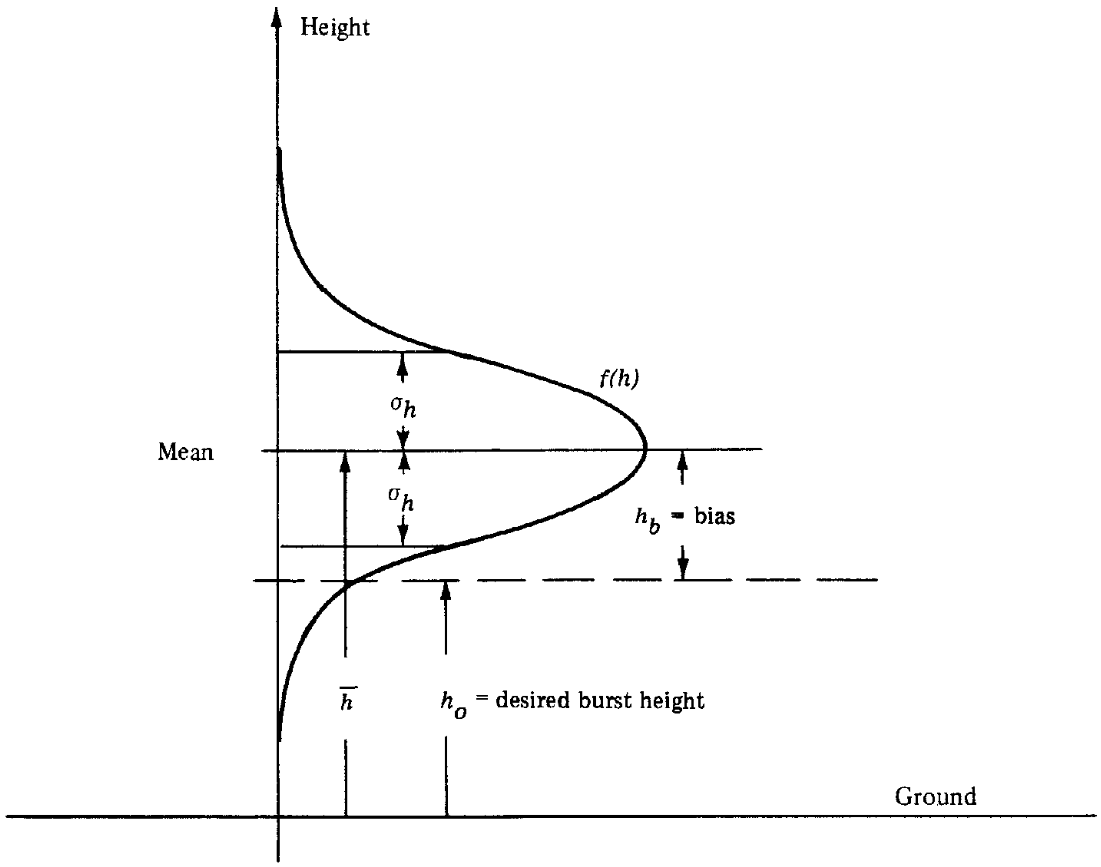
19-13 EXAMPLES OF OTHER FUZE STUDIES

Other typical fuzing system studies are presented in AMSAA Technical Memorandum No. 2 (Ref. 20) and AMSAA Technical Memorandum No. 17 (Ref. 21).

Waldon and McCarthy (Ref. 20) present a study of the degree of precision required of time fuzes employed with 175mm artillery ammunition in the attack of personnel targets. The study includes target size, personnel posture, size of fire unit, type of fire, fire delivery technique, and target defeat criteria. Results are presented in terms of the number of rounds required to attain a desired level of casualties against two arrays of targets, and costs are included also.

In Ref. 21, Scungio made a study of dud rate variations on the effectiveness of the LANCE Missile system. The report presents the relative effectiveness of three candidate warheads for the LANCE missile system against typical targets. Meteorological conditions, target size, target location errors, and system accuracy for the three candidate warheads are taken into account.

Both of these studies would be of some general interest to analysts engaged in fuze optimization problems.



$f(h)$ = probability density function for distribution of burst heights
σ_h = standard deviation of height of burst for individual rounds
h_o = desired or optimum height of burst
$\bar{h}$ = average height of burst (observed)

Figure 19-14. Density of Fuze Function as a Function of Height Above Ground

19-14 SUMMARY

By way of summary, we have elaborated somewhat in this chapter on the types of fuzes, their general characteristics, and some of the reliability and statistical type of problems related with their operation. Moreover, we have introduced the very strong connection between the fuzing problem and that of general reliability studies. The analyst will find that the fuzing problem is indeed one of his prime concerns, for the fuze system is often the "brains" which determines overall weapon system effectiveness. Accordingly, an inadequate fuzing arrangement will ultimately have to be redesigned toward optimality, or at least a cost-effectiveness basis. Hopefully, this general introduction to the fuzing problem will help to get the young analyst on the right course for his more penetrating analyses and developments.

We have not dealt with the overall problem of reliability for fuze systems which generally operate on a continuous scale, such as time, to the extent desired since this extensive subject is being reserved for presentation in Chapter 21 on Reliability, Life Testing, Reliability Growth, Availability, and Maintainability.

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CHAPTER 20

MULTIPLE ROUND HIT PROBABILITIES, TARGET COVERAGE, AND TARGET DAMAGE

Methods for calculating multiple round hit probabilities (usually the chance of at least one hit), the fractional coverage of targets for salvos of rounds, and the fractional damage to targets (casualties) for salvos are covered in this chapter. The models used necessarily must take into account the round-to-round ballistic dispersion, the aiming errors for multiple rounds, the correlation between rounds for automatic or target tracking weapons, target size and characteristics, target vulnerability, and warhead lethality. Moreover, suitably accurate approximations must be used.

20-0 LIST OF SYMBOLS

A = Bråndli's notation for length of a side of a square
A = target area
A_{AP} = single round damage pattern area
A_c = average casualty area
A_L = lethal area (usually of a round)
A_T = target area
$\dot{A}_v$ = vulnerable area of target
A_{VP} = volley damage of pattern area
A/S = $1.31\{R_T/[\sigma_x(1 + \sigma_\mu^2/\sigma_x^2)^{1/2}]\}$, notation for Bråndli's tables (Ref. 3) for chance of at least one hit
a = constant in Eq. 20-37
a_{rn} = coefficient for Jacobi polynomials
a,b = constants in Eqs. 20-85 and 20-86
a,b = exponents—see Eqs. 20-16 and 20-17
b = $(\sigma_k^2 + \sigma_T^2)/\sigma_u^2$, parameter in Washburn's optimum aiming problem
b_{min} = $(\sigma_k^2 + \sigma_x^2)/\sigma_u^2$, minimum value of b
$b^*(r)$ = value of $b(r)$ that makes $Q(b,r)$ a minimum
c = adjustment factor (Eqs. 20-11 and 20-12 only)
c = one-half the side of a rectangle, or an adjustment factor when no confusion is likely
D = density of target elements
$[D_i(j)]$ = ith order symmetrical correlation and shot type variance ratio matrix
d = one-half the side of a rectangle
E = expected value
$E(k)$ = expected number of target elements killed
E_C = expected fraction of coverage
E_{CD} = expected fractional coverage of target by weapon pattern in deflection
E_{CR} = expected fractional coverage of target by weapon pattern in range
F_D = expected fraction damage to target
f = fraction or coefficient
f = fractional coverage
f_1 = $\sigma_x/\sqrt{\sigma_x^2 + \sigma_\mu^2}$

$\bar{f}$ = $1 - m$, expected fraction of target covered or damaged
$\bar{f}(n)$ = fraction of target damaged for n rounds
$\bar{f}_{max}$ = maximum value of expected fractional target damage
$\bar{f}_{min}$ = minimum value of expected fractional target damage
$g(\mu,\nu)$ = density function of C of I
H = burst height
$h(u,v)$ = target element density
h_i = notation for hit on ith round
$I(s)$ = quantity defined by Eq. 20-100
i = running discrete variable = $1, 2, 3, \ldots, n$
i,j,r = running variables: $1, 2, 3, \ldots$
k = lag
M = upper limit on a sum—see Eqs. 20-80 and 20-84
M_i = notation for a miss on ith round
m = expected fraction of target not covered
N = number of submissiles
$N(r)$ = $r/b^*(r)$, parameter of convenience (Washburn)
$N(v)$ = number of volleys
n = number of rounds
n = number of rounds per volley
$\binom{n}{i}$ = combination of n things taken i at a time
OF = nA_{AP}/A_{VP}, overlap factor
P = probability of hit distribution
$P(R_T/\sigma_x,\sigma_\mu/\sigma_x,n)$ = $A(R,T,N)$, notation for chance of at least one hit in n rounds using Thomas' notation (Ref. 2)
P_H = hit probability (Helgert's equation)
P_L = hit probability (Lind's equation)
$\overline{P_k}$ = average kill probability in a pattern
p = fraction in Eq. 20-83
p = transition probability
p = constant defined by Eq. 20-94
$p(k|h)$ = conditional chance that a hit is a kill
$p(R,r)$ = circular coverage function
p_h = single shot hit probability
$p_h(n)$ = P, chance of at least one hit in n rounds
p_{hi} = chance of hit for ith round
$p_k(x,y)$ = kill chance or damage at point (x,y)
p_0 = chance of hit given a miss with previous round
p_1 = chance of hit given a hit with previous round
$Q(b,r)$ = incomplete gamma function—see Eq. 20-105
q = $2\sigma_X^2/(2\sigma_x^2 + R_T^2)$, variance ratio—see Eq. 20-22
q = constant defined by Eq. 20-94
R = R_T/σ, standardized target radius
R_L = lethal radius
R_T = circular target radius

R_v = vulnerable radius of target
r = radial offset
r = R_L/σ, standardized lethal radius
r = $nc\sigma_k^2/\sigma_u^2$, parameter in Washburn's optimum aiming problem—otherwise a running variable
r_R = round reliability
r,s = variables
S = constant to enter Brândli's tables—Eq. 20-40
$S(R_T/\sigma,r/\sigma)$ = polar integral of circular coverage function
SB = constant to enter Brândli's tables—Eq. 20-39
SZ = constant to enter Brândli's tables—Eq. 20-38
SZ/SB = σ_μ/σ_x, notation for using Brândli's tables (Ref. 3) for chance of at least one hit
s = $c\sigma_k^2/(\sigma_k^2 + \sigma_x^2)$, parameter in Washburn's optimum aiming problem
t_i = time for ith shot
u,v = variables
(u,v) = coordinates of a target element
v = variance of fractional target area actually covered
X_i = total miss distance in x-direction for ith shot
x_i = component of miss distance caused by round-to-round ballistic deviation—see Eq. 20-27
y = r/σ_x
Z = Brândli's notation for $p(k\,|\,h)$
α = semiaxis of ellipse in x-direction
β = semiaxis of ellipse in y-direction
η = standard normal deviate
$\mu(b_i)$ = component of miss distance caused by random nonsystematic changes in aim point—see Eq. 20-27
$\mu(t_i)$ = time varying component of miss distance—see Eq. 20-27
ν = number of degrees of freedom in chi-square distribution
χ^2 = $\chi^2(\nu)$ = chi-square random variable
Π = product sign
ρ_a = average value of ρ_k
ρ_k = autocorrelation coefficients
$\rho_{k\mu t}$ = ρ_k, autocorrelation coefficient of lag k in the x-direction
$\rho_{k\nu t}$ = ρ_k, autocorrelation coefficient of lag k in the y-direction
σ = standard deviation
σ_k = standard deviation of lethality when $\sigma_{kx} = \sigma_{ky} = \sigma_k$
σ_{kx} = standard deviation of lethality pattern in x-direction
σ_{ky} = standard deviation of lethality pattern in y-direction
σ_x = round-to-round ballistic standard deviation in x(range)-direction
σ_y = round-to-round ballistic standard deviation in y(deflection)-direction
σ_μ = standard deviation of aim errors (C of I) in x-direction
σ_ν = standard deviation of aim errors (C of I) in y-direction
$\sigma_{k\mu t}^2$ = autocorrelation of lag k

σ_T^2 = total variance of dispersion
σ_X^2 = total variance
σ_0^2 = $\sigma_x^2 + \sigma_\mu^2$, total variance for round-to-round dispersion and aim errors
$\sigma_{\mu b}^2$ = variance of x-aim point due to bias alone
$\sigma_{\mu t}^2$ = autocovariance of x-aim point due to firing time correlation

20-1 INTRODUCTION

A frequent and important problem facing the weapon systems analyst is that of making calculations of hit probabilities for multiple rounds, and the computation of expected target coverage and target damage for multiple rounds or salvos. It will not be possible to destroy or neutralize many targets with single rounds because hit probabilities may be too low, some targets may be large in size or relatively invulnerable, and the lethality per round is often insufficient. In those cases for which the kill probability per round is too low, several or many rounds must be fired at targets if suitable damage is to be accomplished. Also, there exist several sources of error in weapon-target engagements that must be reckoned with. For example, the aiming error* may be several times the ordinary ballistic dispersion, and it is not easy, therefore, to place the center of impact (C of I) of the rounds on the target center. In addition, some weapons—particularly the automatic ones for air defense, or even machine guns or automatic cannons—may exhibit patterns of aim wander which result in correlation between adjacent, alternate, etc., rounds, or the rounds of a salvo. Accordingly, models used for calculating hit probabilities, or especially the chance of at least one hit for multiple rounds, must be appropriate for each particular application. Thus, the appropriate model for calculating the chance of at least one hit, or the chances of target coverage, or target damage must be selected with much care indeed, depending on the particular application for the weapon employed.

Single shot hit probabilities were discussed in Chapter 14, and various methods of estimating such hit chances were covered in detail for targets of different shapes. Therefore, the analyst should review Chapter 14 before proceeding with this chapter.

20-2 CHANCES OF AT LEAST ONE HIT FOR INDEPENDENT BALLISTIC AND AIM ERRORS

As already indicated, it will not always be possible to place the C of I of the rounds on the center of the target or the desired aim point. However, if that could be done for each round fired in sequence of the rounds, then one would have to consider only the round-to-round ballistic dispersion in calculating the chance of at least one hit for several rounds fired.

In Chapter 13 we defined the round-to-round ballistic dispersions or standard deviations in the x- and y-directions as σ_x and σ_y, respectively. Also, the standard deviations describing the movement of the aim point or C of I of the rounds in the x- and y-directions were given the notation σ_μ and σ_ν, respectively. For the general case of unequal sigmas or delivery errors in the two directions, all four of the component standard deviations must be used in applicable models. However, where it is assumed the $\sigma_x = \sigma_y$ and $\sigma_\mu = \sigma_\nu$ it will be convenient to use only σ_x and σ_μ, with the understanding that the condition of "circularity" exists.

If we deal with a circular target of radius R_T and only ballistic errors for the case $\sigma_y = \sigma_x$, then if the C of I is on the target center (which is located at the origin), we have the chance p_h of hitting with a single round to be, as in Eq. 14-2, simply

$$p_h = 1 - \exp[R_T^2/(2\sigma_x^2)] \tag{20-1}$$

*Aiming error as defined here includes target location error.

and if $\sigma_x \neq \sigma_y$, the Polya-Williams approximation could be used to obtain a sufficiently accurate chance of hitting given by

$$p_h \approx \langle\{1 - \exp[-R_T^2/(2\sigma_x^2)]\}\{1 - \exp[-R_T^2/(2\sigma_y^2)]\}\rangle^{1/2} . \quad (20\text{-}2)$$

Now let us assume, as is nearly always likely, that there exists some aiming error, which has the effect of moving the C of I of the rounds about or over the target from shot to shot. In this case, and if the aiming and ballistic errors are independent, and the gunner reaims between rounds, trying to place the C of I of rounds on the target center each time, then for $\sigma_\mu = \sigma_\nu$ the single shot hit probability will depend on the total variance σ_0^2

$$\sigma_0^2 = \sigma_x^2 + \sigma_\mu^2 \quad (20\text{-}3)$$

and will be given by

$$p_h = 1 - \exp[-R_T^2/(2\sigma_0^2)] = 1 - \exp\{-R_T^2/[2(\sigma_x^2 + \sigma_\mu^2)]\} . \quad (20\text{-}4)$$

On the average for many occasions, when using Eq. 20-4, it is assumed that the gunner is able to place the C of I on the target center, i.e., he accomplishes zero aim error on the average, but commits an aim error from shot to shot as described by σ_μ, in addition to σ_x. For these assumptions, then the chance $p_h(n)$ of at least one hit in n rounds fired, with reaiming between rounds, will be

$$p_h(n) = 1 - (1 - p_h)^n \quad (20\text{-}5)$$

i.e., we simply subtract the chance of no hits in n rounds from unity. Furthermore, if the chance of hit per round is suitably small ($p_h \leq 0.10$), then very nearly

$$p_h(n) \approx 1 - \exp(-np_h) . \quad (20\text{-}6)$$

The generalization of Eq. 20-5 for different hit chances per round p_{hi} is

$$p_h(n) = 1 - \prod_{i=1}^{n} (1 - p_{hi}) . \quad (20\text{-}7)$$

We should emphasize immediately that Eqs. 20-5 and 20-6 do not apply in general for multiple rounds. In spite of their almost universal use, they can be subject to serious errors in many applications not involving the rather strict assumptions that on the average the gunner has zero aim error but commits a shot-to-shot aim error described by σ_μ, as we will see. Walsh (Ref. 1) indicates that for relatively small hit probabilities per shot, formulas of the type of Eqs. 20-5 and 20-6, the latter being of the Poisson type, may still apply with suitable accuracy even for occasions involving dependent events. Hence, such uses of Eqs. 20-5 and 20-6 should be checked independently as the occasion may require.

Single shot hit probabilities for some different target shapes were discussed in Chapter 14. Therefore, they will not be repeated here, although the effect of target shape will be commented on as may be appropriate in this chapter.

20-3 SALVO HIT PROBABILITIES FOR CIRCULAR TARGETS—AXISYMMETRIC CASE

Next, we discuss the case where n rounds are fired in a salvo. Again the round-to-round ballistic dispersion is assumed to be circular ($\sigma_y = \sigma_x$), the rounds are fired from n weapons which are aimed to have a common C of I (or a salvo of n rounds is fired from a weapon such that the shots have their ordinary ballistic dispersion about the same C of I), and the C of I of the salvo of n rounds (or the aim error) moves around independently with $\sigma_\nu = \sigma_\mu$. In this case, one notes that when the C of I is off the target center, then the common aim point of the n rounds in the salvo is biased usually and subject to a standard error of σ_μ, in addition to the fact that the individual rounds in the salvo disperse according to the standard deviation σ_x. There is no reaiming between individual rounds of the salvo, and the assumption of par. 20-2 regarding reaiming between rounds of a salvo would not apply.

Thomas (Ref. 2) gives a table of exact probabilities of at least one hit (his kill probabilities being equivalent to hit chances) for salvos of size $n^* = 1(1)14(2)20$, $R_T/\sigma_x = 0.1(0.1)3.0(0.2)5.0$, and $\sigma_\mu/\sigma_x = 0.1(0.1)3.0(0.2)5.0$. The method of computation is also given in Ref. 2. Thomas' notation for the chance of at least one hit on the circular target is labeled as $A(R,T,N)$ in his tables, where

Thomas'	Our
R	$= R_T/\sigma_x$
T	$= \sigma_\mu/\sigma_x$
N	$= n$
A	$= P = p_h(n) =$ chance of at least one hit .

Hence, the chance of at least one hit from the salvo of n rounds fired at the circular target of radius R_T is

$$P(R_T/\sigma_x, \sigma_\mu/\sigma_x, n) = A(R,T,N) . \tag{20-8}$$

The equation for computing the chance of at least one hit is given by Thomas' Eq. 14, Ref. 2, and involves the circular coverage function $p(R,r)$ of Chapter 14, i.e., the probability of one or more hits is

$$P = A = 1 - (\sigma_x^2/\sigma_\mu^2)\int_0^\infty [1 - P(R_T/\sigma_x, y)]^n \exp[-y^2/(2\sigma_\mu^2/\sigma_x^2)]dy \tag{20-9}$$

where

$y = r/\sigma_x$.

We will illustrate with an example of Thomas (Ref. 2), where the circular target concept may be replaced with the equivalent problem of firing at a point target and requiring that the rounds fall within the damage radius of the warhead to destroy the point or small target.

Example 20-1:

Given a salvo of artillery rockets with round-to-round "ballistic" dispersion of $\sigma_x = 100$ ft, an aiming error of $\sigma_\mu = 210$ ft and warhead damage radius of 220 ft. How large a salvo of such artillery

*$n = 1(1)14(2)20 \triangleq n = 1, 2, 3, \ldots, 14, 16, 18, 20$

rockets would be required to guarantee a chance of kill against a point target (i.e., at least one "hit") equal to 0.70?

We have

Thomas' $R = R_T/\sigma_x = 220/100 = 2.2$

Thomas' $T = \sigma_\mu/\sigma_x = 210/100 = 2.1$

Thomas' $N = n$.

Hence, we look up $A(2.2,2.1,n)$ in Thomas' tables (Ref. 2) for n until a value of at least 0.70 is found. It is observed for $n = 6$ and $n = 7$ rounds that

$$A(2.2,2.1,6) = 0.6836$$

and

$$A(2.2,2.1,7) = 0.7033\,.$$

Therefore, the answer is $n = 7$ rounds, or a seven-round salvo will be required to obtain a kill chance of 0.70 (at least one round within 220 ft of the point target).

Suppose we had incorrectly selected for use Eqs. 20-4 and 20-5 for this example. Now we would have from Eq. 20-4 that

$$p_h = 1 - \exp\{-220^2/[2(100^2 + 210^2)]\} = 0.3607$$

and since n may be found from Eq. 20-5 as

$$n = \ln[1 - p_h(n)]/\ln(1 - p_h) \tag{20-10}$$

then $n = \ln(1 - 0.70)/\ln(1 - 0.3607) = 2.7$, or no more than three rounds! Hence, the error in the wholesale use of Eq. 20-5 may be substantial since the aim error for a salvo results in moving all rounds of the salvo away from the target center.

Thomas' tables are limited to $n = 20$ rounds. However, the Stadelmann/Isenring tables in Brândli's book (Ref. 3) may be used to find the chance of at least one hit for $n = 5$, 10(10)50(25)100, and are reproduced here in our Table 20-1. These tables are entered with

$$A/S = 1.31\{R_T/[\sigma_x(1 + \sigma_\mu^2/\sigma_x^2)^{1/2}]\}$$

and

$$SZ/SB = \sigma_\mu/\sigma_x\,.$$

There are several approximations in closed form to Eq. 20-9 for the chance of at least one hit P in a salvo of n rounds, although they may be inaccurate for some isolated cases by as much as 10%. One is

$$P = p_h(n) = \frac{1}{c}\sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\frac{1}{i}\left(\frac{\dfrac{R_T^2}{\sigma_x^2}}{2c + \dfrac{R_T^2}{\sigma_x^2}}\right)^{i-1}\left[1 - \exp\left(-\frac{\dfrac{R_T^2}{\sigma_x^2}}{\dfrac{2c + \dfrac{R_T^2}{\sigma_x^2}}{ci} + \dfrac{2\sigma_\mu^2}{\sigma_x^2}}\right)\right] \tag{20-11}$$

TABLE 20-1. PROBABILITY OF AT LEAST ONE HIT (Ref. 3)

N = 5

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.00090	0.00090	0.00090	0.00090	0.00090	0.00090	0.00090	0.00090	0.00090	0.00089	0.00087	0.00084	0.00018
0.050	0.00361	0.00361	0.00361	0.00361	0.00360	0.00359	0.00357	0.00355	0.00347	0.00337	0.00312	0.00285	0.00072
0.075	0.00811	0.00811	0.00813	0.00809	0.00807	0.00800	0.00791	0.00780	0.00745	0.00702	0.00609	0.00526	0.00163
0.100	0.01438	0.01437	0.01435	0.01430	0.01423	0.01403	0.01377	0.01344	0.01245	0.01134	0.00933	0.00787	0.00289
0.125	0.02237	0.02236	0.02230	0.02219	0.02202	0.02155	0.02093	0.02020	0.01811	0.01602	0.01276	0.01076	0.00451
0.150	0.03205	0.03203	0.03192	0.03168	0.03134	0.03040	0.02920	0.02782	0.02415	0.02088	0.01644	0.01397	0.00649
0.200	0.05626	0.05621	0.05584	0.05512	0.05411	0.05141	0.04816	0.04471	0.03685	0.03115	0.02470	0.02140	0.01151
0.250	0.08650	0.08637	0.08551	0.08383	0.08151	0.07565	0.06909	0.06271	0.05016	0.04240	0.03427	0.03019	0.01793
0.300	0.12212	0.12187	0.12016	0.11687	0.11246	0.10185	0.09091	0.08120	0.06428	0.05479	0.04513	0.04028	0.02571
0.350	0.16243	0.16198	0.15896	0.15328	0.14586	0.12901	0.11398	0.10006	0.07936	0.06834	0.05722	0.05164	0.03483
0.400	0.20661	0.20590	0.20104	0.19207	0.18073	0.15647	0.13541	0.11939	0.09545	0.08303	0.07051	0.06421	0.04523
0.450	0.25386	0.25278	0.24549	0.23233	0.21624	0.18386	0.15792	0.13931	0.11253	0.09879	0.08492	0.07795	0.05689
0.500	0.30331	0.30177	0.29146	0.27325	0.25174	0.21103	0.18069	0.15986	0.13054	0.11555	0.10040	0.09278	0.06973
0.600	0.40551	0.40278	0.38472	0.35435	0.32100	0.26466	0.22722	0.20284	0.16908	0.15180	0.13432	0.12549	0.09878
0.700	0.50699	0.50278	0.47530	0.43139	0.38650	0.31757	0.27506	0.24794	0.21044	0.19120	0.17169	0.16179	0.13190
0.800	0.60259	0.59678	0.55928	0.50230	0.44776	0.36989	0.32380	0.29455	0.25401	0.23314	0.21191	0.20109	0.16853
0.900	0.68857	0.68123	0.63438	0.56642	0.50494	0.42141	0.37288	0.34205	0.29914	0.27699	0.25437	0.24279	0.20810
1.000	0.76265	0.75410	0.69967	0.62389	0.55829	0.47175	0.42173	0.38981	0.34522	0.32213	0.29845	0.28628	0.25000
1.100	0.82414	0.81472	0.75528	0.67516	0.60795	0.52050	0.46978	0.43727	0.39166	0.36795	0.34354	0.33098	0.29363
1.200	0.87323	0.86349	0.80192	0.72080	0.65399	0.56724	0.51656	0.48391	0.43790	0.41390	0.38907	0.37631	0.33838
1.300	0.91108	0.90153	0.84061	0.76129	0.69639	0.61164	0.56165	0.52926	0.48345	0.45943	0.43451	0.42172	0.38369
1.400	0.93930	0.93037	0.87246	0.79705	0.73516	0.65343	0.60470	0.57295	0.52785	0.50410	0.47938	0.46672	0.42899
1.500	0.95966	0.95168	0.89852	0.82848	0.77034	0.69242	0.64543	0.61465	0.57072	0.54747	0.52323	0.51083	0.47380
1.750	0.98706	0.98198	0.94414	0.89033	0.84319	0.77710	0.73600	0.70870	0.66925	0.64813	0.62606	0.61480	0.58086
2.000	0.99647	0.99385	0.97046	0.93247	0.89659	0.84364	0.80964	0.78671	0.75313	0.73498	0.71598	0.70629	0.67676
2.250	0.99918	0.99805	0.98502	0.95990	0.93402	0.89369	0.86691	0.84857	0.82133	0.80648	0.79092	0.78295	0.75844
2.500	0.99984	0.99941	0.99271	0.97698	0.95921	0.92987	0.90970	0.89564	0.87451	0.86289	0.85068	0.84438	0.82491
2.750	0.99997	0.99983	0.99658	0.98720	0.97552	0.95506	0.94048	0.93014	0.91439	0.90568	0.89647	0.89166	0.87681
3.000	1.00000	0.99994	0.99845	0.99309	0.98573	0.97201	0.96187	0.95454	0.94325	0.93694	0.93025	0.92672	0.91580
3.250	1.00000	0.99998	0.99932	0.99638	0.99191	0.98305	0.97624	0.97123	0.96341	0.95901	0.95431	0.95181	0.94406
3.500	1.00000	0.99998	0.99971	0.99816	0.99554	0.99001	0.98559	0.98229	0.97706	0.97409	0.97088	0.96919	0.96385

N = 10

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.00181	0.00181	0.00181	0.00181	0.00181	0.00180	0.00180	0.00179	0.00177	0.00174	0.00166	0.00156	0.00018
0.050	0.00721	0.00721	0.00721	0.00719	0.00717	0.00712	0.00704	0.00694	0.00662	0.00622	0.00532	0.00449	0.00072
0.075	0.01616	0.01615	0.01612	0.01605	0.01595	0.01567	0.01530	0.01484	0.01346	0.01195	0.00926	0.00739	0.00163
0.100	0.02854	0.02853	0.02842	0.02821	0.02790	0.02706	0.02596	0.02468	0.02115	0.01785	0.01310	0.01038	0.00289
0.125	0.04424	0.04420	0.04395	0.04344	0.04272	0.04077	0.03833	0.03564	0.02896	0.02359	0.01702	0.01365	0.00451
0.150	0.06308	0.06300	0.06248	0.06146	0.06003	0.05626	0.05176	0.04705	0.03660	0.02925	0.02119	0.01724	0.00649
0.200	0.10936	0.10913	0.10758	0.10457	0.10049	0.09042	0.07966	0.06970	0.05146	0.04093	0.03046	0.02544	0.01151
0.250	0.16551	0.16499	0.16144	0.15473	0.14597	0.12598	0.10699	0.09137	0.06653	0.05356	0.04101	0.03497	0.01793
0.300	0.22933	0.22834	0.22154	0.20908	0.19349	0.16075	0.13305	0.11241	0.08233	0.06732	0.05281	0.04578	0.02571
0.350	0.29847	0.29678	0.28536	0.26501	0.24071	0.19384	0.15813	0.13340	0.09906	0.08219	0.06582	0.05784	0.03483
0.400	0.37054	0.36794	0.35049	0.32038	0.28615	0.22523	0.18273	0.15470	0.11674	0.09813	0.07997	0.07108	0.04523
0.450	0.44327	0.43957	0.41486	0.37364	0.32903	0.25523	0.20720	0.17649	0.13531	0.11507	0.09522	0.08545	0.05689
0.500	0.51462	0.50967	0.47681	0.42380	0.36912	0.28423	0.23176	0.19879	0.15471	0.13295	0.11149	0.10090	0.06973
0.600	0.64658	0.63857	0.58895	0.51323	0.44151	0.34029	0.28138	0.24483	0.19570	0.17119	0.14682	0.13472	0.09878
0.700	0.75694	0.74701	0.68192	0.58845	0.50547	0.39463	0.33154	0.29230	0.23907	0.21225	0.18541	0.17198	0.13190
0.800	0.84207	0.83073	0.75566	0.65167	0.56303	0.44747	0.38178	0.34061	0.28419	0.25551	0.22663	0.21207	0.16853
0.900	0.90301	0.89143	0.81275	0.70534	0.61540	0.49857	0.43154	0.38911	0.33042	0.30035	0.26987	0.25440	0.20810
1.000	0.94369	0.93293	0.85652	0.75136	0.66320	0.54758	0.48028	0.43724	0.37715	0.34614	0.31450	0.29834	0.25000
1.100	0.96907	0.95990	0.89009	0.79105	0.70671	0.59418	0.52752	0.48446	0.42382	0.39230	0.35992	0.34331	0.29363
1.200	0.98393	0.97667	0.91592	0.82530	0.74611	0.63806	0.57287	0.53032	0.46991	0.43827	0.40558	0.38875	0.33838
1.300	0.99209	0.98672	0.93587	0.85478	0.78154	0.67903	0.61598	0.57442	0.51495	0.48357	0.45094	0.43413	0.38369
1.400	0.99632	0.99257	0.95130	0.88002	0.81315	0.71694	0.65661	0.61646	0.55853	0.52773	0.49555	0.47895	0.42899
1.500	0.99837	0.99589	0.96321	0.90150	0.84113	0.75174	0.69458	0.65617	0.60031	0.57039	0.53898	0.52277	0.47380
1.750	0.99983	0.99909	0.98228	0.94148	0.89674	0.82529	0.77729	0.74427	0.69526	0.66853	0.64021	0.62561	0.58086
2.000	0.99999	0.99980	0.99183	0.96653	0.93515	0.88394	0.84267	0.81571	0.77487	0.75221	0.72806	0.71554	0.67676
2.250	1.00000	0.99995	0.99640	0.98153	0.96058	0.92132	0.89220	0.87118	0.83869	0.82038	0.80077	0.79050	0.75844
2.500	1.00000	0.99998	0.99847	0.99013	0.97676	0.94952	0.92829	0.91258	0.88780	0.87364	0.85838	0.85030	0.82491
2.750	1.00000	0.99999	0.99937	0.99489	0.98669	0.96853	0.95364	0.94235	0.92418	0.91367	0.90223	0.89615	0.87681
3.000	1.00000	0.99999	0.99975	0.99742	0.99259	0.98092	0.97086	0.96304	0.95020	0.94268	0.93441	0.93002	0.91580
3.250	1.00000	0.99999	0.99990	0.99874	0.99598	0.98875	0.98218	0.97694	0.96819	0.96300	0.95721	0.95418	0.94406
3.500	1.00000	0.99999	0.99995	0.99939	0.99788	0.99354	0.98940	0.98601	0.98023	0.97676	0.97284	0.97082	0.96385

Enter table with $SZ/SB = \sigma_\mu/\sigma_x$ and $A/S = 1.31\{R_T/[\sigma_x(1 + \sigma_\mu^2/\sigma_x^2)^{1/2}]\}$

TABLE 20-1 (Cont'd.)

N = 20

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.00361	0.00361	0.00361	0.00361	0.00360	0.00359	0.00357	0.00354	0.00345	0.00333	0.00303	0.00269	0.00018
0.050	0.01438	0.01437	0.01434	0.01429	0.01420	0.01397	0.01365	0.01326	0.01207	0.01072	0.00822	0.00637	0.00072
0.075	0.03206	0.03203	0.03189	0.03161	0.03121	0.03009	0.02864	0.02698	0.02249	0.01841	0.01274	0.00958	0.00163
0.100	0.05627	0.05621	0.05577	0.05491	0.05370	0.05046	0.04652	0.04230	0.03252	0.02525	0.01697	0.01289	0.00289
0.125	0.08652	0.08637	0.08534	0.08333	0.08055	0.07345	0.06542	0.05752	0.04162	0.03159	0.02131	0.01649	0.00451
0.150	0.12218	0.12188	0.11982	0.11587	0.11054	0.09759	0.08406	0.07183	0.05007	0.03781	0.02591	0.02042	0.00649
0.200	0.20676	0.20590	0.20004	0.18914	0.17526	0.14509	0.11832	0.09756	0.06630	0.05066	0.03604	0.02928	0.01151
0.250	0.30363	0.30177	0.28920	0.26678	0.23999	0.18818	0.14853	0.12094	0.08277	0.06447	0.04742	0.03945	0.01793
0.300	0.40607	0.40276	0.38049	0.34248	0.30002	0.22640	0.17620	0.14349	0.09998	0.07938	0.06004	0.05089	0.02571
0.350	0.50785	0.50271	0.46832	0.41224	0.35362	0.26090	0.20262	0.16599	0.11806	0.09535	0.07382	0.06354	0.03483
0.400	0.60378	0.59661	0.54886	0.47430	0.40094	0.29292	0.22850	0.18877	0.13701	0.11233	0.08871	0.07735	0.04523
0.450	0.69005	0.68090	0.61999	0.52850	0.44296	0.32332	0.25421	0.21194	0.15676	0.13024	0.10466	0.09227	0.05689
0.500	0.76441	0.75356	0.68107	0.57557	0.48077	0.35263	0.27989	0.23550	0.17726	0.14901	0.12158	0.10823	0.06973
0.600	0.87509	0.86246	0.77527	0.65261	0.54720	0.40901	0.33131	0.28360	0.22013	0.18885	0.15810	0.14298	0.09878
0.700	0.94092	0.92893	0.83986	0.71318	0.60493	0.46307	0.38258	0.33255	0.26500	0.23122	0.19769	0.18105	0.13190
0.800	0.97506	0.96542	0.88429	0.76246	0.65626	0.51489	0.43320	0.38172	0.31120	0.27549	0.23972	0.22180	0.16853
0.900	0.99059	0.98386	0.91551	0.80344	0.70228	0.56424	0.48268	0.43053	0.35812	0.32103	0.28357	0.26464	0.20810
1.000	0.99683	0.99263	0.93797	0.83790	0.74354	0.61086	0.53053	0.47842	0.40517	0.36724	0.32862	0.30894	0.25000
1.100	0.99904	0.99665	0.95440	0.86698	0.78038	0.65450	0.57634	0.52494	0.45180	0.41355	0.37427	0.35412	0.29363
1.200	0.99974	0.99847	0.96656	0.89146	0.81307	0.69501	0.61981	0.56968	0.49752	0.45942	0.41997	0.39963	0.33838
1.300	0.99994	0.99929	0.97557	0.91198	0.84185	0.73228	0.66068	0.61232	0.54192	0.50438	0.46522	0.44495	0.38369
1.400	0.99999	0.99966	0.98226	0.92908	0.86700	0.76629	0.69878	0.65259	0.58461	0.54801	0.50955	0.48959	0.42899
1.500	1.00000	0.99984	0.98720	0.94322	0.88881	0.79709	0.73402	0.69031	0.62530	0.58996	0.55257	0.53314	0.47380
1.750	1.00000	0.99997	0.99451	0.96832	0.93069	0.86073	0.80948	0.77284	0.71688	0.68575	0.65233	0.63487	0.58086
2.000	1.00000	0.99998	0.99775	0.98297	0.95823	0.90739	0.86775	0.83852	0.79269	0.76661	0.73834	0.72344	0.67676
2.250	1.00000	0.99999	0.99911	0.99115	0.97561	0.94025	0.91092	0.88862	0.85273	0.83189	0.80906	0.79696	0.75844
2.500	1.00000	0.99999	0.99966	0.99554	0.98617	0.96256	0.94172	0.92539	0.89842	0.88246	0.86479	0.85540	0.82491
2.750	1.00000	0.99999	0.99987	0.99782	0.99238	0.97719	0.96294	0.95142	0.93192	0.92018	0.90702	0.90005	0.87681
3.000	1.00000	0.99999	0.99995	0.99896	0.99591	0.98648	0.97707	0.96924	0.95564	0.94732	0.93787	0.93287	0.91580
3.250	1.00000	0.99999	0.99997	0.99951	0.99787	0.99220	0.98620	0.98105	0.97188	0.96618	0.95964	0.95614	0.94406
3.500	1.00000	0.99999	0.99998	0.99977	0.99891	0.99562	0.99192	0.98864	0.98266	0.97887	0.97450	0.97210	0.96385

N = 30

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.00541	0.00541	0.00541	0.00540	0.00539	0.00536	0.00531	0.00525	0.00505	0.00479	0.00418	0.00355	0.00018
0.050	0.02149	0.02148	0.02141	0.02128	0.02109	0.02057	0.01987	0.01904	0.01661	0.01412	0.01008	0.00750	0.00072
0.075	0.04770	0.04765	0.04733	0.04670	0.04580	0.04338	0.04037	0.03706	0.02897	0.02256	0.01480	0.01087	0.00163
0.100	0.08321	0.08307	0.08210	0.08020	0.07757	0.07082	0.06312	0.05546	0.03978	0.02969	0.01924	0.01435	0.00289
0.125	0.12693	0.12661	0.12434	0.12001	0.11417	0.10008	0.08546	0.07236	0.04925	0.03629	0.02381	0.01813	0.00451
0.150	0.17755	0.17690	0.17249	0.16419	0.15339	0.12899	0.10610	0.08747	0.05801	0.04281	0.02863	0.02224	0.00649
0.200	0.29351	0.29175	0.27975	0.25825	0.23233	0.18150	0.14188	0.11394	0.07492	0.05627	0.03921	0.03145	0.01151
0.250	0.41889	0.41530	0.35111	0.34999	0.30435	0.22583	0.17261	0.13798	0.09211	0.07070	0.05104	0.04196	0.01793
0.300	0.54228	0.53632	0.45639	0.43210	0.36614	0.26403	0.20077	0.16125	0.11003	0.08620	0.06407	0.05372	0.02571
0.350	0.65474	0.64623	0.58931	0.50209	0.41846	0.29834	0.22773	0.18448	0.12880	0.10273	0.07826	0.06668	0.03483
0.400	0.75059	0.73980	0.66730	0.56056	0.46338	0.33023	0.25415	0.20795	0.14838	0.12024	0.09354	0.08080	0.04523
0.450	0.82744	0.81506	0.73056	0.60942	0.50279	0.36054	0.28036	0.23175	0.16872	0.13865	0.10985	0.09600	0.05689
0.500	0.88565	0.87260	0.78089	0.65072	0.53814	0.38975	0.30648	0.25587	0.18975	0.15788	0.12711	0.11222	0.06973
0.600	0.95585	0.94415	0.85204	0.71713	0.60017	0.44578	0.35851	0.30485	0.23353	0.19851	0.16424	0.14746	0.09878
0.700	0.98564	0.97732	0.89726	0.76888	0.65390	0.49916	0.41002	0.35435	0.27909	0.24152	0.20433	0.18594	0.13190
0.800	0.99606	0.99112	0.92724	0.81073	0.70138	0.54994	0.46053	0.40377	0.32577	0.28626	0.24676	0.22703	0.16853
0.900	0.99909	0.99653	0.94790	0.84525	0.74359	0.59792	0.50955	0.45252	0.37296	0.33213	0.29091	0.27013	0.20810
1.000	0.99982	0.99861	0.96253	0.87396	0.78106	0.64288	0.55666	0.50011	0.42008	0.37851	0.33616	0.31461	0.25000
1.100	0.99997	0.99942	0.97305	0.89788	0.81417	0.68465	0.60149	0.54608	0.46660	0.42485	0.38191	0.35989	0.29363
1.200	1.00000	0.99975	0.98067	0.91775	0.84323	0.72312	0.64376	0.59008	0.51206	0.47062	0.42762	0.40543	0.33838
1.300	1.00000	0.99989	0.98621	0.93417	0.86854	0.75826	0.68328	0.63181	0.55604	0.51536	0.47278	0.45070	0.38369
1.400	1.00000	0.99994	0.99022	0.94765	0.89042	0.79010	0.71992	0.67104	0.59820	0.55866	0.51695	0.49524	0.42899
1.500	1.00000	0.99997	0.99311	0.95864	0.90920	0.81872	0.75363	0.70764	0.63826	0.60020	0.55974	0.53861	0.47380
1.750	1.00000	0.99998	0.99722	0.97767	0.94462	0.87720	0.82518	0.78712	0.72798	0.69469	0.65871	0.63975	0.58086
2.000	1.00000	0.99999	0.99893	0.98837	0.96733	0.91938	0.87975	0.84975	0.80174	0.77403	0.74369	0.72762	0.67676
2.250	1.00000	0.99999	0.99960	0.99414	0.98132	0.94863	0.91973	0.89709	0.85980	0.83778	0.81336	0.80039	0.75844
2.500	1.00000	0.99999	0.99985	0.99714	0.98962	0.96820	0.94794	0.93153	0.90373	0.88695	0.86812	0.85811	0.82491
2.750	1.00000	0.99999	0.99994	0.99864	0.99439	0.98085	0.96718	0.95572	0.93575	0.92347	0.90952	0.90206	0.87681
3.000	1.00000	0.99999	0.99997	0.99937	0.99705	0.98878	0.97986	0.97214	0.95831	0.94964	0.93968	0.93427	0.91580
3.250	1.00000	0.99999	0.99998	0.99971	0.99848	0.99360	0.98798	0.98295	0.97369	0.96776	0.96091	0.95709	0.94406
3.500	1.00000	0.99999	0.99998	0.99987	0.99924	0.99645	0.99302	0.98984	0.98384	0.97991	0.97537	0.97276	0.96385

Enter table with $SZ/SB = \sigma_\mu/\sigma_x$ and $A/S = 1.31\{R_T/[\sigma_x(1 + \sigma_\mu^2/\sigma_x^2)^{1/2}]\}$

TABLE 20-1 (Cont'd.)

N = 40

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.00721	0.00721	0.00720	0.00719	0.00717	0.00711	0.00702	0.00691	0.00657	0.00613	0.00515	0.00422	0.00018
0.050	0.02855	0.02853	0.02841	0.02818	0.02785	0.02693	0.02573	0.02434	0.02047	0.01679	0.01143	0.00831	0.00072
0.075	0.06308	0.06300	0.06244	0.06133	0.05977	0.05565	0.05072	0.04554	0.03389	0.02555	0.01627	0.01178	0.00163
0.100	0.10938	0.10914	0.10744	0.10416	0.09971	0.08867	0.07678	0.06568	0.04500	0.03284	0.02085	0.01538	0.00289
0.125	0.16556	0.16499	0.16112	0.15379	0.14418	0.12212	0.10091	0.08325	0.05465	0.03963	0.02556	0.01928	0.00451
0.150	0.22943	0.22834	0.22091	0.20724	0.19003	0.15360	0.12230	0.09862	0.06362	0.04634	0.03054	0.02350	0.00649
0.200	0.37077	0.36793	0.34870	0.31530	0.27700	0.20778	0.15844	0.12544	0.08099	0.06021	0.04143	0.03295	0.01151
0.250	0.51507	0.50962	0.47304	0.41334	0.35093	0.25195	0.18936	0.14989	0.09865	0.07505	0.05355	0.04368	0.01793
0.300	0.64725	0.63882	0.58236	0.49539	0.41142	0.28980	0.21780	0.17363	0.11704	0.09094	0.06686	0.05566	0.02571
0.350	0.75779	0.74670	0.67191	0.56177	0.46149	0.32386	0.24507	0.19731	0.13625	0.10784	0.08132	0.06884	0.03483
0.400	0.84301	0.83021	0.74197	0.61542	0.50416	0.35557	0.27181	0.22121	0.15624	0.12570	0.09685	0.08315	0.04523
0.450	0.90393	0.89069	0.79549	0.65948	0.54158	0.38573	0.29830	0.24539	0.17696	0.14442	0.11340	0.09854	0.05689
0.500	0.94450	0.93202	0.83613	0.69647	0.57515	0.41478	0.32464	0.26984	0.19832	0.16395	0.13089	0.11454	0.06973
0.600	0.98440	0.97562	0.89129	0.75577	0.63404	0.47037	0.37696	0.31932	0.24268	0.20510	0.16841	0.15050	0.09878
0.700	0.99651	0.99167	0.92544	0.80187	0.68494	0.52311	0.42849	0.36911	0.28866	0.24852	0.20884	0.18925	0.13190
0.800	0.99938	0.99712	0.94784	0.83899	0.72971	0.57301	0.47880	0.41860	0.33562	0.29356	0.25153	0.23057	0.16853
0.900	0.99991	0.99896	0.96314	0.86942	0.76926	0.61992	0.52741	0.46725	0.38296	0.33962	0.29587	0.27383	0.20810
1.000	0.99999	0.99960	0.97386	0.89454	0.80414	0.66365	0.57393	0.51456	0.43009	0.38610	0.34124	0.31843	0.25000
1.100	1.00000	0.99984	0.98148	0.91528	0.83474	0.70407	0.61801	0.56011	0.47651	0.43243	0.38705	0.36378	0.29363
1.200	1.00000	0.99993	0.98692	0.93236	0.86141	0.74111	0.65942	0.60356	0.52176	0.47811	0.43276	0.40933	0.33838
1.300	1.00000	0.99996	0.99081	0.94633	0.88448	0.77477	0.69799	0.64463	0.56544	0.52269	0.47785	0.45457	0.38369
1.400	1.00000	0.99998	0.99359	0.95770	0.90427	0.80513	0.73361	0.68314	0.60722	0.56577	0.52191	0.49902	0.42899
1.500	1.00000	0.99998	0.99556	0.96687	0.92114	0.83231	0.76627	0.71895	0.64684	0.60702	0.56454	0.54226	0.47380
1.750	1.00000	0.99999	0.99828	0.98249	0.95260	0.88739	0.83519	0.79636	0.73528	0.70061	0.66295	0.64304	0.58086
2.000	1.00000	0.99999	0.99936	0.99108	0.97243	0.92670	0.88733	0.85695	0.80766	0.77893	0.74724	0.73044	0.67676
2.250	1.00000	0.99999	0.99977	0.99560	0.98445	0.95368	0.92523	0.90248	0.86441	0.84166	0.81621	0.80269	0.75844
2.500	1.00000	0.99999	0.99991	0.99785	0.99148	0.97156	0.95179	0.93541	0.90716	0.88990	0.87034	0.85989	0.82491
2.750	1.00000	0.99999	0.99996	0.99901	0.99545	0.98301	0.96978	0.95841	0.93823	0.92561	0.91118	0.90335	0.87681
3.000	1.00000	0.99999	0.99998	0.99955	0.99763	0.99012	0.98156	0.97395	0.96003	0.95114	0.94089	0.93518	0.91580
3.250	1.00000	0.99999	0.99998	0.99975	0.99880	0.99441	0.98905	0.98412	0.97485	0.96878	0.96175	0.95774	0.94406
3.500	1.00000	0.99999	0.99998	0.99990	0.99940	0.99692	0.99367	0.99058	0.98459	0.98059	0.97592	0.97322	0.96385

N = 50

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.00901	0.00901	0.00899	0.00897	0.00894	0.00884	0.00871	0.00854	0.00801	0.00737	0.00599	0.00477	0.00018
0.050	0.03555	0.03553	0.03535	0.03499	0.03448	0.03307	0.03126	0.02920	0.02377	0.01896	0.01248	0.00893	0.00072
0.075	0.07822	0.07810	0.07722	0.07552	0.07315	0.06702	0.05993	0.05278	0.03779	0.02788	0.01741	0.01249	0.00163
0.100	0.13480	0.13443	0.13184	0.12689	0.12027	0.10442	0.08821	0.07390	0.04906	0.03529	0.02209	0.01617	0.00289
0.125	0.20247	0.20162	0.19580	0.18496	0.17107	0.14063	0.11326	0.09175	0.05884	0.04221	0.02692	0.02016	0.00451
0.150	0.27803	0.27642	0.26548	0.24572	0.22163	0.17343	0.13490	0.10722	0.06795	0.04906	0.03201	0.02448	0.00649
0.200	0.43959	0.43558	0.40853	0.36289	0.31279	0.22794	0.17111	0.13427	0.08566	0.06324	0.04312	0.03409	0.01151
0.250	0.59533	0.58808	0.53945	0.46275	0.38631	0.27171	0.20215	0.15903	0.10367	0.07839	0.05546	0.04500	0.01793
0.300	0.72815	0.71769	0.64724	0.54214	0.44484	0.30922	0.23078	0.18309	0.12241	0.09456	0.06899	0.05714	0.02571
0.350	0.83008	0.81740	0.73014	0.60430	0.49287	0.34305	0.25826	0.20710	0.14193	0.11174	0.08364	0.07047	0.03483
0.400	0.90118	0.88781	0.79160	0.65372	0.53376	0.37458	0.28519	0.23129	0.16222	0.12984	0.09936	0.08493	0.04523
0.450	0.94652	0.93400	0.83672	0.69409	0.56964	0.40458	0.31185	0.25573	0.18321	0.14881	0.11609	0.10046	0.05689
0.500	0.97306	0.96245	0.87011	0.72793	0.60186	0.43346	0.33833	0.28041	0.20482	0.16855	0.13374	0.11699	0.06973
0.600	0.99449	0.98851	0.91463	0.78214	0.65835	0.48863	0.39079	0.33022	0.24959	0.21008	0.17156	0.15279	0.09878
0.700	0.99915	0.99649	0.94195	0.82422	0.70707	0.54078	0.44229	0.38017	0.29586	0.25378	0.21223	0.19174	0.13190
0.800	0.99990	0.99886	0.95976	0.85798	0.74976	0.58996	0.49238	0.42969	0.34302	0.29904	0.25511	0.23322	0.16853
0.900	0.99999	0.99960	0.97185	0.88552	0.78731	0.63599	0.54064	0.47821	0.39044	0.34523	0.29959	0.27661	0.20810
1.000	1.00000	0.99985	0.98025	0.90813	0.82026	0.67875	0.58666	0.52528	0.43756	0.39177	0.34505	0.32130	0.25000
1.100	1.00000	0.99994	0.98616	0.92667	0.84901	0.71811	0.63014	0.57048	0.48389	0.43809	0.39089	0.36669	0.29363
1.200	1.00000	0.99997	0.99034	0.94183	0.87393	0.75406	0.67088	0.61350	0.52897	0.48370	0.43659	0.41224	0.33838
1.300	1.00000	0.99998	0.99329	0.95415	0.89538	0.78662	0.70871	0.65407	0.57241	0.52815	0.48164	0.45745	0.38369
1.400	1.00000	0.99998	0.99538	0.96409	0.91369	0.81587	0.74357	0.69201	0.61390	0.57104	0.52561	0.50183	0.42899
1.500	1.00000	0.99999	0.99683	0.97206	0.92921	0.84196	0.77543	0.72722	0.65318	0.61207	0.56812	0.54499	0.47380
1.750	1.00000	0.99999	0.99881	0.98547	0.95790	0.89456	0.84239	0.80307	0.74065	0.70499	0.66610	0.64550	0.58066
2.000	1.00000	0.99999	0.99957	0.99271	0.97577	0.93180	0.89274	0.86216	0.81201	0.78255	0.74987	0.73254	0.67676
2.250	1.00000	0.99999	0.99984	0.99645	0.98647	0.95717	0.92913	0.90636	0.86777	0.84451	0.81834	0.80440	0.75844
2.500	1.00000	0.99999	0.99994	0.99832	0.99266	0.97386	0.95451	0.93819	0.90967	0.89205	0.87200	0.86118	0.82491
2.750	1.00000	0.99999	0.99997	0.99922	0.99612	0.98448	0.97160	0.96034	0.94002	0.92718	0.91242	0.90428	0.87681
3.000	1.00000	0.99999	0.99998	0.99965	0.99800	0.99103	0.98275	0.97524	0.96128	0.95224	0.94178	0.93586	0.91580
3.250	1.00000	0.99999	0.99998	0.99984	0.99899	0.99495	0.98980	0.98495	0.97568	0.96953	0.96236	0.95824	0.94406
3.500	1.00000	0.99999	0.99998	0.99992	0.99950	0.99723	0.99413	0.99110	0.98513	0.98109	0.97631	0.97357	0.96385

Enter table with $SZ/SB = \sigma_\mu/\sigma_x$ and $A/S = \{R_T/[\sigma_x(1 + \sigma_\mu^2/\sigma_x^2)^{1/2}]\}$

TABLE 20-1 (Cont'd.)

N = 75

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.01348	0.01348	0.01345	0.01340	0.01332	0.01311	0.01282	0.01246	0.01135	0.01008	0.00765	0.00580	0.00018
0.050	0.05285	0.05279	0.05239	0.05160	0.05048	0.04748	0.04379	0.03980	0.03029	0.02299	0.01439	0.01007	0.00072
0.075	0.11501	0.11473	0.11283	0.10918	0.10421	0.09201	0.07900	0.06700	0.04495	0.03211	0.01948	0.01376	0.00163
0.100	0.19523	0.19443	0.18898	0.17879	0.16567	0.13663	0.11014	0.08907	0.05640	0.03972	0.02434	0.01761	0.00289
0.125	0.28777	0.28604	0.27423	0.25298	0.22723	0.17621	0.13588	0.10710	0.06641	0.04687	0.02937	0.02175	0.00451
0.150	0.38655	0.38342	0.36224	0.32565	0.28406	0.20985	0.15755	0.12269	0.07578	0.05399	0.03466	0.02622	0.00649
0.200	0.58047	0.57351	0.52671	0.45231	0.37739	0.26359	0.19371	0.15012	0.09407	0.06869	0.04616	0.03614	0.01151
0.250	0.74257	0.73167	0.65791	0.54821	0.44709	0.30638	0.22491	0.17538	0.11268	0.08436	0.05887	0.04733	0.01793
0.300	0.85826	0.84501	0.75230	0.61882	0.50119	0.34322	0.25381	0.19997	0.13200	0.10102	0.07276	0.05975	0.02571
0.350	0.92996	0.91675	0.81715	0.67220	0.54546	0.37657	0.28159	0.22450	0.15206	0.11866	0.08776	0.07335	0.03483
0.400	0.96893	0.95772	0.86159	0.71419	0.58322	0.40771	0.30881	0.24916	0.17285	0.13720	0.10380	0.08807	0.04523
0.450	0.98763	0.97928	0.89277	0.74843	0.61641	0.43733	0.33570	0.27402	0.19429	0.15656	0.12083	0.10384	0.05689
0.500	0.99558	0.98998	0.91536	0.77713	0.64623	0.46582	0.36235	0.29904	0.21630	0.17668	0.13877	0.12060	0.06973
0.600	0.99959	0.99757	0.94513	0.82308	0.69846	0.52005	0.41491	0.34932	0.26174	0.21883	0.17709	0.15681	0.09878
0.700	0.99998	0.99934	0.96323	0.85861	0.74332	0.57102	0.46621	0.39948	0.30849	0.26302	0.21817	0.19610	0.13190
0.800	1.00000	0.99979	0.97493	0.88693	0.78235	0.61876	0.51583	0.44895	0.35594	0.30863	0.26137	0.23786	0.16853
0.900	1.00000	0.99992	0.98277	0.90983	0.81640	0.66316	0.56334	0.49719	0.40347	0.35503	0.30608	0.28147	0.20810
1.000	1.00000	0.99997	0.98813	0.92843	0.84603	0.70412	0.60843	0.54377	0.45055	0.40165	0.35168	0.32629	0.25000
1.100	1.00000	0.99998	0.99185	0.94351	0.87165	0.74160	0.65082	0.58831	0.49668	0.44793	0.39759	0.37175	0.29363
1.200	1.00000	0.99998	0.99442	0.95569	0.89366	0.77560	0.69032	0.63051	0.54143	0.49339	0.44326	0.41731	0.33838
1.300	1.00000	0.99999	0.99621	0.96546	0.91242	0.80621	0.72683	0.67016	0.58444	0.53759	0.48822	0.46244	0.38369
1.400	1.00000	0.99999	0.99744	0.97326	0.92829	0.83355	0.76032	0.70710	0.62539	0.58016	0.53202	0.50672	0.42899
1.500	1.00000	0.99999	0.99828	0.97942	0.94163	0.85778	0.79079	0.74126	0.66407	0.62080	0.57430	0.54976	0.47380
1.750	1.00000	0.99999	0.99938	0.98959	0.96593	0.90618	0.85435	0.81439	0.74984	0.71252	0.67152	0.64979	0.58086
2.000	1.00000	0.99999	0.99978	0.99491	0.98074	0.93998	0.90166	0.87088	0.81940	0.78874	0.75444	0.73617	0.67676
2.250	1.00000	0.99999	0.99992	0.99759	0.98943	0.96271	0.93553	0.91281	0.87347	0.84937	0.82204	0.80727	0.75844
2.500	1.00000	0.99999	0.99997	0.99888	0.99436	0.97747	0.95892	0.94279	0.91390	0.89571	0.87486	0.86335	0.82491
2.750	1.00000	0.99999	0.99998	0.99949	0.99707	0.98675	0.97454	0.96349	0.94304	0.92984	0.91454	0.90593	0.87681
3.000	1.00000	0.99999	0.99998	0.99977	0.99851	0.99242	0.98464	0.97733	0.96336	0.95411	0.94326	0.93710	0.91580
3.250	1.00000	0.99999	0.99998	0.99990	0.99926	0.99578	0.99098	0.98630	0.97706	0.97081	0.96335	0.95913	0.94406
3.500	1.00000	0.99999	0.99998	0.99995	0.99964	0.99771	0.99485	0.99194	0.98602	0.98194	0.97697	0.97418	0.96385

N = 100

SZ/SB= A/S	0.0	0.5	1.0	1.5	2.0	3.0	4.0	5.0	7.5	10.0	15.0	20.0	∞
0.025	0.01793	0.01793	0.01788	0.01779	0.01766	0.01728	0.01677	0.01616	0.01432	0.01234	0.00890	0.00653	0.00018
0.050	0.06985	0.06974	0.06904	0.06766	0.06573	0.06069	0.05474	0.04859	0.03517	0.02589	0.01575	0.01088	0.00072
0.075	0.15032	0.14985	0.14660	0.14042	0.13222	0.11300	0.09392	0.07754	0.05002	0.03510	0.02094	0.01466	0.00163
0.100	0.25143	0.25011	0.24104	0.22448	0.20393	0.16144	0.12604	0.09982	0.06159	0.04285	0.02593	0.01861	0.00289
0.125	0.36395	0.36117	0.34228	0.30935	0.27134	0.20191	0.15177	0.11787	0.07174	0.05017	0.03109	0.02286	0.00451
0.150	0.47876	0.47396	0.44157	0.38776	0.33005	0.23526	0.17331	0.13352	0.08129	0.05745	0.03651	0.02749	0.00649
0.200	0.68594	0.67638	0.61204	0.51409	0.42106	0.28796	0.20940	0.16119	0.09997	0.07252	0.04828	0.03756	0.01151
0.250	0.83624	0.82325	0.73318	0.60279	0.48689	0.33003	0.24067	0.18677	0.11898	0.08852	0.06124	0.04895	0.01793
0.300	0.92610	0.91273	0.81228	0.66600	0.53779	0.36636	0.26972	0.21171	0.13868	0.10552	0.07537	0.06156	0.02571
0.350	0.97113	0.96005	0.86328	0.71341	0.57951	0.39932	0.29766	0.23655	0.15910	0.12347	0.09060	0.07534	0.03483
0.400	0.99023	0.98250	0.89718	0.75070	0.61515	0.43012	0.32503	0.26151	0.18021	0.14229	0.10686	0.09023	0.04523
0.450	0.99714	0.99240	0.92069	0.78111	0.64652	0.45943	0.35203	0.28660	0.20194	0.16192	0.12410	0.10616	0.05689
0.500	0.99927	0.99663	0.93765	0.80661	0.67470	0.48758	0.37874	0.31182	0.22421	0.18227	0.14222	0.12307	0.06973
0.600	0.99997	0.99925	0.95995	0.84740	0.72401	0.54105	0.43128	0.36237	0.27007	0.22483	0.18088	0.15956	0.09878
0.700	1.00000	0.99980	0.97344	0.87885	0.76621	0.59109	0.48235	0.41260	0.31712	0.26934	0.22223	0.19908	0.13190
0.800	1.00000	0.99993	0.98209	0.90380	0.80277	0.63776	0.53156	0.46197	0.36473	0.31516	0.26565	0.24103	0.16853
0.900	1.00000	0.99997	0.98784	0.92384	0.83448	0.68097	0.57851	0.50997	0.41232	0.36169	0.31051	0.28477	0.20810
1.000	1.00000	0.99998	0.99173	0.94000	0.86191	0.72066	0.62291	0.55617	0.45934	0.40835	0.35619	0.32968	0.25000
1.100	1.00000	0.99998	0.99439	0.95300	0.88549	0.75681	0.66450	0.60022	0.50532	0.45460	0.40213	0.37519	0.29363
1.200	1.00000	0.99999	0.99622	0.96341	0.90561	0.78948	0.70313	0.64185	0.54983	0.49994	0.44779	0.42074	0.33838
1.300	1.00000	0.99999	0.99746	0.97170	0.92266	0.81877	0.73873	0.68085	0.59251	0.54397	0.49267	0.46584	0.38369
1.400	1.00000	0.99999	0.99831	0.97825	0.93700	0.84482	0.77127	0.71709	0.63310	0.58630	0.53634	0.51006	0.42899
1.500	1.00000	0.99999	0.99888	0.98339	0.94898	0.86782	0.80079	0.75052	0.67135	0.62667	0.57848	0.55301	0.47380
1.750	1.00000	0.99999	0.99961	0.99175	0.97058	0.91346	0.86207	0.82180	0.75594	0.71757	0.67519	0.65270	0.58086
2.000	1.00000	0.99999	0.99986	0.99604	0.98357	0.94504	0.90737	0.87656	0.82429	0.79287	0.75753	0.73857	0.67676
2.250	1.00000	0.99999	0.99995	0.99815	0.99109	0.96610	0.93959	0.91698	0.87723	0.85260	0.82453	0.80916	0.75844
2.500	1.00000	0.99999	0.99998	0.99916	0.99530	0.97966	0.96170	0.94574	0.91667	0.89814	0.87676	0.86484	0.82491
2.750	1.00000	0.99999	0.99998	0.99962	0.99758	0.98813	0.97638	0.96551	0.94501	0.93160	0.91592	0.90708	0.87681
3.000	1.00000	0.99999	0.99998	0.99983	0.99878	0.99325	0.98582	0.97867	0.96471	0.95536	0.94422	0.93796	0.91580
3.250	1.00000	0.99999	0.99998	0.99992	0.99940	0.99626	0.99171	0.98716	0.97796	0.97167	0.96401	0.95972	0.94406
3.500	1.00000	0.99999	0.99998	0.99996	0.99971	0.99798	0.99529	0.99248	0.98660	0.98251	0.97742	0.97455	0.96385

Enter table with $SZ/SB = \sigma_\mu/\sigma_x$ and $A/S = \{R_T/[\sigma_x(1 + \sigma_\mu^2/\sigma_x^2)^{1/2}]\}$

Eq. 20-11 is obtained by equating a lethal area of

$$2\pi c\sigma_{kx}\sigma_{ky} = 2\pi c\sigma_k^2, \quad 0<c<1 \tag{20-12}$$

where

σ_{kx} = standard deviation of lethality pattern in x-direction
σ_{ky} = standard deviation of lethality pattern in y-direction
σ_k = standard deviation when $\sigma_{kx} = \sigma_{ky}$

to the target area πR_T^2 in Eq. 13 of Ref. 4 for damage models. The best value of c, an adjustment factor, seems to be about $c = 0.9$ although $c = 1$ is about as good.

If we use the data of Example 20-1 and calculate the chance of at least one hit for $n = 7$ rounds and $c = 1$, then Eq. 20-11 gives a chance of at least one hit of $p_h(7) = 0.68$, or slightly less than the exact value of 0.70.

Another approximation due to Lind (Ref. 5) and also obtainable as a special case of a formula on page 677 of Helgert (Ref. 6), is

$$p_h(n) = \sum_{i=1}^{n} (-1)^{i+1}\binom{n}{i}\left[\frac{R_T^2}{\sigma_x^2\left(2+\dfrac{R_T^2}{\sigma_x^2}\right)}\right]^{i-1}\left[\frac{\dfrac{R_T^2}{\sigma_x^2}}{2+\dfrac{R_T^2}{\sigma_x^2}+i\left(\dfrac{2\sigma_\mu^2}{\sigma_x^2}\right)}\right]. \tag{20-13}$$

It should be noted that this equation of Lind may be duplicated exactly by using the target damage model of Grubbs (Eq. 20, Ref. 4) and putting $\sigma_k = R_T/\sqrt{2}$, and $\sigma_u = 0$. (For $n = 1$, use Eq. 20-1 or 20-2; but not Eq. 20-11 or 20-13.)

Mr. Ralph Shear of the BRL has prepared a program for the HP65 pocket calculator to solve Eqs. 20-11 and 20-13. If we again use the data of Example 20-1 and $n = 7$, then Eq. 20-13 gives $p_h(n) = 0.756$, which is somewhat high. We emphasize that these are approximations, and have not been checked throughout all ranges of practical interest. Also, the alternating series of Eqs. 20-11 and 20-13 involving the binomial are notoriously bad for computation since the calculations for large n turn out to be a "battle of big alternating binomial coefficients". For this reason, Breaux and Mohler (Ref. 7) have recommended a transformation using Jacobi polynomials for such series.

A point of some interest associated with Eqs. 20-11 and 20-13 is that each R_T^2 may be replaced by A/π, where A is the area of a "fairly circular or square" target, thus giving the chance of at least one hit on a more or less "regular" target area.

The chance of a hit for a single round (i.e., $n = 1$) is given by p_h of Eq. 20-4, and if there were reaiming between rounds, then Eq. 20-5 could be used. In this connection, we remark that the salvo hit probability is between these two values, i.e.,

$$p_h < p_h(n) \text{ for salvo} \leq 1 - (1 - p_h)^n\,. \tag{20-14}$$

In fact, p_h(single round) = 0.3607, and $1 - (1 - p_h)^7 = 0.956$, giving a wide interval with 0.70 well between, as might be expected.

Eq. 20-11 may be generalized to give the chance of at least one hit for the case $\sigma_x \neq \sigma_y$, $\sigma_\mu \neq \sigma_\nu$, and an elliptically shaped target with semiaxis in the x-direction of α, semiaxis in the y-direction of β, and

with the target center at the origin. In this general case, the chance of at least one hit (for $c = 1$) is very nearly

$$P = p_n(h) = \sum_{i=1}^{n} \binom{n}{i} (-1)^{i+1}(1/i) \left[\frac{(\alpha/\sigma_x)(\beta/\sigma_y)}{\sqrt{(2 + \alpha^2/\sigma_x^2)(2 + \beta^2/\sigma_y^2)}}\right]^{i-1} (1 - e^{-a})^{1/2}(1 - e^{-b})^{1/2} \tag{20-15}$$

where

$$a = (\alpha/\sigma_x)^2/[(2 + \alpha^2/\sigma_x^2)/i + 2\sigma_\mu^2/\sigma_x^2] \tag{20-16}$$

$$b = (\beta/\sigma_y)^2/[(2 + \beta^2/\sigma_y^2)/i + 2\sigma_\nu^2/\sigma_y^2] \; . \tag{20-17}$$

Furthermore, if one needs to consider the case of a rectangular target centered at the origin of side $2c$ in the x-direction and side $2d$ in the y-direction, then we may find the chance of one or more hits on the rectangle by equating the area of the ellipse in Eq. 20-15 to the area of the rectangle, i.e.,

$$\pi\alpha\beta = (2c)(2d) \tag{20-18}$$

and simply replace α by $2c/\sqrt{\pi}$ and β by $2d/\sqrt{\pi}$ in Eq. 20-15. Thus, we have approximations for salvo hit probabilities, which cover circular, square, elliptical, or rectangular targets.

We see clearly from the previous discussion that the chance of at least one hit based on the often used formula of the type $1 - (1 - p)^n$ gives too optimistic a performance for the weapon, unless there is reaiming for each and every round to try and bring the C of I onto the target center. Ordinarily, however, this cannot be done—since rate of fire may be important for suitable kill rates, or because automatic weapons wander off the target in a rather systematic manner, or an artillery battery or battalion may fire their cannons simultaneously for surprise effect, etc. Also, for rapid fire weapons or salvo firing, the aiming errors may add up to move the C of I in a systematic pattern so that consecutive aim points are correlated. This condition is often and loosely referred to as "correlated rounds", although such correlation exists because of a rather systematic movement of the aim points due to usage of weapons in combat engagements. In our account of salvo firing—i.e., Eqs. 20-9, -11, -12, and -13)—we considered random, nonsystematic movements of aim points and have not included the possible existence of autocorrelation effects between aim points; however, we must do so for some applications. First, however, it will be of interest to discuss a very simple equation for a chain-like series of shots.

20-4 A SIMPLIFIED EQUATION FOR A CHAIN-LIKE SERIES OF SHOTS

Since models for calculating the chance of at least one hit vary with weapon usage and also can become very complex, there should be some interest in simplified models or equations. Indeed, this is precisely what Zahle (Ref. 8) apparently had in mind in proposing a simplified hit probability model for an aim-wandering, chain-like series of shots. It is not expected that such a condition would arise from very high rate of fire weapons, for then the case of salvo firing might be applicable. Rather, the

weapon may be firing relatively slowly as the gunner tries to keep his weapon aimed at the target center during the firing of several or many rounds, as for example in Fig. 20-1. Zahle's suggested equation for the chance of at least one hit in n rounds fired is

$$p_h(n) = 1 - (1 - fp_h)^n \tag{20-19}$$

where for the fraction f, we must have $0<f<1$.

The idea here is to assure that if there is a trend in the shots, which fall chain-like about the target, and if the distance between shots is smaller than the maximum width of the target, then perhaps a simple equation of the form of Eq. 20-10 with an appropriate f might apply. Of course, the problem of selecting the fraction f so that Eq. 20-19 would apply with generality becomes critical. Zahle's recommendation is to determine f from

$$f = (\text{total firing area covered})/(nA) \tag{20-20}$$

where

n = number of shots

A = target area.

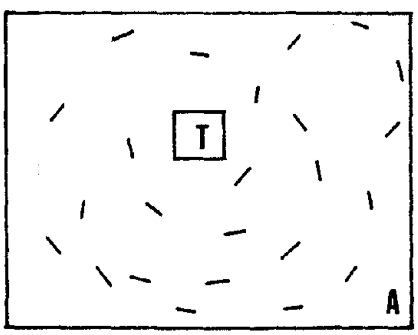
Alternatively, for shot-pictures from aiming camera studies, then f is estimated from

$$f = (\text{total area covered on all pictures})/(nA)\,. \tag{20-21}$$

Although one can see that the demoninator of Eq. 20-20 or Eq. 20-21 is easily determined, the numerator may present some difficulty where concrete data may not exist for the weapon system under study.

As a hunch, or an alternative to Zahle's suggestion, one might consider estimating f by requiring that the numerator of Eqs. 20-20 and 20-21 be equal to the expected area that will be covered by the shots based on the delivery errors, i.e.,

$$\left.\begin{aligned} f &\approx [2\pi(\sigma_x^2 + \sigma_\mu^2)]/(nA) \\ &= [2(\sigma_x^2 + \sigma_\mu^2)]/(nR_T^2) \end{aligned}\right\} \tag{20-22}$$



T = area of target

A = hitting area

Figure 20-1. Typical Pattern for a Chain-Like Series of Shots.

for a circular target. Using the data of Example 20-1, we find that f from Eq. 20-22 is

$$f = 2[(100)^2 + (210)^2]/[(7)(220)^2] = 0.319$$

which when substituted into Eq. 20-19 gives—with $p_h = 0.3607$—a $p_h(n) = 0.58$. This value, however, is rather low for our salvo firing problem; hence such an approach shows little promise.

Another possible suggestion is to take f as the ratio of the round-to-round standard error to the total aiming and ballistic error, i.e.,

$$f = \sigma_x/(\sigma_x^2 + \sigma_\mu^2)^{1/2} . \tag{20-23}$$

If this were done, then the data of Example 20-1 give $f = 0.430$, and then Eq. 20-19 gives a chance of one or more hits equal to 0.693, which shows excellent agreement indeed. However, the generality of application of such a simple equation remains in doubt until a very extensive study uncovers its real worth.

Moreover, one is tempted also to weight the extremes in Eq. 20-14, i.e., the single shot chance p_h from Eq. 20-4 and the n round (optimistic) hit chance of Eq. 20-5. For example, perhaps a sensible method of weighting would be

$$\left.\begin{aligned} p_h(n) &\approx [\sigma_x^2/(\sigma_x^2 + \sigma_\mu^2)]p_h + [\sigma_\mu^2/(\sigma_x^2 + \sigma_\mu^2)][1 - (1 - p_h)^n] \\ &= 1 - [\sigma_x^2/(\sigma_x^2 + \sigma_\mu^2)](1 - p_h) - [(\sigma_\mu^2/(\sigma_x^2 + \sigma_\mu^2)](1 - p_h)^n \\ &= 1 - (1 - p_n)^n - [\sigma_x^2/(\sigma_x^2 + \sigma_\mu^2)][(1 - p_n) - (1 - p_h)^n] \end{aligned}\right\} \tag{20-24}$$

where we have exhibited equivalent forms. Thus, again using the data from Example 20-1, we have

$$p_h = 0.3607$$

$$\sigma_x^2/(\sigma_x^2 + \sigma_\mu^2) = 0.185$$

$$\sigma_\mu^2/(\sigma_x^2 + \sigma_\mu^2) = 0.815$$

and then the last equation of Eq. 20-24 gives

$$p_h(7) = 0.846$$

which is a poor approximation indeed and hence unworthy of any further consideration.

On the other hand, had we used $f_1 = \sigma_x/\sqrt{\sigma_x^2 + \sigma_\mu^2} = 0.430$ instead of $\sigma_x^2/(\sigma_x^2 + \sigma_\mu^2)$ in the last equation of Eq. 20-24, i.e.,

$$p_h(n) \approx 1 - (1 - p_h)^n - [\sigma_x/\sqrt{\sigma_x^2 + \sigma_\mu^2}][(1 - p_h) - (1 - p_h)^n] \tag{20-25}$$

then using Eq. 20-25 gives $p_h(n) = 0.70$, or the exact value, although justification is not clear for such a value.

In summary, therefore, an f calculated from Eq. 20-23, i.e., $f_1 = \sigma_x/\sqrt{\sigma_x^2 + \sigma_\mu^2}$, seems to give good results for a simplified equation, although extended checks over all ranges of interest would be in order to develop perhaps a useful approximation.

It is not difficult to estimate σ_x, σ_y, σ_μ, and σ_ν. Experimentally, one may fire salvos of rounds on a suitable number of occasions and use a one-way classification in the analysis of variance to estimate them. Alternatively, aiming-camera records may be used, during some trials, to estimate σ_μ and σ_ν.

20-5 AUTOCORRELATED AIM POINTS

The case involving calculation of the chance of at least one hit when the aim points (or movement of the C of I) follow an autocorrelated process is of special interest, particularly for rapid fire weapons in the air defense role, but also for other applications. Modern air defense weapons employ radar tracking and gun direction aiming to improve the engagement kill probability, and in fact involve a continuous time correlated random process which is sampled at the instants of firing. For such weapon firing applications, one must consider the ordinary round-to-round ballistic or inherent dispersion, σ_x and σ_y; the distribution of any C of I biases, or systematic errors, expressed previously in terms of σ_μ and σ_ν; and also a third component of variance describing the time varying or autocorrelated aim points since the system aim error may wander along the path of the target in a somewhat systematic manner as the engagement time increases. Depending on system parameters and weapon employment in an engagement, then the gun slewing effectiveness and other characteristics of the overall aiming process may be such that serial type correlation among rounds may have a decided effect on system delivery accuracy. Thus, as contrasted to the previous case where the aim error may be randomly (and normally) distributed due to inaccuracies in determining the correct gun elevation and azimuth for existing combat conditions, the time dependent error process or autocorrelation effect often will overcome and dominate the variable biases or systematic errors that would have occurred otherwise. Hence, for this case we require the use of a model describing the autocorrelation effect to evaluate properly hit chances for serially fired rounds. The inherent or random round-to-round sigmas, σ_x and σ_y, will not be affected by this time varying or aim wandering process, although the usual aiming errors described by σ_μ and σ_ν clearly will be changed, and the new total aiming error will be reevaluated in terms of the serial correlation encountered. As we will see, it is still convenient to consider the problem in terms of two components of variance—i.e., the new overall aiming error variance resulting from the autocorrelation process, but including any variable biases that may exist, and the ballistic errors as before.

The problem we refer to here is a rather difficult one that has been under study for many years. Therefore, the curious reader will be interested in some of the basic or key papers on the subject. In 1946, Cunningham and Hynd (Ref. 9) undertook a comprehensive study of random processes in air warfare in a basic paper on the subject, and in 1951 and 1953 Fraser (Refs. 10 and 11) approached the problem from the multivariate statistical analysis standpoint. As is well known for such statistical undertakings, one invariably is dealing with truncated normal integrals for finite or small targets and, therefore, simple analytically useful approximations thereto for square targets, for example, are most difficult or impossible to attain. It is for this reason that the von Neumann-Carlton diffuse target approximation (par. 14-8) had to be used for most of the multiple round hit probability studies.

Let us recall that in Chapter 14 we used μ and ν for the coordinates of the (unknown) C of I of the rounds, and these are the quantities that would be affected by the aim errors resulting from rounds fired serially in time. Thus, we may ignore ballistic errors for the moment and consider aiming error characteristics which would move the C of I, with coordinates (μ,ν), in some fashion. For the time

varying sources of gun pointing errors, then we can say that μ and ν may become functions primarily of time, i.e.,

$$\mu_i = \mu(t_i), \ \nu_i = \nu(t_i) \ . \tag{20-26}$$

In fact, it would be informative to indicate that movement of the C of I of the rounds may result from a total aiming error consisting of two components—one for a time varying aim wander process and the other for a variable bias or systematic error of the C of I from round to round, or salvo to salvo, if it exists also. Thus, the total error causing the miss distance X_i in the x-direction for the ith shot might be described as

$$X_i = \mu(t_i) + \mu(b_i) + x_i \ . \tag{20-27}$$

Here, x_i is a round to round ballistic deviation with variance σ_x^2; $\mu(b_i)$ a deviation due to random, nonsystematic changes in aim point which we described heretofore as σ_μ^2, but will now label as $\sigma_{\mu b}^2$ to avoid confusion with the pure time component of aim error. Then finally, we have the component $\mu(t_i)$, whose variance might be referred to as $\sigma_{\mu t}^2$, but involves an autocorrelation or serial covariance with time. Hence, the expected values $E[\mu(b_i)\bullet\mu(b_j)] = E(x_i\bullet x_j) = 0$ for $i \neq j$, but $E[\mu(t_i)\bullet\mu(t_j)] \neq 0$ for $i \neq j$, indicating a dependence relation for the time-varying component of error. Otherwise components of Eq. 20-27 can be assumed to be independent, have expectations of zero, and variances as indicated, although the $\mu(t_i)$ are such that the serial or autocovariance of lag* k is nonzero and given by

$$\sigma_{k\mu t}^2 = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \mu_i \mu_{i+k} \prod_{i=1}^{n} f(\mu_i) d\mu_1 d\mu_2 \cdots d\mu_n \tag{20-28}$$

where $\mu_i = \mu(t_i)$ and $k = 1, 2, \ldots, n$. If $k = 0$, $\sigma_{k\mu t}^2 = \sigma_{\mu t}^2$. Similar relations hold for the ν component.

Thus, the normalized autocorrelation or serial correlation coefficients of lag k for the aim error component dependent on time may be defined as

$$\rho_{k\mu t} = \sigma_{k\mu t}^2/(\sigma_{\mu t}\sigma_{\mu t+k}) = \sigma_{k\mu t}^2/\sigma_{\mu t}^2 \ . \tag{20-29}$$

Hence, $\rho_1 = \rho_{1\mu t}$ describes the serial or autocorrelation between adjacent aim points in the μ-direction, and $\rho_1\sigma_{\mu t}^2$ would represent the effective variability or covariance arising therefrom. ρ_2 would describe the autocorrelation between every other aim point, and $\rho_2\sigma_{\mu t}^2$ the corresponding covariance term for variability, etc. Whereas the autocorrelation between adjacent aim points may be high and even approach unity, then that (the autocorrelation) for k sufficiently large, or say every 5th, 10th, etc., aim point, then ρ_k would be expected to approach zero. The dependence of the autocorrelation ρ_k on time (or k) may be as indicated on Fig. 20-2. It will not ordinarily be possible to estimate all of the ρ_k—$k = 1, 2, \ldots$—and use them in hit probability models for multiple rounds. In fact, one usually can estimate only ρ_1 or ρ_2 with good accuracy, depending on available data, and therefore it will be convenient to use some average value ρ_a of the ρ_k in hit probability models. Moreover, we usually will have to employ the assumption $\rho_{k\mu t} = \rho_{k\nu t} = \rho_k$, for the two directions and then use an average $\rho_k = \rho_a$ for both. We should also point out that well designed experiments and analysis of variance techniques are

*lag ≡ interval between recorded observations of aim points (when $k = 1$, every aim point recorded; $k = 2$ every other aim point recorded; etc.)

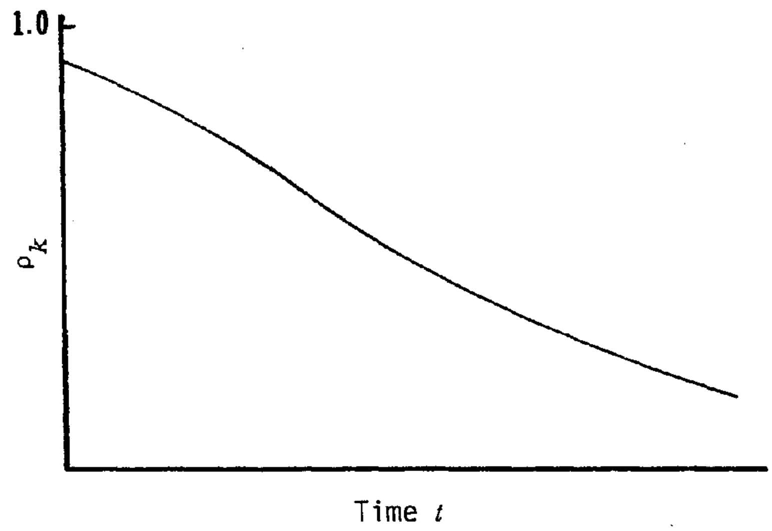


Figure 20-2. Typical Pattern of the Autocorrelation Coefficient ρ vs Time

mandatory in estimating the components of variance that enter into hit probability models for the time varying, random processes as described here.

With this somewhat scanty background on autoregressive firing schemes, we may indicate some of the accomplishments to date in calculating the chance of at least one hit. Lind (Ref. 12) investigated multiple round hit probabilities for automatic antiaircraft guns in 1964 and suggested use of the following model for calculating the chance of hitting (and hence killing) a circular target of radius R_T for n rapidly-fired rounds. (In air defense studies, the circular area ordinarily considered in evaluations is the vulnerable area A_v, of the aircraft or missile, or a component thereof.)

Lind's recommended equation for the chance of at least one hit P_L is

$$P_L = \sum_{i=1}^{n} (-1)^{i+1} \left[\frac{R_T^2}{\sigma_X^2 \left(2 + \frac{R_T^2}{\sigma_X^2} \right)} \right]^i \left[\sum_{j=1}^{\binom{n}{i}} [D_i(j)]^{-1} \right] \tag{20-30}$$

where the total variance $\sigma_X^2 = \sigma_{\mu t}^2 + \sigma_{\mu b}^2 + \sigma_x^2$ and $[D_i(j)]$ is the ith order symmetrical correlation and single shot type variance ratio matrix given by

$$D_i(j) = \begin{bmatrix} 1 & q\rho_1 & q\rho_2 & \cdots & q\rho_{i-1} \\ q\rho_1 & 1 & q\rho_1 & \cdots & q\rho_{i-2} \\ q\rho_2 & q\rho_1 & 1 & \cdots & q\rho_{i-3} \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ q\rho_{i-1} & q\rho_{i-2} & q\rho_{i-3} & \cdots & 1 \end{bmatrix} \tag{20-31}$$

with

$$q = 2\sigma_X^2/(2\sigma_X^2 + R_T^2) = 2/(2 + R_T^2/\sigma_X^2)\,. \quad (20\text{-}32)$$

Unless one is evaluating a weapon system for which all of the autocorrelations ρ_i are known, then as we have already indicated an average value for all i, i.e., $\rho_i = \rho_a$, will have to be employed—and that value itself will often be difficult to estimate accurately. In such case, Eq. 20-30 reduces to the simplified closed form:

$$P_L = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \frac{\left[\dfrac{R_T^2}{\left(2 + \dfrac{R_T}{\sigma_X^2}\right)}\right]^2}{[1 + (i-1)q\rho_a][1 - q\rho_a]^{i-1}}\,. \quad (20\text{-}33)$$

Helgert (Ref. 6) derives a very similar equation for the chance of at least one hit P_H, it being

$$P_H = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} [\pi R_T^2/(\pi R_T^2 + 2\pi\sigma_x^2 + 2\pi\sigma_{\mu t}^2 - 2\pi\rho_a\sigma_{\mu t}^2)]^{i-1}$$

$$\times\ [\pi R_T^2/(\pi R_T^2 + 2\pi\sigma_x^2 + 2\pi\sigma_{\mu t}^2 - 2\pi\rho_a\sigma_{\mu t}^2 + 2\pi i\sigma_{\mu b}^2 + 2\pi i\rho_a\sigma_{\mu t}^2)]\,. \quad (20\text{-}34)$$

For the case of very high average autocorrelation, or $\rho_a = 1$, then Eq. 20-34 reduces to

$$P_H = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \{R_T^2/[\sigma_x^2(2 + R_T^2/\sigma_x^2)]\}^{i-1}$$

$$\times\ R_T^2/\{\sigma_x^2[2 + (R_T^2/\sigma_x^2) + 2i(\sigma_{\mu t}^2/\sigma_x^2) + 2i(\sigma_{\mu b}^2/\sigma_x^2)]\}\,. \quad (20\text{-}35)$$

We emphasize that both the Lind formulas, Eqs. 20-30 and 20-33, and also those of Helgert, Eqs. 20-34 and 20-35, are approximations, and in particular involve the von Neumann-Carlton diffuse target approximation which for many situations would be quite adequate but otherwise would not necessarily have general or universal accuracy for all applications.

A rather comprehensive study of the chance of at least one hit for stochastic error processes and autocorrelated aim error for air defense gunnery is that of Brândli (Ref. 3), who reviews the work of Lind (Ref. 12) and that of Helgert (Ref. 6) also. Borowsky (Ref. 13) summarizes many of the key results of Brândli and Lind, and in addition indicates details of generating miss distances by Monte Carlo methods for time varying, random, and bias or systematic errors. Brândli in his Eqs. 5.50 and 5.51 recommends what is claimed to be an "exact" or accurate model, "freed of the almost unavoidable Gaussian distribution", for hit probability calculations, but does not develop results in much practical detail. Rather, he turns to several methods of approximation, and in particular recommends an approximation attributed to "'Stadelmann/Isenring". The chance of at least one hit for the Stadelmann/Isenring method is not in closed form, but Brândli has provided some tables. The chance of a

hit is given in terms of probable errors (1.349 = 2 × 0.6745), which we express in terms of our standard deviations:

$$P = 0.91/[\pi(SZ/SB)^2] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle 1 - \{1 - 0.91/\pi) \int_{r-a}^{r+a}\int_{s-a}^{s+a} \exp[-0.91(u^2 + v^2)]dudv\}^n\rangle$$

$$\times \exp[-0.91(r^2 + s^2)/(SZ/SB)^2]drds \qquad (20\text{-}36)$$

where

$$a = (A/2S)[1 + (SZ/SB)^2]^{1/2} \qquad (20\text{-}37)$$

A = length of an assumed square target

$$SZ = 1.349(\rho_a\sigma_{\mu t}^2 + \sigma_{\mu b}^2)^{1/2} \qquad (20\text{-}38)$$

$$SB = 1.349[(1 - \rho_a)\sigma_{\mu t}^2 + \sigma_x^2]^{1/2} \qquad (20\text{-}39)$$

$$S = (SZ^2 + SB^2]^{1/2} \qquad (20\text{-}40)$$

and circular ballistic and aim errors are assumed. It is easy to convert Eq. 20-36 to an engagement kill probability simply by replacing the coefficient $0.91/\pi$ for the inner double integral by

$$0.91\ p(k|h)/\pi$$

where of course Brândli's Z = our $p(k|h)$ is the conditional chance that a hit on the target results in a kill. Brândli's tables cover the following ranges of the parameters:

n = 5, 10(10)50(25)100
Z = $p(k|h)$ = 0.2(0.2)1.0
SZ/SB = 0.0(0.5)2.0(1)5,7.5,10,15,20,∞
A/S = 0.025(0.025)0.200(0.050)0.500(0.100)1.500(0.250)3.500 .

The quantity ρ_a, or average autocorrelation coefficient, is approximated by

$$\rho_a = (1/n)\langle 1 + [2\rho_1/(1 - \rho_1)]\{1 - (1 - \rho_1^n)/[n(1 - \rho_1)]\}\rangle \qquad (20\text{-}41)$$

for a Markovian process (where each aim error depends on the immediately preceding aim error); otherwise ρ_a is given by

$$\rho_a = (1/n^2)[n + 2\sum_{k=1}^{n-1}(n - k)\rho_k] \qquad (20\text{-}42)$$

if all the ρ_k are estimable. If all the ρ_k in Eq. 20-42 are taken equal to ρ, then

$$\rho_a = (1/n) + (n - 1)^2\rho/n^2 . \qquad (20\text{-}43)$$

Summarizing, we see that Brândli's tables based on Eq. 20-36 cover some important practical ranges of interest, and the entries may be used as reference values to check various approximations to multiple round hit probabilities.

Of course, Eq. 20-36 or the methods of this paragraph (par. 20-5) may be used also when $\rho = 0$, and/or $\sigma_{\mu t} = 0$, i.e., when the more general autocorrelation case reduces to the assumptions of par. 20-3.

As a further point of some interest, Helgert (Ref. 6) shows that if the serially fired rounds are Markovian dependent, i.e., the chance result for each round can be expressed in terms of the state of the immediately preceding round only, then transition probabilities from one state to another one may be simply expressed. In fact, the Markov chain becomes stationary in the sense of a constant single-shot hit probability. If the chance of hit for the ith round, given a hit for the $(i-1)$st round, is p_1, and the chance that the ith round is a hit, given that the $(i-1)$st round misses, is p_0, then the k-step transition probability is simply

$$p(h_i|h_{i-1}) = p + (1-p)(p_1 - p_0)^k \tag{20-44}$$

where

$p(h_i|h_{i-1})$ = chance of hit on ith round if the $(i-1)$st round is a hit, $(k = 1, 2, \ldots, i-1)$

$$p = p_0/(1 - p_1 + p_0) \tag{20-45}$$

and for

$p(h_i|M_{i-1})$ = chance of hit on ith round if the $(i-1)$st round is a miss

we have

$$|p(h_i|h_{i-1}) - p(h_i|M_{i-1})| < 1\,. \tag{20-46}$$

Moreover, for this case the probability of hit distribution P for the rounds $i_1, i_2, \ldots,$ and i_k becomes

$$P = p \prod_{j=2}^{k} [p + (1-p)(p_1 - p_0)^{i_j - i_{j-1}}]\,. \tag{20-47}$$

We give an example based on 40 rounds and hence some rather involved computations, but we nevertheless illustrate some of the principles just covered.

Example 20-2:

During an engagement against an enemy aircraft, a friendly air defense gun system can shoot some 40 rounds. Assume that at the crossing range, the ballistic dispersion amounts to $\sigma_x = 3$ m, the random systematic or bias error is $\sigma_{\mu b} = 5$ m, analysis of the time varying aim points otherwise indicate $\sigma_{\mu t} = 6$ m, and the autocorrelation coefficient is estimated as $\rho_a = 0.8$. Thus for a vulnerable area of 6 m^2 for the target, what is the engagement kill probability?

We have that:

$A = \sqrt{6} = 2.449$

SZ = 9.895 from Eq. 20-38

SB = 5.430 from Eq. 20-39

S = 11.287 from Eq. 20-40

$SZ/SB = 1.822$
$A/S = 0.217$
$n = \mathcal{N} = 40$
$p(k|h) = \mathcal{Z} = 1$.

Hence, from Brândli's tables, we find $P \approx 0.32$. Further use of the tables indicates that 100 rounds would have to be fired during the engagement to obtain a kill probability of 0.48. The basic input data here could also be used in Eq. 20-33 which gives 0.31, and Eq. 20-34 which gives 0.32 for 40 rounds. Hence, good agreement is obtained.

As a calculation of some interest, suppose one uses the total variance σ_X^2 which is equal to $3^2 + 5^2 + 6^2 = 70$, ignores the autocorrelation among aim points, and assumes reaiming between rounds. Then from a formula similar to Eq. 20-4, the single shot hit probability is approximately,

$$p_h \approx 0.0135$$

and the chance of at least one hit in 40 rounds is

$$p_h(40) = 1 - (1 - 0.0135)^{40} = 0.420$$

and that for 100 rounds is

$$p_h(100) = 1 - (1 - 0.0135)^{100} = 0.743 .$$

Hence, the loss in hit probability by not being able to reaim after each round, even for the total dispersion, is $0.42 - 0.32 = 0.10$ for 40 rounds, and $0.74 - 0.48 = 0.26$ for the 100 rounds. Put another way, 29 reaimed rounds would have given the chance of at least one hit equal to 0.32, and 48 reaimed rounds the 0.48 hit chance. These calculations, therefore, give some idea of the loss in effectiveness due to time varying aim problems. (Had there been only the ballistic dispersion and the C of I placed on the target center for each round, then the single shot hit probability would have been 0.10 and the chance of at least one hit in 40 would be 1.00)

For attempts at some simplification of the autocorrelated aim error process of this paragraph, one might try to develop some simplified models similar to the ones commented on in par. 20-4. For example, for a circular target of radius R_T, and the assumption of equal delivery errors in the two directions, the chance of at least one hit in n rounds fired in an autocorrelated aim error process may be approximated by (Ref. 13, Eq. 41)

$$P \approx 1 - \exp[- nR_T^2/(2\sigma_X^2)] - f\{\exp[- R_T^2/(2\sigma_X^2)] - \exp[- nR_T^2/(2\sigma_X^2)]\} \tag{20-48}$$

where

$$\sigma_X^2 = \sigma_{\mu t}^2 + \sigma_{\mu b}^2 + \sigma_x^2 \tag{20-49}$$

and

$$f = (\rho_a \sigma_{\mu t}^2 + \sigma_{\mu b}^2)/\sigma_X^2 \tag{20-50}$$

However, for the data of Example 20-2 and $n = 40$, $R_T = 1.38$, $\sigma_X^2 = 70$, $f = 53.8/70 = 0.769$ and $P \approx 0.11$, which is clearly of no value as a suitable approximation. Again, we remark that had we taken f to involve the ballistic σ_x in the numerator, i.e.,

$$f = \sigma_x/\sigma_X = 0.359$$

and substituted this value of f into Eq. 20-48, then $P \approx 0.275$, which is low by about 0.08. However, we must point out that there seems to be little logic to recommend such a simplified equation as a useful approximation.

From our discussion, the reader sees that the evaluation of multiple round hit probabilities can become complex, depending on the particular application. Often some particular assumptions and quick "ball park" approximations also may be required; hence some further study of the general problem is still called for.

One of the prime areas of use of the autocorrelated aim wander process would, of course, be that of air defense studies using rapidly firing guns. In this connection, the weapon systems analyst would be engaged primarily in calculations of the chance of at least one hit in n rounds fired during an engagement, and not in target damage models discussed in pars. 20-7 and 20-8. The reason for this is that the approach for air defense evaluations involves estimation of the vulnerable areas of aerial targets for expected end-engagement geometry of projectile and aircraft. Hence, it is natural to use the vulnerable area of the aerial target instead of the presented area. Thus, instead of the target area defined above as $A_T = \pi R_T^2$, one uses the vulnerable area of the target, i.e., $A_v = \pi R_v^2$, or perhaps an elliptically shaped vulnerable area as required. Hence, it certainly seems reasonable to use hit probability models such as Eqs. 20-36, 20-33, and 20-34 with R_T replaced by R_v. Nevertheless, at the time of preparation of this handbook, the autocorrelated aim wander process apparently is not often used by some system analysts. Rather, approximations are used which take $\sigma_{\mu t} = 0$, or include such variability in the variable bias $\sigma_{\mu b}$.

20-6 COMMENT ON SMALL ARMS EVALUATIONS

For rigorous evaluations, it becomes of considerable importance to model weapon performance on the basis of its actual operation in various firing modes. For example, for bursts of fire from a machine gun, the first (aimed) round may usually strike somewhere near the aim point, but succeeding rounds of the burst jump off the aim point into a pattern of rounds by themselves, and consequently the aiming error must be corrected—see Fallin (Ref. 14) and Simmons et al. (Ref. 15).

In small arms evaluation, it has not been the practice to use vulnerable areas of personnel type targets. Rather, wound ballistic $p(k|h)$'s are employed (Chapter 15).

It becomes very desirable also to evaluate weapons in their expected combat role, or at least conduct realistic simulations in order to obtain an analysis approaching the overall performance of the weapon under study because computations of hit probability alone may leave much to be desired. In a study of some mathematical models and computer programs for small arms analyses, Simmons et al. (Ref. 15) developed some standard methods for evaluating small arms in various roles. They cover the necessary interior and exterior ballistic input models for evaluation, and then discuss target engagement effectiveness in terms of the individual soldier model, the squad model, the bunker model, the hemisphere model, the heavy machine gun emplacement model, the hidden point target in area model, and the brush penetration model in order to cover properly the details of weapon usage in the field. We cannot go into all the details of such evaluations in this handbook, however.

20-7 TARGET COVERAGE AND ITS RELATION TO DAMAGE

20-7.1 INTRODUCTION

We will distinguish between target coverage and target damage models in our account here—although it will be seen that the expected fraction of target kills or incapacitations may be practically equivalent—and either method of assessing damage may be used with perhaps suitable accuracy. Target coverage has to do with the part of the target actually covered once or more (or not covered at all) by, say, n areas dropped at random onto the general target area. Thus, our interest here centers around the fraction of the target covered at least once. Of course, the manner in which the n areas are dropped at random onto the target area will affect the part and also the fraction of the target which is covered at least once. Indeed, the n areas may be dropped so that the distribution of rounds is uniform, for example, or normal, etc. Also, the location of the point of aim with respect to the target center will affect the fraction of coverage of the target, as well as will the dispersion of dropped areas within the overall pattern of droppings.

The basic problem under study here is that of determining the measure of two-dimensional random sets which arose from analytical evaluations of bombing problems during World War II. Initially, some of the original contributions were made by Robbins (Refs. 16 and 17), Bronowski and Neyman (Ref. 18), and Garwood (Ref. 19). In practical terms, it becomes desirable to know the effectiveness of n bombs dropped at random, which occurs due to inherent dispersion and aiming errors in attacking a target area. That is to say, we want to estimate the fractional area of the target covered by the lethal effect of n bombs when their effective areas may overlap.

Suppose we have a target whose area we will designate by A_T and we drop n "discs" or "cookie cutters", each of area A_L, onto the target, and further we assume a bivariate *uniform* delivery distribution of dropped areas. (We will use normal delivery distributions in par. 20-7.3.) The approximate chance of the target being covered, i.e., the expected fraction of the target covered for a single dropped area A_L, will clearly be A_L/A_T (except for edge effects). Generally, the effective or lethal area per round A_L is many times smaller than the target area A_T. Moreover, the fraction of the target not covered by a single drop will be $1 - A_L/A_T$. Thus, the fraction m of the target not covered in n rounds or bombs dropped will be given very nearly by

$$m = (1 - A_L/A_T)^n . \tag{20-51}$$

Here m designates the mean fraction not covered or, put alternatively, m is the chance of the target not being covered in dropping n discs onto the target area. (We ignore edge effects or assume that they average out.) In case each of the dropped areas A_L is small compared to the target area, then the expected fraction of the target not covered is approximately

$$m \approx \exp(-\, nA_L/A_T) . \tag{20-52}$$

Further, if for each dropped area A_L, we have the condition that the target elements will be damaged, killed, or incapacitated if they happen to lie within any lethal area pattern A_L, then the expected fraction $\bar{f}(n)$ of the target that will be damaged by n rounds is given by

$$\bar{f}(n) = 1 - m \approx 1 - (1 - A_L/A_T)^n \approx 1 - \exp(-\, nA_L/A_T) . \tag{20-53}$$

20-7.2 RELATION OF COOKIE CUTTERS TO DAMAGE

Before we proceed with further discussions of target coverage, we should indicate its relation to damage and just how to convert from one to the other, especially the cookie cutter concept and its relation to damage.

Consider for the moment that a target element is located at the origin and that a round falls at the point (x,y) on the ground plane, or bursts in the air at a height of H above (x,y). Then, the chance that such a target element is damaged at such a distance from the burst point may be designated by

$$p_k(x,y,H) = p_k(x,y) \tag{20-54}$$

if the height of burst is $H = 0$. Furthermore, the "lethal" area of a single round or drop may be found by integrating the kill function Eq. 20-54 over the ground area, i.e.,

$$A_L = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_k(x,y)dxdy \tag{20-55}$$

which therefore becomes one of the key parameters in evaluating target coverage or target damage. (Ordinarily, A_L is determined apart from any systems analysis and requires some experimental investigation.) Moreover, if the density of target elements is D, then the expected number $E(k)$ of target elements killed is

$$E(k) = DA_L \tag{20-56}$$

for each round.

Of course, the kill function $p_k(x,y)$ may take on any of a variety of forms, and one of the most useful or convenient forms for it to follow is a negative exponential square fall-off law. With this type of assumption, we may easily relate the damage parameters to the lethal area or the lethal radius of a round. In fact, we have immediately that

$$\begin{aligned} A_L = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_k(x,y)dxdy &= c\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp[-x^2/(2\sigma_{kx}^2) - y^2/(2\sigma_{ky}^2)]dxdy \\ &= 2\pi c\sigma_{kx}\,\sigma_{ky} \end{aligned} \tag{20-57}$$

where $0 < c \leq 1$, and σ_{kx} and σ_{ky} are measures of the dispersion in lethality in the x- and y-directions. If the kill pattern or lethal area is circular and we use the lethal radius R_L of a round, then

$$\begin{aligned} A_L &= \pi R_L^2 = 2\pi c\sigma_k^2 \\ &= 2\pi\sigma_k^2 \end{aligned} \tag{20-58}$$

for $c = 1$, and $\sigma_{kx} = \sigma_{ky} = \sigma_k$.

Thus, we have the relation

$$\sigma_k = [A_L/(2\pi)]^{1/2} = R_L/\sqrt{2} . \tag{20-59}$$

We also note for a circular target of radius R_T, then the ratio A_L/A_T in Eq. 20-53 may be replaced by

$$A_L/A_T = \pi R_L^2/(\pi R_T^2) = R_L^2/R_T^2 . \tag{20-60}$$

These considerations are often sufficient to connect or determine the relation between target coverage using the cookie cutter concept on one hand and damage functions on the other. Both approaches represent only approximations to the true chances of target kills, although they possess sufficient accuracy for practically all target damage problems of interest.

Hence, we may say that a considerable amount of work on target coverage problems, or the dropping of discs or cookie cutters on target areas, has been carried out by various investigators because coverage can be related to target damage. In this connection, a considerable number of early contributions were made by Germond (Refs. 20 through 25) when he was with the Rand Corporation. In fact, the reader may recall that Germond's circular coverage function (Ref. 25) was used to advantage in connection with single shot hit probabilities (Chapter 14).

Example 20-3:

A circular target of radius 50 m is being attacked by 36 artillery rounds, each having a lethal area of 100 m^2. What is the expected fractional coverage or damage to the target?

We will assume here that the guns are aimed so that the rounds will be distributed uniformly over the target area, and that each "lethal" area (per round) is a "cookie cutter", for which a target element is killed if it is within the lethal pattern of the round, but is otherwise unharmed.

Hence, using Eq. 20-53, we have

$n = 36$
$A_L = 100 \text{ m}^2$
$A_T = \pi(50)^2 = 7854 \text{ m}^2$

$$\bar{f}(36) \approx 1 - (1 - 100/7854)^{36} = 0.37 .$$

Thus, the expected or average fraction of the target covered, or expected percentage of casualties would be 37%.

20-7.3 TARGET COVERAGE FOR GAUSSIAN DELIVERY ERRORS

In par. 14-6, we found that for circular normal delivery errors $\sigma_x^2 = \sigma_y^2 = \sigma^2$ the chance of hitting an offset circle of radius R_T which is located at (μ,ν), so that $\mu^2 + \nu^2 = r^2$, is given by the normalized Germond circular coverage function

$$p(R_T/\sigma, r/\sigma) = (1/2\pi)\int\int \exp[-(u^2 + v^2)/2]dudv \tag{20-61}$$

where this integral is taken over the offset circular region

$$(u - \mu/\sigma)^2 + (v - \nu/\sigma)^2 \leq r^2/\sigma^2 . \tag{20-62}$$

The probability given by Eq. 20-61 is also the chance that a circular disk, or cookie cutter, of normalized radius R_T/σ will cover a point located at the normalized radial distance r/σ if the probability follows a bivariate Gaussian law with standard deviation σ.

Furthermore, the polar integral S of the circular coverage function Eq. 20-61 over the area of a circle of radius r/σ with center at the origin is given by

$$S(R_T/\sigma, r/\sigma) = \int_0^{r/\sigma} 2\pi\rho\, p(R_T/\sigma, \rho) d\rho \tag{20-63}$$

and this represents the expected area of overlap of two circles, one of radius R_T/σ centered at the origin, and the other of radius r/σ which is fired at or dropped on the plane with aim point at the origin and Gaussian delivery error equal to σ. As the normalized radius r/σ of the dropped disc becomes large, it can be seen that $S(R_T/\sigma, r/\sigma)$ approaches the normalized target area $\pi R_T^2/\sigma^2$. Hence, if we now put $r = R_L$, the lethal radius of a cookie cutter delivered according to the Gaussian distribution, then the expected *fraction* of the circular target which will be covered by the one round is simply

$$\bar{f}(1) = S(R_T/\sigma, R_L/\sigma)/(\pi R_T^2/\sigma^2)\,. \tag{20-64}$$

Germond's Table 2.1, Ref. 25, gives values of Eq. 20-64 for

$$R = R_T/\sigma = 0(0.5)3.0(1)6.0$$

and

$$r = R_L/\sigma = 0.1(0.1)6.5\,.$$

Also, Germond's Table 2.2, Ref. 25, gives values of Eq. 20-64 for

$$R = R_T/\sigma = 3(1)6(2)20$$

and

$$r - R = (R_L - R_T)/\sigma = -3.2(0.1)3.5\,.$$

Example 20-4:

A circular target of radius 20 m is attacked by a weapon with circular normal delivery error of $\sigma = 10$ m and lethal radius of 15 m. What is the expected fraction of coverage or damage to the target?

We have $R = R_T/\sigma = 2$ and $r = R_L/\sigma = 1.5$. With these values, Table 2.1 of Germond (Ref. 25) gives directly without interpolation

$$\bar{f}(1) = 0.4031$$

or 40% of the target would be damaged.

For n rounds with bivariate normal delivery standard deviation σ, and all rounds (or bombs) aimed at the target center, then Jarnagin (Refs. 26 and 27) shows that the expected fraction of target coverage is

$$\bar{f}(n) = 1 - (2/R^2) \int_0^R [1 - p(a,r)]^n r dr \tag{20-65}$$

where Jarnagin's E = our $\bar{f}(n)$ the fractional coverage for n rounds, $R = R_T/\sigma$, $a = R_L/\sigma$, and $p(a,r)$ is the circular coverage function. In fact, $1 - [1 - p(a,r)]^n$ is the probability that a specified point at the radial distance r from the origin will be within the lethal area of at least one of n rounds, and the contribution to the expected damage area resulting from a polar element of area is the product of this and $rdrd\theta$. This product is integrated over the circular target and divided by πR^2, giving Eq. 20-65. Ref. 27 gives only two example pages of Jarnagin's tables, whereas Ref. 26 gives two extensive tables. One is a direct table for Jarnagin's E = our $\bar{f}(n)$ as a function of R, a, and n; and the other a very useful inverse table giving the minimum number of rounds n required to give at least a specified fractional coverage for given values R and a. A total of 57 values of R, and 68 values of a are listed for both the direct and inverse tables, as follows:

$$a = R_L/\sigma = 0.005(0.005)0.05(0.01)0.10(0.02)0.20(0.05)1.0(0.1)2(0.2)4(0.5)10$$

and

$$R = R_T/\sigma = 0.05,0.01(0.1)4(0.5)12 .$$

For the direct tables, $n = 1(1)20$, and for the inverse tables $E = \bar{f}(n) = 0.50(0.50)0.95$. These are valuable tables for the weapon systems analyst and could provide a standard for checking approximations.

Groves (Ref. 28) gives some curves of expected fractional target coverage for $n = 1(1)8$. His graphs are entered with the same parameters R_T/σ and R_L/σ.

For interested readers, Di Donato and Jarnagin (Ref. 29) give a method of computing the circular coverage function $p(R,r)$ of Eq. 20-61, which is used in Eq. 20-65. They also give valuable tables of R for $n = 1$ in Eq. 20-65 as a function of the probability $\bar{f}(1) = 0.01(0.01)0.99$ and $r = 0(0.1)5(0.2)10(2)20(5)120$.

An evaluation of frequent interest to the analyst is that of cluster type warheads, or warheads with many submissiles, which may be used to attack ground or aerial targets. Some studies on this particular subject would seem to indicate that it is desirable for the submissiles to be distributed uniformly over a disc-like area in attacking targets. On the other hand, and as usual, it may be assumed with some assurance that the center of the uniform pattern of submissiles follows a circular bivariate normal distribution with standard deviation σ. Jarnagin and Di Donato (Ref. 30) have also investigated this problem thoroughly and give some 160 graphs for determining the average coverage (or casualty or damage) area divided by the normal or Gaussian delivery variance σ^2, and the expected fraction of the circular target covered or damaged by the N submissiles.

To use the Jarnagin-Di Donato curves (Ref. 30), we need the following parameters to find expected target damage:

	Jarnagin-Di Donato Parameter	Our Parameter
	$\mathcal{N}$ =	n
	R_T =	R_T/σ
The lethal area of a bomblet	$= A_B$ =	$\pi R_L^2/\sigma^2 = A_L/\sigma^2$
Damage radius of the $\mathcal{N}$ bomblets	$= R_D$ =	R_D/σ
Expected fraction of target area covered or damaged	$= E_c$ =	$\bar{f}(n)$
	A_c/σ^2 =	(Not used)

where A_c is the average casualty area.

Example 20-5:

Given a circular target of radius 200 m which is being attacked with 20 rounds, each having a lethal radius of 30 m. What is the expected fraction of target coverage or damage if (a) each round has a delivery error of σ = 20 m, and (b) the rounds are considered uniformly distributed submissiles over a circular area of radius 150 m which were ejected from an artillery warhead having normal delivery error σ = 20 m?

For (a) we have

$n = 20$

$R_T/\sigma = 10$

$R_L/\sigma = 1.50$; and read directly from page 89 of Jarnagin's tables (Ref. 26) that

$$\bar{f}(20) = 0.101 \ .$$

For (b) we have in addition that $R_D/\sigma = 7.5$. We enter the curves of Ref. 30 with Jarnagin's R_D = 7.5, n = 20, and from page 131 read approximately that

$$E_c = \bar{f}(20) = 0.325 \ .$$

We note in connection with this example that for (b) the whole pattern of 20 submissiles spread over a circular area of radius 150 m is delivered with error σ = 20 m, whereas in (a) each round has the same delivery error of σ = 20 m.

For a more complex case, the fractional coverage or kill probability of a normally distributed "cookie-cutter" type lethal area against a random uniformly distributed element or point target within an elliptical area, even inclined to the delivery axes, has been investigated by Di Donato, Jarnagin, and Hageman (Ref. 31), and some tables of practical interest given. Some eight variables are involved, which may be reduced to seven by normalizing with respect to one of the preferred parameters, although such detailed studies obviously become rather involved. The analyst may have to break new ground, so to speak, in determining the most suitable model for his particular problem, as evidenced by such an evaluation as this one.

20-7.4 THE VARIANCE OF OVERLAP AND AN APPROXIMATE DISTRIBUTION FOR FRACTIONAL COVERAGE

In par. 20-7.1, and Eq. 20-53 in particular, we gave an account of the mean fraction of the target not covered for "cookie-cutters" dropped according to a uniform distribution over the target area. In this

connection, the variance of the fraction of overlap has been studied by Robbins (Ref. 17), Bronowski and Neyman (Ref. 18), and Garwood (Ref. 19). The work of Garwood (Ref. 19) is of particular interest to the weapon systems analyst since Garwood studied both theoretically and experimentally the mean area not covered and the variance of the area not covered for:

1. Circles dropped on circles.
2. Circles dropped on squares.
3. Squares dropped on squares (sides parallel).
4. Circles dropped on rectangles (of different size).

This interesting work of Garwood verified the use of Eqs. 20-51 and 20-53 for the fraction of target area covered by such figures, but also established that the variance v of the fraction of target area not covered—and hence the variance of the fractional target area actually covered (since they differ by only the constant one)—is very nearly or approximately

$$v \approx 2.3m(1 - m)(A_L/A_T)^{3/2} . \tag{20-66}$$

For an example on the use of Eq. 20-66, we may return to Example 20-3, where we had $n = 36$, $A_L = 100$ m^2, and $A_T = 7854$ m^2, and obtained $\bar{f}(36) = 1 - m = 0.37$. To find the variance v of the fraction of overlap or target damage, we substitute into Eq. 20-66 and get

$$v = 0.00077$$

so that the standard deviation is $\sqrt{v} = 0.028$. Hence, the expected fractional damage is 0.37 with a sigma of about 0.03, indicating the precision of the estimate.

As a computation of even further interest, we use the data of Example 20-5. Here, we made a comparison based on each of the 20 rounds having a delivery error of $\sigma = 20$ m, the result being $\bar{f}(20) = 0.101$, and a circular ring of 20 uniformly distributed rounds or submissiles the center of which was delivered with $\sigma = 20$, giving the higher $\bar{f}(20) = 0.325$. Now, if we consider the same data, but assume that 20 rounds each with lethal radius of 30 m are uniformly distributed over the target with no centers of the lethal areas falling off the target, then we may use Eq. 20-53 and obtain

$$\bar{f}(20) \approx 0.37$$

which is a still higher value. This might be expected since we recall from hit probability calculations that the simplified forms of models such as Eqs. 20-5 and 20-53 invariably give values that are too optimistic for most weapon delivery error situations. We see, therefore, that for fractional target coverage the simplified forms may be used to obtain an upper bound, although the actual delivery distribution for the rounds—or the more realistic models—will have to be applied appropriately.

Finally, and now that we have available estimates of the mean and variance of the fraction of the target covered with n uniformly distributed rounds, what can be said of the probability distribution of coverage? Clearly, this is a complex problem even for uniformly distributed (lethal) areas, although we might conjecture along the lines of Ref. 32 that an approximate probability distribution for fractional target coverage may be obtained as follows. We note that the fractional coverage, call it f (where $\bar{f}$ is the mean value of f), is a random variable, and we also have that its mean $\bar{f} = 1 - m$ is given by Eq. 20-53, and variance v by Eq. 20-66. Hence, we conjecture that the weighted random variable

$$2(1 - m)f/v \approx \chi^2[2(1 - m)^2/v] \tag{20-67}$$

is distributed approximately as chi-square. This means really that

$$(A_T/A_L)^{3/2}f/(1.15m) \approx \chi^2[(1-m)(A_T/A_L)^{3/2}/(1.15m)] \tag{20-68}$$

or the quantity on the left follows approximately the chi-square distribution with

$$\nu = (1-m)(A_T/A_L)^{3/2}/(1.15m) \tag{20-69}$$

degrees of freedom (df).

Suppose, for example, we desire the chance that the fractional target coverage (or damage) f will exceed 40%. Then we have the following equivalent probability statements:

$$\left.\begin{aligned} P &= Pr(f > 0.40) \\ &= Pr[A_T^{3/2}f/(1.15m\,A_L^{3/2}) \geq 0.4\,A_T^{3/2}/(1.15m\,A_L^{3/2})] \\ &= Pr[\chi^2(\nu) \geq 0.4\,A_T^{3/2}/(1.15m\,A_L^{3/2})]\,. \end{aligned}\right\} \tag{20-70}$$

However, since the degrees of freedom are fractional, we may use the Wilson-Hilferty transformation (Ref. 33) of chi-square to approximate normality, which means that we calculate

$$\eta = \frac{\sqrt[3]{\gamma/\bar{f}} - [1 - v/(9\bar{f}^2)]}{\sqrt{v}/(3\bar{f})} \tag{20-71}$$

where from Eq. 20-53

$$\bar{f} = 1 - m = 1 - (1 - A_L/A_T)^n$$

and from Eq. 20-66

$$v = 2.3m(1-m)(A_L/A_T)^{3/2}$$

and η is referred to a table of the standard normal probability distribution to find the chance of the coverage exceeding $\gamma = 40\%$.

Example 20-6:

Use the data of Example 20-3 to find the probability that the fractional target coverage (damage) for uniformly distributed rounds will exceed 40%.

We have already calculated $\bar{f} = 0.37$, $v = 0.00077$, and given $\gamma = 0.40$ we find from Eq. 20-71

$$\eta = 1.078$$

and from a table of the standard normal integral the chance is about 0.14 that the fractional coverage of the target will exceed 40%. (The chance that the coverage will be less than the expected value of 0.37 turns out to be about 0.51.) It should be noted that this is only for uniformly distributed discs or lethal areas, the centers of which do not spill off the circular target. Distribution properties for Gaussian delivered rounds and salvos remain to be investigated fully.

20-8 TARGET DAMAGE MODELS

Having covered the "cookie-cutter" type of damage areas, or "lethal area" discs dropped onto targets, we will discuss briefly some of the methods which involve analytical damage functions. Different Army installations have developed a variety of computer models for determining expected target damage, some of them using cell kill probabilities for the pattern of fragments from detonating projectiles. The computer model of Einbinder (Ref. 34), for example, was referenced in Chapter 15, and has been associated with investigations for the *Joint Munitions Effectiveness Manual*. We will not, however, include any detailed discussions of available computer models here since the investigation of the available models is the responsibility of the analyst who elects to use them and because some of the particular details are not always given. Instead, we will cover in this account the use of analytical models which include in a very clear manner the round-to-round delivery errors, the C of I distribution, the lethality or damage function, target sizes and shapes, and the distribution of target elements. This coverage of target damage models should be very useful to the weapon systems analyst, although it is certainly recognized that variations from our listed results here may have to be made, or in some applications the analyst may have to develop some models more appropriate to his own problems. Actually, we are discussing an area where exact computations are difficult, or almost impossible, to obtain and it is always a good idea to compare models or check one against the other.

Some of the more significant ground-breaking work with useful results on estimation of target damage was accomplished by Weiss in *Methods for Computing the Effectiveness of Area Weapons* (Ref. 35). Weiss' work is very definitely recommended for the young analyst entering the field of the analysis of weapon systems. Also, an excellent account and survey of many of the mathematical models of target coverage and missile allocation up to about 1972 is that of Eckler and Burr (Ref. 36), which has been published by the Military Operations Research Society of America. The work of Svesnikov et al. (Ref. 37), which is a collection of articles on the theory of firing, gives a very informative account of the analytical problems involved in artillery firing procedures, and hence is highly recommended reading and study. See Ref. 38 also.

In our approach here, we first assume a circular target of radius R_T with center located at the origin and a target element represented by the point (u,v). The target elements are assumed to be uniformly distributed so that the probability density of elements is given by

$$h(u,v) = \begin{cases} 1/(\pi R_T^2), & \text{for} \quad u^2 + v^2 \le R_T^2 \\ 0, \text{ otherwise} \end{cases} \tag{20-72}$$

or the target elements are assumed to be distributed normally, i.e., by

$$h(u,v) = [1/(2\pi\sigma_u\sigma_v)]\exp\{-[u^2/(2\sigma_u^2) + v^2/(2\sigma_v^2)]\} . \tag{20-73}$$

The damage function for the ith round that lands at the point (x_i,y_i) against the target element at (u,v) is taken as

$$p_k[(u - x_i)/\sigma_{kx},(v - y_i)/\sigma_{ky}] = \exp\{-[(u - x_i)^2/(2\sigma_{kx}^2) + (v - y_i)^2/(2\sigma_{ky}^2)]\} . \tag{20-74}$$

Here, we use this damage function as compared to the "cookie-cutter" damage function of par. 20-7. Also, the analyst is cautioned that lethality data (or lethal areas) will vary with the posture of enemy troops.

The noncircular normal delivery function for the aim point or C of I of the rounds—or the target location error, or both—is taken to be

$$g(\mu,\nu) = [1/(2\pi\sigma_\mu\sigma_\nu)]\exp\{- [\mu^2/(2\sigma_\mu^2) + \nu^2/(2\sigma_\nu^2)]\} \tag{20-75}$$

and the round-to-round delivery density for any round of the n-round salvo is taken as

$$f[(x_i - \mu)/\sigma_x,(y_i - \nu)/\sigma_y] = [1/(2\pi\sigma_x\sigma_y)]\exp\{-[(x_i - \mu)^2/(2\sigma_x^2) + (y_i - \nu)^2/(2_y^2)]\}\,. \tag{20-76}$$

Fig. 20-3 gives a sketch of the target damage geometry for our model here. Note that the abscissa of the point at which the round detonates is x, and the abscissas of the target element location u and that of the C of I at μ are referred to the same x-axis. For the ordinates, we use the y-axis for the fall of shot y, the target element location v, and the ordinate location ν of the C of I. The target T is circular with center located at the origin, and radius R_T. The lethal radius R_L of a round about the location of a target element at (u,v) is depicted, and if a round lands within that lethal radius at (x,y) for example, it will damage the target element at (u,v). The side figure indicates merely the relationship to polar coordinates.

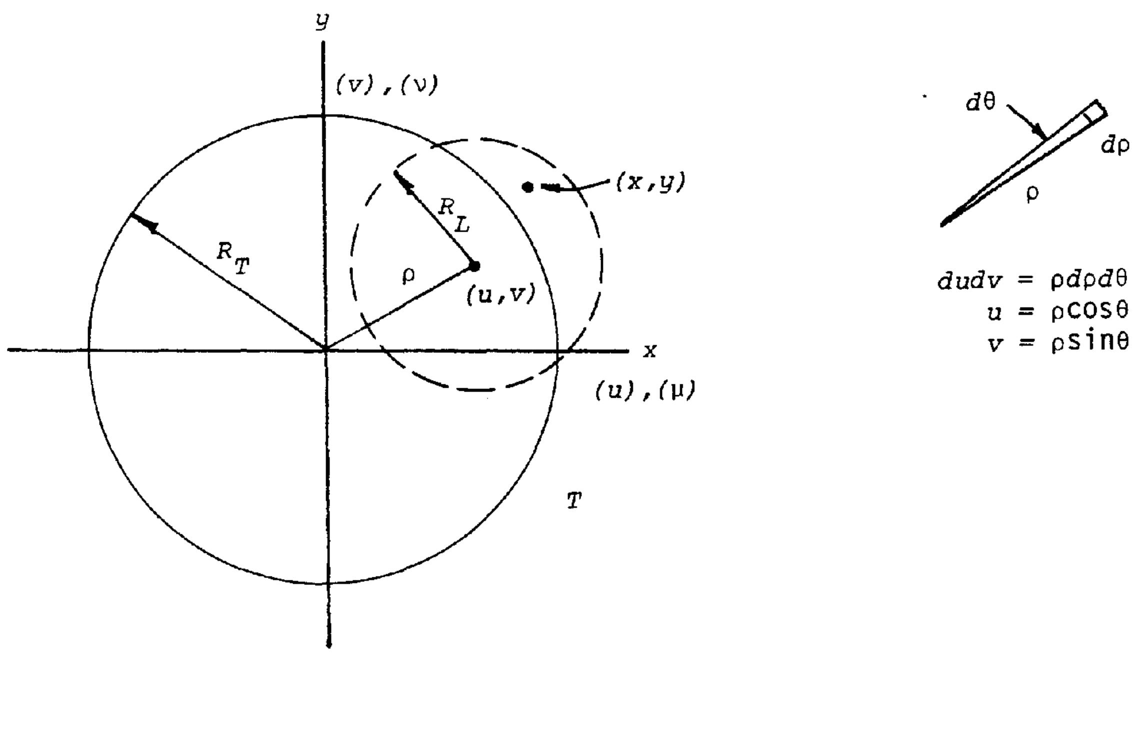


Figure 20-3. Sketch of Target Damage Geometry

With this background, it can be shown that for uniformly distributed target elements over the circular target of radius R_T, the expected fraction of damage is given by Eq. 13 of Ref. 4 and Eq. 4 of Ref. 39:

$$\bar{f}(n) \approx [2\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)}/R_T^2] \sum_{i=1}^{n} \binom{n}{i} (-1)^{i+1}(1/i)$$
$$\times \left\{\sigma_{kx}\,\sigma_{ky}/\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)}\right\}^i$$
$$\times \langle 1 - \exp\{- iR_T^2/[2(\sigma_{kx}^2 + \sigma_x^2 + i\sigma_\mu^2)]\}\rangle^{1/2}$$
$$\times \langle 1 - \exp\{- iR_T^2/2(\sigma_{ky}^2 + \sigma_y^2 + i\sigma_\nu^2)]\}\rangle^{1/2} \quad (20\text{-}77)$$

for noncircular delivery and damage functions. In the case of equal delivery errors in x and y, and a circular kill function, $\sigma_{kx} = \sigma_{ky} = \sigma_k$, then Eq. 20-77 reduces to

$$\bar{f}(n) \approx [2(\sigma_k^2 + \sigma_x^2)/R_T^2] \sum_{i=1}^{n} \binom{n}{i} (-1)^{i+1}(1/i)[\sigma_k^2/(\sigma_k^2 + \sigma_x^2)]^i$$
$$\times \langle 1 - \exp\{-iR_T^2/[2(\sigma_k^2 + \sigma_x^2 + i\sigma_\mu^2)]\}\rangle \,. \quad (20\text{-}78)$$

In case we are dealing with a single round, i.e., $n = 1$, then Eq. 20-77 becomes

$$\bar{f}(n) = [2\pi\sigma_{kx}\,\sigma_{ky}/(\pi R_T^2)]\langle 1 - \exp\{- R_T^2/[2(\sigma_{kx}^2 + \sigma_x^2 + \sigma_\mu^2)]\}\rangle^{1/2}$$
$$\times \langle 1 - \exp\{- R_T^2/[2(\sigma_{ky}^2 + \sigma_y^2 + \sigma_\nu^2)]\}\rangle^{1/2} \,. \quad (20\text{-}79)$$

As is well known, the alternating binomial series Eq. 20-77 is notoriously bad for computation. For this reason, Breaux and Mohler (Ref. 7) have transformed Eq. 20-77 to the following computational form using Jacobi polynomials:

$$\bar{f}(n) \approx - \sum_{r=1}^{M} \{[(n - r + 1)(r - 0.99)^2(2r + 0.01)]$$
$$\times [r^2(n + r + 0.01)(2r - 1.99)]^{-1}\}a_{r-1,n}$$
$$\times \sum_{i=1}^{r} \prod_{j=1}^{i} \{(r + j - 0.99)(- r + j - 1)/[j(j - 0.99)]\}(1/i)$$
$$\times 2[(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)]^{1/2}/R_T^2$$
$$\times \left\{\sigma_{kx}\,\sigma_{ky}/[(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)]^{1/2}\right\}^i$$
$$\times \langle 1 - \exp\{- iR_T^2/[2(\sigma_{kx}^2 + \sigma_x^2 + i\sigma_\mu^2)]\}\rangle^{1/2}$$
$$\times \langle 1 - \exp\{- iR_T^2/[2(\sigma_{ky}^2 + \sigma_y^2 + i\sigma_\nu^2)]\}\rangle^{1/2} \quad (20\text{-}80)$$

where

$$a_{0n} = n!/[(1.01)(2.01) \cdots (n + 0.01)] \tag{20-81}$$

while for general a_{rn} we have

$$\begin{aligned} a_{rn} &= \binom{n}{r} \frac{(2r + 0.01)\Gamma^2(r + 0.01)\Gamma(n + 1)}{\Gamma(0.01)(n + r + 1.01)\Gamma(r + 1)} \\ &= \left[\frac{(n - r + 1)(r - 0.99)^2(2r + 0.01)}{r^2(n + r + 0.01)(2r - 1.99)} \right] a_{r-1,n} \end{aligned} \tag{20-82}$$

and $M \leq n$ is the variable upper limit on the sum (we terminate the series upon arriving at the number of decimal places desired in the expected fraction of target damage).

Mr. Harold Breaux of the BRL has programmed the Ballistic Research Laboratories' Electronic Scientific Computer (BRLESC), which has a word length of 60 binary digits ($\approx$ 17 decimal digits), for the solution of Eq. 20-80. Accurate computations have been performed without difficulty using Eq. 20-77 for n up to about 75 or 80 and

$$p = \sigma_{kx}\sigma_{ky}/\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)} \leq 0.5 \tag{20-83}$$

although an exact region on p and n has not been actually determined. For $p < 0.30$, no computational difficulties arose for n as large as about 100, and, when $p = 0.1$, an n as large as 200 gave no computational difficulties. Also, for large numbers of rounds, it might be of more interest in actual firing to consider battery volley fire or battalion volley fire, which has the effect of dividing n by 6 or 18, respectively. Such firing involves lethality or lethal areas for battery or battalion volleys (6 or 18 rounds) as discussed in par. 15-6.

On the other hand, the Jacobi series of Eq. 20-80 has rather general advantage and gives four-decimal-place accuracy on the BRLESC for $p \leq 0.5$ and $n \leq 300$, for example. For smaller values of $p(p \leq 0.01)$, either the Jacobi series (Eq. 20-80) or the binomial series (Eq. 20-77) may be used for n up into many hundreds of rounds.

Mr. Ralph Shear has achieved excellent results in programming an HP 65 pocket calculator to solve Eq. 20-77.

20-8.1 NORMAL DENSITY OF TARGET ELEMENTS

For the case of a bivariate normal density of target elements with standard deviations σ_u and σ_v, instead of the previously assumed uniform density of target elements, then the expected fraction of target damage becomes

$$\begin{aligned} \bar{f}(n) \approx &- \sum_{r=1}^{M} \{[(n - r + 1)(r - 0.99)^2(2r + 0.01)]\,[r^2(n + r + 0.01)(2r - 1.99)]^{-1}\}a_{r-1,n} \\ &\times \sum_{i=1}^{r} \prod_{j=1}^{i} \{(r + j - 0.99)(-r + j - 1)/[j(j - 0.99)]\} \\ &\times \left[\sigma_{kx}\,\sigma_{ky}/\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)}\right]^i \\ &\times \left(\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)}\left\{\sqrt{[(\sigma_{kx}^2 + \sigma_x^2) + i(\sigma_u^2 + \sigma_\mu^2)]\,[(\sigma_{ky}^2 + \sigma_y^2) + i(\sigma_v^2 + \sigma_\nu^2)]}\right\}^{-1}\right) \\ &\times \{[1 - \exp(-a^2/2)]\,[1 - \exp(-b^2/2)]\}^{1/2} \end{aligned} \tag{20-84}$$

where

$$\left.\begin{aligned} a^2 &= R_T^2[\sigma_{kx}^2 + \sigma_x^2 + i(\sigma_u^2 + \sigma_\mu^2)]/[\sigma_u^2(\sigma_{kx}^2 + \sigma_x^2 + i\sigma_\mu^2)] \\ b^2 &= R_T^2[\sigma_{ky}^2 + \sigma_y^2 + i(\sigma_v^2 + \sigma_\nu^2)]/[\sigma_v^2(\sigma_{ky}^2 + \sigma_y^2 + i\sigma_\nu^2)] \end{aligned}\right\} \tag{20-85}$$

for the Jacobi-polynomial series expansion.

The corresponding series expansion for fractional target damage for a bivariate normal density of target elements and alternating binomial coefficients is Eq. 24 of Ref. 4:

$$\begin{aligned} f(n) \approx & \sum_{i=1}^{n} \binom{n}{i} (-1)^{i+1} \left[\sigma_{kx}\sigma_{ky}/\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)}\right]^i \\ & \times \left\langle \sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)} \left\{\sqrt{[(\sigma_{kx}^2 + \sigma_x^2) + i(\sigma_u^2 + \sigma_\mu^2)][(\sigma_{ky}^2 + \sigma_y^2) + i(\sigma_v^2 + \sigma_\nu^2)]}\right\}^{-1}\right\rangle \\ & \times \{[1 - \exp(-a^2/2)][1 - \exp(-b^2/2)]\}^{1/2} \end{aligned} \tag{20-86}$$

where a^2 and b^2 are as defined in Eq. 20-85.

We remark that, for Eqs. 20-84 and 20-86, and for Eq. 24 of Ref. 4, it is to be understood that the standard deviations σ_u and σ_v of the normal density of target elements should be determined from the size of the target radius R_T so that there is practically no truncation of the target-element distribution. Otherwise, for sizable truncation of the target element distribution by target boundaries, all the given equations should be divided by the proportion of elements actually within target boundaries for strict fraction of target damage, i.e., the divisor would be very nearly

$$\langle\{1 - \exp[-R_T^2/(2\sigma_u^2)]\}\{1 - \exp[-R_T^2/(2\sigma_v^2)]\}\rangle^{1/2} . \tag{20-87}$$

In such cases of significant truncation, one may properly have more interest in the expected true fraction of damage to all enemy elements in the general target area. One should, therefore, preferably use Eq. 20-91 for "indeterminable" target radius or size since strictly cut off target boundaries may be considered somewhat academic in combat. In addition, if the target elements are actually "peaked" at some location instead of being uniformly distributed over an area, then one would natually aim weapons at the higher target densities.

20-8.2 ELLIPTICALLY SHAPED TARGETS, RECTANGULAR TARGETS, AND UNIFORM DENSITY OF TARGET ELEMENTS

When the target has an elliptical shape, with semiaxes α and β, respectively, in the x- and y-directions, and also a uniform density of target elements, then it is easily shown as indicated in Ref. 4 that the expected fraction of damage is

$$\begin{aligned} \bar{f}(n) \approx & [2\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)}/(\alpha\beta)] \sum_{i=1}^{n} \binom{n}{i} (-1)^{i+1} \\ & \times (1/i)\left[\sigma_{kx}\sigma_{ky}/\sqrt{(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)}\right]^i \langle 1 - \exp\{-i\alpha^2/[2(\sigma_{kx}^2 + \sigma_x^2 + i\sigma_\mu^2)]\}\rangle^{1/2} \\ & \times \langle 1 - \exp\{i\beta^2/[2(\sigma_{ky}^2 + \sigma_y^2 + i\sigma_\nu^2)]\}\rangle^{1/2} \end{aligned} \tag{20-88}$$

which obviously reduces to Eq. 20-77 if $\alpha = \beta = R_T$. It is thus seen that only a slight change in the exponent and the coefficient is involved. Furthermore, because of the great accuracy of the Polya-Williams approximation

$$[1/(\sqrt{2\pi})] \int_{-x}^{x} \exp(-t^2/2)dt \approx [1 - \exp(-2x^2/\pi)]^{1/2} \tag{20-89}$$

a square target may be replaced by a circular target of the same area, a rectangular target by a corresponding elliptical target of the same area—using $\pi\alpha\beta = (2c)(2d)$ or vice versa—with little overall effect on expected target damage. We therefore have appropriate and suitably accurate equations for working with a variety of target shapes.

20-8.3 ELLIPTICALLY SHAPED TARGETS AND NORMAL DENSITY OF TARGET ELEMENTS

For an elliptically shaped target and normal density of target elements, we may use Eq. 20-86 with a and b replaced as follows:

$$\left.\begin{aligned} a^2 &= \alpha^2[\sigma_{kx}^2 + \sigma_x^2 + i(\sigma_u^2 + \sigma_\mu^2)]/[\sigma_u^2(\sigma_{kx}^2 + \sigma_x^2 + i\sigma_\mu^2)] \\ b^2 &= \beta^2[\sigma_{ky}^2 + \sigma_y^2 + i(\sigma_v^2 + \sigma_\nu^2)]/[\sigma_v^2(\sigma_{ky}^2 + \sigma_y^2 + i\sigma_\nu^2)] \,. \end{aligned}\right\} \tag{20-90}$$

20-8.4 INDETERMINABLE TARGET SIZE

Very frequently clearly defined boundaries do not exist for many targets that are attacked in combat. Hence, it often may be of interest to consider targets that are of indeterminable size and let the model involving a normal density of target elements take care of such situations, especially since the peakedness or dispersion of individual elements may be accounted for by the standard deviations σ_u and σ_v. For such cases, the expected fractional damage is given by Eqs. 18 and 19 of Ref. 4:

$$\begin{aligned} \bar{f}(n) \approx \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \left\{\sigma_{kx}\sigma_{ky}/[(\sigma_{kx}^2 + \sigma_x^2)(\sigma_{ky}^2 + \sigma_y^2)]^{1/2}\right\}^i \\ \times \{[1 + i(\sigma_u^2 + \sigma_\mu^2)/(\sigma_{kx}^2 + \sigma_x^2)][1 + i(\sigma_v^2 + \sigma_\nu^2)/(\sigma_{ky}^2 + \sigma_y^2)]\}^{-1/2} \,. \end{aligned} \tag{20-91}$$

If in Eq. 20-91 we put

$$\sigma_{kx} = \sigma_{ky} = \sigma_k, \quad \sigma_y = \sigma_x, \quad \sigma_\nu = \sigma_\mu \quad \text{and} \quad \sigma_v = \sigma_u$$

then Eq. 20-91 for the circular normal case becomes

$$\begin{aligned} \bar{f}(n) \approx \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} [\sigma_k^2/(\sigma_k^2 + \sigma_x^2)]^i (\sigma_k^2 + \sigma_x^2) \\ \times [(\sigma_k^2 + \sigma_x^2) + i(\sigma_\mu^2 + \sigma_u^2)]^{-1} \,. \end{aligned} \tag{20-92}$$

This series expansion, Eq. 20-92, for the circular case was derived by H. K. Weiss (Ref. 35) in Ballistic Research Laboratories Report No. 879, *Methods for Computing the Effectiveness of Area Weapons* (1953).

Harold Breaux of the Ballistic Research Laboratories has shown that Eq. 20-92 may be put in the form of the Incomplete Beta Function:

$$\bar{f}(n) = 1 - qp^{-q}\int_0^p u^{q-1}(1-u)^n du = 1 - qp\beta_p(q,n+1) \quad (20\text{-}93)$$

where

$$\left.\begin{aligned} p &= \sigma_k^2/(\sigma_k^2 + \sigma_x^2) \\ q &= (\sigma_k^2 + \sigma_x^2)/(\sigma_\mu^2 + \sigma_u^2)\,. \end{aligned}\right\} \quad (20\text{-}94)$$

When $n = 1$, then Eq. 20-92 or Eq. 20-93 becomes simply

$$\bar{f}(1) \approx \sigma_k^2/(\sigma_k^2 + \sigma_x^2 + \sigma_\mu^2 + \sigma_u^2) = A_L/[A_L + 2\pi(\sigma_x^2 + \sigma_\mu^2 + \sigma_u^2)] \quad (20\text{-}95)$$

since the lethal area of a round A_L may be approximated by $2\pi\sigma_k^2$. Eq. 20-95 is of course a simple extension of the well-known von Neumann-Carlton equation.

With this given background of equations for expected fractional target damage, we see that the key parameters involve the target dimensions (usually the target radius R_T), the round to round ballistic dispersions σ_x and σ_y, the aiming errors or movement of the C of I σ_μ and σ_ν for the n rounds, the density of target elements (uniformly or normally distributed), and the lethal area of the round or the salvo, or a suitably accurate kill function. Furthermore, it is seen that the given equations for calculating fraction of target damage are quite clear in displaying the parameters involved, which is not the case for many of the "simple" equations as we will see subsequently. An important point concerning use of appropriate damage models is that the lethality pattern usually stretches out more in the deflection direction, so that σ_{ky} is greater than σ_{kx}, and the dispersion σ_x in range for artillery weapons is almost always greater than that σ_y in the deflection direction. Thus, the selection of appropriate equations has to be made accordingly, and the models given herein clearly cover all such cases.

20-8.5 SUMMARY OF APPROXIMATIONS USED

As a record for the reader, the approximations used in Refs. 4 and 39 to obtain the analytical expansions of expected target damage for this paragraph are:

$$\begin{aligned} &\iint_{x^2/\alpha^2 + y^2/\beta^2 \leq 1} \exp[-(1/2)(x^2/\sigma_x^2 + y^2/\sigma_y^2)]dx\,dy/(2\pi\sigma_x\sigma_y) \\ &\approx \int_{-c}^{c}\int_{-d}^{d} \exp[-(1/2)(x^2/\sigma_x^2 + y^2/\sigma_y^2)]dx\,dy/(2\pi\sigma_x\sigma_y) \\ &\approx \{1 - \exp[-2c^2/(\pi\sigma_x^2)]\}^{1/2}\{1 - \exp[-2d^2/(\pi\sigma_y^2)]\}^{1/2} \\ &\approx \{1 - \exp[-\alpha^2/(2\sigma_x^2)]\}^{1/2}\{1 - \exp[-\beta^2/(2\sigma_y^2)]\}^{1/2} \end{aligned} \quad (20\text{-}96)$$

where the ellipse of area $\pi\alpha\beta$ has been replaced by the rectangle of the same area (or vice versa) with sides $2c$ and $2d$, so that $\pi\alpha\beta = (2c)(2d) = 4cd$, and the Polya-Williams approximation (Eq. 20-89) is also involved. Eq. 20-96 has been found to be quite good to two (and nearly three) decimal places, which is suitable for our weapon systems analysis applications, and therefore may also be of sufficient accuracy for many other applied problems.

20-8.6 EXAMPLES

Example 20-7:

Suppose a battalion volley of 18 artillery rounds would give the kill- or damage-probability pattern of Table 15-3, i.e., a total lethal area of 14002 m^2, say, with round to round (precision) standard deviation in range of $\sigma_x = 70$ m, sigma in deflection of $\sigma_y = 40$ m, and aiming errors of $\sigma_\mu = \sigma_\nu = 200$ m. For a target of radius $R_T = 175$ m, what is the expected fraction of personnel casualties, assuming uniform distribution of target elements?

From par. 15-6, we have:

$A_L = 14002$ m^2

σ_{kx} (range) $= 110$ m

σ_{ky} (deflections) $= 61$ m.*

Then, we will assume that the lethal area pattern for the volley, $A_L = 14002$, does not include the precision sigmas or the aiming errors, but that all 18 rounds will be fired "time on target" (simultaneously). Hence, in this case we may use Eq. 20-79 for $n = 1$, obtaining

$$\bar{f}(1) \approx 0.12$$

or therefore there would be about 12% casualties.

Example 20-8:

Use the data of Example 20-7. What would be the expected fraction of casualties if the total lethal area of the salvo or volley is broken up into 18 parts (rounds)?

Now we have

$$\begin{aligned} n &= 18 \\ R_T &= 175\ \text{m} \\ \sigma_x &= 70\ \text{m} \\ \sigma_y &= 40\ \text{m} \\ \sigma_\mu = \sigma_\nu &= 200\ \text{m} \\ A_L\ \text{per round} &= 14002/18 = 778\ \text{m}^2 \\ 2\pi c \sigma_{kx}\sigma_{ky} &= 778\ \text{m}^2. \end{aligned}$$

Since $c \approx 0.35$, and $\sigma_{ky} = 1.8\sigma_{kx}$, we have $\sigma_{kx} = 14$ and $\sigma_{ky} = 25.2$. Hence, substituting in Eq. 20-77, we find that

$$\bar{f}(18) = 0.082$$

so that there is some, but not a very great change in fraction of casualties, for alteration of the delivery model.

*The range direction x here is parallel to the shorter dimension of the kill probability pattern.

20-9 OPTIMUM AIMING AND ARTIFICIAL DISPERSION

It should be clear to the reader that in attacking some targets there may not exist an optimum relation, from the standpoint of the greatest percentage of casualties, among the parameters R_T, σ_x, σ_y, σ_μ, σ_ν, and A_L. Thus, the normal lethality pattern may be too widely dispersed or too compact, depending on the target size and lethality per round A_L. Hence, it becomes desirable to study whether artificial dispersion—introduced by firing rounds of a battery in a particular pattern or by purposely increasing the aim errors σ_μ and σ_ν—would increase the expected fraction of casualties $\bar{f}(n)$ to a maximum value. Clearly, in some cases it might be advantageous; in other cases it may not be worthwhile. This is a very complex problem that has been investigated by Kolmogoroy (Ref. 37), von Neumann (Ref. 40), Merritt and King (Ref. 41), Sangal (Ref. 42), Washburn (Ref. 43), and others. The exact relation between all of the parameters of interest to maximize the expected fraction of casualties is likely to be a complex one for which a general solution is not yet available for all cases of interest. For three rounds, for example, and a "regular" shaped target, then "obviously" the aim points should be a triangle; whereas for four shots the case for aim points being the corners of a square may not be so clear. Then again for long thin targets, such as a railroad, for example, linear releases in the form of "train bombing" would be in order, etc. We cannot develop detailed solutions to various cases for the problem of optimum aiming in this handbook, although the systems analyst would, no doubt, have some particular interest in upper and lower bounds on the expected fraction of casualties, especially as developed by Washburn (Ref. 43). In this connection, Washburn's treatment does not give definite patterns of aim points for many cases of interest, but he does give an indication as to whether the normal and random dispersion of aim points likely to be encountered should be increased. Moreover, he is able to indicate the relative sizes of errors in expected damage involved when artificial dispersion is not introduced, and the difference between the maximum and minimum damage is clearly of interest. In Washburn's work, there is no treatment of the case for a clear cut target boundary, or target radius R_T. Instead, this is taken care of by a normal density of target elements such as in Eq. 20-73 for the "indeterminable" target size condition.)

The maximum value of fractional target damage derived by Washburn (Ref. 43) for n rounds, circular delivery errors, and normal density of target elements ($\sigma_v = \sigma_u$) is

$$\bar{f}_{max}(n) = 1 - [1 + \sqrt{2rI(s)}]\exp[-\sqrt{2rI(s)}] \tag{20-97a}$$

$$\leq 1 - (1 + \sqrt{r\pi^2/3})\exp(1 - \sqrt{r\pi^2/3}) \tag{20-97b}$$

which depends on r and s, where

$$r = 2\pi nc\sigma_k^2/(2\pi\sigma_u^2) = nc\sigma_k^2/\sigma_u^2 \tag{20-98}$$

may be interpreted as the ratio of the "expected lethal area covered with widely spaced shots" to the equivalent "target area" for Gaussian target element density, and

$$s = c\sigma_k^2/(\sigma_k^2 + \sigma_x^2) \ . \tag{20-99}$$

The quantity $I(s)$ is the following integral:

$$I(s) = (1/s)\int_0^s [-(1/y)\ln(1-y)]dy \ . \tag{20-100}$$

Since $0 < s < 1$, the quantity $I(s)$ will be bounded by

$$1 = I(0) \leq I(s) \leq I(1) = \pi^2/6 = 1.645 \tag{20-101}$$

which is a relatively narrow range. $I(s)$ is plotted on Fig. 20-4.

For the lower bound on expected target damage, Washburn (Ref. 43) found it convenient to use an approximate approach to simplify calculations. We should recall that we are trying to find the optimum aiming errors $\sigma_\mu = \sigma_\nu$—given the number of rounds n, $\sigma_x = \sigma_y$, $\sigma_u = \sigma_v$, and $\sigma_k = \sigma_{kx} = \sigma_{ky}$. Washburn uses the concept of a standard deviation of "total" dispersion σ_T which includes the ballistic dispersion and the aiming error dispersion to maximize expected target damage. Now his parameter b, or

$$b = (\sigma_k^2 + \sigma_T^2)/\sigma_u^2 \tag{20-102}$$

which takes on a minimum value

$$b_{min} = (\sigma_k^2 + \sigma_x^2)/\sigma_u^2 \tag{20-103}$$

when there is only ballistic dispersion, becomes of importance. Washburn then shows that the lower bound $\bar{f}_{min}(n)$ or fraction of target damage is

$$\bar{f}_{min}(n) \geq 1 - Q[b^*(r),r] \tag{20-104}$$

where

$$\begin{aligned} Q(b,r) &= \exp(-r/b) + (b/r)^b \int_0^{r/b} u^b \exp(-u)du \\ &\approx \sum_{i=0}^{\infty} (-r/b)^i b/[i!(b+i)] \end{aligned} \tag{20-105}$$

and is related to Karl Pearson's Incomplete Gamma Function (Ref. 44).

The lower bound given by Eq. 20-104 is valid only when the unique value $b = b^*$ which makes $Q(b,r)$ a minimum is such that

$$b^*(r) \geq b_{min} \tag{20-106}$$

where b_{min} is given by Eq. 20-103. Otherwise the lower bound on $\bar{f}(n)$ must be taken as

$$1 - Q(b_{min}, r) = 1 - Q[(\sigma_k^2 + \sigma_x^2)/\sigma_u^2, r]\,. \tag{20-107}$$

Washburn defines the quantity $\mathcal{N}(r)$ as

$$\mathcal{N}(r) = r/b^*(r) \tag{20-108}$$

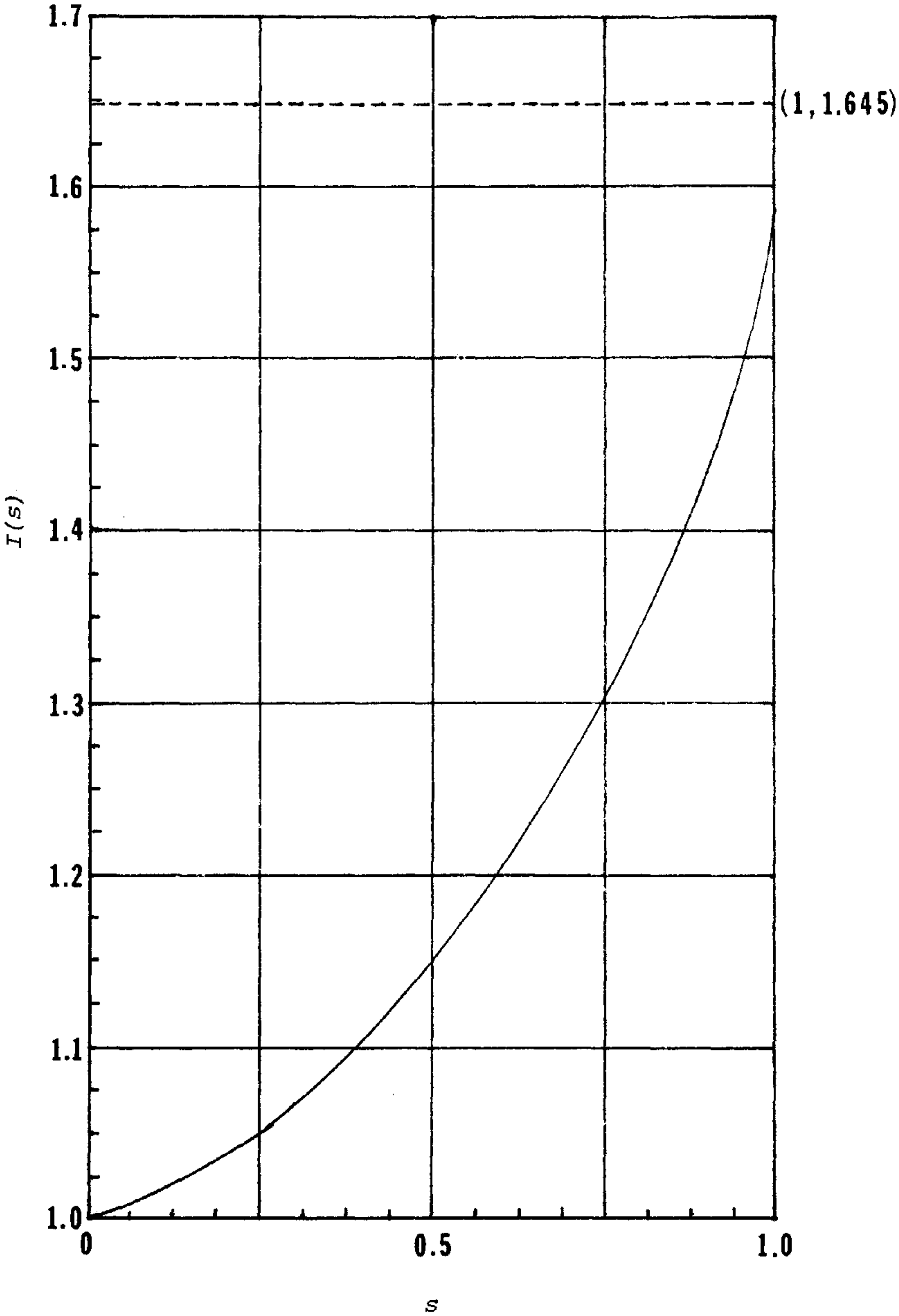


Figure 20-4. The Function $I(s)$

with r given by Eq. 20-98 and $b^*(r)$ is, as already indicated, the value of b which makes $Q(b,r)$ in Eq. 20-105 a minimum. $\mathcal{N}(r)$ is plotted on Fig. 20-5.

Then, since

$$r/b_{min} = nc\sigma_k^2/(\sigma_k^2 + \sigma_x^2) \tag{20-109}$$

the lower bound is valid provided

$$r/b_{min} \geq \mathcal{N}(r)\ . \tag{20-110}$$

Otherwise, one must use Eq. 20-107.

For the reader's information, the following indicates the correspondence between our notation and Washburn's parameters:

Our		Washburn's
n	=	$\mathcal{N}$
c	=	R
σ_k	=	c
σ_x	=	σ_I
σ_u	=	σ_T
r	=	a
s	=	p
σ_T	=	σ

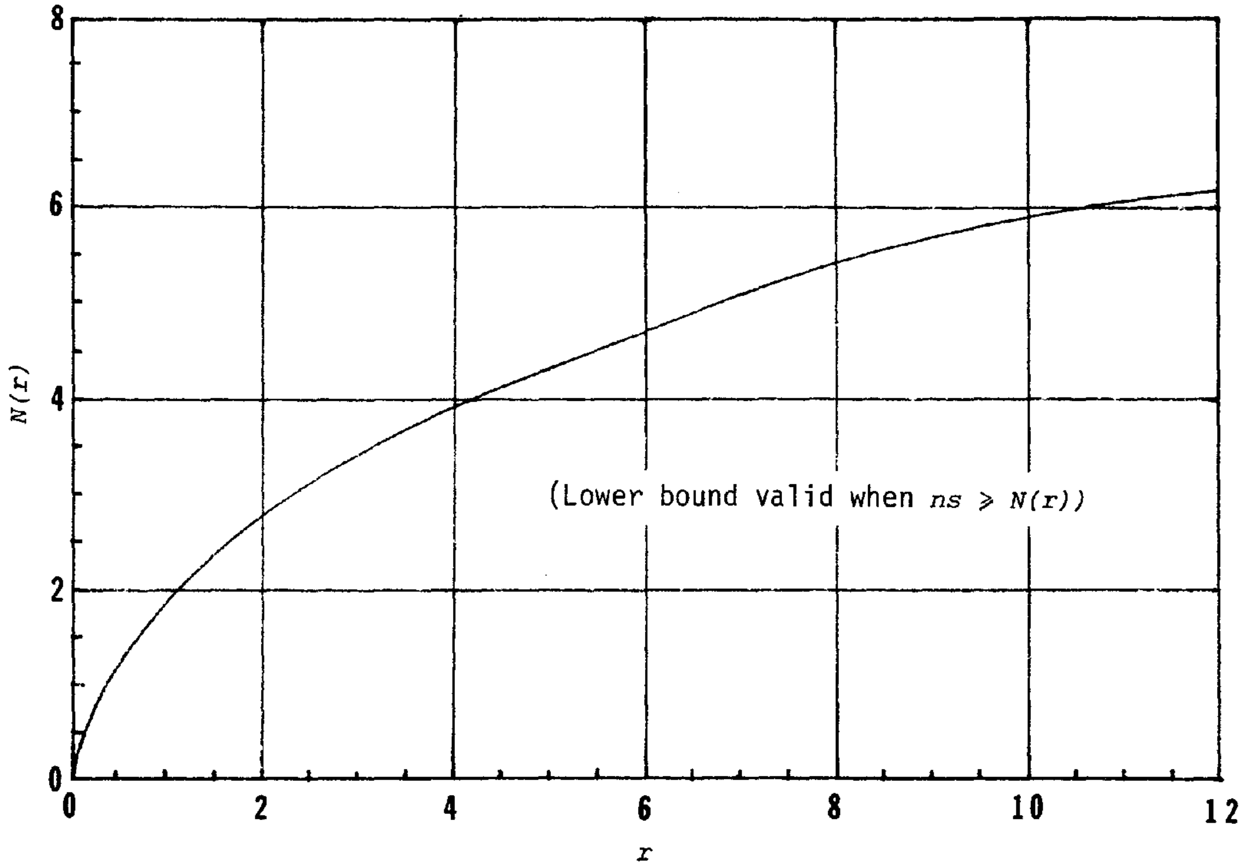


Figure 20-5. The Function $\mathcal{N}(r) = r/b^*(r)$
[Lower Bound Valid When $ns \geq \mathcal{N}(r)$]

Both the upper bounds for $s = 0$ and $s = 1$ and the lower bound on the expected fractional target damage are shown on Fig. 20-6 as a function of the parameter (see Eq. 20-98)

$$r = nc\sigma_k^2/\sigma_u^2 \ .$$

Note that the upper bound is very dependent on s and the difference between the two bounds for s approaching one can be as high as about 0.15, especially for expected fractions of target damage less than about 0.7 or so, whereas the differences above such a value to be no greater than about 0.1 or less. Nevertheless, since perhaps some 20%-30% casualties in battle may bring about withdrawal of a side, i.e., result in a battle "breakpoint", optimum aiming procedures may be very worthwhile and in fact may save many rounds. The problem of optimum aiming, therefore, continues to demand further study, especially to determine if simple rules could be developed for firing doctrines.

As pointed out by Washburn, the upper bound corresponds to an optimal "nonfeasible" pattern, while the lower bound corresponds to a nonoptimal "feasible" pattern. This is accomplished by either aiming all rounds at the same point or degrading delivery accuracy when needed so that shots fall in a random pattern of increased dispersion.

Example 20-9:

Forty artillery rounds with fuzes set for superquick action are fired at a large enemy tank park. The normal target element density of the tanks amounts to standard deviations $\sigma_u = \sigma_v = 100$ m. The round-to-round standard deviations due to widened deflection firing may be considered to be about equal and amount to $\sigma_x = \sigma_y = 20$ m. The kill function can be taken as circular normal with $\sigma_k = 50$ m; however, the maximum kill probability in the damage pattern for HE projectiles against tanks is no greater than about 0.1. What can be said about the bounds on the expected fraction of tanks destroyed, and the need for optimum aiming procedures (artificial dispersion)?

With the given data, we have

$$n = 40$$
$$\sigma_u = 100\,\text{m}$$
$$\sigma_x = 20\,\text{m}$$
$$\sigma_k = 50\,\text{m}$$
$$c = 0.1.$$

Therefore,

$$r = 1.0 \text{ (Eq. 20-98)}$$

$$s = 0.086 \text{ (Eq. 20-99)} \ .$$

$I(s) \approx 1.02$ from Fig. 20-4 or Eq. 20-100, and hence from Eq. 20-97a the upper bound on $\bar{f}(n)$ is

$$\bar{f}_{max}(n) \approx 0.417 \ .$$

Further, (see Eqs. 20-98, 20-99, and 20-103) $r/b_{min} = ns = 3.44 > \mathcal{N}(r) = 1.91$ from Fig. 20-5, so that the lower bound is valid, or

$$\bar{f}_m(n) \approx 0.40 \ .$$

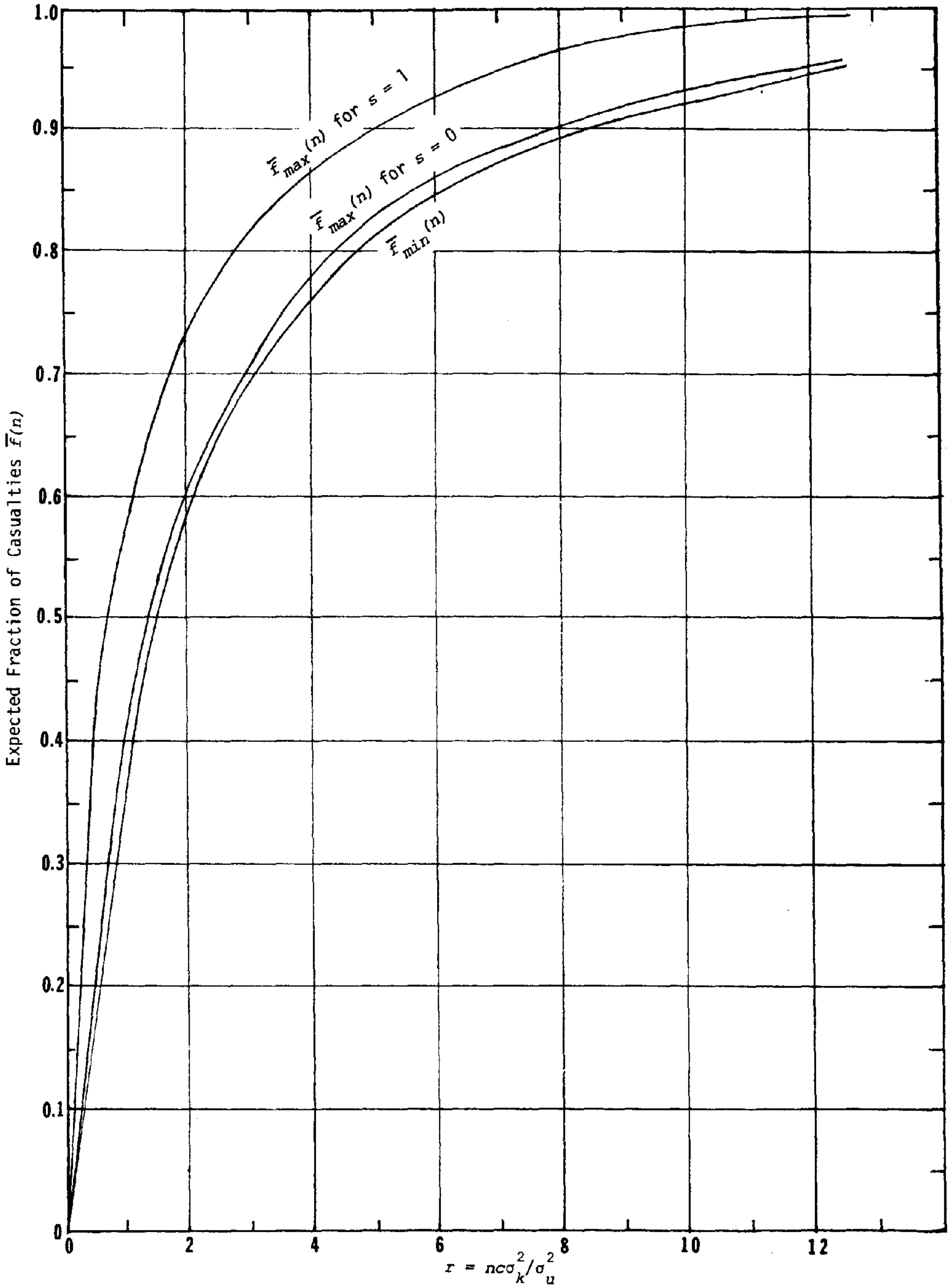


Figure 20-6. Upper and Lower Bounds on $\overline{f}(n)$

Hence the bounds are close since the parameters match well in this case. Had the parameters been somewhat near one, by an increase in c or $c\sigma_k^2$, then diverging the aim points would have increased the fraction of tanks killed significantly.

20-10 SIMPLIFIED TARGET DAMAGE MODELS

Over the years, there have been many, many attempts to develop simplified models for the expected fraction of target damage, kills, or casualties. With the beginning of Army weapon systems analyses at the BRL, various types of simple models were investigated. Fort Sill later developed some damage models, as did Honeywell and Picatinny Arsenal, and a simplified damage model was used in studies as recently as the "Battleking" study (Ref. 45). In one way or the other, all of these models depend on the basic idea of using the "optimistic" type of equation

$$\bar{f}(n) \approx 1 - (1 - \overline{P_k})^n \tag{20-111}$$

for an average pattern kill probability $\overline{P_k}$, and making variations on it. Invariably, however, such models usually run into some difficulties over parameter ranges of interest, and they are not clear cut in displaying the key parameters involved, as in, for example, Eqs. 20-72 through 20-95. We cannot recommend such models without reservation in spite of their rather wide use, although simple accurate models would obviously be worthwhile, and "Edisonian" approaches to establish them could indeed prove to advantage insofar as quick and easy estimates are concerned.

In this handbook, we will present only one of the more recent "simplified" models, which has been checked against Monte Carlo calculations, and used with *Joint Munitions Effectiveness Manual* work, which Mr. John Blomquist of the Army Materiel Systems Analysis Activity kindly brought to our attention. This particular model for calculating the expected fraction of casualties is given in Refs. 46 and 47 as

$$F_D = \bar{f}(n) = (EC_R)(EC_D)\left(1 - \{1 - A_L n r_R/[A_{VP}(OF)]\}^{N_v(OF)}\right) \tag{20-112}$$

where

F_D = expected fractional target damage
EC_R = expected fractional coverage of the target by weapon pattern in range
EC_D = expected fractional coverage of the target by weapon pattern in deflection
A_L = lethal area per round
n = number of rounds per volley
r_R = round reliability
A_{VP} = volley damage pattern area
OF = overlap factor
= nA_{AP}/A_{VP}
A_{AP} = single round damage pattern area
N_v = number of volleys.

All of the listed quantities must be calculated and then substituted into Eq. 20-112. The details of the calculations are given in Refs. 46 and 47. One can note that Eq. 20-112 is a variation of the basic form of Eq. 20-111 and, while not clearly expressed in terms of the statistical parameters needed, nevertheless, gives very good results, representing a substantial improvement over previous "simplified"

models. Note that the expected fractions of coverage, EC_R and EC_D, are calculated for each (univariate) direction (Ref. 47), whereas in par. 20-7 we treat the general bivariate problem, or two dimensional coverage. We will illustrate with an example.

Example 20-10:

A volley of 24 artillery projectiles is fired against an enemy personnel target of 50-m radius. The lethal area per round is about 650 m², although the lethality pattern, $A_L \approx 2\pi\sigma_{kx}\sigma_{ky}$, is such that σ_{kx}(range) = 5.5 m and σ_{ky}(deflection) = 18.9 m. The round-to-round dispersion in range is 49 m and that in deflection is only 3 m. Ordinarily, the aiming errors would amount to about 53.4 m in range and 28.2 m in deflection; however, for this target, doctrine is to fire in a "Lazy W" pattern of 240 m frontage and 80 m range. What is the expected fraction of casualties?

We have

$$\begin{aligned} n &= 24 \\ R_T &= 50\text{ m} \\ \sigma_{kx} &= 5.5\text{ m} \\ \sigma_{ky} &= 18.9\text{ m} \\ \sigma_x &= 49\text{ m} \\ \sigma_y &= 3\text{ m} \\ \sigma_\mu &= [(53.4)^2 + (80)^2/12]^{1/2} = 58.2\text{ m} \\ \sigma_\nu &= [(28.2)^2 + (240)^2/12]^{1/2} = 74.8\text{ m} \end{aligned}$$

where we estimated the sigma for the "Lazy W" sheaf by dividing each dimension of the pattern by $\sqrt{12}$*, assuming approximately a uniform distribution of aim points, and adding the results to the usual aim errors.

With Eq. 20-77, we obtain

$$\bar{f}(n) = \bar{f}(24) \approx 0.21\ .$$

For this particular problem, we also have available a Monte Carlo value of $\bar{f}(24) = 0.21$ from a computer program (Ref. 34), and the simplified model of Eq. 20-112 also gives $\bar{f}(24) = 0.21$—thus indicating excellent agreement.

In this particular case, the use of diverged aim points is harmful, for if all the weapons were fired at the target center with normal aim errors of $\sigma_\mu = 53.4$ and $\sigma_\nu = 28.2$, then from Eq. 20-77 the expected fraction of casualties would be nearly 0.40 or 40%, representing quite a gain indeed. Thus, we again see the importance of optimal relations among the major statistical parameters and projectile lethality.

20-11 SUMMARY

We have covered some of the more promising methods of calculating the chance of at least one hit in firing n rounds under various policies and have indicated the relative sizes of errors which may arise by not using the most appropriate model for the particular application. Also, the use of models which include the case of correlated aim points with time of firing may be of great importance, perhaps especially in air defense situations, but elsewhere also. The weapons systems analyst makes frequent use of target coverage and target damage models, especially for surface-to-surface weapons, or the attack of ground targets. We have indicated the relationship between the two concepts and how to convert

*$\sigma = \dfrac{b - a}{\sqrt{12}}$ for a uniform distribution; here $b - a$ = 240 or 80

from one to the other approximately. The general problem of target damage and the probability distribution of the fraction of casualties represents a complex area of investigation, and one that becomes tractable only through various approximation procedures. The weapon systems analyst will continue to face such problems, and it will always be a good idea to make a direct comparison of the models or approximations involved, especially by means of computations if necessary.

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CHAPTER 21

RELIABILITY, LIFE TESTING, RELIABILITY GROWTH, AVAILABILITY, AND MAINTAINABILITY

Due to the ever increasing complexity of materiel and the demand for high quality, we can state that reliability, life testing, maintainability, and availability now represent some of the more important characteristics of weapon systems requiring accurate evaluation. The weapon systems analyst must be thoroughly familiar with certain life-time or failure-time distributions. Therefore, we cover here the exponential, the lognormal, the Weibull, the gamma, and the binomial reliability distributions, and how they are applied to the evaluations of weapons. We cover also the estimation of population parameters, the system reliability, and how to determine confidence bounds on system reliability from component test data. Some considerations of the analytical aspects of high reliability are discussed for the analyst, as well as the concept of tolerance limits for distributions. On occasions, availability and maintainability analyses will be required of the analyst and are therefore introduced.

21-0 LIST OF SYMBOLS

A = availability, i.e., chance system is ready
$A(n,r,i)$ = coefficients for determining (Mann's) linear invariant estimates
A_a = achieved availability
A_o = operational availability
A_0,A_1,A_2 = coefficients for determing $k_{r,n}$
a = special symbol for $n_{(1)} \not\Sigma (1/n_j)$
B_0,B_1,B_2 = coefficients for determining $c_{r,n}$
$Be(u,v)$ = beta variate
b = $1/\beta$ = scale parameter for Gumbel extreme value distribution
b^* = Bain-Engelhardt simplified estimator of b
$\tilde{b}$ = linear invariant estimator (Gumbel extreme value parameter)
$C(n,r,i)$ = coefficients for determining (Mann's) linear invariant estimates
$c_{r,n}$ = constant for determining u^*
$E(x)$ = expected or mean value of x
$F(t)$ = cumulative distribution function = $1 - R(t)$
$F(u,v)$ = Fisher-Snedecor F-variate
$f(r)$ = chance of r failures
$f(t)$ = $F'(t)$ = derivative of $F(t)$ = probability density function
$h(t)$ = conditional failure rate
$h(t)$ = hazard or intensity function
$I(u,v)$ = incomplete gamma function
$I_x(u,v)$ = Karl Pearson's incomplete beta function
K = constant of proportionality
k = number of series or parallel components (or number of subsystems in the system)
k = kth moment
$k_{r,n}$ = constant, depending on amount of censoring for the estimator b^*

$\ell_{r,n}$ = constant for Mann-Fertig simplified estimator
M = mean active maintenance downtime
MDT = mean downtime
$MTBF$ = mean time between failures
$MTBM$ = mean time between maintenance
$MTTF$ = mean time to fail
$MTTR$ = mean time to repair
m = mean of the waiting time required to get the rth failure in a sample of n items put on test
m, m_s, m_p = mean value in general, mean value for a series system, and mean value for a parallel system, respectively
n = sample size
n = number of fuzes for a k-out-of-n system
n_i = sample size of ith component
$n_{(1)}$ = smallest n
$\bar{n}$ = average sample size
n^0 = $n_{(1)}(1 - 0.5\hat{P}^2)(1 - 0.5\hat{P})$—see Eq. 21-55
n_i^0 = $z_i/t_m - 1$ — for conversion of exponential to binomial data
n_i' = $(t_{0i}/t_m)n_i$—for conversion of exponential to binomial data
n_s^*, n_p^* = equivalent failures for series and parallel systems, respectively
$\binom{n}{r}$ = combination of n things taken r at a time
$P(c,n,p)$ = binomial probability of c or more failures in n when chance of failure on a single trial equals p
$P[\ ]$ or $Pr[\ ]$ = probability of
$\hat{P}_s, \hat{P}_p$ = estimated chance of system failure for a series system and parallel system, respectively
p = chance of failure in a single trial $= 1 - q = 1 - R$
p_i = chance of failure for ith component
p_L = lower limit on p
p_U = upper limit on p
$\hat{p}$ = estimate of p
$\hat{p}_{ML}$ = maximum likelihood estimate
q = chance of success in a single trial
R = reliability (true, unknown reliability of a system)
$R(t_m)$ = reliability at mission time t_m
R_a = assumed value of R_i
R_i = reliability of ith component
R_L = lower limit on R
R_p = parallel system reliability
R_s = series system reliability
$\hat{R}$ = estimate of R
r = number of failures
r_i = number of failures for ith component
$r_{(1)}$ = smallest r

r_i^0 = $r_i - 1$—for conversion of exponential to binomial data (Type I censoring—fixed number of failures)
r_i' = $r_i - 1$—for conversion of exponential to binomial data (Type II censoring—fixed time of testing)
r_s^*, r_p^* = equivalent failures for a series and parallel system, respectively
T = time interval
$T(t_i)$ = total life observed in getting the ith failure
T^* = preassigned total life
$\bar{t}$ = average of t_i
t_i = time of ith failure
t_{ij} = jth ordered failure time for ith component test
t_m = mission time
t_r = time of rth failure
t_{0i} = preset test time—for conversion of exponential to binomial data
t_α = reliable life
t_γ = value of t at probability level γ
$u = \ln \delta$ = location parameter for Gumbel extreme value distribution
u^* = Bain-Engelhardt estimate of u
$\tilde{u}$ = linear invariant estimator (Gumbel extreme value parameter)
v, v_s, v_p = variance in general, variance for series system, variance for parallel system, respectively
v = variance of the waiting time required to get the rth failure in a sample of n items put on test
w = $[t_r/(m\theta)]^{1/\beta}$ (see Eq. 21-96)
X = number of random failures
$\bar{x}$ = average
y_r = $t_r - t_{r-1}$
$z = r\theta$ = total (failure) time on test (see Eq. 21-84)
$z_{(1)}$ = smallest z
α_3 = skewness measure = μ_3/σ^3
σ_4 = kurtosis measure = μ_4/σ^4
$1 - \alpha$ = confidence level (α = 0.01, 0.05, etc.)
β = shape parameter for Weibull or gamma distribution
$\hat{\beta}$ = maximum liklihood estimate of shape parameter for the Weibull distribution
$\hat{\beta}_1$ = particular sample estimate of $\hat{\delta}$ (see Eq. 21-188)
$\Gamma(x)$ = gamma function of x
γ = percentage level; i.e., 0.01, 0.05, etc.
δ = $\theta^{1/\beta}$, characteristic life
δ_1 = particular sample estimate of $\hat{\delta}$ (see Eq. 21-188)
$\eta_{1-\alpha}$ = $100(1 - \alpha)$% probability level or percentage point for a standard normal deviate
$\theta = 1/\lambda$ = $MTTF$ (mean time to fail) usually for an exponential distribution
θ = scale population parameter
$\hat{\theta}$ = sample estimate of population parameter θ

λ = total number of failures for Poisson, or failure rate for exponential distribution
λ_i = expected number of failures for ith component (Poisson)
μ = maintenance rate for a system
μ_i = ith central moment
μ_ℓ = mean for lognormal distribution
ν = location parameter for two-parameter negative exponential distribution
ν = number of degrees of freedom for chi-square distribution
Π = product symbol
ρ = repair rate
σ = standard deviation
σ^2 = variance
$\phi = \sum_{i=1}^{k} \lambda_i = \sum_{i=1}^{k} (1/\theta_i)$ = series system failure rate
$\chi^2(x)$ = chi-square variate for x degrees of freedom
$\chi^2_{1-\alpha}$ = $100(1 - \alpha)\%$ probability level of chi-square
$\psi(x)$ = digamma function of x
$\psi'(x)$ = trigamma function of x
$|$ = symbol for "given"
$\not\Sigma$ = restricted sum

21-1 INTRODUCTION

Without doubt, the fastest growing and some of the most important fields of technical interest during the past 20 yr relate to reliability, life testing, maintainability, and availability. It can be argued that such development was a rather natural occurrence, for it would be demanded that more reliable and longer-life products be available as soon as technology and quality control would permit it. In the military, reliability came into being in a very prominent fashion in the very early 1950's, due especially to rather full-scale effort on the design and development of guided missiles, and the need to analyze and improve electronic reliability which was designed into the guidance and fuzing packages of the weapon system. The term "reliability" itself became a magic word and spread like "wild-fire" in all directions and to all items in military usage, demanding the best in performance. Some preliminary descriptive definitions will be presented before we give models of reliability, maintainability, and availability.

Reliability is the probability that an item, component, or system operates successfully during its mission. In other words, it is a chance of success or proper operation of an item for a given application and usually for a stated period of time called the "mission" time. To this definition, some usually add, "given that the item was in proper condition at the beginning of the mission". However, we must balk at including such a phrase in the definition, for one cannot look at an item and always tell that "it was in proper condition" at the start of its usage, especially for applications, such as in the military, where the continued use of the item invariably results in its destruction! We could, however, reword the usual definition to state that "reliability is the probability that an item operates successfully during its assigned mission, given that it is a randomly selected item from an accepted lot". Also, we might add that it is to be expected that the lot was manufactured under acceptable industrial practices of quality control, but we must stop here, perhaps!

Life testing represents a field directly related to reliability since it is a study of life times of items or products and the models which describe the probability distributions of failure times. A simple example is the life of a light bulb. Surely, the customer is interested in the expected or mean life of the light bulbs he purchases, and perhaps he is sufficiently aware of the problem to desire small variability in light bulb life, except for longer lives beyond a minimum guaranteed life of, for example, 2000 h.

Maintainability is a characteristic of design and installational usage of equipment, and is defined as the probability that an item or system will be retained in a specified operational condition, or restored to that condition within a given period of time, when maintenance is performed according to prescribed procedures and resources. Maintenance consists of those actions and corrections to the system needed to retain the designed-in characteristics throughout system lifetime. Maintainability, like reliability, must be designed into the system initially.

Availability is the probability that, at any random instant, an item or piece of equipment is in the proper condition to begin a mission. Thus, it could be said that availability is the probability that a system is operating satisfactorily at any point in time when used under specified or desired conditions—and some add "where total time considered includes operating time, active repair time, administrative time, and logistic time". In short, availability is simply the chance that a system is ready to operate when demanded.

We see that reliability, maintainability, and availability all relate to probabilities, or chances of occurrence, and we will define these concepts in terms of the proper equations or mathematical terms as we proceed. In any event, it is useful to keep in mind that in dealing with such terms, we are referring to the evaluation of chance events.

For the interested reader, some useful definitions are given in MIL-STD-721 (Ref. 1) which covers definitions of effectiveness terms for reliability, maintainability, human factors, and safety. Some other useful definitions are given in Ref. 2, and Refs. 3 and 4 are of general interest for the analyst working in the areas of reliability, life testing, maintainability, and availability.

AMCP 706-196, AMCP 706-197, AMCP 706-198, and AMCP 706-200 (Refs. 5, 6, 7, and 8) are devoted to the fields of reliability, maintainability, and availability and give good background material for this chapter. It is our purpose here not to duplicate any of that material except where absolutely necessary; rather, we will cover topics of some special interest to the weapon systems analyst, or at least consider them in a different light than the referenced series of handbooks.

Before we proceed, it is well to point out that there are perhaps two types or kinds of "reliability". First, (discrete) there is the type of article that either functions properly or fails to function. An example is a point-detonating fuze; either it operates or it fails on impact. Then (continuous) there is the concept of time-to-fail, i.e., the item or piece of equipment will operate in service for some period of time and fail. Then the item has to be repaired or replaced. A military vehicle, for example, will operate for a random number of miles and then fail for various reasons. Thus, we have the concepts of time-to-fail, miles-to-failure, cycles-to-fail, etc. In these latter cases, the reliability is the probability that the piece of equipment will survive or live to or beyond a desired or mission time, mileage, etc., and our interest is in analyzing such probability distributions.

For the case where the item simply fails or operates, such cases are covered by the binomial reliability distribution. For the concept of time-to-failure, miles-to-failure, etc., then it is convenient to deal with time-to-fail probability distributions such as the exponential, the Weibull, the lognormal, the gamma, and other probability distributions. Therefore we will cover these types of distributions in sufficient detail for the analyst to use them as required in the evaluation of weapons.

Due to its special importance, we also will cover the problems of determining confidence bounds on the true, unknown reliability of components, series systems, parallel systems, and some complex systems. The subject of reliability growth also will be introduced.

To introduce the general subject, we will first indicate computations of system reliability when the true (or large-sample estimate of) component reliabilities are known.

21-2 COMPUTATION OF SYSTEM RELIABILITY FROM KNOWN COMPONENT RELIABILITY

Some concepts of system reliability were introduced in Chapter 19 on fuzing since for the fuzing circuits it was important to guarantee high reliability of fuze action, especially for the expensive or nuclear type warheads. Here, we will give a few examples of some simple systems or types of circuits, and calculate analytically the reliability of them for the case where the true reliability of each component is known. This is contrasted to the case where we usually have only an estimate of reliability of each component based on a small sample.

As we indicated, reliability is defined as the probability of proper functioning of an item, component, circuit, or system. For discrete type data, i.e., "success" or "failure", we may analyze reliabilities by the use of binomial probabilities. On the other hand, for "time-to-fail" type data, we will be interested in the continuous reliability distributions such as the negative exponential, the lognormal, the Weibull, and gamma densities, for example. In this case the chance that an item, component or system lives or functions properly to or beyond a given mission time is also the reliability of the circuit involved. Hence, if the probability density function of any time-to-fail distribution is integrated from the mission time to infinity, then the result is the chance that the unit functions properly, or it is the reliability. (The integral from time zero to the mission time, of course, gives the chance of failure of the unit involved.) In either the discrete or continuous case, however, we may develop equations for determining the reliability of several types of systems by designating R_i as the (true) reliability of the ith component.

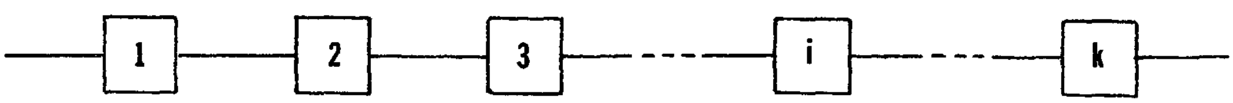
If we observe Fig. 21-1, we see a series system of k components or elements. Obviously, there is only a single path through the system which will result in successful operation, i.e., we must have proper functioning of each and every element or component. Hence, it is easy to see that the reliability R of the series System I is simply

$$R = R_s = \prod_{i=1}^{k} R_i \tag{21-1}$$

where

R_s = series system reliability = R, the overall system reliability

R_i = reliability of ith component.



$$R = R_S = \prod_{i=1}^{k} R_i$$

Figure 21-1. System I: Series System of k Elements or Components

Fig. 21-2 shows System II composed only of (redundant) elements or components in parallel. Clearly, in this case the systems will function properly if any one of the k parallel elements operates as intended, and the reliability of the parallel system may be determined by subtracting from unity the chance that all elements fail. The parallel system reliability R_p is

$$R = R_p = 1 - \prod_{i=1}^{k} (1 - R_i) \,. \tag{21-2}$$

If there are only two (redundant) components in parallel, then the system reliability is given by

$$R = R_1 + R_2 - R_1R_2 \,. \tag{21-3}$$

For System III, Fig. 21-3, we have a series system of two subsystems, with each subsystem having two parallel elements. In this case, the reliability R of the overall system is easily seen to be

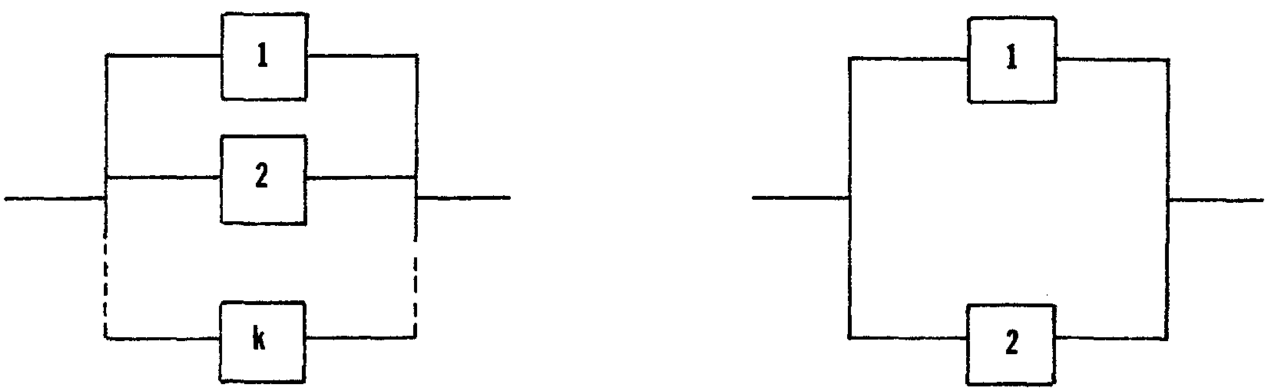
$$\left.\begin{aligned} R &= [1 - (1 - R_1)(1 - R_2)][1 - (1 - R_3)(1 - R_4)] \\ &= (R_1 + R_2 - R_1R_2)(R_3 + R_4 - R_3R_4) \,. \end{aligned}\right\} \tag{21-4}$$

If each of the $R_i = R_a$, the same reliability value, then the system reliability for this case is

$$R = [1 - (1 - R_a)^2]^2 = R_a^2(2 - R_a)^2 \tag{21-5}$$

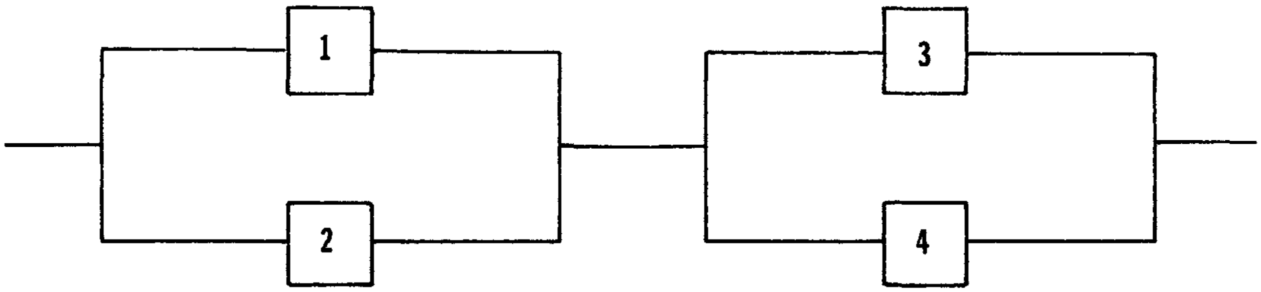
where

R_a = assumed value of R_i .



$$R = R_p = 1 - \prod_{i=1}^{k}(1 - R_i) \qquad\qquad R = R_1 + R_2 - R_1R_2$$

Figure 21-2. System II: Parallel System of k Elements or Components



$$R = [1 - (1 - R_1)(1 - R_2)][1 - (1 - R_3)(1 - R_4)]$$
$$= (R_1 + R_2 - R_1R_2)(R_3 + R_4 - R_3R_4)$$
$$= [1 - (1 - R_a)^2]^2 \text{ if } R_i = R_a$$
$$= 4R_a^2 - 4R_a^3 + R_a^4 = R_a^2(2 - R_a)^2$$

Figure 21-3. System III: Combination Parallel-Series System

For System IV, Fig. 21-4, we have a parallel bank system, with each bank consisting of two series elements. Here, the system reliability is easily determined to be

$$\left.\begin{aligned} R &= 1 - (1 - R_1R_2)(1 - R_3R_4) = R_1R_2 + R_3R_4 - R_1R_2R_3R_4 \\ &= R_a^2(2 - R_a^2), \text{ if } R_i = R_a \, . \end{aligned}\right\} \tag{21-6}$$

Considering Eq. 21-5 (System III) and Eq. 21-6 (System IV), we see for components of equal reliability that System III will be more reliable than System IV provided

$$R_{III} - R_{IV} > 0$$

or

$$2R_a^2(1 - R_a)^2 > 0$$

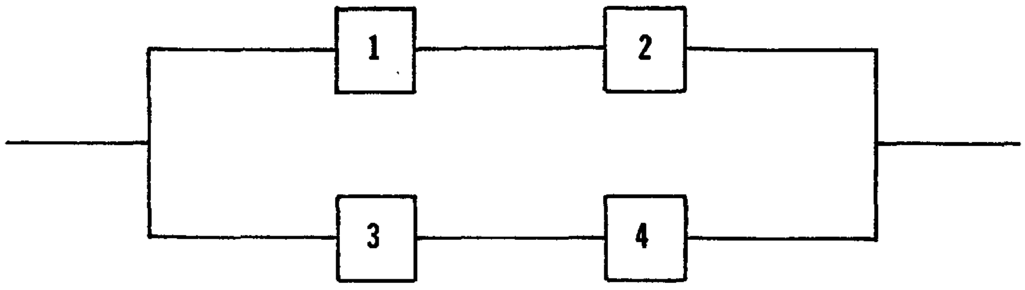
i.e., if $R_a < 1$, which is very likely!

Finally, let us observe the cross-strapped System V, Fig. 21-5. This system, due to component 5 and the two new added paths to System IV, is bound to be more reliable than System IV. In fact, the reliability of System V can easily be seen to be

$$R = R_{IV} + R_5[R_1R_4(1 - R_2)(1 - R_3) + R_2R_3(1 - R_1)(1 - R_4)] \tag{21-7}$$

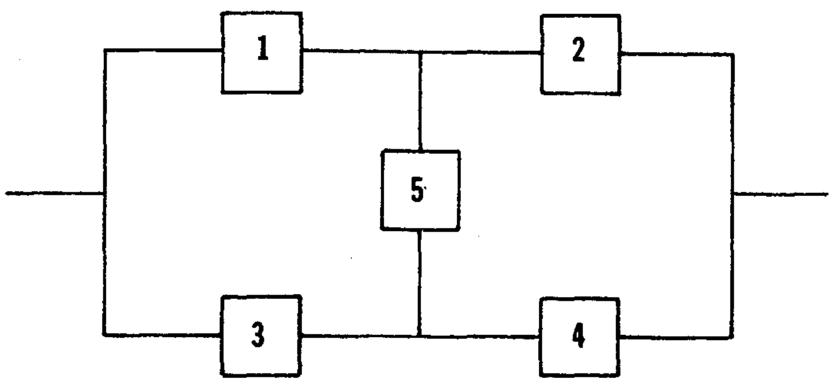
where R_{IV} is the reliability of System IV.

The "k-out-of-n" type system and its reliability for fuze structures were mentioned in par. 19-9.3.



$$R = 1 - (1 - R_1R_2)(1 - R_3R_4)$$
$$= R_1R_2 + R_3R_4 - R_1R_2R_3R_4$$
$$= 2R_a^2 - R_a^4 \text{ if } R_i = R_a$$
$$= R_a^2(2 - R_a^2)$$

Figure 21-4. System IV: Combination Series-Parallel System



$$R = R_{IV} + R_5[R_1R_4(1 - R_2)(1 - R_3)]$$
$$+ R_2R_3(1 - R_1)(1 - R_4)$$
where R_{IV} = reliability of System IV

Figure 21-5. System V: Example of a Cross-Strapped Series-Parallel System

These brief system reliability determinations should give the young systems analyst some working knowledge of methods for calculating the reliability of other networks. Hence, we will not pursue this subject any further here, for our major problem really involves the cases where we have only sample data or trials of component reliability, and we are required to find confidence bounds on the true, but unknown system reliability. We cover this subject next.

21-3 THE BINOMIAL RELIABILITY DISTRIBUTION AND ITS USES

For the success or failure type of operation of a system or component, we may model chances of occurrence and reliabilities with the binomial probability distribution, the most widely used distribution for a discrete random variable. We introduce the notation:

n = number of items tested
r = number of failures observed
p = chance of failure in a single trial or test $= 1 - q$
q = chance of success in a single trial or test
i = running variable $= 0, 1, 2, \ldots, n$
$R = 1 - p = q =$ *reliability* of the item. $(p + q = p + R = 1)$.

Then the chance $f(r)$ of obtaining r failures in n items put on test is the binomial term or probability

$$f(r) = \binom{n}{r} p^r (1-p)^{n-r} \tag{21-8}$$

which is simply the $(r + 1)$st term in the binomial probability expansion

$$[(p) + (1-p)]^n = \sum_{r=0}^{n} \binom{n}{r} p^r (1-p)^{n-r} = 1 \,. \tag{21-9}$$

The quantity p is the unknown population parameter of the binomial reliability distribution, and must be estimated. Also, $R = 1 - p$, the true reliability, is obviously unknown and requires estimation or the placing of confidence bounds on it.

The mean or expected number of failures $E(r)$ in n trials is

$$E(r) = \sum_{r=0}^{n} r f(r) = np \,. \tag{21-10}$$

The variance σ^2, and standard deviation σ of the binomial reliability distribution are

$$\sigma^2 = npq = npR, \text{ and } \sigma = \sqrt{npq} \,. \tag{21-11}$$

The skewness α_3 of the binomial distribution is the third central moment divided by the standard deviation cubed and is

$$\alpha_3 = (q - p)/\sqrt{npq} \,. \tag{21-12}$$

Hence, if the true reliability $R = q > 0.5$, the distribution is skewed to the right (positive skewness) and if $R < 0.5$, the distribution has negative skewness.

The measure of "kurtosis" α_4, or peakedness of the distribution, is given by the fourth central moment divided by the square of the variance and is

$$\alpha_4 = 3 + (1 - 6pq)/(npq) \,. \tag{21-13}$$

Any general central moment μ_t may be determined from the preceding two lower central moments with the aid of Eq. 21-14.

$$\mu_i = pq[n(i-1)\mu_{i-2} + \frac{d}{dp}(\mu_{i-1})]. \tag{21-14}$$

As is well known, if the reliability is not too far from 0.5, or for reliability departing from 0.5 for the case of large sample sizes, then the binomial distribution approaches the normal distribution, i.e., $\alpha_3 \to 0$ and $\alpha_4 \to 3$. Since we will be interested primarily in systems which are highly reliable, i.e., for the unreliability p being small ($\leq$ about 0.10), then it will be necessary and convenient to develop suitably accurate approximations for binomial reliability confidence bounds, especially for complex systems.

The maximum likelihood estimates of the chance of failure $\hat{p}$ and the reliability $\hat{R}$ are

$$\hat{p} = r/n \quad \text{and} \quad \hat{R} = \hat{q} = (n-r)/n \tag{21-15}$$

where we have used "^" for the *estimate* of the parameter based on a sample. $\hat{p}\hat{q}/(n-1)$ is an unbiased estimate of $\sigma^2 = pq/n$.

With this preliminary background, we are now in a position to determine confidence bounds on the reliability R, given a component or system for which r failures in n items, components, etc., were obtained in a testing procedure.

21-3.1 CONFIDENCE BOUNDS ON BINOMIAL RELIABILITY

The usual procedure to obtain a confidence bound on the unknown parameter p (or R) would involve finding a function of the estimate $\hat{p} = r/n$ and the unknown p, which is distributed independently of p. However, this is not possible for the discrete binomial distribution. Nevertheless, we may proceed in the manner that follows to determine binomial reliability bounds.

To obtain an *upper* confidence bound p_U on the true (unknown) chance of failure in a single trial, or the unreliability p at confidence level $(1-\alpha)$ for $\alpha = 0.01, 0.05$, etc., we find the value of p for which $P(n-r,n,1-p)$ of Ref. 9 is

$$\sum_{i=0}^{r} \binom{n}{i} p^i (1-p)^{n-i} = \alpha \quad , \qquad (\alpha = 0.01, 0.05, \text{etc.}) . \tag{21-16}$$

[See, for example, Ref. 10 by Mood and Graybill (1963)].
The solution of this equation gives us $p = p_U$, and we may state with $100(1-\alpha)\%$ assurance that the true unknown chance of component (system) failure will not exceed p_U. The lower $100(1-\alpha)\%$ confidence bound on the reliability R therefore, is $R = 1 - p_U$.

For r failures observed in n tests, a *lower* limit on p, or upper limit on R, at confidence level $(1-\alpha)$ is found by obtaining the value of p (call it p_L) for which $P(r,n,p)$ of Ref. 9 is

$$P(r,n,p) = \sum_{i=r}^{n} \binom{n}{i} p^i (1-p)^{n-i} = \alpha . \tag{21-17}$$

When $r = 0$, the lower confidence limit p_L on the unreliability p is taken to be 0, and if and when we ever have $r = n$ (all failures) the upper limit p_U on p is taken to be 1.

One might wonder why an "upper" limit on the unreliability p is found from the "lower" or "left-hand" sum of terms, and why a "lower" confidence bound is found from the upper or right-hand sum of binomial terms. The reason for this is that we are dealing with a discrete probability and want

$$Pr[p \le p_U] \ge 1 - \alpha \qquad (21\text{-}18)$$

that is, we desire to find p_U, an upper bound on p, such that for the chance for r failures observed in n trials the statement can be made with at least confidence level $(1 - \alpha)$. Thus, if $\alpha = 0.05$, we desire p_U such that

$$Pr[p \le p_U] \ge 0.95\,. \qquad (21\text{-}19)$$

This means that the *smallest* value of p (call it p_U) for which some random number of failures (call it X) exceeds r, given p, i.e.,

$$Pr[X > r|p] = \sum_{i=r+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \ge 1 - \alpha \qquad (21\text{-}20)$$

or the equivalent probability is found from

$$Pr[X \le r|p] = \sum_{i=0}^{r} \binom{n}{i} p^i (1-p)^{n-i} \le \alpha\,. \qquad (21\text{-}21)$$

In passing, we note here from the definition of the incomplete beta function (Ref. 11), or the cumulative beta distribution, its relation with the sum of terms of the binomial distribution. It is

$$\sum_{i=0}^{r} \binom{n}{i} p^i (1-p)^{n-i} = 1 - \frac{\Gamma(n+1)}{\Gamma(r+1)\Gamma(n-r)} \int_0^p u^r (1-u)^{n-r-1}\,du \qquad (21\text{-}22)$$

$$= 1 - I_p(r+1, n-r) = I_{1-p}(n-r, r+1) = P(n-r, n, 1-p)$$

where

$I_x(u,v)$ = Karl Pearson's incomplete beta function (Ref. 11).

We say in connection herewith that the number of failures r has a beta distribution, $Be(r + 1, n - r)$, with parameters $r + 1$ and $n - r$.

For the often important practical case of zero failures ($r = 0$) observed in n trials or tests, note that we take

$$\sum_{i=0}^{r=0} \binom{n}{i} p^i (1-p)^{n-i} = (1-p)^n = \alpha \qquad (21\text{-}23)$$

so that the $100(1 - \alpha)\%$ upper confidence bound p_U on the unreliability p is simply

$$p_U = 1 - \alpha^{1/n} \qquad (21\text{-}24)$$

or the $100(1 - \alpha)\%$ lower confidence bound R_L on the reliability R is given simply by

$$R_L = 1 - p_U = \alpha^{1/n}\,. \qquad (21\text{-}25)$$

The corresponding bound R_L based on the Poisson approximation to the binomial distribution is given by

$$R_L = 1 + (1/n)\ln\alpha \ . \tag{21-26}$$

We illustrate with an example.

Example 21-1:

Suppose we test 20 primers selected randomly from a lot and find zero failures. What is the 95% lower confidence bound on the reliability of the lot of primers, assuming a binomial distribution?

Here $n = 20$, $\alpha = 0.05$, $r = 0$, and from Eq. 21-24 we have with 95% confidence that

$$R_L = \alpha^{1/n} = (0.05)^{1/20} = 0.86 \ .$$

In addition to the important case of zero failures in practical situations, the case of only a single failure is also of wide interest. Here, we return to Eq. 21-16 and obtain

$$\left.\begin{aligned} &\sum_{t=0}^{1} \binom{n}{t} p^t (1-p)^{n-t} = (1-p)^n + np(1-p)^{n-1} = \alpha \\ \text{or} \quad & \\ &R^n + nR^{n-1}(1-R) = \alpha \end{aligned}\right\} \tag{21-27}$$

which may be solved (by cut and try) for R if we know n and α. In fact, we may construct the very informative Table 21-1, which gives the sample sizes n necessary to obtain lower 90% and 95% confidence bounds for the reliabilities of R = 0.90, 0.95, 0.975, and 0.99—i.e., some desirably high levels.

From the definition of the incomplete beta function in Eq. 21-22, we reiterate that its relation to the binomial distribution is as follows, and hence we may find R_L at confidence level $(1 - \alpha)$ from

$$I_{R_L}(n-r, r+1) = I_{1-p}(n-r, r+1) = \sum_{t=0}^{r} \binom{n}{t} p^t (1-p)^{n-t} = \alpha \tag{21-28}$$

TABLE 21-1. SAMPLE SIZES n NEEDED FOR ZERO OR ONE FAILURE IN n TESTS TO OBTAIN 90% AND 95% LOWER CONFIDENCE BOUNDS ON THE RELIABILITY

Lower Limit on R	Number of Failures Observed	Degree of Confidence	
		$1 - \alpha = 90\%$	$1 - \alpha = 95\%$
	r	n	
0.90	0	22	29
	1	38	46
0.95	0	45	59
	1	77	93
0.975	0	91	119
	1	155	188
0.990	0	230	299
	1	388	473

that is, we can find confidence bounds from tables of percentage points of the incomplete beta function, such as in Ref. 12, the *Biometrika Tables for Statisticians*. We give an example using the incomplete beta function approach.

Example 21-2. For the 20 primers tested with zero failures in Example 21-1, we have

$$n = 20,\ r = 0,\ n - r = 20,\ r + 1 = 1\,.$$

Hence, for a lower confidence bound on reliability, we want the value of $I_x(u,v)$ such that

$$I_{1-p}(n - r, r + 1) = I_{1-p}(20,1) = 0.05\,.$$

To find the $1 - p$, or the lower 95% confidence bound on the reliability R, we use Ref. 12, page 148, and enter the table, as directed, with $\nu_2 = (2)(20) = 40$ and $\nu_1 = (2)(1) = 2$. The value of $R = 1 - p = 0.861$ is read directly and checks with Example 21-1, which was calculated by another less general method, i.e., Eq. 21-25. The incomplete beta function may be used to obtain confidence bounds on reliability for any values of r and n, although some interpolation will be required for sample size gaps in the tables. We nevertheless proceed to other methods since the incomplete beta function tables of percentage points are somewhat complex to use, and not widely available. Ref. 9 discusses the relationship with binomial probabilities rather extensively.

21-3.2 BINOMIAL CONFIDENCE BOUNDS FOR A SINGLE COMPONENT SYSTEM

Let us now consider a simple system of only one component for which we had r failed components in n similar ones tested. Then, we would like to determine confidence bounds on the true unknown reliability of a system composed of one such component. We could use the incomplete beta function just discussed, but other more convenient techniques are available, and we should look ahead for methods which will be applicable to complex systems. First, we record the method of using the Fisher-Snedecor F-statistic and the Poisson-chi-square technique. As is well known, there is a very definite relation between a beta variate and the Fisher-Snedecor F-statistic. In fact, the inverted beta distribution is actually the F probability distribution, and the relation is

$$Be(u/2, v/2) = \frac{u}{u + vF(v,u)} \tag{21-29}$$

where Be and F stand for beta and F variates, and u and v are arguments. It can be shown in this connection for r failures in n items tested that the $100(1 - 2\alpha)\%$* confident bounds on the true reliability R may be found from

$$Pr\left[\frac{2(n-r)}{2(n-r) + 2(r+1)F_{1-\alpha}(2r+2, 2n-2r)} \le R \le \frac{2(n-r+1)F_{1-\alpha}(2n-2r+2, 2r)}{2r + 2(n-r+1)F_{1-\alpha}(2n-2r+2, 2r)}\right] = 1 - 2\alpha\,. \tag{21-30}$$

*2α refers to the two-sided bounds

Thus, to find the $100(1 - 2\alpha)\%$ confidence bounds on R, one substitutes the sample size n, the number of failures r, and the values of the $100(1 - \alpha)\%$ points of the F-statistic, which may be found in nearly all textbooks on statistics—Table 6-4 of AMCP 706-200 (Ref. 8), or Ref. 12, for example.

Since the Poisson-chi-square distribution may be used as a good approximation to the binomial distribution for high reliability (or small $p \leq$ about 0.10), then the $100(1 - 2\alpha)\%$ confidence bounds on R may also be determined from

$$Pr\left[1 - \frac{1}{2n}\chi^2_{1-\alpha}(2r + 2) \leq R \leq 1 - \frac{1}{2n}\chi^2_{\alpha}(2r)\right] \approx 1 - 2\alpha \tag{21-31}$$

where the chi-square levels may also be found in any stastitics textbook, Table 6-1 of AMCP 706-200 (Ref. 8), or Ref. 12, for example. Eq. 21-31 gives a lower bound even if $r = 0$ failures! We illustrate with an example.

Example 21-3:

A performance test of a new fuze was carried out by assembling 50 of the fuzes to projectiles and firing the rounds for ground impact. Two duds were observed in the test. Find the 95% confidence bounds on the true reliability of the new type of fuzes.

We have $n = 50$, $r = 2$, $n - r = 48$, and $r + 1 = 3$, and using Eq. 21-30 we calculate

$$Pr\left[\frac{96}{96 + 6F_{0.975}(6,96)} \leq R \leq \frac{98F_{0.975}(98,4)}{4 + 98F_{0.975}(98,4)}\right]$$

$$= Pr[0.8629 \leq R \leq 0.9952] = 0.95$$

where we put $\alpha = 0.025$ to get $1 - 2\alpha = 0.95$.

To obtain (approximate) confidence bounds based on the Poisson-chi-square Eq. 21-31, we calculate

$$Pr\left[1 - \frac{1}{100}\chi^2_{0.975}(6) \leq R \leq 1 - \frac{1}{100}\chi^2_{0.025}(4)\right]$$

$$= Pr[0.8555 \leq R \leq 0.9952] \approx 0.95 .$$

Thus, to two decimal places the 95% confidence bounds on the new fuze reliability are 0.86 and 1.00 (0.995). The lower 97.5% confidence bound on reliability for the fuze is 0.86. The best estimate of the new fuze reliability is $\hat{R} = 48/50 = 0.96$.

For such a simple system (the single component one), there are many good tables of percentage points of the binomial distribution, which may be used immediately to find confidence bounds on reliability—e.g., see Table 3-2 of AMCP 706-200 (Ref. 8); or the tables of Cook, Lee, and Vanderbeck (Ref. 13); the excellent tables of Harter (Ref. 14); and others. Table 3-4 of AMCP 706-200 (Ref. 8) gives the Neyman-shortest unbiased confidence intervals for the unknown parameter, the bounds being made unbiased for the discrete binomial by drawing a uniform random number and using it with the observed data to calculate the confidence intervals. A very useful table of binomial probabilities with an informative introduction may be found in AMCP 706-109 (Ref. 9), which involves much the same notation used here.

Our main approach for calculating confidence bounds on series, parallel, and more complex systems will be to use suitably accurate approximations and models which can be applied with the aid of many scientific type pocket calculators only, and hence not having to resort to any tables at all.

21-3.3 CONFIDENCE BOUNDS FOR PARALLEL COMPONENTS, ZERO OBSERVED FAILURES, AND EQUAL SAMPLE SIZES

It will be instructive for frequent applications to discuss some special cases of confidence bounds on parallel (and series) systems before we proceed to more complex systems. An example is that of k components in parallel, where the true unknown component unreliabilities are $p_1, p_2, \ldots, p_k$, respectively, and we observe $r = 0$ failures in n tests on each of the components. In this particular case, we can see by a generalization of Eq. 21-23, or the work of Buehler (Ref. 15), that the likelihood of this k-sample occurrence is

$$(1 - p_1)^n(1 - p_2)^n \cdots (1 - p_k)^n \tag{21-32a}$$

and we equate this to $\alpha = 0.01$ or 0.05, etc., to find an upper confidence bound on the parallel system unreliability

$$1 - R_p = \prod_{i=1}^{k} p_i \,. \tag{21-32b}$$

By equating the product (21-32a) to α and solving for R_P, along with the use of Eq. 21-32b, we have a lower $(1 - \alpha)$ confidence bound on the parallel system reliability R_p.

Now the maximum value of $p_1p_2p_3 \cdots p_k$ occurs when $p_1 = p_2 = \cdots = p_k = p$, say. Hence, the upper $100(1 - \alpha)\% = (1 - \alpha)$ confidence bound on the unreliability is found by solving

$$(1 - p)^{kn} = \alpha \tag{21-33}$$

for p, which is

$$p = 1 - \alpha^{1/(kn)} \,. \tag{21-34}$$

The upper $(1 - \alpha)$ confidence limit on $p_1p_2p_3 \cdots p_k$ is then found by raising Eq. 21-34 to the kth power, i.e.,

$$p^k = [1 - \alpha^{1/(kn)}]^k \tag{21-35}$$

or finally the lower $(1 - \alpha)$ confidence bound on parallel system reliability is

$$R_p = 1 - [1 - \alpha^{1/(kn)}]^k \,. \tag{21-36}$$

The corresponding lower $(1 - \alpha)$ confidence bound based on the Poisson approximation is

$$R_p = 1 - \left[- \frac{\ln(1 - \alpha)}{kn} \right]^k \,. \tag{21-37}$$

It is well known that circuits can be made highly reliable by using redundant components. Also, one may be able to guarantee high reliability for parallel components by really testing only a few items. We illustrate with an example.

Example 21-4:

To insure high reliability of a fuzing system, it has been decided to use two (dissimilar) component elements in parallel. It is known from past experience that each of the two types of components has been tested, and zero failures in 10 tests observed for each. What reliability for this arrangement can be guaranteed with 90% confidence?

Here, we see that $r_1 = r_2 = 0$, and $n_1 = n_2 = n = 10$. Then a lower 90% confidence bound on the reliability of such a parallel or redundant arrangement would be from Eq. 21-36:

$$R_L = 1 - [1 - (1 - 0.90)^{1/20}]^2 = 0.9882$$

whereas the Poisson approximation, Eq. 21-37, gives

$$R_L \approx 1 - \left[\frac{-\ln(1 - 0.90)}{20}\right]^2 = 0.9867 .$$

We could thus say with 90% confidence that a lower bound on reliability would be at least 0.98—a very high value for $n = 10$.

21-3.4 APPROXIMATE BINOMIAL CONFIDENCE BOUNDS FOR A SERIES SYSTEM BASED ON LARGE NEARLY EQUAL SAMPLE SIZES AND SMALL NUMBERS OF FAILURES

For binomial sampling and a series system of k components with high reliability per component and approximately equal numbers of tests per component, we may for many applications develop a suitably accurate Poisson-chi-square confidence bound for system reliability. Here, the true reliability R_s of the series system is

$$R_s = \prod_{i=1}^{k} R_i \tag{21-38}$$

where R_i is the reliability of the ith component.

The chance of failure for the ith component is $p_i = 1 - R_i = 1 - q_i$, and since the p_i are relatively small (≤ 0.10) for high reliability, we have approximately that

$$R_s = \prod_{i=1}^{k} (1 - p_i) \approx 1 - \sum_{i=1}^{k} p_i . \tag{21-39}$$

Thus, for this application where the p_i are small and n_i the number of tests for the ith component relatively large, the expected number of failures λ_i for the ith component is $\lambda_i = n_i p_i$, and the chance of r_i failures for the ith component may be determined from the Poisson distribution

$$f(r_i) = Pr[r_i] = \exp(-\lambda_i) \cdot \lambda_i^{r_i} / r_i! . \tag{21-40}$$

As is well known, there exists an exact relation between the Poisson distribution and the chi-square distribution, so that percentage points (or probability levels) of the Poisson distribution may be obtained from the percentage points of chi-square. The relation between the cumulative distribution of Poisson probabilities and the upper tail areas of the chi-square distribution is

$$\sum_{x=0}^{r} \exp(-\lambda)\lambda^x/x! = [2^{r+1}\Gamma(r+1)]^{-1} \int_{2\lambda}^{\infty} (\chi^2)^{\nu/2-1}\exp(-\chi^2/2)\, d\chi^2 \tag{21-41}$$

where values of $\chi^2 = 2\lambda$ can be selected to equate the tail areas to $\alpha = 0.01, 0.05$, etc., and the number of degrees of freedom ν of chi-square is $\nu = 2(r+1)$. Further, when the results from the component tests are statistically independent, as they would likely be, then the degrees of freedom for chi-square, i.e., twice the failures plus one, may be used to give total series system equivalent degrees of freedom. Thus, the total number of failures, call it r, for k components in series

$$r = \sum_{i=1}^{k} r_i \tag{21-42}$$

also follows a Poisson distribution with parameter or expected total failures λ given by

$$\lambda = \sum_{i=1}^{k} \lambda_i = \sum_{i=1}^{k} n_i p_i \, . \tag{21-43}$$

If the n_i or number of tests per component are nearly equal, and failure rates p_i small as assumed, then we may use an average sample size $\bar{n}$ given by

$$\bar{n} = \sum_{i=1}^{k} n_i/k \tag{21-44}$$

and obtain an upper $(1-\alpha)$ confidence bound on the unknown Poisson parameter λ by finding that value of λ for which

$$\sum_{i=0}^{r} \exp(-\lambda)\lambda^i/i! = \alpha \tag{21-45}$$

where r = total number of failures from all the k components. Since the value of λ from Eq. 21-45, which we may call $\lambda_{1-\alpha}$, is an upper $(1-\alpha)$ confidence bound on $\lambda = \bar{n}\Sigma p_i$, then $\lambda_{1-\alpha}/\bar{n}$ is a $(1-\alpha)$ upper confidence bound on Σp_i. But by Eq. 21-39

$$1 - \Sigma p_i \approx R_1 \bullet R_2 \bullet \cdots \bullet R_k = R_s$$

the series system reliability. Therefore, a lower $(1-\alpha)$ confidence bound on R_s is given by

$$1 - \lambda_{1-\alpha}/\bar{n} = 1 - [1/(2\bar{n})]\chi^2_{1-\alpha}(2r+2)$$

or

$$Pr\{R_s \geq 1 - [1/(2\bar{n})]\chi^2_{1-\alpha}(2r+2)\} \approx 1 - \alpha \, . \tag{21-46}$$

Moreover, the $(1 - \alpha)$ upper confidence bound on R_s is similarly found to be

$$Pr\{R_s \leq 1 - [1/(2n)]\chi^2_\alpha(2r)\} \approx 1 - \alpha \; . \tag{21-47}$$

We illustrate with an example.

Example 21-5:

In tests on components of artillery projectiles, 197 in 200 primers tested functioned as intended, the propellant ignited properly in 185 out of 190 tests, 220 of 225 fuzes functioned, and the high explosive projectiles detonated "high order" in 204 of 210 cases. Since these actions are of a serial nature, what is the 95% confidence bounds on the "series" system or complete round reliability?

We have:

(primers)	$r_1 = 3$	$n_1 = 200$	$r = 3 + 5 + 5 + 6 = 19$
(propellant)	$r_2 = 5$	$n_2 = 190$	$k = 4$
(fuzes)	$r_3 = 5$	$n_3 = 225$	$\Sigma n_i = 825$
(projectiles)	$r_4 = 6$	$n_4 = 210$	$\bar{n} = \Sigma n_i/k = 825/4 = 206.25$.

By substituting these data into Eqs. 21-46 and 21-47, we find that

$$Pr[1 - \frac{1}{2\bar{n}}\chi^2_{0.975}(40) \leq R_s \leq 1 - \frac{1}{2\bar{n}}\chi^2_{0.025}(38)] \approx 0.95$$

or

$$Pr[0.856 \leq R_s \leq 0.944] \approx 0.95 \; .$$

Note also that the lower 97.5% confidence bound on complete round reliability is 0.856, and the best estimate of complete round reliability $\hat{R}_s$ is

$$\hat{R}_s = (197/200)(185/190)(220/225)(204/210) = 0.911 \; .$$

In connection with the methodology of this paragraph for series system confidence bounds and that of par. 21-3.3, we reemphasize that we have used available theory or approximations for only some very special cases. The general problem of determining confidence bounds on series, parallel, and complex system reliability based on binomial component test data is indeed a very involved one on which much effort has been placed in recent years by many investigators. Some of the investigations are listed in the references or bibliography with titular identifiable applications. For the interested reader, a survey report by Cox and Downs (Ref. 16) would be pertinent since they discuss and compare some of the more recent, promising methods. As a result of many, many comparisons with exact bounds, wherever available, it is becoming increasingly clear that the approximately optimum procedures of Mann and Grubbs (Ref. 17) can be applied to a majority of the systems under study—the procedures generally possess two decimal accuracy (which is sufficient for practice); they are relatively easy to calculate with a scientific type pocket calculator; and data for exponential time-to-fail cases may be easily converted to the binomial case or vice versa, thus indicating a rather unified approach to the whole subject. Moreover, the computation of confidence bounds by exact methods is almost impossible for all but very specialized models, and for these there are invariably problems such as the use of large amounts of computer time or the loss of precision in the computations.

21-3.5 APPROXIMATELY OPTIMUM CONFIDENCE BOUNDS FOR SERIES SYSTEMS WITH BINOMIAL DATA

For a rather general approach to the problem of placing confidence bounds on series (and parallel) systems with binomial (or exponential) failure data see Refs. 18, 19, 20, 21, 22, 23, 24[†], and 25[†] for the approximately optimum Mann-Grubbs techniques. The highlights of these procedures for binomial data are described.

Consider a series system of k subsystems or components, and let

n_i = number of tests on ith component

r_i = number of failures observed for ith component.

Then, Refs. 18-25 show that the negative logarithm $-\ln R_s$ of series system reliability follows a noncentral chi-square distribution which may be approximated by a central chi-square distribution. Furthermore, one needs only estimates of the mean and variance of that distribution to find confidence bounds on the true, unknown series system reliability R_s. There are two procedures to calculate the mean m_s and variance v_s for the series system case. The first is the simpler procedure and involves the determination of the *equivalent* sample size and *equivalent* number of failures for the *system*, due to Mann and Fertig (Ref. 23). The effective system sample size n_s^* is

$$n_s^* = \left[n_{(1)} \sum_{i=1}^{k} (1/n_i)\right] \Big/ \left[(0.5/n_{(1)}) + 0.5 \sum_{i=1}^{k} (1/n_i)\right] \tag{21-48}$$

and the equivalent system failures r_s^* is

$$r_s^* = n_s^* \hat{P}_s \tag{21-49}$$

where $n_{(1)}$ is the *smallest* of all the numbers of component tests $n_1, n_2, \ldots, n_k$; and $\hat{P}_s$ is the estimate series system unreliability

$$\hat{P}_s = 1 - \prod_{i=1}^{k} \hat{R}_i \tag{21-50}$$

and $\hat{R}_i = (n_i - r_i)/n_i$. Then, the mean m_s and variance v_s are determined from

$$m_s \approx \ln(n_s^* + 0.5) - \ln(n_s^* - r_s^* - 0.5) \tag{21-51}$$

and

$$v_s \approx -1/(n_s^* + 0.5) + 1/(n_s^* - r_s^* - 0.5)\,. \tag{21-52}$$

Eqs. 21-51 and 21-52 are recommended if m_s is greater than about 0.5. Otherwise, better estimates of m_s and v_s are determined from

$$m_s = 0.5(1 + 1/a)/n^0 + \hat{P}_s/(1 - 0.5\hat{P}_s) \tag{21-53}$$

[†]These references provide some background theory.

and

$$v_s = 0.5(1 + 1/a)m_s/n^0 \quad (21\text{-}54)$$

where

$$n^0 = n_{(1)}(1 - 0.5\hat{P}_s^2)(1 - 0.5\hat{P}_s) \quad (21\text{-}55)$$

$n_{(1)}$ is the smallest of the $n_1, n_2, \ldots, n_k$ and $\hat{P}_s = 1 - \prod_{i=1}^{k} \hat{R}_i$ is the maximum-likelihood estimate of the probability of failure of the series system. The quantity a is taken as

$$a = n_{(1)} \textstyle\sum\!\!\!\!/\, (1/n_i) \quad (21\text{-}56)$$

a restricted sum', whereas in calculating a confidence bound by the method of Buehler (Ref. 15), if a lower confidence bound on series system reliability R_s obtained by including a zero-failure subsystem is larger than that obtained by excluding the subsystem, the latter bound (the one excluding the zero-failure subsystem) is the correct one to use. For this reason, the n_i's for all zero-failure subsystems are omitted in calculating the value of $a = n_{(1)}\sum\!\!\!\!/\,(1/n_i)$ except for a single zero-failure subsystem with its sample size equal to $n_{(1)}$. It, too, is ignored if at least one of the other subsystems which exhibits some failures has the smallest sample size $n_{(1)}$, or if the lower confidence bound obtained by not ignoring it is larger than that obtained by ignoring it. The exclusion of some subsystems in calculating series-system confidence bounds also is necessitated when all sample sizes are large and most of the failures pertain to a single subsystem. In this case the bound obtained based on all subsystem failures can be larger than that based on the single subsystem exhibiting nearly all the failures.

Finally, the approximately optimum nonrandomized confidence bound on R_s is obtained by fitting the posterior distribution of $-\ln R_s$ with a noncentral chi-square distribution approximated by a central chi-square distribution and then using the Wilson-Hilferty transformation (Ref. 25) of chi-square to normality, i.e., obtaining the expression

$$Pr(R_s \geq \exp\{-m_s[1 - v_s/(3m_s)^2 + \eta_{1-\alpha}\sqrt{v_s}/(3m_s)]^3\}) \approx 1 - \alpha \quad (21\text{-}57)$$

where $\eta_{1-\alpha}$ is the $100(1 - \alpha)\%$ level of the standard normal distribution.

For the series system of Example 21-5, we have $n_{(1)} = 190$ and from Eq. 21-48 we calculate $n_s^* = 299.14$. Then $r_s^* = n_s^*\hat{P}_s = (299.14)(1 - 0.911) = 26.62$. Hence, the series system of four components tested is equivalent to obtaining 26.62 failures in 299 test trials for the entire system, and one might note here that the estimated system reliability is $1 - (26.62/299.14) = 0.911$, as before. Continuing, the system mean and variance from Eqs. 21-51 and 21-52 are

$$m_s = \ln(299.64) - \ln(272.02) = 0.09674$$

and

$$v_s = -1/299.64 + 1/272.02 = 0.000339 .$$

'A restricted sum is one in which the zero-failure subsystems are excluded.

However, we get an m_s smaller than 0.5, and hence check with $m_s = 0.09666$ from Eq. 21-53 and $v_s = 0.0003395$ from Eq. 21-54. Thus, in this particular case we get equivalent means and variances anyway, so that using either set and Eq. 21-57 with $\eta_{0.975} = 1.96$, we get

$$Pr[R_s \geq 0.873] \approx 0.95 \; .$$

Hence, the lower 97.5% confidence bound on series system reliability is 0.873, which is slightly higher and perhaps more accurate than the corresponding value of 0.856 obtained by the Poisson-chi-square approximation procedure of par. 21-3.4.

We begin to see a great advantage of this approach in that the equivalent sample size and number of failures for the whole system are determined, and indeed this gives an idea for analyzing more complex systems, as we will see.

21-3.6 APPROXIMATELY OPTIMUM CONFIDENCE BOUNDS FOR PARALLEL SYSTEMS (BINOMIAL FAILURE DATA)

For a system consisting only of parallel components, one proceeds to calculate the mean m_p and variance v_p for the system (Ref. 17), if one or more failures are observed for each component, from

$$m_p = -\ln[\hat{P}_p(1 + 0.5/r_p^*)/(1 + 0.5\,\hat{P}_p/r_p^*)] \tag{21-58}$$

and

$$v_p = [1 - \exp(-m_p)]/(r_p^* + 0.5) \tag{21-59}$$

where

$$\hat{P}_p = \prod_{i=1}^{k} (1 - \hat{R}_i) = \prod_{i=1}^{k} \frac{r_i}{n_i} \tag{21-60}$$

is the maximum likelihood estimate of the probability of parallel system failure, and r_p^*

$$r_p^* = 1/\sum_{i=1}^{k} (1/r_i) \tag{21-61}$$

is the equivalent number of failures for the parallel system. The equivalent sample size n_p^* for the parallel system, incidentally, is

$$n_p^* = r_p^*/\hat{P}_p \; . \tag{21-62}$$

(Should any subsystem or component be so unreliable that $r_i = n_i$, then such a component is ignored in calculating r_p^*.)

As for the series system, we see also for the parallel system that the equivalent or effective *system* failures and sample size are given by Eqs. 21-61 and 21-62, respectively, which may be used to build up component test data to the complex system level. Hence, we now have a means to obtain equivalent system parameters for combinations of series and parallel circuits.

If $\hat{P}_p = r_{(1)} = 0$, then

$$v_p = \psi'(r_{(1)} + 1) - 1/n'$$
$$= \psi'(1) - 1/n'$$
$$= 1.6449 - 1/n'$$

where

$$n' = \ell \left(\frac{\prod_{i=1}^{k} n_i}{\prod_{r_i \neq 0} r_i} \right)^{1/\ell} \quad \text{, using } r_i\text{'s} \neq 0$$

$$n' = \ell \left(\prod_{i=1}^{k} n_i \right)^{1/\ell} \quad \text{, if all } r_i = 0$$

$\psi'(y) \approx 1/y + 0.5/y^2 - 0.1\dot{6}/y^3 - 0.0\dot{3}/y^5 + 0.0238/y^7$
$\approx 1/(y - 0.5)$, trigamma function of y
ℓ = number of r_i's equal to zero
$r_{(1)}$ = smallest r.

In this case the mean value m_p for a parallel system is

$$m_p = \ln \left[2\ell \left(\frac{\prod_{i=1}^{k} n_i}{\prod_{r_i \neq 0} r_i} \right)^{1/\ell} \right] \quad \text{, using } r_i\text{'s} \neq 0$$

or

$$m_p = \ln \left[2\ell \left(\prod_{i=1}^{k} n_i \right)^{1/\ell} \right] \text{, if all } r_i = 0 .$$

The approximate equivalent or effective number of system failures and system sample size for a parallel system when $\hat{P}_p \neq 0$ are given by Eqs. 21-61 and 21-62, respectively. If $\hat{P}_p = 0$, then the effective system sample size is

$$n_p^* = \ell^\ell \left(\frac{\prod_{i=1}^{k} n_i}{\prod_{r_i \neq 0} r_i} \right) \text{, using } r_i\text{'s} \neq 0$$

*The "•" over a number indicates a repeating number.

or

$$n_p^* = \ell^{\ell}\left(\prod_{i=1}^{k} n_i\right) \text{, if all } r_i = 0 \text{ .}$$

An upper confidence bound on $(1 - R_p)$ is obtained for $\hat{P}_p > 0$ by making use of the fact that $-2m_p\ln(1 - R_p)/v_p$ has a distribution which is approximately distributed as chi square with $2m_p^2/v_p$ degrees of freedom, i.e., $-2\ln(1 - R_p)$ has a noncentral chi-square distribution. The Wilson-Hilferty transformation of chi square to normality can then be used by substituting values of m_p and v_p in

$$Pr\langle 1 - R_p \le \exp\{-m_p[1 - v_p/(3m_p)^2 - \eta_{1-\alpha}\sqrt{v_p}/(3m_p)]^3\}\rangle \approx 1 - \alpha \text{ .} \quad (21\text{-}63)$$

When $\hat{P}_p$ is equal to zero, the AO (approximate optimum) upper confidence bound for $(1 - R_p)$ obtained by substitution of calculated values of m_p and v_p in Eq. 21-63 must be raised to the ℓth power before subtracting from unity to obtain the corresponding and correct lower confidence bound on R_p.

These methods for estimating lower confidence bounds for series systems or parallel systems have been found to give very accurate results by checks with available bounds based on exact methods.

For illustrative purposes, we give two examples of nonrandomized confidence bounds for parallel system reliability which can be compared with known optimum lower confidence bounds. First, consider two elements in parallel and the data set of Example 21-4: $n_1 = n_2 = 10$, $r_1 = r_2 = 0$ (therefore $\ell = 2$). Here $\hat{P}_p = \prod_{i=1}^{2} (1 - \hat{R}_i) = 0$, so that $m_p = \ln[(2)(2)(10)] = \ln 40 = 3.689$, and $v_p = 1.645 - 0.05 = 1.595$. Hence the 90% upper AO confidence bound on $(1 - R_p)$ for $\ell = 2$ is

$$\langle\exp\{-3.688[1 - 0.0130 - 1.282(0.114)]^3\}\rangle^2 = 0.0126$$

for $\eta_{0.90} = 1.282$. (The quantity was squared because $\ell = 2$, i.e., two zero failure rates.) Thus, we see that the AO lower 90% confidence bound on R_p is 0.987, while the optimum bound is 0.988, indicating excellent agreement.

For a rather severe small sample case, consider two elements in parallel with $n_1 = n_2 = 2$, and $r_1 = 0$, $r_2 = 1$. For this case, the optimum lower 90% confidence bound is 0.54. Here, for the approximate bound, we have that the posterior mean of $-\ln(1 - R_p)$ is $m_p = \ln 8 = 2.0794$ and the posterior variance is $1.645 - 0.25 = 1.395$, so that the 90% upper confidence bound on $(1 - R_p)$ is $(0.46)^1 = 0.46$, thus agreeing exactly with the known value. (Here $\ell = 1$).

21-3.7 INTRODUCTION TO PROCEDURES FOR DETERMINING CONFIDENCE BOUNDS ON RELIABILITY FOR A COMPLEX SYSTEM (BINOMIAL DATA)

Since we learned for the series system of par. 21-3.5 and the parallel system structures of par. 21-3.6 how to determine the system overall or equivalent sample size n^* for the number of equivalent tests and number of failures r^*, we may now step forward to the procedure for calculating the parameters of a complex system from data on the subsystem series and parallel circuits. We illustrate with an example.

Example 21-6:

We will consider the relatively simple series-parallel arrangement of Fig. 21-6 for a power supply. In this application, battery actuator No. 1 worked properly in 49 of 50 tests, and battery actuator No. 3 had an observed success ratio of 47/50. Battery No. 2 and No. 4 had success rates of 49/50 and 50/50,

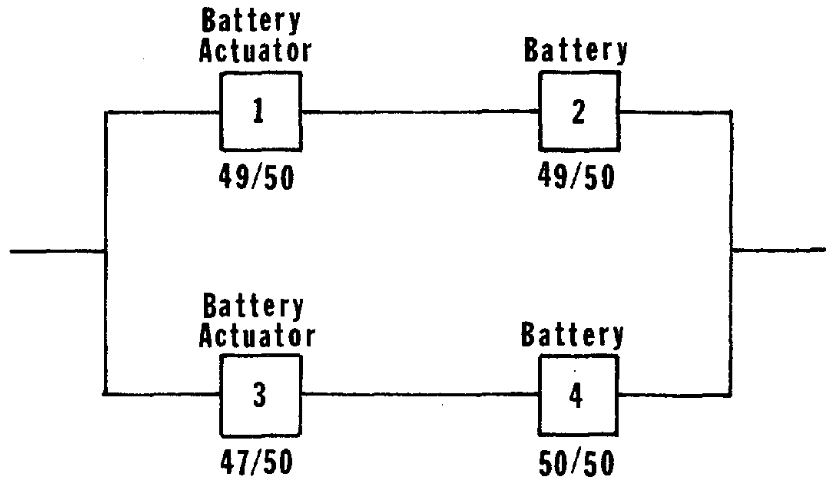


Figure 21-6. Series-Parallel Power Supply With Success Ratios

respectively. Our primary interest here is that of finding the lower 95% confidence bound on overall system reliability.

The computations are as follows.

Series Components 1 and 2:

$$\hat{P} = 1 - (49/50)^2 = 0.0396 \text{ (from Eq. 21-50)}$$

$$n^0 = 50[1 - 0.5(0.0396)^2][1 - 0.5(0.0396)] = 48.97 \text{ (from Eq. 21-55)}$$

$$a = 50(1/50 + 1/50) = 2 \text{ (from Eq. 21-56)}$$

$$n_s^* = 50/(0.5 + 0.5/2) = 66.67 \text{ (from Eq. 21-48)}$$

$$r_s^* = 66.67(0.0396) = 2.64 \text{ (from Eq. 21-49)}.$$

Equivalent subsystem for 1 and 2: $n_s^* = 66.67$, $r_s^* = 2.64$.

$$\hat{R}_s = (n_s^* - r_s^*)/n_s^* = 64.03/66.67 = 0.9604$$

$$m_s = 0.5(1 + 1/2)/48.97 + 0.0396/[1 - 0.5(0.0396)] \text{ (from Eq. 21-53)}$$

$$= 0.0557$$

$$v_s = 0.5(1 + 1/2)0.0557/48.97 = 0.000853 \text{ (from Eq. 21-54)}.$$

For $\eta_{0.95} = 1.645$ by Eq. 21-57:

$$Pr\left\langle R_s \geq \exp\left\{-0.0557\left[1 - \frac{0.000853}{(3 \times 0.0557)^2} + \frac{1.645\sqrt{0.000853}}{3 \times 0.0557}\right]^3\right\}\right\rangle$$

$$\approx 0.95$$

$$Pr[R_s \geq 0.895] \approx 0.95\,.$$

Summary for components 1 and 2: $r_1^* = 2.64$ and $n_1^* = 66.67$.

Series Components 3 and 4:

$$\hat{P} = 1 - (47/50)(50/50) = 0.06 \text{ (from Eq. 21-50)}$$

$$a = 50(1/50) = 1 \text{ (the component with } r = 0 \text{ is not included in restricted sum) (from Eq. 21-56)}$$

$$n^0 = 50[1 - 0.5(0.06)^2][1 - 0.5(0.06)] = 48.41 \text{ (from Eq. 21-55)} .$$

But the equivalent subsystem for 3 and 4 is $r_2^* = 3$, $n_2^* = 50$, using only the smallest size, 50, exhibiting failures.

Whole System: $r_1 = 2.64$, $n_1 = 66.67$ in parallel with $r_2 = 3$, $n_2 = 50$. For the equivalent parallel system of two "new" components we have:

$$r_p^* = 1/[(1/2.64) + (1/3)] = 1.404 \text{ (from Eq. 21-61)}$$

$$\hat{P}_p = (2.64/66.67)(3/50) = 0.00238 \text{ (from Eq. 21-60)}$$

$$n^* = 1.404/0.00238 = 590 \text{ (from Eq. 21-49)}$$

$$\hat{R}_{sys} = (n_p^* - r_p^*)/n_p^* = 0.9976$$

$$m_p = -\ln\left[\frac{0.00238\left(1 + \dfrac{0.5}{1.404}\right)}{1 + \dfrac{0.5(0.00238)}{1.404}}\right] = 5.738 \text{ (from Eq. 21-58)}$$

$$v_p = [1 - \exp(-5.738)]/(1.404 + 0.5) = 0.5235 \text{ (from Eq. 12-59)} .$$

As a check on the values of m_p and v_p:

$$m_p = \ln(n^* + 0.5) - \ln(r^* + 0.5) =$$

$$= \ln(590 + 0.5) - \ln(1.404 + 0.5) = 5.738$$

$$v_p = -1/(n^* + 0.5) + 1/(r^* + 0.5)$$

$$= -1/(590 + 0.5) + 1/(1.404 + 0.5) = 0.5235 .$$

Therefore, $\eta_{0.95} = 1.65$, by Eq. 21-63,

$$Pr\left\langle R_{sys} \geq 1 - \exp\left\{-5.738\left[1 - \frac{0.5235}{(3 \times 5.738)^2} - \frac{1.645\sqrt{0.5235}}{3 \times 5.738}\right]^3\right\}\right\rangle \approx 0.95$$

$$Pr\,[R_{sys} \geq 0.990] \approx 0.95 .$$

Thus, we have given a very accurate and useful procedure for converting series and parallel component binomial type data to equivalent system parameters, and fractional failures or sample sizes may be handled with ease for this general approach to complex system analyses. We will also show that exponential failure data—whether truncated for a fixed number of failures or for a fixed time—can be converted to binomial type data using the Mann-Grubbs approach (Ref. 17), and hence, the generality

of determining confidence bounds on system reliability for binomial or exponential data with this approach.

21-3.8 THE NEGATIVE BINOMIAL (WAITING TIME) RELIABILITY DISTRIBUTION

A very important distribution for the analysis of reliability "fail" or "succeed" type data, and one not used enough in practice, is the negative binomial "waiting time" distribution—often called the binomial waiting time distribution. For the ordinary binomial approach, the sample size n is fixed and on the basis of testing exactly n items we observe some random number of failures r, depending on the underlying p of the population or lot sampled. In contrast to this procedure of sampling and testing, we could specify in advance that items will be tested until a preset number of failures r are observed. Thus, we sample at random from the lot and test until $r = 1$ failure, premature, etc., occurs, or until $r = 2$ failures occur, etc. In this case, the number n of items tested will be a random variable—a form of sequential sampling. This procedure leads to the negative binomial waiting time distribution, which was used for target damage assessment in Chapter 10, par. 10-5, and target acquisition distribution in Chapter 11, par. 11-9. There must be at least r trials to obtain r failures, and the chance that the number of trials n will be $x \geq r$—where $x = r, r + 1, \ldots$ is a random variable—is given by

$$Pr[n = x] = f(x) = \binom{x-1}{r-1} p^r (1 - p)^{x-r} \, . \tag{21-64}$$

The minimum variance, unbiased estimator $\hat{p}$ of the unknown population failure chance p is (Ref. 26)

$$\hat{p} = (r - 1)/(n - 1) \, .$$

For $r > 1$, this estimate should be used, although clearly for $r = 1$, it makes no sense, and consideration should be given when $r = 1$ to the geometric distribution and the use of the maximum likelihood estimate $\hat{p}_{ML}$

$$\hat{p}_{ML} = r/n \tag{21-65}$$

which is biased toward too high a value (Ref. 27).

In order to obtain an upper confidence bound on the unknown p for waiting time sampling, and hence a lower bound on the reliability $R = 1 - p$, it is easy to show that the chance of n or more negative binomial trials to get r failures is actually equal to the chance of $(r - 1)$ or less failures in $(n - 1)$ trials for ordinary binomial sampling (see, for example, Ref. 9). Hence,

$$\sum_{t=n}^{\infty} \binom{t-1}{r-1} p^r (1 - p)^{t-r} = \sum_{t=0}^{r-1} \binom{n-1}{t} p^t (1 - p)^{n-1-t} \tag{21-66}$$

$$= I_{1-p}[(n - 1) - (r - 1), r] = I_{1-p}(n - r, r) \tag{21-67}$$

where

$I_{1-p}(n - r, r)$ = Karl Pearson's incomplete beta function (Ref. 11).

Thus, we might say that sampling until we get exactly r failures in n random trials for the negative binomial sampling scheme acts like ordinary binomial sampling for $(r - 1)$ failures in $(n - 1)$ fixed

trials. Moreover, to obtain a lower confidence bound on reliability for negative binomial sampling we replace r and n for ordinary binomial sampling by $(r - 1)$ and $(n - 1)$, respectively.

A lower $(1 - \alpha)$ confidence bound on R may be obtained by finding the value of $(1 - p)$ in Eq. 21-67 for which

$$I_{1-p}(n - r, r) = \alpha \tag{21-68}$$

or, that is, by a procedure similar to that for the straight binomial sampling as in par. 21-3.1.

For the important practical case where we sample until a single failure is observed, the lower $(1 - \alpha)$ confidence bound on the reliability $R = 1 - p$ can be shown to be

$$R = \alpha^{1/(n-1)} \tag{21-69}$$

and is therefore a higher lower-confidence bound on reliability than that given by Eqs. 21-27, 21-28, 21-30, or 21-31 for a single failure in n and ordinary binomial sampling. We illustrate with an example.

Example 21-7:

Suppose for a fixed sample size of 20 items tested we obtained a single failure on one hand, and on the other we happened to test until a single failure occurred and the random number of trials also turned out to be 20. Find 95% lower confidence bounds on the reliability of the item.

For ordinary binomial sampling with fixed $n = 20$ and $r = 1$, we find from Eq. 21-30, for example, that

$$Pr\left[\frac{38}{38 + 4F_{0.95}(4,38)} = 0.78 \le R\right] = 0.95$$

where we look up $F_{0.95}(4,38) = 2.619$ in Ref. 28, for example.

On the other hand, had it happened that we started to sample and tested until one failure was found in 20 trials, we get from Eq. 21-69 that

$$Pr[\alpha^{1/(n-1)} = (0.05)^{1/19} = 0.85 \le R] = 0.95$$

a much higher lower-95% confidence bound for the reliability. This indicates the efficiency of sequential sampling in saving on the number of tests—on the *average*. In fact, for the negative binomial sampling, we could have reached the decision that $Pr(R \ge 0.78)$ at the 95% confidence level in $n = 13$ trials, which is easily found by solving Eq. 21-69 for n, i.e.,

$$n = 1 + \ln \alpha / \ln R\ . \tag{21-70}$$

We repeat nevertheless that for negative binomial trials the sample size is a random variable to get r(fixed) failures, or prematures, etc., and that the expected number of trials is r/p. However, there will be occasions on which we might have to sample much beyond r/p trials to get r occurrences.

In any event, we cannot ordinarily afford to test huge sample sizes to guarantee high reliability or safety. Hence, one might adopt the policy of starting testing and continuing until a failure is observed, in which case the estimate of reliability $\hat{R}$ for n trials is taken as

$$\hat{R} \approx 1 - 1/n \tag{21-71}$$

and a lower $(1 - \alpha)$ confidence bound on R is found from Eq. 21-69. On the other hand, if after some "reasonable" amount of testing, one has not observed any occurrences in n random trials, he might consider stopping any further (expensive) testing and calculate a lower $(1 - \alpha)$ confidence bound on reliability from the left-hand side of Eq. 21-31, which would be

$$Pr[R \geq 1 - \frac{1}{2n}\chi^2_{1-\alpha}(2)] \approx 1 - \alpha \ . \qquad (21\text{-}72)$$

Table 21-2 gives some lower 95% confidence bounds for the case of zero failures in n (fairly large) trials.

For one failure in 100, for example, we would estimate the reliability to be 0.99; however, by Table 21-2 it requires zero failures in 300 to state with 95% assurance that the reliability is at least 0.99! For one failure in 100, we can state with 95% confidence that the lower bound on reliability by Eq. 21-72 is very nearly

$$1 - (1/200)\chi^2_{0.95}(4) \approx 0.953 \ .$$

In treating high reliability and safety problems, one invariably has to consider design and engineering details along with the statistical aspects of the problem. There are many papers on the general subject, and for some limited amount of orientation the reader might be interested in Simon's paper (Ref. 29) and that of Wuerfful and Dunphy (Ref. 30).

21-4 THE EXPONENTIAL RELIABILITY DISTRIBUTIONS

21-4.1 GENERAL CONSIDERATIONS

As contrasted to the success or failure type reliability analyses, we now turn to time-to-fail data or measurements on a continuous scale—such as hours to fail, miles to fail, cycles to fail, rounds to failure. For such data, the single parameter negative exponential distribution is widely used. It is based on the assumption that the failure rate is the same—i.e., constant over the entire life of the item—and yet the exponential density still describes many failure processes with sufficient accuracy for failure processes which are not exactly constant. The probability density function (pdf), or $f(t)$, easily is

TABLE 21-2. LOWER 95% CONFIDENCE BOUNDS ON RELIABILITY BASED ON ZERO FAILURES IN n TRIALS

Number of Tests n	Lower 95% Bound on R
50	0.940
100	0.970
200	0.985
300	0.990
400	0.993
500	0.994
1000	0.997
2000	0.9985
3000	0.9990
4000	0.9993
5000	0.9994
29957	0.9999

derived from the conditional chance that given an item has lived to any age t, the chance of failure in the next small time interval, t to $t + dt$, is a constant λ, the failure rate, multiplied by dt, i.e.,

$$\frac{f(t)dt}{1 - F(t)} = \lambda dt \tag{21-73}$$

where $F(t)$ is the cumulative distribution function. Since for the left-hand-side the numerator is the negative of the differential of the denominator, we may integrate easily and get

$$\left.\begin{aligned} \ln[1 - F(t)] &= -\lambda t \\ \text{or} \qquad F(t) &= 1 - \exp(-\lambda t) \end{aligned}\right\} \tag{21-74}$$

and

$$f(t) = \lambda \exp(-\lambda t) \, . \tag{21-75}$$

The quantity λ is the failure rate—e.g., one failure per thousand hours of operation, one per ten thousand cycles—and is the exponential population parameter. Also, for general usage we often will need the relation

$$\lambda = 1/\theta, \tag{21-76}$$

where θ is the true mean "time-to-fail" ($MTTF$).

For the exponential distribution, the chance $R(t_m)$ that an item lives to or beyond some mission time t_m is

$$R(t_m) = 1 - F(t_m) = \exp(-\lambda t_m) \tag{21-77}$$

and is the *reliability* of the item measured on the continuous scale.

The important moment characteristics of the exponential distribution to remember are:

$$\text{Mean} = E(t) = 1/\lambda = \theta \tag{21-78}$$

$$\text{Variance} = \sigma^2(t) = 1/\lambda^2 = \theta^2 \tag{21-79}$$

$$\text{Skewness} = \alpha_3(t) = 2 \tag{21-80}$$

$$\text{Kurtosis} = \alpha_4(t) = 9 \, . \tag{21-81}$$

We note that the distribution is highly peaked and skewed to the right (positive skewness only), and peaks at time zero, i.e., $f(0) = \lambda$. The exponential distribution is also the same as the chi-square distribution for 2 degrees of freedom ($\nu = 2$).

21-4.2 LIFE TESTING AND PARAMETER ESTIMATION

We will now develop the concept of life testing and also give the optimum procedure for estimating the single parameter $\theta = 1/\lambda$. Suppose we place n items, components or systems, on a life-test and let the time to failure of the ith designated item be t_i. Then consider testing until the rth item ($r \leq n$) fails,

where we may, for example, stop testing when the tenth item of 20 on test fails. In other words, we truncate the life test at the rth (10th) item since testing until the nth (20th) item fails may take too long or be too expensive in many practical applications. Now rank the r observed life-times in increasing time, or order of magnitude, i.e.,

$$t_1 \le t_2 \le t_3 \le \cdots \le t_r \le \cdots \le t_n \,. \tag{21-82}$$

An excellent account of exponential life testing is given in a basic paper of Epstein and Sobel (Ref. 31). They show that the maximum likelihood, minimum variance, unbiased estimator (MLMVUE) $\hat{\theta}$ of the mean time to fail θ is given by

$$\hat{\theta} = [\sum_{i=1}^{r} t_i + (n-r)t_r]/r = 1/\hat{\lambda} \,. \tag{21-83}$$

The numerator of Eq. 21-83, which we will denote by z, i.e.,

$$z = r\hat{\theta} = \sum_{i=1}^{r} t_i + (n-r)t_r \tag{21-84}$$

is the total time on test (TTOT) and a very important concept and characteristic indeed.

If testing continues until all n items placed on test have failed ($r = n$), then

$$\hat{\theta} = \sum_{i=1}^{n} t_i/n \tag{21-85}$$

but ordinarily, in applications, testing will be stopped for economical reasons at the rth ($r < n$) failure, which is called Type I censoring, or at a fixed time—often the mission time—which is referred to as Type II censoring.

It is well known (Ref. 31) that the quantity $2r\hat{\theta}/\theta$ is distributed in probability as chi-square with $2r$ degrees of freedom, or

$$2r\hat{\theta}/\theta = \chi^2(2r) \,. \tag{21-86}$$

With this knowledge, it becomes easy to obtain a confidence bound on the true unknown θ and the reliability $R(t_m)$ for a given mission time t_m. In fact, the following probability statements hold:

$$Pr[\chi^2_{\alpha}(2r) \le 2r\hat{\theta}/\theta \le \chi^2_{1-\alpha}(2r)] = 1 - 2\alpha \tag{21-87}$$

from which by simple algebraic manipulation we obtain the confidence bound on θ, i.e.,

$$Pr[2r\hat{\theta}/\chi^2_{1-\alpha}(2r) \le \theta \le 2r\hat{\theta}/\chi^2_{\alpha}(2r)] = 1 - 2\alpha \,. \tag{21-88}$$

Furthermore, since the true, unknown reliability at mission time t_m is from Eq. 21-77

$$R(t_m) = \exp(-\lambda t_m) = \exp(-t_m/\theta)$$

then Eq. 21-88 is easily manipulated into

$$Pr\{\exp[-t_m\chi^2_{1-\alpha}(2r)/(2r\hat{\theta})] \le R(t_m) \le \exp[-t_m\chi^2_{\alpha}(2r)/(2r\hat{\theta})]\} = 1 - 2\alpha \qquad (21\text{-}89)$$

which gives a confidence interval on the true, unknown reliability based on test data for a truncated or complete sample.

The mean and variance of the MLMVUE $\hat{\theta}$ are

$$E(\hat{\theta}) = \theta, \text{ and Var } (\hat{\theta}) = \theta^2/r\,. \qquad (21\text{-}90)$$

Thus, the precision of the estimator $\hat{\theta}$ depends only on the value of the parameter θ and the point of truncation r—hence not on the sample size n.

If we consider the random time to the first failure and the random times between failures, i.e.,

$$y_1 = t_1 \quad y_2 = t_2 - t_1, \ldots, \text{ and } y_r = t_r - t_{r-1};\ 2 \le r \le n \qquad (21\text{-}91)$$

then $y_1 = t_1$ and each of the quantities,

$$(n - r + 1)y_r = (n - r + 1)(t_r - t_{r-1}) \qquad (21\text{-}92)$$

follows the exponential distribution (Eqs. 21-74 and 21-75), as shown in Ref. 31, or each of the quantities y_1 and

$$2(n - r + 1)y_r/\theta = \chi^2(2) \qquad (21\text{-}93)$$

has an independent chi-square distribution with 2 degrees of freedom. Also, the times between failures—$y_2, y_3, \ldots, y_r$—can be considered as a random variable which is the smallest value in a random sample of size $(n - i + 1)$ from Eq. 21-74.

The random, ordered time t_r is the waiting time required to get the rth failure in a sample of n items put on test, and its mean and variance are, respectively,

$$E(t_r) = \theta \sum_{i=1}^{r} [1/(n - i + 1)] = m \qquad (21\text{-}94)$$

and

$$\text{Var}(t_r) = \theta^2 \sum_{i=1}^{r} [1/(n - i + 1)^2] = v\,. \qquad (21\text{-}95)$$

Epstein and Sobel (Ref. 31) give the probability density function of t_r in their equation (14); however, Mann and Grubbs (Ref. 32) give a simple, excellent (cube root) approximation

$$w = [t_r/(m\theta)]^{1/3} \qquad (21\text{-}96)$$

which for all practical purposes is normally distributed with mean

$$E(w) = 1 - v/(9m^2) \qquad (21\text{-}97)$$

and variance

$$\text{Var}(w) = v/(9m^2) \tag{21-98}$$

with m and v from Eqs. 21-94 and 21-95. Hence, one may use only any rth order statistic of the sample and obtain a confidence bound on the unknown θ, and also on the unknown reliability at mission time.

The approximation is very good for the degrees of freedom, $\nu = 2m^2/v$, equal to three or more, and the $100(1 - \alpha)\%$ percentile or probability level of t_r/θ is given very accurately by

$$m[1 - v/(9m^2) + \eta_{1-\alpha}\sqrt{v}/(3m)]^3 \tag{21-99}$$

with m given by Eq. 21-94, v by Eq. 21-95, and $\eta_{1-\alpha}$ the $(1 - \alpha)$ level of the standard normal deviate, as derived in Ref. 32.

Finally, the reliability at the rth waiting time t_r, or

$$R(t_r) = \exp(-t_r/\theta) \tag{21-100}$$

has a beta distribution, $I_R(n - r + 1, r)$, as in Eq. 21-22, and the unreliability or failure chance at the rth failure,

$$F(t_r) = 1 - R(t_r) = 1 - \exp(-t_r/\theta) \tag{21-101}$$

has a beta distribution with parameters r and $(n - r + 1)$. This indicates, therefore, a conversion relation between the continuous exponential failure time distribution and the discrete binomial probability distribution.

Epstein and Sobel (Ref. 31) indicate the waiting time advantages of placing more items on test than the expected number of failures an experimenter might aim for. For example, a test to ten failures in 20 items will last only 23% of the time to get ten failures from only ten items placed on test.

These details have been covered here to give the weapon systems analyst a sufficient background in exponential life-testing and reliability theory. In particular, there is wide-spread confusion on the use of terms "mean-time-to-fail" (*MTTF*) and "mean-time-between-failures" (*MTBF*); however, the prime interest should center around the population parameter θ, or $\lambda = 1/\theta$. The time to the *first* (*MTTF*) failure multiplied by the sample size n, and the times between successive (*MTBF*) failures multiplied by proper factors, $n - r + 1$, as in Eq. 21-92, each follow an exponential distribution with parameter θ as in Eq. 21-74. On the other hand, the distribution of the rth ordered time t_r is more complicated as indicated in Eqs. 21-94 to 21-101. Then again, one should be careful in not mixing or confusing sample values with population parameters. The terms *MTTF* and *MTBF* often do and should refer to the unknown parameter θ for the exponential distribution, whereas such a simple single characteristic or parameter is completely inadequate in describing a nonexponential (or two or more parameters) distribution such as the Weibull time-to-fail distribution. We give an example covering a typical application of the exponential theory.

Example 21-8:

Nineteen 1/4-ton trucks were given a grueling durability test until ten failed for one reason or the other. The mileages at failure were 46, 84, 155, 186, 277, 392, 423, 513, 590, and 661. Assume the failure data are exponentially distributed; estimate the parameter θ (the *MTTF*), the failure rate, and find the 95% lower confidence bound on vehicle reliability at 100 mi.

We have $n = 19$, $r = 10$. From Eq. 21-83 the estimate of θ is

$$\hat{\theta} = [\sum_{i=1}^{10} t_i + (19 - 10)(661)]/10 = 927.6 \text{ mi}.$$

The estimated failure rate $\hat{\lambda}$ is 1/927.6 or one failure per 928 miles. (Note that nine vehicles had not failed at 661 miles.)

To find the 95% lower confidence bound on vehicle reliability at 100 mi, we have $r = 10$, $t_m = 100$, $\hat{\theta} = 927.6$, and $\chi^2_{0.95}(20) = 31.41$. Hence, from Eq. 21-89 we obtain

$$Pr\langle R(100) \geq \exp\{-(100)(31.41)/[(2)(10)(927.6)]\} = 0.844\rangle \approx 0.95$$

or at 100 mi we could state with 95% confidence that 84.4% or 16 vehicles would still be functioning properly under such a "grueling" test. (Perhaps such accelerated life data could be transformed to normal operating conditions.)

21-4.3 CONFIDENCE BOUNDS ON RELIABILITY OF SERIES AND PARALLEL SYSTEMS WITH EXPONENTIAL FAILURE TIME DATA

21-4.3.1 Series System Approximate Theory

Now consider a system of k components in series for which we obtain failure-time data, exponentially distributed, for each component. We let

n_i = sample size for ith component

r_i = number of failures for the ith component, with $r_i \leq n_i$ and testing truncated at r_i failures

t_{ij} = jth ordered time-to-fail for data on the ith component.

where $i = 1, 2, \ldots, k$; and $j = 1, 2, \ldots, r_i \leq n_i$.

Now the total time on test for the ith component is

$$z_i = r_i\hat{\theta}_i = \sum_{j=1}^{r_i} t_{ij} + (n_i - r_i)t_{r_i} \qquad (21\text{-}102)$$

and we will let $z_{(1)}$ be the smallest time on test for all k component tests.

As is well known, the overall series system failure rate ϕ for the exponentially distributed failure data is the sum of component failure rates and is therefore

$$\phi = \sum_{i=1}^{k} \lambda_i = \sum_{i=1}^{k} 1/\theta_i \qquad (21\text{-}103)$$

where the ith component in series may have a failure rate λ_i, differing from the others.

With this background, Mann and Grubbs (Ref. 17) have shown that a lower $(1 - \alpha)$ confidence bound on series system reliability at mission time t_m is obtained accurately from the expression

$$Pr\langle R(t_m) = \exp(-t_m\phi) \geq \exp\{-t_m m[1 - v/(9m^2) + \eta_{1-\alpha}\sqrt{v}/(3m)]^3\}\rangle \approx 1 - \alpha \qquad (21\text{-}104)$$

where the simplified expressions of Mann (Ref. 20) for the mean m and variance v are

$$m = \sum_{i=1}^{k} (r_i - 1)/z_i + 1/z_{(1)} \tag{21-105}$$

and

$$v = \sum_{i=1}^{k} (r_i - 1)/z_i^2 + 1/z_{(1)}^2 \, . \tag{21-106}$$

(Athough we might well refer to the mean and variance for the series system as m_s and v_s, we drop the subscripts for simplicity here.)

We illustrate with an example.

Example 21-9:

Consider a system of three components in series for which $n_1 = 10$, $n_2 = 15$, and $n_3 = 20$ tests were conducted with failures of $r_1 = 2$, $r_2 = 3$, and $r_4 = 4$, and the total times on test were $z_1 = 23.059$, $z_2 = 32.504$, and $z_3 = 37.188$. What is the lower 95% confidence bound on the reliability for this series system for a mission time $t_m = 1$?

We find m and v from Eqs. 21-105 and 21-106, respectively, noting that $z_{(1)} = 23.059$, to be

$$m = 0.229 \quad \text{and} \quad v = 0.00782.$$

Finally, using Eq. 21-104 with $t_m = 1$ and $\eta_{0.95} = 1.645$, we obtain the lower 95% confidence bound on system reliability to be 0.676, whereas the exact value is known to be 0.671 (Ref. 17), giving excellent agreement.

21-4.3.2 Conversion Of Exponential Parameters To Equivalent Binomial Parameters

An advantage of the approach of Ref. 17 is that the exponential failure time data for each component may be converted to the equivalent binomial parameters or vice versa. In fact, Ref. 17 shows that for the Type I censoring (fixed number of failures r), the equivalence relations for binomial n_i^0 and r_i^0 for the ith component are

$$n_i^0 = z_i/t_m - 1 \text{ (equivalent test sample size)} \tag{21-107}$$

and

$$r_i^0 = r_i - 1 \text{ (equivalent failures)} \, . \tag{21-108}$$

In other words, for each component of a system with exponential failure data, we simply subtract one from the total time on test divided by the mission time to get the equivalent binomial test sample size n_i^0, and the equivalent binomial number of failures r_i^0 is that for the exponential data reduced by one. For the data of Example 21-9, the exponential to binomial conversions for $t_m = 1$ are:

$z_1 = 23.059$, $r_1 = 2$ transform to $n_1^0 = 22.059$, $r_1^0 = 1$

$z_2 = 32.504$, $r_2 = 3$ transform to $n_2^0 = 31.504$, $r_2^0 = 2$

$z_3 = 37.188$, $r_3 = 4$ transform to $n_3^0 = 36.188$, $r_3^0 = 3$.

Then the equivalent binomial parameters on the right may be used to find the system $n^* = 30.79$ from Eq. 21-48 and equivalent system failures $r^* = 5.54$ from Eq. 21-49 where

$$\hat{P}_s = 1 - \left(\frac{21.059}{22.059}\right)\left(\frac{29.504}{31.504}\right)\left(\frac{33.188}{36.188}\right) = 0.18 .$$

Further, from Eq. 21-51 we get $m_s = 0.234$ and from Eq. 21-52 $v_s = 0.00844$, so that the lower 95% confidence bound on the series system reliability using converted exponential to binomial data and Eq. 21-57 is 0.669—very close agreement with the values at the end of par. 21-4.3.1. Eqs. 21-53 and 21-54 could have been used to obtain $m_s = 0.234$ and $v_s = 0.00849$, which give the lower bound 0.668, or nearly the same result.

It should be noted that Eqs. 21-107 and 21-108 apply to individual component data. Hence, this means that the theory and approximations used heretofore for series, parallel, or complex system analyses based on binomial parameters can be used for the same system configurations and exponential failure data, once the conversion from exponential to binomial parameters has been made. In fact, if we use the same component exponential failure data of Example 21-9 and the transformed binomial parameters of the preceding paragraph, then for the same three components in *parallel*, we may find the mean $m_p = 7.584$ from Eq.* 21-58, variance $v_p = 1.224$ from Eq. 21-59, and a lower 50% confidence bound on parallel system reliability from Eq. 21-63 to be 0.9995. The interested reader may consult Ref. 17 for details.

As a matter of record, we give here other equations for the mean m_p and variance v_p to be used in Eq. 21-63 for the determination of the lower $(1 - \alpha)$ confidence bound on parallel system reliability, once the exponential total time on test z_i and number of failures r_i have been converted to the equivalent binomial n_i^o and r_i^o, respectively, by Eqs. 21-107 and 21-108. The expressions for the (parallel) mean and variance are approximately

$$m_p \approx \sum_{i=1}^{k} [\psi(n_i^o + 1) - \psi(r_i^o + 1)] + 1/(n_{(k)}^o + 1)$$

$$\approx \sum_{i=1}^{k} [\ln(n_i^o + 0.5) - \ln(r_i^o + 0.5)] + 1/(n_{(k)}^o + 1) \tag{21-109}$$

and

$$v_p \approx \sum_{i=1}^{k} [-\psi'(n_i^o + 1) + \psi'(r_i^o + 1) + 1/(n_{(k)}^o + 1)^2$$

$$\approx \sum_{i=1}^{k} [-1/(n_i^o + 0.5) + 1/(r_i^o + 1)] + 1/(n_{(k)}^o + 1)^2 \tag{21-110}$$

where $n_{(k)}^o$ is the *largest* of the n_i^o, and ψ and ψ' are, respectively, the digamma and trigamma functions which may be found in many tables of mathematical functions. The approximations in logarithms and reciprocals are usually quite accurate.

These results, it will be recalled, are for Type I censoring or truncation at a fixed (preset) number of failures. If the exponential life tests are stopped for each component test at preset and even unequal

*$\hat{P}_p$ and r_p^* are determined by Eqs. 21-60 and 21-61, respectively.

times t_{0i}, say, and by such times one observes r_i random failures for the ith component in n_i tests, then for a system mission time equal to t_m again, the binomial equivalents of the *fixed-time* (Type II) exponential failure data truncations are simply (Ref. 17)

$$n_i' = (t_{0i}/t_m)n_i \tag{21-111}$$

and

$$r_i' = r_i - 1\,. \tag{21-112}$$

These equivalent binomial parameters, n_i' and r_i', may be used to analyze series, parallel, or complex systems as already indicated, and the conversions are again simple to apply.

In summary, therefore, with the approach presented here, we may analyze complex system reliability for exponential failure data (either fixed number of failures or fixed time truncation) by reducing or transforming component test data to the equivalent binomial parameters. Indeed, it is clear that we may even work with systems involving a mixture of exponential and binomial type failure data—a considerable advantage also.

The exponential failure-time distribution is of central importance in reliability and life testing just as the normal distribution has been of central importance in other fields of statistical analysis. The exponential distribution is widely used for many applications of failure data, especially where insufficient data exist to indicate the form of the distribution sampled. The exponential assumption often is used for even large, repairable systems or for failure due to occasional, unpredictable environmental extremes. An excellent account of some general uses and types of applications where the exponential density applies is that of Davis (Ref. 33), which is highly recommended reading.

There are many applications involving failure-time data for which the smallest failure time cannot start at time $t = 0$, and thus it is necessary to consider the case where distribution starts at some positive value of failure time. For example, an army truck or tank may not fail as soon as delivered, but rather the minimum failure mileage may be 50, 100, 200, etc. For such cases, we must consider the two-parameter negative exponential distribution discussed in par. 21-4.3.3.

21-4.3.3 The Two-Parameter Negative Exponential Distribution

The probability density function for the two-parameter (negative) exponential distribution is given by

$$f(t) = (1/\theta)\exp[-(t - \nu)/\theta] \tag{21-113}$$

where the minimum life is $\nu > 0$, $\theta = MTTF$ as before, and $t \geq \nu$. If $\nu = 0$, then Eq. 21-113 becomes the single parameter exponential distribution of Eq. 21-75.

Since Eq. 21-113 involves only a shift in origin, or change of the location parameter, then only the mean of the exponential distribution Eq. 21-75 is moved, and it is now

$$\text{Mean} = E(t) = \nu + \theta\,. \tag{21-114}$$

The variance, skewness, and kurtosis of Eq. 21-113 are θ^2, 2, and 9, respectively, as in Eqs. 21-79, 21-80, and 21-81 for the single-parameter exponential distribution.

For the observed ordered failure times in a sample of n items placed on test, i.e.,

$$\nu \le t_1 \le t_2 \le t_3 \le \cdots \le t_r \le \cdots \le t_n \qquad (21\text{-}115)$$

then Epstein and Sobel (Ref. 34) show that the best estimates of θ and ν in the sense that they are unbiased with minimum variance are given by

$$\hat{\theta} = [\sum_{i=1}^{r} t_i + (n-r)t_r - nt_1]/(r-1) \qquad (21\text{-}116)$$

and

$$\hat{\nu} = t_1 - \hat{\theta}/n \ . \qquad (21\text{-}117)$$

Furthermore, the smallest observed time t_1 and the estimator $\hat{\theta}$ are statistically independent, and the following quantities follow chi-square distributions with the degrees of freedom (df) indicated:

$$2n(t_1 - \nu)/\theta = \chi^2(2), \qquad \text{(chi-square with 2 df)} \qquad (21\text{-}118)$$

and

$$2(r-1)\hat{\theta}/\theta = \chi^2(2r-2), \quad \text{(chi-square with } 2r-2 \text{ df)} \ . \qquad (21\text{-}119)$$

The true unknown reliability at mission time t_m is

$$R(t_m) = \exp[-(t_m - \nu)/\theta] \qquad (21\text{-}120)$$

and Grubbs (Ref. 35) gives the lower $(1-\alpha)$ confidence bound on $R(t_m)$ as

$$Pr\{R(t_m) \ge \exp[-v\chi^2_{1-\alpha}(2m^2/v)/(2m)]\} \approx 1-\alpha \qquad (21\text{-}121)$$

where the mean m and variance v are found from

$$m = (1/n) + (t_m - t_1)/\hat{\theta} \qquad (21\text{-}122)$$

$$v = (1/n^2) + (t_m - t_1)^2/[(r-1)\hat{\theta}^2] \qquad (21\text{-}123)$$

with $\hat{\theta}$, the estimate of the scale parameter, as in Eq. 21-116.

An example will be instructive.

Example 21-10. Nineteen military personnel carriers failed in service for one reason or the other at the following mileages: 162, 200, 271, 302, 393, 508, 539, 629, 706, 777, 884, 1008, 1101, 1182, 1463, 1603, 1984, 2355, and 2880 mi. (It is easy to show by plotting that these data follow a two-parameter negative exponential distribution.) Assume from experience that a critical mission mileage of 200 mi ordinarily is needed; estimate the 95% lower bound on the true unknown reliability, $R(t_m) = R(200) = \exp[-(200-\nu)/\theta]$. For this problem, we find from Eq. 21-116 that $\hat{\theta} = 881.6$ and from Eq. 21-117

that $\hat{\theta} = 115.6$, so that from Eq. 21-122, $m = 0.09573$, and from Eq. 21-123 $v = 0.002873$. Finally, by using Eq. 21-121 we get $Pr[R(200 > 0.821] \approx 0.95$, or we state with 95% confidence that 82.1% of the vehicles will not fail in 200 mi (here $\chi^2_{0.95}(2m^2/v) = 13.168$).

The reader may note that confidence bounds can be placed on the unknown minimum life ν by using Eq. 21-118, and on the unknown mean-time-to-fail parameter θ by using Eq. 21-119. Mann and Grubbs (Ref. 32) also show that by using only two of the 19 ordered statistics, i.e., $t_1 = 162$, and $t_{18} = 1603$, then approximate confidence bounds with suitable accuracy may be calculated for each ν and θ.

21-5 EXPONENTIAL LIFE TESTING AND THE POISSON PROCESS

We have seen that in exponential life testing, the items on test have a "memoryless" or forgetfulness of age property, or, that is, no matter what the age of an item is, the failure rate is constant and equal to λ or $1/\theta$. Thus, if we divide the time-on-test axis or the life of an item into equal time intervals T, then the expected number of failures in any one of the equal periods T is λT. Furthermore, it is easily seen that the observed number of failures r in time interval T is a random variable which follows a Poisson distribution with mean $m = \lambda T$, i.e.,

$$Pr(r) = f(r) = \exp(-m)m^r/r! \ . \qquad (21\text{-}124)$$

The characterization just described often is referred to as a homogeneous Possion Process. Had the expected number of occurrences in each equal time interval been different, as for example for a Weibull density, then the term "nonhomogeneous" Poisson Process would apply.

The moment properties of interest for the Poisson distribution are

$$\text{Mean} = E(r) = m \qquad (21\text{-}125)$$

$$\text{Variance} = \text{Var}(r) = m \qquad (21\text{-}126)$$

$$\text{Skewness} = \alpha_3 = 1/m^{1/2} \qquad (21\text{-}127)$$

$$\text{Kurtosis} = \alpha_4 = 3 + 1/m \ . \qquad (21\text{-}128)$$

Hence, the mean and variance are equal (which helps to detect the existence of a Poisson distribution), and for the Poisson distribution to approach normality a large value of the expected number of failures are required. Finally, we have observed in par. 21-3.4 the relation between the Poisson and chi-square distributions, i.e.,

$$Pr(r \le x) = Pr[\chi^2(2x + 2) \ge 2m] \ . \qquad (21\text{-}129)$$

It can be shown that if one observes a Poisson process for a fixed length of time T and if r events occur in $[0,T]$ at times $t_1 \le t_2 \le \cdots t_{r-1} \le t_r \le T$, then these times (after being subjected to a random permutation) can be considered as r independent observations on a random variable uniformly distributed over $(0,T)$. For r even moderately large, $\sum_{i=1}^{r} t_i$ is approximately normally distributed with mean $rT/2$ and variance $rT^2/12$. These properties, therefore, can be used to test whether or not the data are drawn from a Poisson process.

One also can show that if one observes a Poisson process until exactly r events (failures) occur (where r is now a preassigned integer) and if the events occur at $t_1 \le t_2 \le \cdots \le t_r$, then the $(r-1)$ random variables $t_1, t_2, \ldots, t_{r-1}$ can be considered, when unordered, as $(r-1)$ independent observations on a random variable which is uniformly distributed over $(0,t_r)$. For r moderately large, $\sum_{i=1}^{r-1} t_i$ is approximately normally distributed with mean $(r-1)t_r/2$ and variance $(r-1)t_r^2/12$. As before, these properties can be used to test whether or not the underlying distribution is Poisson.

In the context of life testing, it is clear that the comments just made for Poisson processes apply directly in case the items on test are replaced immediately by new items. This gives rise, of course, to a Poisson process with constant failure rate and we return to the situation just treated. If it happens that we are dealing with a nonreplacement situation (where failed items are not replaced), then all we need to do is to use total lives $T(t_i)$, rather than the failure times t_i. That is, if $T(t_i)$ is the total life observed in getting the ith failure, then $T(t_1) \le T(t_2) \le \cdots \le T(t_r)$. In the case where the life test starts with n items and is terminated at a preassigned total life T^*, the number of failures observed will be a random variable r with $T(t_1) = nt_1$; $T(t_2) = t_1 + (n-1)t_2; \ldots;$ $T(t_r) = t_1 + t_2 + \cdots + t_{r-1} + (n-r+1)t_r$. As before, one can show that the total lives $T(t_1), T(t_2), \ldots, T(t_r)$ can be considered as being drawn from a density function which is uniform over $[0,T^*]$. If the life test ends as soon as the first r failures occur, then the $(r-1)$ random variables $T(t_1), T(t_2), \ldots, T(t_{r-1})$ can be considered as being drawn from a density function which is uniform over $[0,T(t_r)]$.

The fact that the conditional distribution of total lives is uniform over suitable intervals makes it quite evident that one has a good tool for detecting whether the failure rate is indeed constant (as it must be in the pure exponential case). Thus, for example, the contamination of a purely exponential distribution by early failures would manifest itself in a pronounced tendency to get too many failures clustering together in the early part of the time or total life axis, thus violating uniformity. If the underlying distribution is really described by a two-parameter exponential with $\nu > 0$, then we should expect to get too few failures occurring in the early part of the uniform distribution, thus violating uniformity. If the failure rate changes, for example increases with time, then this should result in a tendency for failures to cluster together as time goes on—again violating uniformity. For a case in point, suppose that half-way through the life test the failure rate increases by a factor of two; then one can expect to find roughly twice as many points in the second half of the interval as in the first, and this would violate uniformity.

If the amount of failure data observed is quite small, then we can expect to detect only fairly large changes from exponentiality by the preceding tests. If we have a substantial amount of data, one can use a χ^2 test to detect whether the conditional distribution of times-to-failure or total lives (whichever is appropriate) deviate excessively from that of being uniform.

These characterizations and properties should give the young systems analyst a good understanding and much appreciation of exponential life-testing and the so-called Poisson Process.

21-6 THE LOGNORMAL RELIABILITY OR TIME-TO-FAIL PROBABILITY DISTRIBUTION

A distribution which has come into considerable usage in reliability and life-testing problems in recent years is the so-called lognormal distribution.

If life-test data on times-to-fail t are such that the logarithm to the base e, or $\ln t$, of the times-to-fail are normally distributed, then $\ln t$ follows a lognormal distribution and the probability density function of $\ln t$ is given by

$$f(\ln t) = (\sqrt{2\pi}\,\sigma_\ell\, t)^{-1} \exp[-(\ln t - \mu_\ell)^2/(2\sigma_\ell^2)] \tag{21-130}$$

where μ_ℓ and σ_ℓ are the true mean and standard deviation, respectively, expressed as logarithms.

Note that

$$\int_{-\infty}^{\infty} (\sqrt{2\pi}\,\sigma_\ell)^{-1} \exp[-(\ln t - \mu_\ell)^2/(2\sigma_\ell^2)]d(\ln t) = 1$$

as it should for a probability distribution.

If the logarithms of the times-to-fail are normally distributed, then we may use all known probability distribution theory for normal variates on the transformed scale ($\ln t$) which often may be an advantage in practice.

An important question centers around the relation between moments on the logarithmic scale and the corresponding moments on the original time scale.

Thus, to find the expected value $E(t)$ or mean on the original scale t, we have

$$E(t) = \int_{-\infty}^{\infty} (\sqrt{2\pi}\,\sigma)^{-1} t \exp[-(\ln t - \mu_\ell)^2/(2\sigma_\ell^2)]d(\ln t)$$

and we let

$$y = (\ln t - \mu_\ell)/\sigma_\ell$$

$$\sigma_\ell\, dy = dt/t$$

$$t = \exp(\sigma_\ell y + \mu_\ell)$$

then

$$\left.\begin{aligned} E(t) &= \int_{-\infty}^{\infty} (\sqrt{2\pi}\,\sigma_\ell)^{-1}\exp(\mu_\ell + \sigma_\ell y)\bullet\exp(-y^2/2)\sigma_\ell dy \\ &= \exp(\mu_\ell + \sigma_\ell^2/2)\ . \end{aligned}\right\} \tag{21-131}$$

In other words, the mean on the original time scale is found by raising e to the power equal to the sum of the mean and half the variance on the logarithmic (lognormal) or transformed scale.

For the second moment about the origin,

$$\left.\begin{aligned} E(t^2) &= \int_{-\infty}^{\infty} (\sqrt{2\pi}\,\sigma_\ell)^{-1} \exp[-(\ln t - \mu_\ell)^2/(2\sigma_\ell^2)]t^2 d(\ln t) \\ &= (\sqrt{2\pi}\,\sigma_\ell)^{-1} \int_{-\infty}^{\infty} \sigma_\ell \exp(-y^2/2)\bullet\exp(2\mu_\ell + 2\sigma_\ell y)dy \\ &= \exp[2(\mu_\ell + \sigma_\ell^2)]\ . \end{aligned}\right\} \tag{21-132}$$

For the variance, we get

$$\mathrm{Var}(t) = \exp[2(\mu_\ell + \sigma_\ell^2)] - \exp(2\mu_\ell + \sigma_\ell^2) . \tag{21-133}$$

The kth moment about the origin is easily found to be

$$E(t^k) = \exp(k\mu_\ell + k^2\sigma_\ell^2/2) \tag{21-134}$$

relating the two lognormal moments with all original moments. Finally, it is seen that one may rather easily transform probability statements made on the lognormal scale back to the original scale of the data, or vice versa.

The lognormal distribution is used often in fatigue life problems, times-to-repair, bearing life, semiconductor life, and generally where the log transform seems natural or easy. The lognormal distribution is somewhat similar in shape to that of the Weibull distribution, especially in the central region. A thorough coverage of the lognormal distribution is given by Aitchison and Brown (Ref. 36), who also discuss its history in science.

The lognormal distribution has a very interesting property in connection with the problem of improving the mean useful life of items by eliminating or screening out those items with short lives. If the items have life times which are lognormally distributed, then it is always possible to increase the mean life of the remaining items to any extent desired by continuing to test all the items until a sufficient large number of articles have failed, as shown by Watson and Wells (Ref. 37). The improvement may be very costly indeed, however, since it depends on the value of the standard deviation of the population, and after screening there may be only a very small fraction of the original population remaining for actual use. Moreover, it is often difficult to distinguish in some applications that the lognormal distribution is a more proper assumption than the gamma or Weibull, for example.

Aitchison and Brown (Ref. 36) discuss truncation and censoring in their book, and Cohen addresses the matter of progressively censored sampling for the case of the three-parameter lognormal distribution (Ref. 38).

For complete samples, then one may estimate the mean and standard deviation based on the transformed data, which is assumed to be normal, and hence obtain an approximate law or distribution, converting probability statements back to the original scale as desired. For complete samples, the estimates of the mean and variances are

$$\hat{\mu}_\ell = \text{estimate of } \mu_\ell = (1/n) \sum_{i=1}^{n} \ln t_i = \overline{\ln t_i} \tag{21-135}$$

$$\hat{\sigma}_\ell = \text{estimate of } \sigma_\ell = \{[n \sum_{i=1}^{n} (\ln t_i)^2 - (\sum_{i=1}^{n} \ln t_i)^2]/[n(n-1)]\}^{1/2} . \tag{21-136}$$

We give an example for a complete sample.

Example 21-11:

Twenty high-velocity antitank guns of a new design were fired at 150% (50% excess) pressure and the tubes ruptured or failed at the following number of rounds: 365, 610, 420, 840, 334, 792, 589, 472, 190, 662, 1102, 850, 1195, 2063, 1240, 900, 1303, 960, 1807, and 1342. These data were plotted and did

not seem to follow an exponential distribution. Hence, fit the rounds at failure with a lognormal distribution, and determine the number of rounds at which 95% of such gun tubes would survive beyond.

The ordered rounds to failure or rupture of the tubes are given in Table 21-3 with logarithms to the Naperian base.

An estimate of the mean $\hat{\mu}_\ell$ on the log scale is given by Eq. 21-135

$$\hat{\mu}_\ell = \frac{1}{n}\sum_{i}^{n} \ln t_i = \frac{1}{20}(132.982) = 6.649 .$$

An estimate of the standard deviation $\hat{\sigma}_\ell$ on the log scale is given by Eq. 21-136.

$$\hat{\sigma}_\ell^2 = \frac{20(891.104) - 17683.946}{(20)(19)}$$

$$\hat{\sigma}_\ell = 0.602 .$$

In general, the reliability at time t_0 is defined as

$$R(t_0) = \int_{t_0}^{\infty} f(t)dt, \text{ which we want to equal } 0.95.$$

Since we assume that the logarithms of the times-to-fail are distributed normally, we look for the point $\ln t_0$ on the log scale such that

$$\frac{\ln t_0 - \hat{\mu}_\ell}{\hat{\sigma}_\ell} = -1.645 .$$

TABLE 21-3. EXAMPLE DATA

i	t_i	$\ln t_i$
1	190	5.247
2	334	5.811
3	365	5.900
4	420	6.040
5	472	6.157
6	589	6.378
7	610	6.413
8	662	6.495
9	792	6.675
10	840	6.733
11	850	6.745
12	900	6.802
13	960	6.867
14	1102	7.005
15	1195	7.086
16	1240	7.123
17	1303	7.172
18	1342	7.202
19	1807	7.499
20	2063	7.632
	Total =	132.982

Solving for $\ln t_0$, we get, therefore, that $\ln t_0 = -1.645\,\hat{\sigma}_\ell + \hat{\mu}_\ell$. Finally, on the original rounds-to-fail scale, this means that

$$t_0 = \exp[-1.645(0.602) + 6.649] = 286.7$$

or we estimate that 95% of such gun tubes would survive beyond 287 rounds at 150% of the normal pressure.

Without going into details here, it is clear that since the logarithms are normally distributed, then one may easily place confidence bounds on the lognormal sigma by using the chi-square statistic, or place confidence bounds on the lognormal mean by using Student's t statistic, as demonstrated in standard textbooks on statistics.

Finally, a remark about censored samples. Since the logarithms of failure times are assumed to be normally distributed, one can use tabulated weights for multiplying the logarithms of the first r-of-n ordered observations or failure times to obtain either the best linear unbiased or best linear invariant estimates of the lognormal mean and lognormal sigma. Such weights are given for the best linear unbiased estimates in Sarhan and Greenburg (Ref. 39), and Section 3.6 of Mann, Schafer, and Singpurwalla (Ref. 40) may be used to determine the best linear invariant estimates. Hence, the theory for linear estimation based on order statistics for the normal population may be applied to many problems for which the lognormal distribution is suitable on practical grounds.

Originally, the lognormal distribution was applied to and seemed to be a rather natural development for fatigue studies and the formation of cracks in materials. However, the Weibull distribution, which captured more attention, also seemed to apply well to such practical problems and many others too. Hence the lognormal distribution will not be pursued further.

21-7 THE TWO-PARAMETER GAMMA RELIABILITY DISTRIBUTION

The two-parameter gamma (or Pearson Type III) reliability distribution has the density function

$$f(t) = t^{\beta-1}\exp(-t/\theta)/[\theta^\beta\Gamma(\beta)] \qquad (21\text{-}137)$$

with $t > 0$, β and $\theta > 0$. Note that if the shape parameter $\beta = 1$, then this special case of the gamma distribution becomes identically the exponential density of Eq. 21-75. An advantage of the gamma distribution in reliability and life testing is that it can take on many different shapes to fit various failure data simply by a change in the shape parameter β.

The moment parameters of interest here are

$$\text{Mean} = E(t) = \beta\theta \qquad (21\text{-}138)$$

$$\text{Variance} = \text{Var}(t) = \beta\theta^2 \qquad (21\text{-}139)$$

$$\text{Skewness} = \alpha_3 = 2/\sqrt{\beta} \qquad (21\text{-}140)$$

$$\text{Kurtosis} = \alpha_4 = 3(\beta + 2)/\beta\,. \qquad (21\text{-}141)$$

We note that as the shape parameter β increases, then the skewness becomes smaller and smaller, and the kurtosis approaches 3, so that the gamma density approaches the normal density fairly rapidly.

For complete, or uncensored samples, the general gamma distribution is easily fitted in terms of the calculated estimate of skewness, or

$$\hat{\alpha}_3 = (1/n) \sum_{i=1}^{n} (t_i - \bar{t})^3/[(1/n) \sum_{i=1}^{n} (t_i - \bar{t})^2]^{3/2} \tag{21-142}$$

where
$\bar{t}$ = average of the t_i.
and "Salvosa's" form of the Type III frequency curve described by Carver (Ref. 41), which is

$$f(t) = \left(\frac{A^{A^2}}{\Gamma(A^2)}\right) e^{-At}\, t^{A^2-1} \tag{21-143}$$

where

$$A = 2/\hat{\alpha}_3 \,. \tag{21-144}$$

Thus, one needs to compute only $\hat{\alpha}_3$ from Eq. 21-142 to get a suitable fit and enter the tables of Ref. 41 with the calculated value of $\hat{\alpha}_3$ to obtain all the desired probabilities. This procedure is very simple indeed as compared to the more complex one of using Pearson's Tables of the incomplete gamma function (Ref. 42). The integral of Eq. 21-143 from zero to t_0, say, may be related to Karl Pearson's incomplete gamma function $I(u,p)$ of Ref. 42, as follows:

$$I(t_0/A, A^2 - 1) = \int_0^{t_0} f(t)dt \,. \tag{21-145}$$

As an exercise, the reader may use the data of Example 21-11, fit the approximate gamma distribution, and estimate the number of rounds beyond which 95% of the gun tubes would survive in order to compare results with that of the lognormal distribution.

Cohen (Ref. 43) discusses censored sampling for the three-parameter gamma distribution.

Again, the gamma distribution, like the lognormal distribution, has not been as widely used in practice as the Weibull distribution, which we discuss next in par. 21-8.

21-8 THE TWO-PARAMETER WEIBULL RELIABILITY DISTRIBUTION

21-8.1 INTRODUCTION

By far, the distribution of most current interest and most widely investigated in recent years for reliability and life testing is the two-parameter Weibull distribution. It is derived easily from a consideration of a variable conditional failure or hazard rate $h(t)$, instead of the constant failure rate λ for the exponential distribution. In fact, for the conditional failure density or hazard function given by

$$h(t) = \frac{f(t)}{1 - F(t)} = \beta\lambda t^{\beta-1} \tag{21-146}$$

we can see that the chance of failure between time t and $(t + dt)$ depends on the life t raised to a power $\beta - 1$, with β and λ positive (greater than zero), and the time $t \geq 0$.

Integration of Eq. 21-146 leads to the cumulative two-parameter Weibull failure law:

$$F(t) = 1 - \exp(-\lambda t^{\beta}) = 1 - \exp(-t^{\beta}/\theta) = 1 - \exp[-(t/\delta)^{\beta}] \tag{21-147}$$

where $\lambda = 1/\theta$ as before, $\theta = \delta^{\beta}$, and the quantity δ is widely referred to as the "characteristic life". (Note that when $t = \delta$, the characteristic life, then $F(t) = F(\delta) = 1 - \exp(-1) = 0.63$; or 63% of the items fail before the characteristic life is reached and 37% survive beyond δ). We have listed the three slightly different forms in Eq. 21-147, since authors have different preferences in the literature for approaches to parameter estimation.

If one deals with logarithms of the failure times, i.e., makes the transformation

$$x = \ln t \tag{21-148}$$

then

$$Pr(T \leq t) = Pr(X \leq x) = F(x) = 1 - \exp\{-\exp[(x - u)/b]\} \tag{21-149}$$

where $u = \ln\delta$ and $b = 1/\beta > 0$. The cumulative distribution Eq. 21-149 is called Gumbel's extreme value distribution, and it is clear that if one can estimate u and b using the extreme value theory, then estimates of the shape parameter β and scale parameter δ (or θ or λ) for the Weibull distribution easily follow, or vice versa.

The probability density function is found by differentiating Eq. 21-147, and in terms of θ is given by

$$f(t) = (\beta/\theta)t^{\beta-1}\exp(-t^{\beta}/\theta)\ . \tag{21-150}$$

The kth moment about the origin easily is found to be

$$E(t^{k}) = \theta^{k/\beta}\Gamma[(k/\beta) + 1] \tag{21-151}$$

so that the mean and variance of the Weibull distribution are

$$\text{Mean} = E(t) = \theta^{1/\beta}\Gamma(1 + 1/\beta) \tag{21-152}$$

$$\text{Var} = \theta^{2/\beta}[\Gamma(1 + 2/\beta) - \Gamma^{2}(1 + 1/\beta)]\ . \tag{21-153}$$

The exponential distribution is a special case of the Weibull distribution, i.e., when the shape parameter $\beta = 1$, and if β is known, then the random variable t^{β} would be exponentially distributed.

The Weibull distribution can take on a very wide variety of shapes as shown in Fig. 21-7. In fact, the shapes vary from the subexponential ($\beta < 1$), to the exponential ($\beta = 1$), to the normal ($\beta \approx 3.\dot{3}$), and even to the more peaked curves. Hence, one has available a whole family of distributions with only two parameters to study reliability and life testing problems. If t is replaced by $(t - \mu)$ in Eq. 21-150, with the failure time $t \geq \mu$, then we have the three parameter Weibull distribution, which has a location parameter μ, a scale parameter θ (or δ), and a shape parameter β.

It is easily seen that by taking logarithms twice, Eq. 21-147 may be put in a linear form

$$\ln\{-\ln[1 - F(t)]\} = -\beta\ln\delta + \beta\ln t\ . \tag{21-154}$$

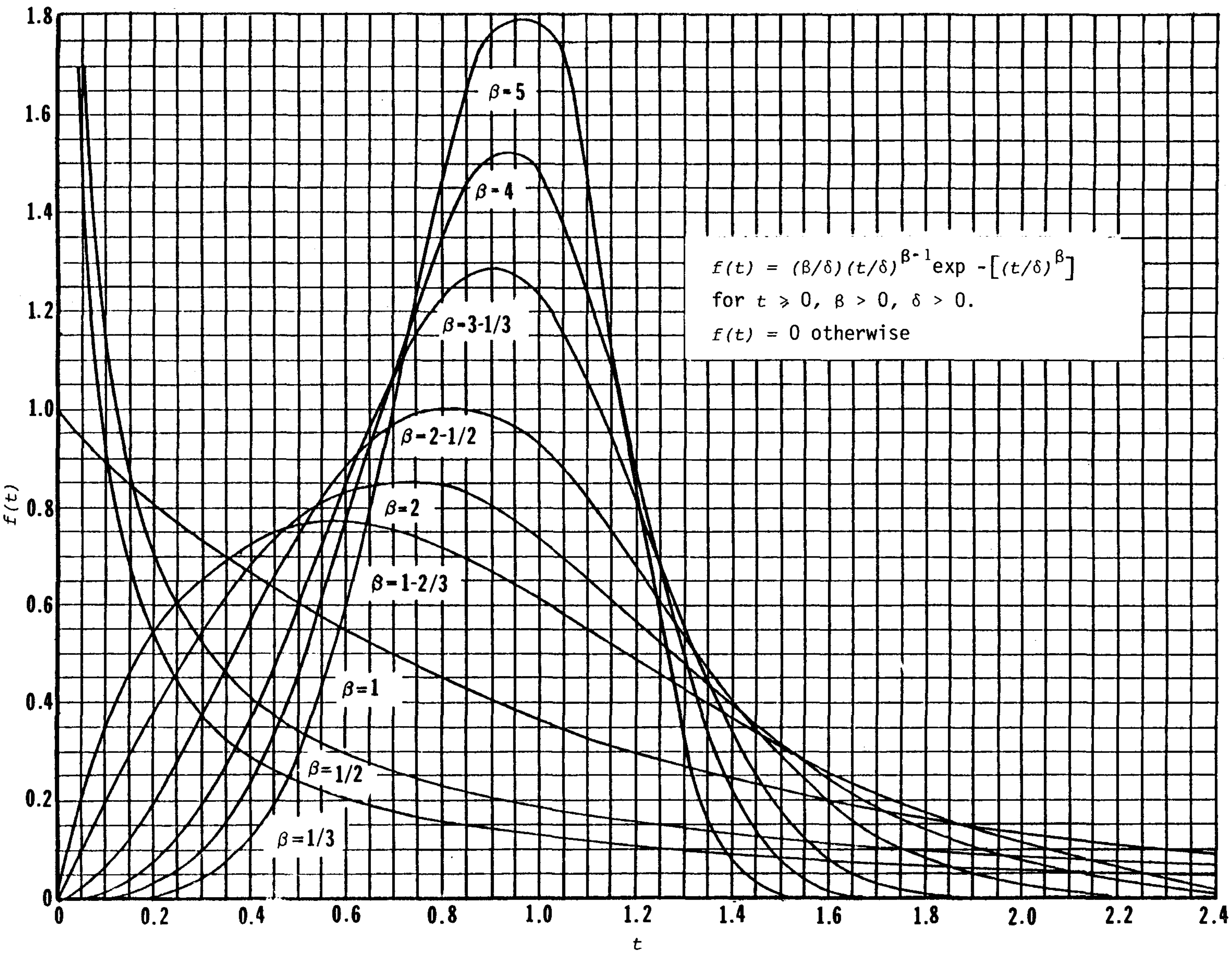


Figure 21-7. Plot of the Weibull Probability Density Function for Various Values of β ($\delta = 1$)

Hence, a plot of y_i = left-hand side of Eq. 21-154 versus $\ln t_i$, the logarithms of the failure times, may for suitably large amounts of data be used to estimate the shape parameter β (now the slope of the line), and also the characteristic life δ from the fitted intercept $-\beta\ln\delta$ of the line. In general, we will be interested in the case of censored samples, where for n items placed on test we observe the first $r < n$ failures and stop testing. Hence, we get the failure times, $t_1 \le t_2 \le \cdots \le t_r$.

21-8.2 POINT AND INTERVAL ESTIMATION OF PARAMETERS

There are a number of procedures for estimating the shape and scale parameters of the Weibull distribution (or the scale and location parameter for the extreme value distribution). First, we discuss a very simple estimate of the shape parameter due to Jaech (Ref. 44), and then some of the more efficient estimators.

21-8.2.1 Jaech's Simple Estimator

Jaech (Ref. 44) suggested a very simple estimate of the Weibull shape parameter based on only the smallest life t_1 and the second smallest life t_2. It is

$$\text{Est. of } \beta = 1/\ln(t_2/t_1) \ . \tag{21-155}$$

An approximate $(1 - 2\alpha)$ confidence bound on β for suitably large sample size n is given by

$$-\ln(1 - \alpha)/\ln(t_2/t_1) \le \beta \le -\ln(\alpha)/\ln(t_2/t_1) \ . \tag{21-156}$$

Jaech recommends "large" sample sizes n to use Eqs. 21-155 and 21-156 successfully. Perhaps n should be at least 20 or 25, or more, for suitable stability.

As an example, if we return to the gun-tube failure data of Example 21-11, we have $t_1 = 190$ and $t_2 = 334$. Hence, Jaech's estimate of the shape parameter is simply

$$\hat{\beta} = 1/\ln(334/190) = 1.77 \ .$$

(We will subsequently find that this estimate is a bit low compared to more efficient estimators).

The 95% confidence band on β from Eq. 21-156 is very wide, being

$$Pr[0.09 \le \beta \le 5.3] \approx 0.95 \ .$$

Since the shape parameter is estimated as 1.77, we conclude immediately that the data are not distributed exponentially. Thus, this simple and easily obtained estimate of Jaech's may be used for a quick judgment or as a starting point for iteration in the maximum likelihood procedure next discussed.

21-8.2.2 Maximum Likelihood Estimation

As is well known from the statistical literature, R. A. Fisher's principle of maximum likelihood (ML) estimation is usually optimum indeed. Cohen (Ref. 45) covers thoroughly the problem of maximum likelihood estimation for the Weibull distribution. Given the ordered failure times $t_1 \le t_2 \le \cdots \le t_r$, he shows that the ML estimators of β and θ are determined from the following two equations by iteration:

$$\frac{\sum_{i=1}^{r} t_i^{\beta} \ln t_i + (n-r) t_r^{\beta} \ln t_r}{\sum_{i=1}^{r} t_i^{\beta} + (n-r) t_r^{\beta}} - 1/\beta = (1/r) \sum_{i=1}^{r} \ln t_i \quad (21\text{-}157)$$

and

$$\hat{\theta} = [\sum_{i=1}^{r} t_i^{\hat{\beta}} + (n-r) t_r^{\hat{\beta}}]/r \quad (21\text{-}158)$$

where $\hat{\beta}$ in Eq. 21-158 is the value of β determined by iteration from Eq. 21-157. Note that Eq. 21-157 is free of the scale parameter θ, and a starting value for the iteration could be Jaech's estimate, or a graphical estimate as from Eq. 21-154.

We should mention that ML estimators, on the average, are somewhat biased.

We illustrate with an example.

Example 21-12:

Conduct the analysis required in Example 21-11, except now fit a two parameter Weibull distribution to the first ten failure times—i.e., 190, 334, 365, 420, 472, 589, 610, 662, 792, and 840.

The reader may verify that by using Eq. 21-157 and starting with Jaech's estimate of $\hat{\beta} = 1.77$—or a value of $\beta = 2$, say—iteration gives

$$\hat{\beta} = 2.144$$

and from Eq. 21-158, we get

$$\hat{\theta} = 2.664 \times 10^6$$

or the characteristic life δ is estimated as

$$\hat{\delta} = \hat{\theta}^{1/\hat{\beta}} = 993 .$$

The number of rounds to failure, at which the "reliable life" is 0.95, may be obtained easily from

$$R(t) = \exp(-t^{\beta}/\theta) = 1 - \alpha = 0.95$$

or

$$t = \{\theta[-\ln(1-\alpha)]\}^{1/\beta} . \quad (21\text{-}159)$$

For $\alpha = 0.05$, $\beta = 2.144$, and $\theta = 2.664 \times 10^6$, we calculate from Eq. 21-159 that

$$t_{0.05} = 249 \text{ rounds}$$

based on only the first ten failures, as compared to 287 rounds for a lognormal fit to all 20 rounds in par. 21-6.

Cohen (Ref. 45) also presents the methodology for estimating the asymptotic ML estimators of the variances and covariances of $\hat{\beta}$ and $\hat{\theta}$. For example, the estimated variance of the shape parameter β for our data here is Var $(\hat{\beta})$ = 0.387, or the standard error is 0.62, and the correlation between $\hat{\beta}$ and $\hat{\theta}$ is estimated as 0.997, indicating very high correlation between estimates as one might expect!

21-8.2.3 Linear Invariant Estimation

A highly efficient and rather straightforward method (due to tabulations) for estimating Weibull parameters is to use tables of coefficients developed by Mann (Ref. 46) for the best linear invariant estimates. The reader should study Ref. 46 for the details, where tables of coefficients are given for n equals 2 through 15, and all amounts of censoring, i.e., $2 \leq r \leq n$. Tables through $n = 25$ are available from the author on request. We indicate the technique for the gun tube failure data of Example 21-11.

It will be convenient to tabulate the data with an extract of Mann's coefficients, $A(n,r,i)$ and $C(n,r,i)$, for estimating first the Gumbel extreme value parameters and then the Weibull shape and scale parameters. The appropriate data for $n = 20$, $r = 10$ are:

i	t_i	$\ln t_i$	$A(n,r,i)$	$C(n,r,i)$
1	190	5.247	−0.052900	−0.090626
2	334	5.811	−0.049115	−0.093031
3	365	5.900	−0.042792	−0.091837
4	420	6.040	−0.034710	−0.088309
5	472	6.157	−0.025087	−0.082842
6	589	6.378	−0.013973	−0.075573
7	610	6.413	−0.00134	−0.066511
8	662	6.495	0.01292	−0.055584
9	792	6.675	0.02894	−0.042651
10	840	6.733	1.17805	0.686964

The logarithms of rounds at failure are used since the Gumbel extreme value parameters $\tilde{u}$ and $\tilde{b}$ are first estimated. They are

$$\tilde{u} = \sum_{i=1}^{r} A(n,r,i)\ln t_i = 6.932 \tag{21-160}$$

and

$$\tilde{b} = \sum_{i=1}^{r} C(n,r,i)\ln t_i = 0.4699 \tag{21-161}$$

from which we estimate β and θ:

$$\tilde{\beta} = 1/\tilde{b} = 2.128 \tag{21-162}$$

and

$$\tilde{\theta} = \exp(\tilde{u}/\tilde{b}) = 2.551\times10^{6} . \tag{21-163}$$

These values are close to the maximum likelihood estimators of 2.144 and 2.664×10^6. The characteristic life is estimated as

$$\tilde{\delta} = \exp(\tilde{u}) = 1025 . \qquad (21\text{-}164)$$

Mann (Ref. 46) also gives values of quantities listed as $E(LU)$ and $E(LB)$ which may be used to estimate the variances of $\tilde{u}$ and $\tilde{b}$. They are used as follows:

$$\mathrm{Var}(\tilde{u}) = b^2E(LU) = 0.1379/\beta^2 \qquad (21\text{-}165)$$

and

$$\mathrm{Var}(\tilde{b}) = b^2E(LB) = 0.08723/\beta^2 \qquad (21\text{-}166)$$

where we use $\beta = \tilde{\beta}$ to get variances of $\tilde{u}$ and $\tilde{b}$ (see Ref. 46).

The $(1 - \alpha)$ reliable life, such as $1 - 0.05 = 95\%$, etc., is given as

$$t_\alpha = \exp(\ln t_\alpha) = \exp\{\tilde{u} + \tilde{b}\ln[\ln(1 - \alpha)^{-1}]\} \qquad (21\text{-}167)$$

which for our data results in

$$t_{0.05} = 254 \text{ rounds}$$

versus 249 rounds for Cohen's ML method.

21-8.2.4 Bain-Engelhardt Simplified Estimators

In spite of the complexity of the problem of Weibull estimation, Bain and Engelhardt (Refs. 47-50) have suggested some rather simple unbiased estimation procedures for Weibull (and extreme-value) parameters not requiring the use of extensive tables. Bain's 1972 estimate (Ref. 47) of the extreme value scale parameter b^* in simplified form for less than 90% censoring is

$$b^* = [(r - 1)\ln t_r - \sum_{i=1}^{r-1} \ln t_i]/nk_{r,n} \qquad (21\text{-}168)$$

where

$k_{r,n} \approx A_0 + A_1(10/n) + A_2(10/n)^2$ (21-169)

t_r = last reading in the ordered censored sample

and the coefficients A_0, A_1, and A_2 depend on the amount of censoring and are given in Table 21-4.

As an example, let us use the data of Example 21-11 again. With the aid of Table 21-4, we use Eq. 21-169 to calculate $k_{r,n} = k_{10,20}$ for $1 - r/n = 50\%$ censoring to be

$$k_{r,n} = k_{10,20} = 0.58937 - 0.12415(10/20) + 0.00145(10/20)^2$$

$$= 0.5277 .$$

TABLE 21-4. TABLE OF COEFFICIENTS OF A_i AND B_i FOR CALCULATING $k_{r,n}$ AND $c_{r,n}$ FOR BAIN-ENGLEHARDT ESTIMATORS FOR VARIOUS VALUES OF r/n†

	r/n								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
A_0	0.10265	0.21129	0.32723	0.45234	0.58937	0.74274	0.92026	1.1382	1.4436
A_1	−0.10274	−0.10622	−0.11060	−0.11634	−0.12415	−0.13540	−0.15313	−0.18567	−0.26929
A_2	0.00001	0.00030	0.00054	0.00089	0.00145	0.00242	0.00433	0.00906	0.02796
B_0	−2.2504	−1.4999	−1.0309	−0.67173	−0.36651	−0.08742	0.18563	0.47589	0.83403
B_1	−0.55419	−0.30740	−0.22859	−0.19301	−0.17619	−0.17114	−0.17727	−0.20110	−0.27773
B_2	−0.07848	−0.01886	−0.00767	−0.00335	−0.00091	0.00111	0.00369	0.00891	0.02825

†See Eqs. 21-169 and 21-171

Then from Eq. 21-168, we get

$$b^* = \frac{9(6.733) - 55.116}{20(0.5277)} = 0.5194$$

and its reciprocal $\beta^* = 1/b^* = 1.925$ is an unbiased estimate of the shape parameter of the Weibull distribution, and somewhat lower than the $\tilde{\beta} = 2.128$ for linear invariant estimation or the 2.144 for ML estimation. The standard deviation of b^* is approximately

$$b/\sqrt{nk_{r,n}} = 1.925/\sqrt{20(0.5277)} = 0.59$$

so that all three estimates are within a standard deviation of each other, and hence β is about 2 perhaps.)

The estimate of the extreme value scale parameter u is from this approximate theory given by

$$u^* = \ln t_r - c_{r,n}b^* \tag{21-170}$$

where

$$c_{r,n} \approx B_0 + B_1(10/n) + B_2(10/n)^2 \tag{21-171}$$

and the coefficients B_0, B_1, and B_2—which also depend on the amount of censoring—are given in Table 21-4 also.

To continue with our example, we calculate by Eq. 21-171

$$c_{r,n} = c_{10,20} = -0.36651 - 0.17619(10/20) - 0.00091(10/20)^2$$

$$= -0.4548$$

and from Eq. 21-170

$$u^* = \ln 840 - (-0.4548)(0.5194) = 6.9696 \cdot$$

Finally, the estimate of the characteristic life is

$$\delta^* = \exp(u^*) = 1064$$

as compared with 1025 for linear invariant estimation.

Bain and Engelhardt also have observed the close agreement between their simple estimators (when transformed) and the maximum likelihood or the linear invariant estimators. Transformations from one to the other are given in Refs. 49-51.

In summary, we have available a variety of methods for estimating Weibull shape and scale parameters with high efficiency. Moreover, the techniques are rather simple, as in the case of Bain's estimation procedure (Ref. 47), or tables of coefficients for linear type estimation are readily available in the literature.

Cohen (Ref. 52), for example, discusses estimation for the three-parameter Weibull model.

21-8.2.5 Distributional Properties of Estimators

The investigations and results of Bain and Engelhardt (Refs. 47-50), and that of Mann and Fertig (Ref. 51) indicated that for 50% censoring or more,

$$2nk_{r,n}b^*/b \approx \chi^2(2nk_{r,n}) \tag{21-172}$$

or the left-hand side follows very approximately a chi-square distribution with $2nk_{r,n}$ degrees of freedom, and for $n = 20$ or more, we have also that

$$2b^*/(b\ell_{r,n}) \approx \chi^2(2/\ell_{r,n}) \tag{21-173}$$

(Ref. 51), where the $k_{r,n}$ are as in par. 21-8.2.3 and the values of $\ell_{r,n}$ are given by Mann and Fertig in Table 1 of Ref. 51. (The reader is cautioned that $nk_{r,n}$ is not precisely equal to $1/\ell_{r,n}$ except for large amounts of censoring.) Thus, Eq. 21-172 or 21-173 may be used to obtain approximate confidence bounds on b or its reciprocal β, the shape parameter. In fact, the approximate $(1 - 2\alpha)$ confidence bounds on β may be obtained from

$$Pr\{[1 - (\ell_{r,n}/9) - \eta_{1-\alpha}\sqrt{\ell_{r,n}}/3]/b^* \le \beta \le [1 - (\ell_{r,n}/9) + \eta_{1-\alpha}\sqrt{\ell_{r,n}}/3]/b^*\} \approx 1 - 2\alpha \tag{21-174}$$

with $\eta_{1-\alpha}$ the standard normal deviate.

Mann and Fertig (Ref. 51, pp. 365-7) give an approximate method for determining confidence bounds on u—and hence on θ or δ. (see also pp. 245-254 of Ref. 40).

Ref. 53 is also of interest for obtaining confidence bounds on Weibull parameters.

21-8.2.6 Estimation of Reliability and Confidence Bounds on Reliability

The reliability at mission time t_m is, as we know, given by

$$R(t_m) = \exp[-(t_m/\delta)^\beta] \ . \tag{21-175}$$

Once we have maximum likelihood estimators $\hat{\beta}$ of β and $\hat{\delta}$ of δ, the maximum likelihood estimate of the reliability is

$$\hat{R}(t_m) = \exp[-(t_m/\hat{\delta})^{\hat{\beta}}] . \tag{21-176}$$

Thoman, Bain, and Antle (Ref. 54) show that $\hat{R}(t_m)$ is very nearly a minimum variance unbiased estimator of the true unknown $R(t_m)$, and that the probability density of $\hat{R}(t_m)$ depends only on $R(t_m)$ and n. This makes it possible to obtain confidence bounds on the true, unknown reliability $R(t_m)$. Ref. 54 gives lower 75%, 90%, 95% and 98% confidence bounds on $R(t_m)$ for uncensored sample sizes n = 8, 10, 12, 15, 18, 20, 25, 30, 40, 50, 75, and 100; and $\hat{R}(t_m)$, the ML estimate, of 0.50(0.02) 0.98.

Billman, Antle, and Bain (Ref. 55) give tables for finding lower 90%, 95%, 98%, and 99% confidence bounds on the reliability $R(t_m)$ for 25% and 50% censoring.

The results of Johns and Lieberman for confidence bounds on $R(t_m)$ (Ref. 56) also may be applied to Weibull data.

21-9 TOLERANCE LIMITS FOR ANY CONTINUOUS DISTRIBUTION

Any coverage of topics on the analyses of reliability or safety problems should include a discussion of "tolerance limits" for any continuous distribution. The initial work on this important subject was originated by Wilks (Ref. 57), who developed the relation between sample sizes, the fractions of an unknown continuous distribution between any two sample order statistics, and the confidence levels involved. A discussion of this appears also in Ref. 9, which indicates that for sample order statistics,

$$t_1 \leq t_2 \leq \cdots \leq t_i \leq \cdots \leq t_j \leq \cdots \leq t_n;\ 1 \leq i < j \leq n$$

from *any* continuous unknown (and hence unspecified parametric form of) distribution, that

$$P(n - j + i + 1,\ n,\ \gamma) \geq 1 - \alpha \tag{21-177}$$

where Eq. 21-177 is the probability that the fraction of the population between the sample order statistics t_i and t_j, i.e.,

$$F(t_j) - F(t_i) = \int_{t_i}^{t_j} f(t)\, dt \tag{21-178}$$

equals or exceeds the quantity $1 - \gamma = 0.95, 0.99$, etc., is at least $1 - \alpha$. The left-hand side of Eq. 21-177 is simply the binomial probability $P(c,n,p)$ from the tables of Ref. 9, for example. The sample order statistics t_i and t_j are called the "tolerance limits" of the distribution, and Eq. 21-177 does not depend on the form of $f(t)$—a remarkable result!

For the reliability problems, we would be particularly interested in the fraction (or reliability) of a failure distribution sampled, which lies above (or below) the lowest sample order statistic t_1. In this connection, if we set $i = 1$ and $j = n + 1$ in Eq. 21-177, we have

$$Pr[R(t_1) = \int_{t_1}^{\infty} f(t)dt \geq 1 - \gamma] = P(1,n,\gamma) \geq 1 - \alpha . \tag{21-179}$$

This says that given a sample of size n from any continuous distribution for which we find the smallest sample value t_1 then the chance that the fraction of the population sampled (or the reliability at t_1) exceeds $(1 - \gamma)$ is at least $(1 - \alpha)$, which may be read directly from a table of binomial probabilities (Ref. 9).

But we know that

$$P(1,n,\gamma) = \sum_{i=1}^{n} \binom{n}{i} \gamma^i (1 - \gamma)^{n-i} = 1 - (1 - \gamma)^n \qquad (21\text{-}180)$$

and we may desire a sample of size n such that this equals or exceeds the confidence level $(1 - \alpha)$. Hence, the chance that the reliability at the least failure time t_1—which is the proportion of the population above t_1—exceeds $(1 - \alpha)$ is given by

$$Pr[R(t_1) \geq 1 - \gamma] = P(1,n,\gamma) = 1 - (1 - \gamma)^n \geq 1 - \alpha \ . \qquad (21\text{-}181)$$

Hence, solving for the required sample size n, we get

$$n = \ln\alpha/\ln(1 - \gamma) \ . \qquad (21\text{-}182)$$

We illustrate with an example.

Example 21-12:

Suppose we know the number of rounds to failure for 40 howitzers. With what confidence can we say that 90% of such howitzers would fail at or above the least number of rounds observed in the 40 howitzers? How many such weapons should be fired to failure to guarantee with 95% confidence that 90% of the howitzers would survive beyond the minimum life observed.

We easily see for this problem that we want

$$P(1,n,\gamma) = P(1,40,0.10) = 0.985 \ .$$

The value 0.985 is obtained directly from page 73 of Ref. 9. Thus, the chance is 0.985 that 90% of the howitzers from the population sampled will survive beyond the minimum life of 40 howitzers randomly selected and tested from that population.

For the second question, we have from Eq. 21-182 that

$$n = \ln\alpha/\ln(1 - \gamma) = \ln(0.05)/\ln(0.90) = 28.43$$

or we need to test only 29 howitzers.

21-10 AN INTRODUCTION TO RELIABILITY GROWTH

In the early 1950's, and with the advent of large-scale efforts on guided missile technology, the Army Ordnance Corps set up a Guided Missile Reliability Committee to study the reliability of missiles in general, and hence to try and improve on what was considered to be rather poor reliability at the time. Perhaps, this led eventually to the field we now know as "reliability growth". In fact, for complex systems, the problem of trying to improve and upgrade the reliability of an item or system through corrective action has developed into a field of considerable importance and interest. There must be

some concentrated effort, of course, during the development stages of systems when failures are relatively frequent and the various failure modes, which have been observed during design inspection and testing, can likely be removed. Whenever design changes are made, or corrective action taken, or even the installation of quality control features, then such efforts may lead to "reliability growth". The analyst should be interested, therefore, in estimating the reliability of a system as a function of the parameters involved, and also the prediction of reliability growth as a function of development time.

The literature on reliability growth in recent years has grown enormously, as one might expect. Therefore, we can give only a rather limited introduction to the subject here, with the hopes that the analyst will proceed to the type of studies needed in his particular investigations. In citing some of the background literature, we mention that Weiss (Ref. 58) considered a system which might have several possible failure modes, each being with an exponential type failure law. In his paper, he developed methods for fitting typical reliability growth curves to experimental data taken during development by the technique of maximum likelihood estimation. His models are for an idealized engineering development process to provide a basis for the selection of a suitable reliability growth curve.

Lloyd and Lipow (Ref. 59) consider a system for which a testing program is carried out to discover and remove the failure mode observed. On each trial, the system is considered to fail with probability p, say, if the failure mode has not been corrected; otherwise, it has a reliability of one. When a failure occurs, an attempt is made to correct that type of failure permanently, and each such attempt is assumed to have a chance ϕ of success. The system is either in a repaired or an unrepaired state after n tests have been performed, and Lloyd and Lipow show that the reliability of the system for the $(n + 1)$st test is $1 - p(1 - p\phi)^n$.

Barlow and Scheuer (Ref. 60) and Wolman (Ref. 61) consider a system for which failures are of two types—the "inherent" or the "assignable cause" types. Each trial on the system could then result in a success, an inherent failure, or an assignable cause failure. It is assumed that the system originally contains k assignable-cause failures, and once a failure mode has been observed that mode is permanently corrected and removed. On each trial, the remaining assignable cause failures occur with a constant probability, and Wolman shows how a Markov-chain model can be used to obtain the reliability of the system after n trials have been made.

Barr (Ref. 62) extends the work of Barlow and Scheuer to a class of general reliability growth prediction models. His paper considers a class of models that accommodates variations in such important factors as the interdependence of assignable cause failure modes, inclusion of an inherent failure mode, the repair policy, and the distribution of the initial states of a system.

Duane (Ref. 63) considered the reliability of several different and rather complex electromechanical and mechanical systems during development programs at the General Electric Company. He plots the cumulative failure rate versus the cumulative hours of operation on a log-log scale and notices a striking linear relation, so that the idea of learning curves for items may be established. He demonstrates that for relatively complex aircraft accessories the reliability growth, as measured by the decreasing failure rate with operating time, "follows a relatively simple and predictable pattern, and is approximately inversely proportional to the square root of the cumulative operating time". Hence, if the failure rate at operating time t is λ, then Duane indicates that

$$\lambda \approx (K/2)(\Sigma t)^{-1/2} \tag{21-183}$$

where Σt is the cumulative total operating time for the system and K is a constant of proportionality. The so-called Duane model, therefore, indicates only that for some rather complex equipment the

failure rate decreases with operating time for repairable systems, and perhaps a reasonable prediction of the (decreasing) failure rate for some future time can be made.

As a pertinent comment, we should point out here that reliability growth could be defined generally as increasing reliability with system development or operating development time. That is to say, the fraction of the failure time distribution above the mission time increases as development time goes on. This means, for example, that if one desires to fit a Weibull distribution to failure time data at a point during a development program, then the Weibull shape and scale parameters could change with development time in such a manner that the fraction of the failure time distribution below the mission time t_m decreases with increasing development time.

Crow (Refs. 64-67) considers the reliability of a complex, repairable system during a development program and allows for changes in trend of the intensity of system failures as a nonhomogeneous Poisson process with Weibull hazard function as given by Eq. 21-146. There have been some suggestions (Ref. 67) at a particular development time to equate Duane's slope, i.e., $\approx -1/2$, to $(\beta - 1)$, where β is the Weibull shape parameter, but this would restrict the Weibull fit. For a complex, repairable system, a failed component is repaired or replaced by one "as good as new", and consequently times between failures are considered independent and follow the so-called "renewal process".

Finkelstein (Ref. 68) also considers the Weibull process which is nonhomogeneous Poisson with hazard or intensity function $h(t)$

$$h(t) = (\beta/\delta)(t/\delta)^{\beta-1}. \quad (21\text{-}184)$$

Crow and Finkelstein both show that in this case the estimates of β and δ are very simple, being

$$\hat{\beta} = n/\sum_{i=1}^{n-1} \ln(t_n/t_i) \quad (21\text{-}185)$$

and

$$\hat{\delta} = t_n/n^{1/\hat{\beta}} \quad (21\text{-}186)$$

where

t_i = ith ordered failure time
t_n = last failure time
n = number of corrected failures for the system .

Crow (Ref. 66) extends the problem of obtaining estimates of the Weibull shape and scale parameters to failure data involving several complex, repairable systems.

The quantity

$$2n\beta/\hat{\beta} = \chi^2(2n)^{**} \quad (21\text{-}187)$$

follows chi-square with $2n$-df, so that a confidence bound on the shape parameter β may be easily obtained.

Crow (Ref. 66) gives confidence bounds on $\lambda = 1/\theta = \delta^{-\beta}$ when β is assumed known, and considers the problem of simultaneous confidence bounds on λ and β.

**$Pr[\hat{\beta}\chi^2_{\alpha}(2n)/2n \leq \beta \leq \hat{\beta}\chi^2_{1-\alpha}(2n)/2n] = 1 - 2\alpha$

Finkelstein establishes the very useful result that the following identity holds:

$$(\hat{\delta}/\delta)^{\hat{\beta}} = (\hat{\delta}_1)^{\hat{\beta}_1} \tag{21-188}$$

where $\hat{\delta}_1$ and $\hat{\beta}_1$ are ML estimates of δ and β when the sample is known to be from the degenerate Weibull (or particular exponential) distribution $\beta = 1$, $\delta = 1$. Further, either side of Eq. 21-188 is distributed independently of the unknown shape parameter β and scale parameter δ for the Weibull process. Consequently, Finkelstein was able to tabulate by Monte Carlo methods the probability distribution and percentage points of $(\hat{\delta}_1)^{\hat{\beta}_1}$, and hence that of $(\hat{\delta}/\delta)^{\hat{\beta}}$, so that a confidence bound may be placed on δ by using Finkelstein's table of standard probability levels in Ref. 68, which is our Table 21-5. We illustrate with an example.

Example 21-13:

An electronic system has been undergoing development for many months, and the Weibull process was considered to apply to reliability growth at the last failure correction period. At the current state of development, suppose ten failure times for the system were observed: 13, 48, 89, 121, 189, 262, 323, 395, 499, and 626. Find the estimates of the Weibull shape and scale parameters, and also the 80% confidence bounds on the true, unknown parameters.

TABLE 21-5. TABLE OF PERCENTAGE POINTS γ FOR $(\hat{\delta}/\delta)^{\hat{\beta}}$

n \ γ	0.02	0.05	0.10	0.20	0.30	0.50	0.70	0.80	0.90	0.95	0.98
2	—	0.002	0.06	0.33	0.60	1.5	8.0	61.0	2380.0	*	*
3	0.09	0.23	0.40	0.58	0.79	1.6	4.7	13.0	113.0	2730.0	*
4	0.26	0.36	0.47	0.65	0.86	1.6	3.7	8.0	35.0	210.0	5140.
5	0.31	0.40	0.49	0.67	0.87	1.5	3.2	6.1	19.0	75.0	629.
6	0.33	0.42	0.51	0.68	0.88	1.5	3.0	5.2	14.0	43.0	220.
7	0.34	0.42	0.51	0.68	0.87	1.4	2.8	4.6	11.0	28.0	126.
8	0.35	0.42	0.51	0.68	0.86	1.4	2.6	4.1	9.2	21.0	69.
9	0.35	0.42	0.51	0.68	0.85	1.3	2.4	3.8	7.8	17.0	49.
10	0.35	0.42	0.51	0.67	0.85	1.3	2.3	3.6	7.1	15.0	42.
12	0.35	0.43	0.52	0.67	0.84	1.3	2.2	3.2	5.9	11.0	27.
14	0.36	0.43	0.52	0.67	0.83	1.3	2.0	2.9	5.1	9.0	19.
16	0.36	0.43	0.52	0.68	0.83	1.2	2.0	2.7	4.8	8.0	16.
18	0.36	0.43	0.53	0.68	0.83	1.2	1.9	2.6	4.3	6.9	13.
20	0.36	0.44	0.53	0.68	0.83	1.2	1.9	2.5	4.1	6.4	11.
22	0.36	0.44	0.53	0.68	0.82	1.2	1.8	2.4	3.9	5.9	9.8
24	0.37	0.44	0.53	0.67	0.81	1.2	1.8	2.3	3.6	5.5	9.2
26	0.37	0.45	0.53	0.67	0.81	1.2	1.7	2.3	3.4	5.1	8.2
28	0.37	0.45	0.53	0.67	0.81	1.2	1.7	2.2	3.3	4.9	7.9
30	0.37	0.45	0.53	0.67	0.81	1.1	1.7	2.2	3.2	4.7	7.1
35	0.38	0.46	0.54	0.68	0.82	1.1	1.6	2.1	3.0	4.3	6.5
40	0.39	0.46	0.55	0.69	0.82	1.1	1.6	2.0	2.8	3.9	5.9
45	0.39	0.47	0.55	0.70	0.83	1.1	1.6	1.9	2.7	3.7	5.4
50	0.39	0.47	0.56	0.70	0.83	1.1	1.5	1.9	2.6	3.5	5.0
60	0.40	0.49	0.57	0.71	0.83	1.1	1.5	1.8	2.4	3.2	4.4
70	0.42	0.49	0.58	0.71	0.82	1.1	1.4	1.7	2.3	3.0	4.0
80	0.43	0.51	0.59	0.71	0.82	1.1	1.4	1.7	2.2	2.8	3.8
90	0.44	0.51	0.60	0.71	0.83	1.1	1.4	1.7	2.1	2.6	3.5
100	0.45	0.52	0.60	0.72	0.83	1.1	1.4	1.6	2.1	2.6	3.3

*Greater than 10^{10}

From "Confidence Bounds on the Parameters of The Weibull Process" by J. M. Finkelstein, *Technometrics*, Vol. 18.

From Eqs. 21-185 and 21-186, we find

$$\hat{\beta} = n/\sum_{i=1}^{n-1} \ln(t_n/t_i) = 10/13.45 = 0.74$$

and

$$\hat{\delta} = t_n/n^{1/\hat{\beta}} = 626/(10)^{1/0.74} = 626/22.46 = 27.87\,.$$

Using the chi-square distribution with $2n = 20$ degrees of freedom, we find the 80% confidence bounds on β from Eq. 21-187 to be (0.460,1.051). Finally, from Table 21-5, we get using $n = 10$, and $\gamma = 0.10$ and 0.90, the values of $t_\gamma = 0.51$ and 7.1, from which

$$\begin{aligned} Pr&[0.51 \le (\hat{\delta}/\delta)^{\hat{\beta}} \le 7.1] \\ &= Pr[\hat{\delta}/(7.1)^{1/\hat{\beta}} \le \delta \le \hat{\delta}/(0.51)^{1/\hat{\beta}}] \\ &= Pr[1.97 \le \delta \le 69.23] = 0.80 \end{aligned}$$

a rather wide confidence band.

For further information on reliability growth, the analyst should study the references and bibliography.

21-11 AVAILABILITY, OPERATIONAL READINESS, AND MAINTAINABILITY

21-11.1 AVAILABILITY OR OPERATIONAL READINESS

A weapon system must be ready or available to start a mission once a demand has been placed upon it. Otherwise, precious time will be lost or some degradation in the mission effectiveness will occur. At the beginning of the chapter we defined availability as the probability that for any random point or instant in time a weapon system, item, or piece of equipment is in proper operating condition to begin a mission. If a weapon is on the average "down" or unready to fire for, say, a third of the occasions demanded and is ready to fire during two-thirds of the occasions, then clearly the overall chance of success otherwise would have to be multiplied by two-thirds. Thus, depending on availability or operational readiness, it can be seen that the overall effectiveness could be degraded rather rapidly.

The chance that a weapon system is ready for a mission or is "available" depends on the average "up time" when the system is operating properly and the average "down time" when the system is not functioning, is being repaired, maintained, etc. The proportion of up time, or chance a system is available, is the total time up divided by the total time in intended use. The widely accepted availability ratio A, or chance that a system is in the readiness state, is given by

$$A = \frac{MTBF}{MTBF + MTTR} = \frac{1/\lambda}{1/\lambda + 1/\rho} = \frac{\rho}{\rho + \lambda} = \frac{1}{1 + (\lambda/\rho)} \tag{21-189}$$

where

λ = $1/\theta$, the failure rate

θ = mean time between failures $MTBF$

ρ = repair rate or reciprocal of the mean time to repair $MTTR$.

A ordinarily is referred to as the intrinsic, inherent, point-wise, or steady state availability. It should be noted in particular that availability is defined in terms of population parameters and is inherently tied in with the concept of exponential distributions. In fact, if we consider an exponential failure time distribution with failure rate λ for the system, and an exponential repair time distribution with repair rate ρ, then the chance that the repair time on the average is less than the failure time turns out to be Eq. 21-189. A similar treatment for the response time of a weapon was considered in Chapter 18, Eq. 18-3.

Availability usually is treated separately from the concept of reliability. A missile system, for example, must be capable of responding immediately to a surprise threat, and it becomes important to measure and treat such a characteristic separately from the probable failure times or reliability during an actual engagement. It must be admitted, however, that the terms reliability, availability, dependability, and maintainability are often confused.

An examination of Eq. 21-189 indicates that the key factor in determining availability is the ratio λ/ρ of failure rate for the equipment to the repair rate (or vice versa). If they are equal then system availability is only 1/2 or 50%, and for very high availability the failure rate should be low relative to the repair rate.

Goldman and Slattery (Ref. 69) define the achieved availability and operational availability. These definitions are:

$$\text{Availability (achieved)} = A_a = \frac{MTBM}{MTBM + M} \tag{21-190}$$

where

$MTBM$ = mean time between maintenance

M = mean active maintenance downtime resulting from both mean preventive and corrective maintenance actions

$$\text{Availability (operational)} = A_o = \frac{MTBM}{MTBM + MDT} \tag{21-191}$$

where

$MTBM$ = mean time between maintenance and ready time during the same interval

MDT = mean downtime, including supply downtime and administrative downtime during the same time interval. When preventive maintenance downtime is zero or not considered, $MTBM$ becomes $MTBF$.

Some examples on availability are given in AMCP 706-196 (Ref. 5).

Some mathematical models for availability or operational readiness are discussed by, for example, Coleman and Abrams (Ref. 70), and Sasaki (Ref. 71).

On occasions, the analyst may be required to compare the availabilities of two systems using sample data. This type of problem has been studied by Nelson (Ref. 72), Gray and Lewis (Ref. 73), and Chase and Hewett (Ref. 74). Much of the accomplishments for this problem appear to be somewhat disappointing in so far as robustness of the tests are concerned (see Refs. 73 and 74).

21-11.2 MAINTAINABILITY

Maintainability is a characteristic of design and installational usage of the system or equipment, and is defined as the chance that the system will be retained in a specified operational condition, or

restored to that condition within a given period of time, when maintenance is performed according to prescribed procedures and resources. Fortunately, there is available an excellent handbook of this series on maintainability, AMCP 706-133 (Ref. 75), so that only an introduction is needed here.

Maintenance and maintainability have different meanings. Maintenance involves activities which are directed toward failure prevention; i.e., preventive and scheduled maintenance, and the correction of failures as they occur. In order to maintain a system at a desired level of operation, the maintenance organization typically has to perform regular routine inspections, or examine the system otherwise to determine whether it is in satisfactory operating condition. Therefore, maintenance is concerned with actions taken by the user to retain a system in, or restore to, an operable condition.

Maintainability is concerned with those actions taken by the designer during development to incorporate design features which will enhance ease of maintenance. (Reliability is the chance that a failure will not occur in a specified time, while maintainability is the chance that successful completion of maintenance will occur in a specific time.) Maintainability is a very important part of the overall process of keeping weapon systems in a state of readiness. It has often been defined as the ratio of the mean down time MDT and the mean time between maintenance $MTBM$. Another way to consider the maintainability problem is to observe the average number of maintenance actions which can be accomplished in the maximum allowable time t, which we will designate as the μt, where the quantity μ is the maintenance action rate. Then, we can describe the chances of 0, 1, 2, etc., maintenance actions in time t by the Poisson distribution. Hence, the probability of completing zero maintenance actions in an increment of time t is $\exp(-\mu t)$, and the probability of completing one or more maintenance actions is

$$1 - \exp(-\mu t) = 1 - \exp(-t/MTTR) \tag{21-192}$$

this latter quantity being the expression for maintainability, it might be said. In essence, the time t here is really the maximum allowable time after a failure occurs during which it is mandatory that a repair or maintenance action be completed, and such time should be only a small fraction of the mission time for high system effectiveness.

For the weapon systems analyst, the problem of maintainability will likely be mostly a peripheral issue, so that complete coverage will not be attempted here, especially since Ref. 75 is available. Nevertheless, the reader also will likely have some interest in studying the book of Goldman and Slattery (Ref. 69), AMCP 706-133 (Ref. 75), the ARINC Research Corporation book on Reliability Engineering (Ref. 76), and others.

21-12 SUMMARY

In this chapter, we have attempted to cover those topics in reliability and related fields of interest which the weapon systems analyst will likely find of use in his evaluations. In particular, the analyst often will have to depend on available sample data and be prepared to make predictions in the form of point or interval estimates of population parameters, or on the confidence he may express concerning the reliability of the equipment he analyzes. Also, the topics covered in this chapter have been selected so as to supplement Refs. 5, 6, 7, 8, and 75.

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CHAPTER 22

MOBILITY, MANEUVERABILITY, AND AGILITY

The concepts of mobility, maneuverability, and agility have defied definition, quanitification, and adequate modeling for many, many years. Nevertheless, the weapon systems analyst must be thoroughly conversant with such measures of effectiveness and often take them into consideration in his evaluation process. The description given in this chapter should give the analyst a good introduction to some of the principles involved.

22-0 LIST OF SYMBOLS

a = acceleration, ft/s^2
a_{AL} = acceleration due to tractive limit, ft/s^2
a_{TE} = acceleration for tractive effort, ft/s^2
b = width of stream, ft
D = vehicle density per mi (depending on vehicle length and speed), vehicles/mi
d = depth of stream, ft
F = force, lb
f = factor for engine inertia = 0.07 (hp)$(GR)^2$, $lb\cdot s^2/ft$
g = acceleration due to gravity, ft/s^2
GR = gear ratio
hp = horsepower (1 hp = 550 ft•lb/s)
K = $n^2/[(1.486)^2 d^{4/3}]$ = constant, s^2/ft^2
ℓ = length of terrain unit, mi
m = mass of vehicle, $lb\cdot s^2/ft$
N = number of vehicles in column, vehicles
n = a constant for flow in a river bed depending on channel walls
P = power, hp or ft•lb/s
P(survival) = chance of survival
$P_{ACQ|LOS}$ = chance of acquisition given a line of sight
P_{LOS} = chance of an uninterrupted line of sight during missile flight
$p(h)$ = chance of a hit
$p(k|h)$ = conditional chance of a kill given a hit
TE = tractive effort, lb
T_c = travel time for vehicle column, h
v = speed of water (or speed of vehicle), ft/s or mph
v_s = speed of single vehicle, mph
W = weight of water (or vehicle), lb
α = an angle (slope), deg
Δt = small time increment, s
θ = angle of inclination of an incline, deg
ν = specific weight of water, lb/ft^3

22-1 INTRODUCTION

Mobility, as defined in Ref. 1 and par. 8-3.2, is a quality or capability of military forces which permits them to move from place to place while retaining the ability to fulfill their primary mission. Maneuverability of a weapon system is an indication of its ability to change its position in a tactical situation in order to secure an advantageous offensive or defensive position. Maneuver is basically the tactical employment of mobility, i.e., the delivering of men and their associated firepower to accomplish a tactical objective by application of combat power at a decisive point and time. Environmental factors such as terrain and weather conditions affect mobility and maneuverability. Agility of a vehicle or weapon system would indicate it is readily able to move quickly and easily. To a great extent, it could be said that mobility, maneuverability, and agility all involve interface problems between the characteristics of the vehicle and the varying environment in which it operates. Environmental factors—just to mention a few—which may have an effect on mobility, maneuverability, and agility include soil type and strength; terrain roughness; slope; obstacle size and spacing; terrain features such as rivers and streams, tree size and spacing; surface cover such as leaves or snow; vegetation; weather; and visibility constraints. Vehicle characteristics of importance include power, speed, range, tractive force as a function of speed, suspension system characteristics, turning radius as a function of speed, acceleration, deceleration, vision and guidance, ground contact pressure, crew protection, reliability, maintainability, and other factors. A vehicle achieves its full potential in mobility, maneuverability, and agility when it possesses the maximum degree possible of these attributes for its weight and volume.

Certain conditions of terrain—such as desert, mountain, swamps, marshes, or arctic—impose great constraints on mobility, but the possessor of mobility holds a considerable tactical advantage over a less mobile opponent. As pointed out in par. 8-3.2, which the analyst might reread to advantage, great gains in strategic and tactical mobility have been achieved in recent years. In particular, tactical mobility has improved markedly with the advent of the helicopter, for example.

The objectives of this chapter will be to expand upon these topics as they relate to Army weapon systems and to discuss their role in a systems analysis. Thus, the chapter introduces topics for the mobility of land, water, and air vehicles, which are considered as platforms for weapon systems. The weapon system may be integral with the platform, or it may be towed or carried to the point of use by the platform. Mobility generally implies the interactions between the vehicle and the environment and is measured in terms of speed—e.g., average speed, speed-made-good, or in terms of freedom of movement, e.g., percent of go-no-go conditions. Maneuverability, on the other hand, generally is accepted as a higher level of abstraction which includes mobility and agility, with agility implying the ability to run an erratic path. For the towed or carried weapon system, the concept of transportability, which is the capability of being moved from one point to another, also comes into consideration. Combinations of integral and towed, or carried weapon systems are possible, as in the case of the aircraft transported tank.

Within the chapter, various methods for attaining weapon system mobility are discussed, as are the vehicles to implement these methods. Vehicle limitations and the effects of environment on mobility are considered also. Means for the evaluation of the mobility of a given weapon system and the use of mobility parameters as inputs to an overall weapon system analysis are also treated in this chapter.

Three basic types of platforms are in current use by the US Army to provide weapon system mobility: land vehicles, water vehicles, and aircraft. In addition, combinations of types are possible as in amphibious and air cushion vehicles. Also included among the types of platforms, though not discussed in detail within this chapter, are human beings and animals.

For detailed information on terrain factors and land locomotion refer to Refs. 25 and 26.

22-2 SOME EXTRACTS FROM THE WORK OF M. G. BEKKER

An excellent treatise on the mechanics of vehicle locomotion and mobility is that of M. G. Bekker, *The Theory of Land Locomotion* (Ref. 2). Bekker calls attention to some analogy between rail and highway vehicle mobility, and geological movements of solids and water over the surface of the earth. For water flowing in a river, the force of gravity supplies the motive power and the resistance to flow is occasioned by the river bed. The power P consumed to overcome the resistance to flow of water in a river bed is given by

$$P = n^2 \nu b v^3 / [(1.486)^2 d^{1/3}], \text{ hp} \tag{22-1}$$

where

n = a constant depending on characteristics of channel walls
ν = specific weight of water, lb/ft^3
b = width of stream, ft
v = speed of water, ft/s
d = depth of water, ft .

The power P per unit weight W required to have the water flow at a given speed v is

$$P/W = Kv^3, \text{ hp/lb} \tag{22-2}$$

where the constant of proportionality K is

$$K = n^2/[(1.486)^2 d^{4/3}], \text{ s}^2/\text{ft}^2. \tag{22-3}$$

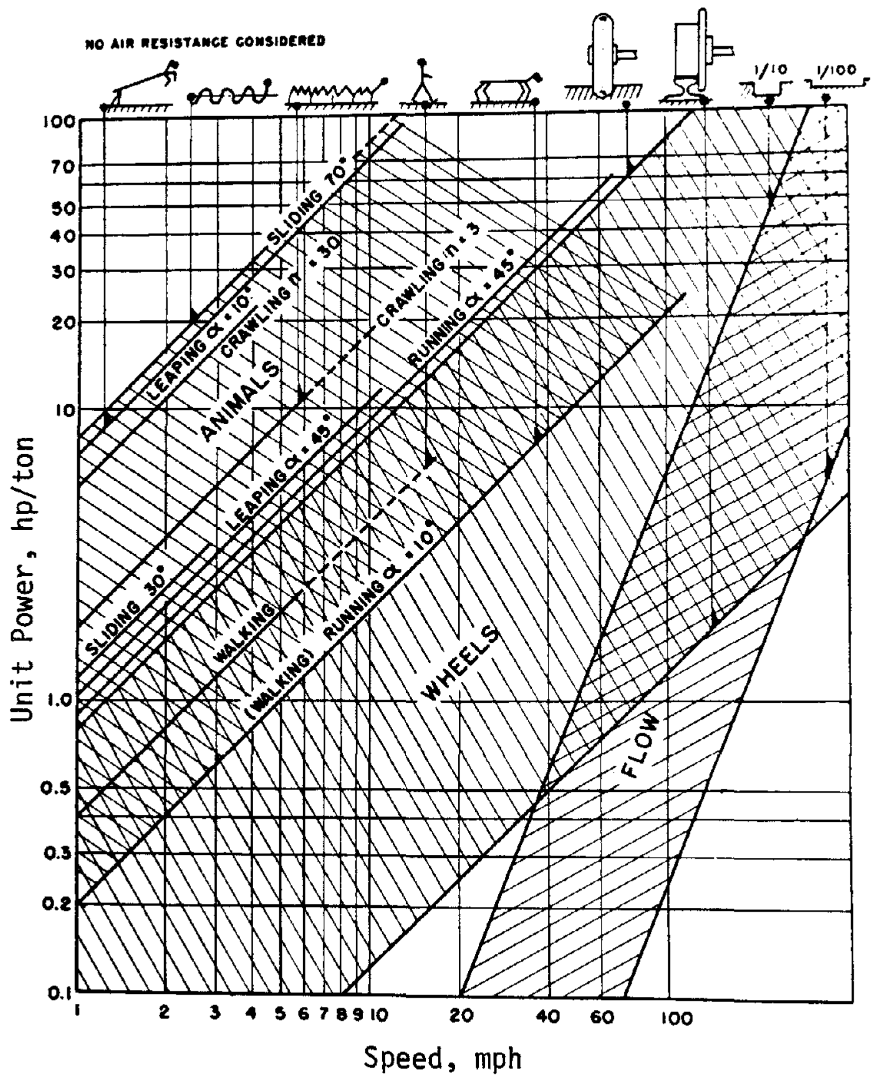
Hence, the power consumed per unit weight to overcome resistance for mobility at speed v depends on the cube of the speed.

The concept of power per unit weight or horsepower per ton expended to attain a speed of v is a widely useful functional relation to compare locomotion for the animal world, cross country vehicles, and road or railway vehicles. This is illustrated in a very informative manner by Bekker's Fig. 12, page 28 and Fig. 32, page 64 (Ref. 2), i.e., our Figs. 22-1 and 22-2, respectively. As Bekker says, "In a long chain of evolution, nature seems to have developed quite a few types of locomotion on the surface of the earth. It seems to have started with the movement of water under the action of gravity and ended with a final type of locomotion in the form of walking, which in an extreme case, may be compared to the rolling of a wheel. Only man has developed the wheel. It would offer no resistance to movement if it were prefectly rigid and if it moved on a rigid surfact in a vacuum. When moving in a soft medium, as Reynolds pointed out in 1876, (then the wheel) a 'soft roller' has to slip . . . . The respective requirements for overcoming resistance are as follows:

$$P(\text{hard surface}) \approx 0.0133v, \text{ hp/ton} \tag{22-4}$$

$$P(\text{soft surface}) \approx 0.800v, \text{ hp/ton} \tag{22-5}$$

where the speed of locomotion v is measured in mph." Fig. 22-1 gives the boundaries of power requirements for animals, wheels, and flow in a channel to attain various speeds. It is evident that the most expensive methods of power for moving are, of course, crawling and sliding, whereas flow and wheel rolling on a hard surface require the same power in certain cases or regions. A wheel driven in soft terrain



From *Theory of Land Locomotion* by M. G. Bekker. Copyright © 1956 by the University of Michigan. Reprinted by permission of the University of Michigan.

Figure 22-1. Unit Power vs Speed for Animals, Man, Wheels, and Flow

may not be as economical as walking or running since it requires more power per unit of weight. Nevertheless, the wheel has become the universal means of locomotion for man-made machines, and this includes tracks.

Fig. 22-2 gives further delineation of boundaries for the animal world, cross country vehicles, and road or railway vehicles. It shows that presently built vehicles when operating off the road form an extension of the animal type of locomotion throughout the realm of higher speeds. Road and railroad vehicles are definitely more efficient and economical, and approach a theoretical border line of power and speed—first plotted by Gabrielli and von Karman (Ref. 3)—which is typical not only of land vehicles, but also of air and water ships. The approximate equation of the Gabrielli-von Karman line is

$$P/(mg) = 0.001v^2 \qquad (22\text{-}6)$$

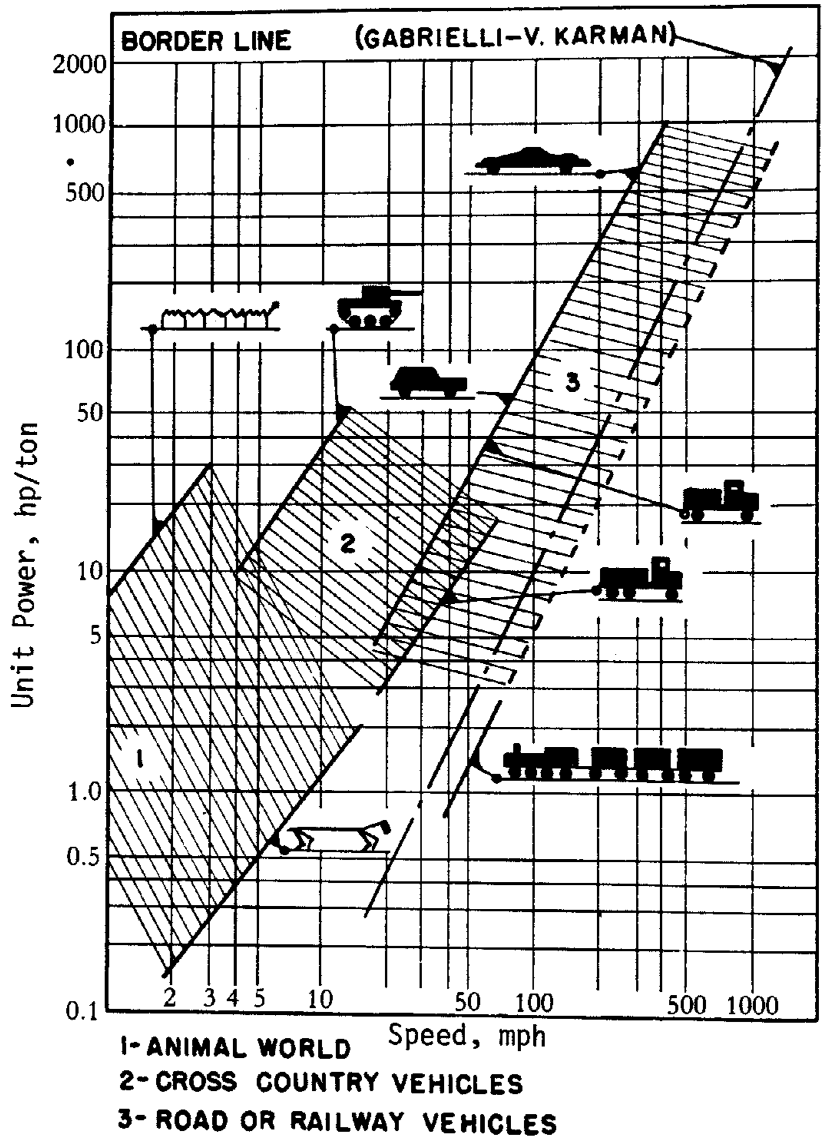


Figure 22-2. Unit Power vs Speed for Animal World, Cross-Country Vehicles, and Road or Railway Vehicles

where

mg = weight, ton

v = speed, mph

or in dimensionless form,

$$P/(mgv) = v/7820 \tag{22-7}$$

where

m = mass, $lb \cdot s^2/ft$

g = acceleration due to gravity, ft/s^2

v = speed, ft/s .

Note the constant 7820 must be in ft/s to make the equation dimensionally correct and, since this value refers to all land, sea and air vehicles, it should be independent of the medium in which a vehicle operates. (Trains, which have less frontal area per unit of weight and thus smaller air resistance, fall below the line as indicated.) The area for cross-country vehicles deserves considerable exploration in the future.

22-3 METHODS FOR LAND-FORCE WEAPON SYSTEM MOBILITY

In order to illustrate some of the means of providing weapon system mobility, consider the situation that follows. Assume the basic mobile platform has an integral weapon system and is a tracked, armored vehicle. For high speed travel over surfaced roads, it is possible for the vehicle to be designed with a removable track and to be towed, the track being replaced at the area of operation. Other means of reaching the area of operations would be to carry the vehicle on another land vehicle capable of high speed travel; airlift the vehicle to the area of operations with it being either offloaded upon landing, parachute dropped, or lowered to the ground (if carried by helicopter); or the vehicle could travel to the area of operations under its own power. Each of these alternatives would possess quantifiable performance measures, and costs, which the analyst would have to examine in establishing the preferred course of action. These would include factors such as transit time and its criticality; availability of the transport vehicles; expected reliability of the transport vehicles, including the probability of damage to the weapon system itself, e.g., if parachuted, and the costs associated with each alternative (both dollar and resource.) Once the vehicle is in the operations area, maneuvering under battlefield conditions, then other mobility considerations may arise. For example, there may be a requirement for a river crossing capability. Some of the possible approaches in this case may be for the vehicle to ford (if depth, current, and bottom conditions permit), to swim, to be picked up by helicopter and placed on the other side, or to wait for a bridge or raft to be built. For the swimming alternative, an additional number of approaches are open. The vehicle itself may be buoyant or a flotation kit may be provided. Propulsion may be by rotation of the track with possible added fins, or a special water propulsion system may be provided. Again, determination of the optimum course of action will be decided by the analysis of the corresponding performance and cost measures, e.g., time delay, availability of transport or appropriate kits, and the tactical and physical environment. Thus, for an in-depth evaluation of the mobility or maneuverability of a weapon system, it is important that the analyst be aware of the potential alternatives, such as those for overcoming obstacles or reducing time to cover a given distance, and use these alternatives in his evaluation model to assess their comparative worth.

22-4 MOBILE PLATFORMS

As noted previously, a number of platform concepts for obtaining weapon system mobility are possible. For example, land vehicles may be wheeled or tracked, self-powered, or towed; air vehicles may be fixed wing or rotary wing with systems being off-loaded, parachute delivered, or lowered. Each of the possible approaches has its inherent advantages and drawbacks which ultimately are reflected in terms of time, distance, and cost parameters, as well as the probabilistic factors of availability, maintainability, and reliability. These factors in turn serve as mobility inputs to the overall weapon system analysis model in determining the overall ability of weapon system alternatives to satisfy stated mission requirements. Some of the considerations facing the analyst in the determination of these mobility factors are treated in subsequent paragraphs.

22-4.1 LAND PLATFORM CONSIDERATIONS

One of the primary means for evaluating the mobility of a land based platform is through the development and use of a mission type model. This model represents the "combat day" of the platforms being investigated. The term "combat day" relates to the sustained operation of the vehicle under evaluation over a given period of time and with speed, maneuvers, and surface requirements as established by the mission to be accomplished. From examination of the mission requirements, the analyst establishes the apportionment of the vehicle operation over primary and secondary roads, cross country, and stationary. In addition, speed requirements within each type of operation are established, as are obstacles and other environmental or operational factors.

A second type of model employed in evaluating vehicle mobility is one which describes its transit performance to the combat area of operations. In this case, the concept of "cruising (or convoy) range" becomes the measure of effectiveness, which relates to the maximum distance covered at a prescribed speed and a given type of surface. This type of model can be used in combination with the combat day model. As with the combat day model, the cruising range model must be refined to reflect mission requirements as to surface or soil conditions, grades, obstacles, general nature of the terrain, weather conditions, ditches, streams and rivers, and turning requirements. Some of these environmental considerations are discussed in the paragraphs that follow.

22-4.1.1 On-Road Mobility

Primary roads are defined as reasonably smooth surfaces, free of vertical obstacles or sudden depressions. They may also have gentle longitudinal undulations, gentle slopes (generally under 10%), and excellent load carrying characteristics which provide minimum rolling resistance. Two other characteristics which ultimately limit speeds are the width and curvature. Primary roads are characterized by widths of 16 ft or more, and tend to have large radii of curvature. Wheeled vehicles are most efficient on roads because of the high speeds that can be achieved with relatively little shock and vibration transmission to the vehicle mass through the suspension system. Wheeled vehicles have the additional advantages of simple power trains and simple steering mechanisms, both of which result in smaller power loss and so require smaller power plants.

Tracked vehicles, while providing lower ground pressure per square inch, exhibit some undesirable characteristics on hard surface roads. As speed is increased, vibration is increased due to the violence with which the track blocks strike the pavement. Tracked vehicles also are plagued with another speed-vibration problem, i.e., the engagement frequency of the track and sprocket often forms the limiting speed condition. Noise levels may become intolerable, and rubber type track blocks may overheat and fail. Thus, while mobility otherwise is good on hard surface roads, tracked vehicles tend to become more difficult to handle at higher speeds as a result of the power losses due to the continuous impacting of track blocks on the road; the resistance of the road to the lateral, skewing motion used by tracked vehicles to change course; and the breakup of road surfaces due to cleat or grouser action.

Secondary roads present all of the problems of hard surfaced roads with some modifications. Since the surface is usually dirt or gravel, it is more readily destroyed by the tracked vehicle, and there also is considerable slippage and lateral slide due to the looseness of the material. Noise levels are somewhat lower, however. The road surface is more affected by the climatic conditions; i.e., it tends to collect water or ice, become muddy, and develop pot holes.

Since secondary roads follow rather than cut through the terrain, grades are generally steeper than for hard surface roads; there are more and sharper curves; and the roads are narrower (8-16 ft widths). Natural obstacles such as fallen rocks, fallen trees, and washouts are common. Furthermore, it is

generally easier to emplace man-made obstacles, such as land mines and barricades, on secondary roads. As a result, wheeled vehicles, while still faster and more maneuverable, begin to encounter conditions on secondary roads which are less favorable for their characteristics. Further, in assessing the mobility or maneuverability of any vehicle over these types of roads, the aforementioned climatic, topographic, and natural and mad-made obstacle factors, must be included, and the resultant operationally effective values reduced accordingly.

22-4.1.2 Off-Road Mobility

Depending upon the vehicle type and its mission, cross-country movement may account for such a large percentage of the combat day that design for this type of operation can become a leading consideration in vehicular-mounted weapon systems. Many factors affect this type of mobility—e.g., natural factors such as topography, soil characteristics, vegetation, climate, streams—as well as man-made structures, e.g., buildings, fences, walls, and culverts. Vehicle requirements vary widely for different situations. The requirements for movement over hard, rocky terrain are quite different from those needed for movement through deep mud, or from those needed for operations in snow, or on ice. In addition, weather and vegetation effects on visibility will further constrain vehicle mobility. Hence, a major consideration facing the analyst is the determination of the desired level of versatility for a proposed vehicle. For example, a completely versatile vehicle may be prohibitively costly to produce and operate, whereas, alternatively less costly, special purpose vehicles or kits may be employed to achieve the desired level of versatility.

22-4.1.2.1 Early Approaches To Vehicle Cross-Country Mobility Evaluation

It might be said that through the late 1960's there were three available approaches to evaluation of the mobility of vehicles in a cross-country situation. These were:

1. A simple *tabulation and comparison of physical and performance characteristics*. The tabulated data included dimensional and other basic information that affect operation, as well as dynamic performance data under different operating conditions—e.g., cross-country, dirt, or paved roads; water obstacles; and such arbitrary standard test obstacles as walls, bumps, and slopes. The mobility rating could be stated as go/no-go finding in terms of speed, fuel consumption, ride quality, gradeability, etc. A variation of this approach employed a figure of merit system to determine the relative rankings of candidate vehicles by summing of empirical superiorities in various types of performance. Ratings by this method clearly lacked realism in that natural combinations of terrain were not used and no account was taken of the frequency of occurrence of specific obstacles. To the extent that the data used were derived from measured observations during proving ground tests, however, the values obtained were directly comparable to prior evaluations of other vehicle models.

2. A second way to evaluate vehicle mobility was by *statistical analysis of obstacles*—i.e., distribution, frequency, or probability. This approach evolved from several years' work by the US Army Tank-Automotive Command (TACOM) and assumed some determinable distribution of obstacles in the field, so that a probability of a go/no-go situation could be defined. The method relied on physical and performance data for the vehicle. Although obstacles were generally considered individually, they could be grouped in combinations. Until recently, this analytical method did not consider speed or fuel consumption, but only whether the vehicle would or would not go.

At TACOM, ability to traverse various soil conditions was based upon soil strength as determined from "bevameter" measurements of sinkage and shear. From the vertical measurement of sinkage there were derived two moduli and an exponent; horizontal shear-deformation measurements yielded values for cohesion, angle of internal friction, and the tangent modules of deformation. The six soil

strength values thus obtained provided sinkage of and resistance to the wheels or tracks under specific conditions in addition to gross traction at different amounts of slip. The resulting data thus are particularly useful to designers who need values for particular soil conditions so that track and wheel dimensions, ground pressures, power, etc., will be within requirements for the described condition.

3. The third method of evaluating mobility until about 1970 was a *quantitative vehicle-terrain analytical approach* developed primarily by the Mobility and Environmental Division of the Waterways Experiment Station (WES). The method was essentially empirical in that the mathematical model was derived principally from performance data for vehicles operated in a variety of terrain conditions. The terrain conditions of a given area were described by (a) identifying the pertinent environmental factors, (b) taking field measurements of these factors, (c) assigning significant ranges of values or classes to each factor, and (d) mapping the areal distribution of the factor classes. The terrain factor-families that are significant to mobility are surface composition, surface geometry, vegetation, and hydrologic geometry. After quantitative data that describe individual factors for each terrain factor-family have been measured *in situ*, or have been reduced from aerial surveys, significant class ranges for each factor could be established. Special techniques, based on correlation between ground services and aerial maps enabled the latter to be used to identify terrain conditions from the geometric, tonal, and textural characteristics of photographic patterns.

The unique indices used in the WES model included a "mobility index" as the input for vehicle characteristics and a "cone index" to reflect soil conditions. From these there was derived a "vehicle cone index" which indicated whether or not a particular vehicle would cross a given terrain.

Operating costs based on POL consumption could be derived from the WES model. Thus, the model provided the basic operating data for evaluating mission effectiveness and cost effectiveness based on payload and speed (time to operate between two points). Costs in no-go situations also were accounted for in terms of time penalties assessed when the model showed that such expedients as winching or bridge construction were required for a particular vehicle. A dollar value also could be placed on these penalties.

The vehicle-terrain analysis, or WES model, was of much value because it used speed (time) as a measure of effectiveness. This had a direct relation to mission definition. In the TACOM model the measure of effectiveness was the probability of mission completion. In tactical situations, it is not sufficient that the vehicle get from point A to point B to be of value; this must be accomplished within some given time. The WES model, therefore, had the additional advantage of providing a means for assessing operating cost. This was done in terms of fuel requirements and in the cost of additional required support equipment and troops. Both models suffered somewhat from limited input terrain data. The TACOM model had only a hypothetical terrain distribution. Until recent years, a highland area of Thailand, for example, was the only area considered in WES model form. How comparable such an area is to other areas was so questionable that its significance was not evaluated. Other areas have recently been surveyed and set up for automatic data processing (ADP) as time and funds permitted. A disadvantage was the relatively high dollar cost, though this is not now prohibitive in specific cases. High costs are one-time investments for a given geographic area, however, because the labor required to interpret aerial photographs and detailed maps, and to conduct on-site surveys need not be repeated. Real-life distributions of obstacles for the TACOM model may be more economical to obtain.

Finally, we remark that these three techniques considered only mobility and acknowledge that other areas must be considered to arrive at overall vehicle evaluations. Typical of these other considerations would be availability, reaction time, maintenance requirements, reliability, detection, and vulnerability as covered in Chapter 15. Because differences in mobility of proposed vehicles for a given time in

history may be small, these other factors therefore may be the more important decision items. Further, all costs must be ultimately included in overall evaluations.

22-4.1.2.2 AMC (DARCOM) '71 Mobility and AMC (DARCOM) '74 Mobility Model

Establishment of surface-to-vehicle interface relations is an essential element in evaluating the off-road mobility of a weapon system. These interface relationships may be established experimentally for a specific vehicle for various surface conditions through a controlled test program. These interface relationships may be defined analytically also. Bekker (Ref. 2) presents the relationships necessary for defining the interface mathematically as we indicated in par. 22-2. Another practical approach to assessing off-road mobility is to apply the procedures developed in this decade, the AMC (DARCOM) '71 Mobility Model (Ref. 4), and the AMC (DARCOM) '74 Mobility Model (Ref. 5). This model is a comprehensive computer simulation which relies on empirical relationships to assess the interactions of the vehicle, the terrain, and the operation in order to predict cross country performance of wheeled and tracked vehicles.

The AMC '71 Mobility Model requires a total of 76 vehicle characteristic inputs, ranging from vehicle size and weight to details of its power train and suspension components. With these data the various mathematical submodels of the overall model predict vehicle performance for both areal and linear type terrain features. Usually, only the areal terrain part of the model is utilized. In brief, the areal mobility prediction part of the AMC '71 Mobility Model shown schematically in Fig. 22-3 operates in the following manner. Detailed areal terrain data are collected from existing terrain data sources such as topographical maps, air photos, terrain studies, agricultural data, and soil maps. Where possible, these data sources are supplemented by actual field surveys. All these data sources then are used to develop a series of individual maps of the area being considered for each of the terrain factors shown in Fig. 22-3.

The terrain input processor accepts these maps and overlays them to define areas in which the terrain is homogeneous with respect to all of the terrain factors simultaneously. The result of this process is an areal terrain unit map as shown in Fig. 22-3, where unit number 98, for example, might reflect an area where the slopes are always between 5 and 10 % and the soil strength in the wet season is always between 40 and 60 cone index, etc. Associated with each map unit number is a range of values for each of 12 terrain factors. For example, areal terrain unit number 14 may have detail factor value listings of 1 5 3 9 4 3 5 1 1 3 1 5 4 3 3 2 1 1 3, where the detail factor values are defined as follows:

Terrain Factor	Factor Value	Description
1	1	Soil type—fine grain
2	5	Soil strength (wet) 61 to 100 remolded cone index (RCI)
3	3	Slope—5.1% to 10%
4	9	Obstacle approach angle 149.1 to 158 deg
5	4	Obstacle vertical height 36 to 45 cm
6	3	Obstacle base width 61 to 90 cm
7	5	Obstacle length 3.1 to 6.0 m
8	1	Obstacle spacing bare > 60 m
9	1	Ostacle spacing type random
10	3	Surface roughness 2.6 to 3.5 rms in.

(continued)

Terrain Factor	Factor Value	Description	
11	Spacing of vegetation stems equal to or greater than		
	1	0 cm dia	Bare
	5	2.5 cm dia	5.6-8 m
	4	6.0 cm dia	8.1-11 m
	3	10.0 cm dia	11.1-20 m
	3	14.0 cm dia	11.1-20 m
	2	18.0 cm dia	> 20 m
	1	22.0 cm dia	Bare
	1	25.0 cm dia	Bare
12	3	Visibility range	12.1-24 m.

Ref. 25 provides additional information on terrain factor values.

Areal terrain unit maps presently exist for five specific transects in various part of the world. The transects are shown in Table 22-1 and were selected to examine mobility performance over a range of terrain conditions that would reflect areas of possible deployment for the scout vehicle. Descriptions of the general terrain and environmental characteristics of each transect are determined and kept available for use.

The 76 vehicle characteristic inputs range from vehicle size and weight to details of its power train and suspension components. With these data the various mathematical submodels of the overall model predict vehicle performance in the terrain factor values established for each map unit. Submodels consider vehicle performance in the following manner:

Terrain Factors Considered	Vehicle Performance Predicted
Soil type	Tractive and resistance
Soil strength	Forces throughout speed range
Slope	Ability to negotiate
Roughness	Ride limited speed
Obstacles	Hangup, traction, dynamic loading, acceleration and braking between obstacles
Vegetation	Traction for overriding, and vehicle size for maneuvering between trees. Driver visibility.

For a given map unit the speed result of each of these submodels is examined, the lesser speed being the limiting value which is selected as the best speed of the vehicle in that map unit. In considering the vegetation factor the model examines various strategies of maneuvering around certain size trees and overriding others to obtain the best vehicle speed. Some terrain factors such as soil strength and slope naturally interact with others and so are considered simultaneously. For example, a vehicle on a soft soil slope will have less tractive force available to climb an obstacle or override a tree than it would on a level hard surface because some of its tractive force capability is used in overcoming the soft soil motion resistance and the grade resistance. The basic output of the model is then a speed map as shown in Fig. 22-3. The areas defined on this map are obtained by combining contiguous terrain unit areas where

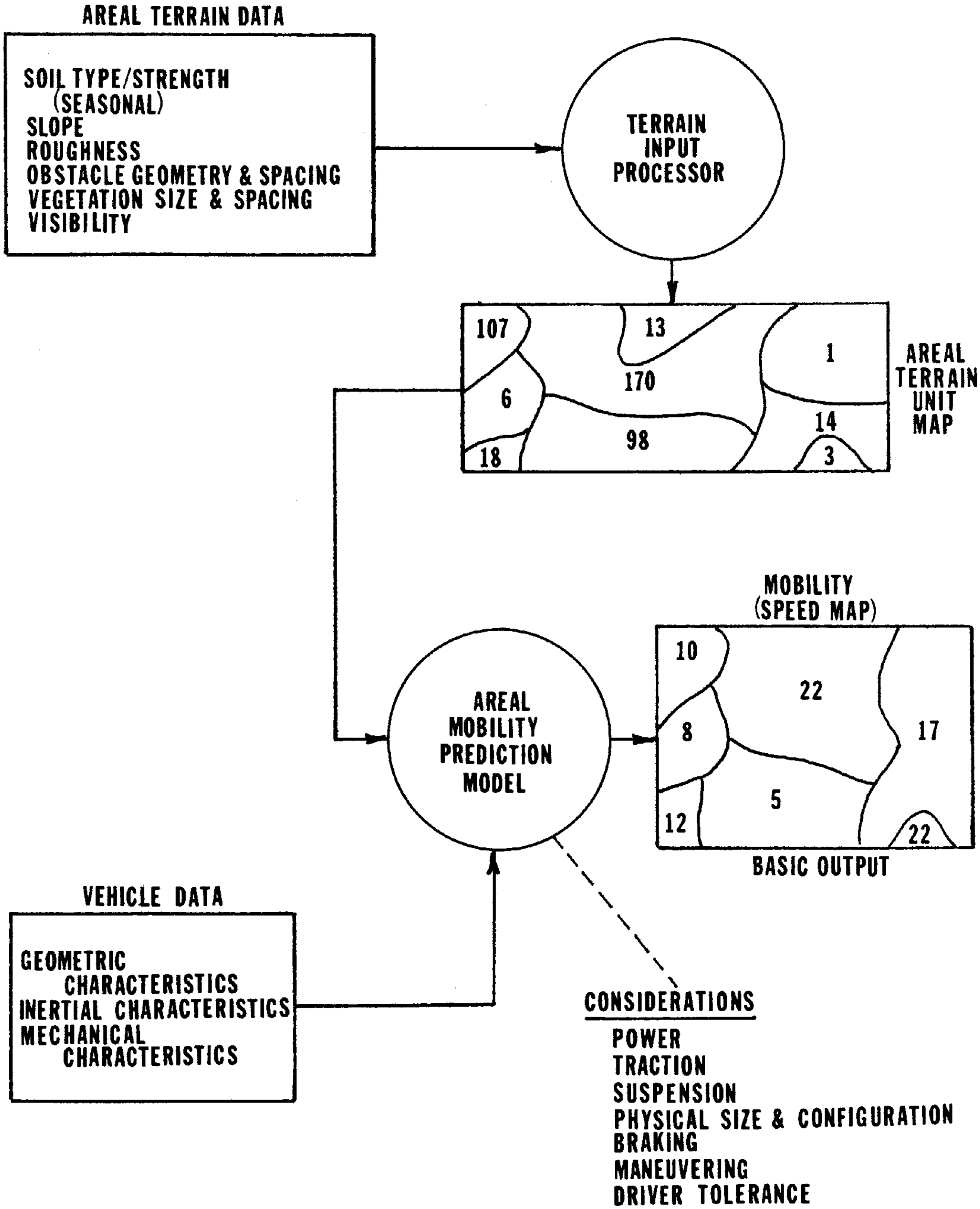


Figure 22-3. AMC Mobility Model (Areal Mobility Prediction)

TABLE 22-1. MOBILITY TRANSECT CHARACTERISTICS

LOCATION	CLASSIFICATION	TOTAL AREA	NUMBER OF TERRAIN UNITS	
			AREAL	LINEAR
West Germany Near town of Heilbronn	Temperate	156 km^2	1408	641
Kentucky Ft. Knox Military Reservation (2 areas)	Temperate	28 km^2	385	43
Arizona Yuma Proving Ground	Arid	143 km^2	405	184
Thailand Bangkok Plain	Tropic	159 km^2	485	211

the vehicle speed is equal. For the example shown on Fig. 22-3, areal terrain units 1 and 14 both show a vehicle speed of 17 mph.

Additional special outputs from the model of the types shown on Fig. 22-4 can be obtained also to aid in quantifying mobility performance. The first output option allows prediction of vehicle speed over a selected traverse covering various areal terrain units. A further modification of the traverse speed output is a routine that selects the best route (minimum time) between any two points.

The third output option is a display of the vehicle speed profile over all of the terrain within the transect area being examined. This output shows the effect of terrain trafficability on vehicle speed. It is developed by ordering the terrain units on the basis of vehicle speed in each unit. Thus vehicle speed as a function of the cumulative percentage of the total transect area can be displayed graphically. If we are interested in overall movement rates, we can calculate a cumulative average speed for all terrain of trafficability equal to or greater than that at each point on the abscissa. If we would like to know vehicle speed performance over the short run, actual vehicle speed would be plotted versus percent of the traverse area. The last special output shown on Fig. 22-4 indicates that frequency with which the various terrain factors control the vehicle speed. This type of output is useful from the standpoint of vehicle design assessment. For example, if terrain roughness limits vehicle speed most frequently, detailed examination of the vehicle suspension system design may be in order.

The AMC '71 Mobility Model can predict the cross-country speed of any wheeled or tracked vehicle operated over any geographic area. The model is a particularly useful tool for:

1. Establishing mobility criteria which ensure a desired level of performance over a specified geographic area
2. Determining and comparing the expected performance of various vehicle concepts in a specified environment
3. Studying the effect of design changes on the cross-country performance of wheeled or tracked vehicles, a requirement for the computer-aided-design and engineering of military vehicles (CAD-E).

The AMC '71 Mobility Model is well suited for (1) the assessment of the adequacy of existing vehicle performance prediction technology and hence for the unveiling of important knowledge gaps, and (2) sensitivity analyses to establish the relative importance of various vehicle characteristics which influence field mobility.

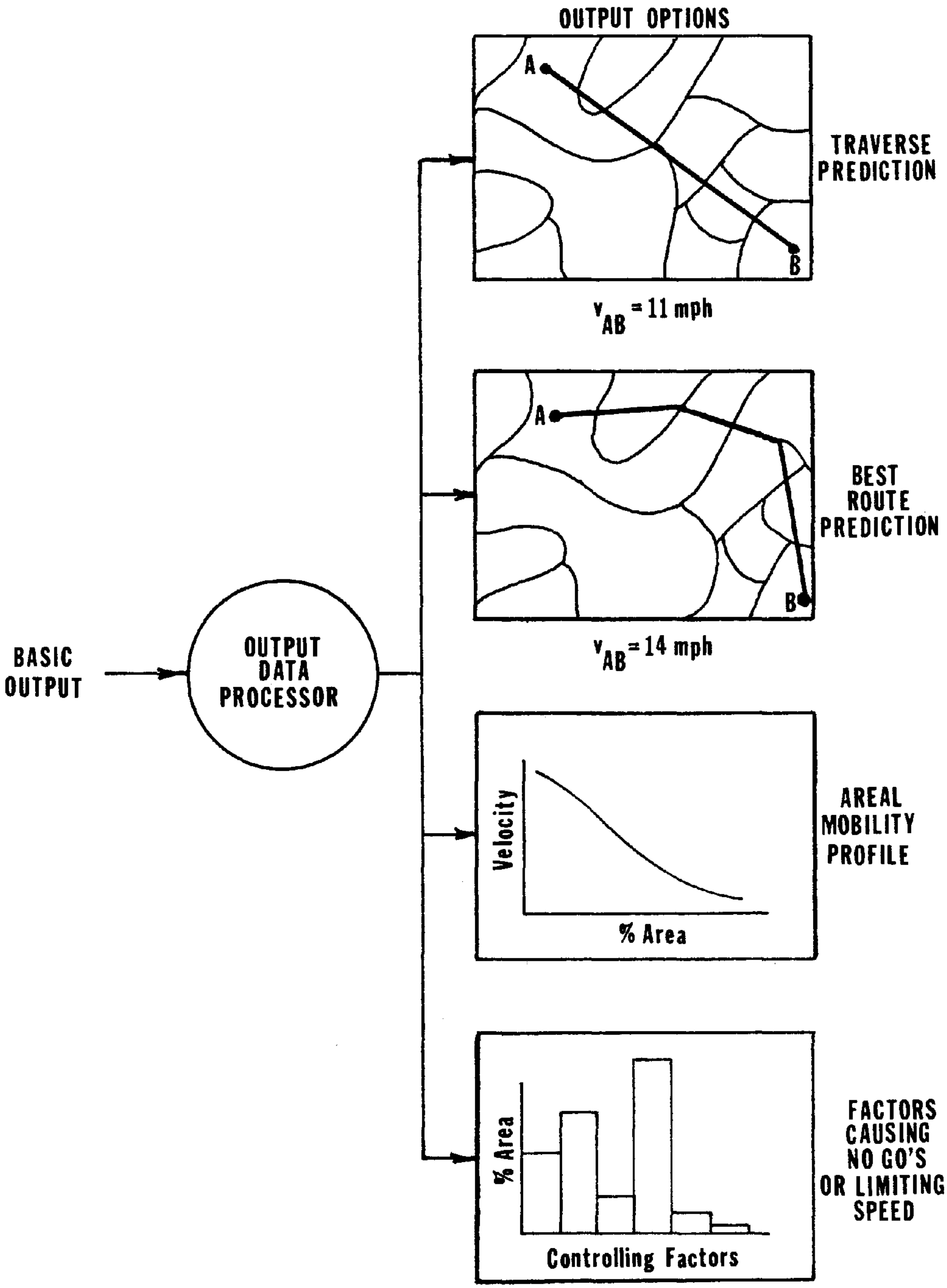


Figure 22-4. AMC '71 Mobility Model Special Outputs (Areal Mobility)

The AMC '74 Mobility Model is an improved, updated and extended revision of the AMC '71 Mobility Model. The main improvements include: specifications for axle-by-axle traction, braking and resistance calculations with recently developed equations to simulate slippery soil, muskeg and snow interaction; a corrected acceleration/deceleration model; enhanced scenario input specifications; a road module simulating travel on primary and secondary roads, and trails; an improved hasty river and dry linear features crossing module; a vehicle preprocessor module and a terrain preprocessor module; an improved obstacle crossing module; and an updated ride dynamics module.

The AMC '71 and AMC '74 Vehicle Mobility Models represent the first generation of a family of mobility models whose descendants will incorporate additional accuracies and ranges of applicability as subsequent research results become available. Hence, the analyst is cautioned to ascertain the current status of such a model prior to any attempt to use it in order to ensure its applicability.

22-4.1.2.3 The MOBTANK Model

A mobility model for tanks and track-laying vehicles called "MOBTANK" has been developed by the British and perhaps should be mentioned here, although we give only a shortened approximation version. The basic assumption is that the tractive effort TE for idealized transmission is equal to the horsepower (hp) divided by the tank speed v i.e.,

$$TE = hp/v, \text{lb} \tag{22-8}$$

where

hp = power, ft•lb/s (550 ft•lb/s = 1 hp)

v = vehicle speed, ft/s.

The tractive effort is the amount of force delivered by the engine and transmission to the drive sprocket. The maximum value of the tractive effort is the weight of the vehicle; no modeling of the gear box, transmission, and engine is considered in MOBTANK.

The program includes maximum speed versus distance, profile (height vs distance); instantaneous velocity for hazards; the surface type versus distance or percent of rolling resistance as a percent of vehicle weight (tractive effort); retardation rate (assuming constant deceleration of about 2 m/s^2 for passenger comfort); and vehicle descriptors of horsepower, maximum speed, efficiency factor, vehicle weight, and vehicle frontal area. The efficiency factor depends on the number of gears and is as follows:

No. Gears	Efficiency Factor
4	≈ 0.85
6	≈ 0.90

The efficiency factor is the efficiency of the transmission. This parameter could be reduced to simulate the tank moving at something less than the maximum speed the MOBTANK model predicts along the path, i.e., one could simulate reduced power.

The following calculations of interest are made in MOBTANK:

1. Calculate minimum gear for each level of tractive effort and velocity. This is indicated on Fig. 22-5 for the portion of the tractive effort vs vehicle velocity curve of interest.

2. Calculate the resistance on gradient to oppose speed. This is given simply by

$$\text{Resistance} = W \sin \theta \tag{22-9}$$

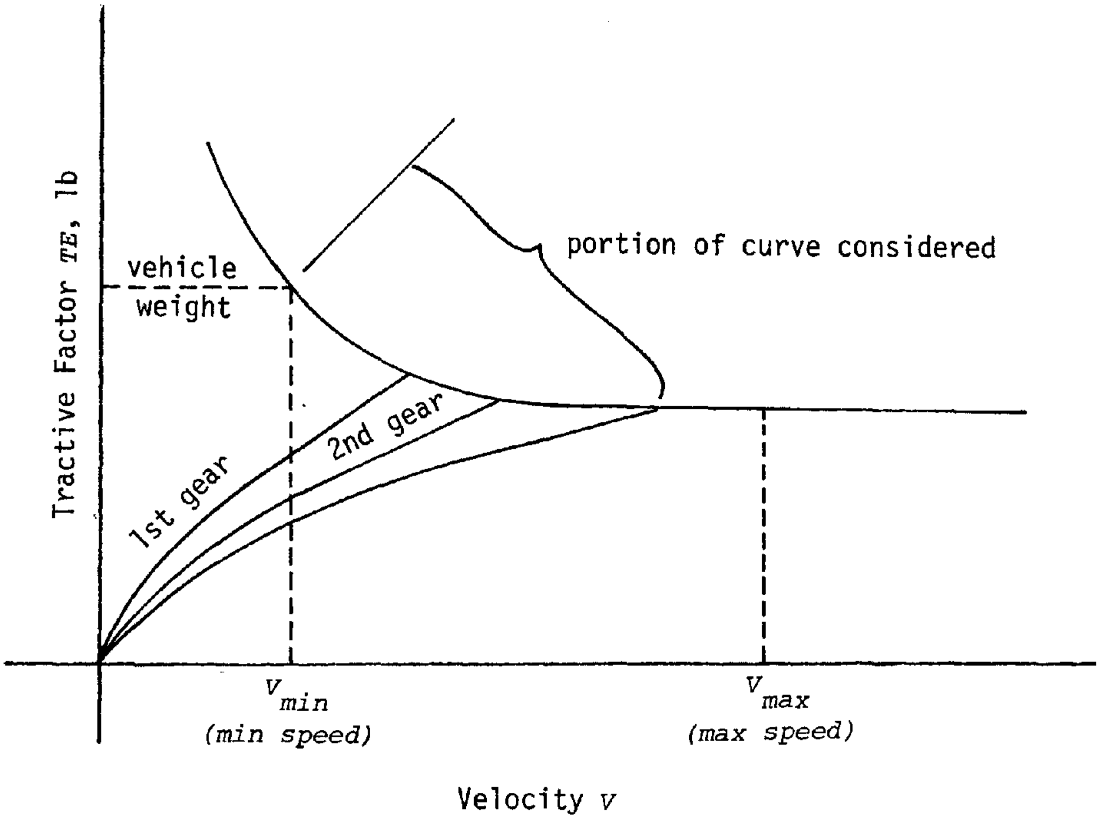


Figure 22-5. Curve of Tractive Effort vs Velocity

where

W = vehicle weight

θ = angle of inclination.

3. Calculate the aerodynamic drag on the vehicle, especially for speeds greater than 40 mph.
4. Specify the coefficient of adhesion, which will be between 0.2 and 0.8. (It is 0.4 to 0.6 for grass).
5. Calculate the total effective mass of the vehicle. This is taken to be:

$$\text{tracked vehicle effective mass} = \text{vehicle mass} + 10\% \text{ vehicle mass} + f \tag{22-10}$$

where the 10% vehicle mass is a partial component for rotating parts, and f is a factor for engine inertia (to accelerate engine) and is proportional to the horsepower and the square of the gear ratio GR. Thus,

$$f = 0.07\,(hp)(GR)^2 \tag{22-11}$$

where

hp = horsepower

GR = gear ratio.

For vehicle speeds up to 0.6 maximum, the minimum gear ratio is used, and for vehicle speeds exceeding the 0.6 maximum a gear ratio of 4 is used. From Newton's first law of motion, force F is equal to mass m multiplied by acceleration a, i.e.,

$$F = ma\,. \tag{22-12}$$

But F is the tractive effort and m the tracked vehicle mass from Eq. 22-10. Hence,

$$hp/v_i = ma_{TE} \tag{22-13}$$

so that the acceleration for tractive effort, i.e., a_{TE} can be calculated. (The speed v is variable but is at least V_{Min} on Fig. 22-5. The subscript i is used for iteration from one vehicle speed to the next one.)

6. Calculate also the acceleration a_{AL} due to tractive limit. This is given by

$$a_{AL} = (\text{coefficient of adhesion}) \times (\text{normal contact force})\,. \tag{22-14}$$

The normal contact force is

$$\text{normal contact force} = W \cos\theta \tag{22-15}$$

where as before W is the vehicle weight and θ the angle of inclination.

Finally, the speed which describes mobility for the path taken between two points is given by

$$v_i = v_{i-1} + a(\Delta t) \tag{22-16}$$

where

a = lesser of a_{TE} or a_{AL} for moving vehicle, and

where a_{TE} is determined from Eq. 22-13, but if $a_{TE} < a_{AL}$, then the vehicle is considered in a no-go state; otherwise the lower limit of a_{TE} is a_{AL}. The v_i iterated from the preceding v_{i-1} in Eq. 22-16 as i proceeds from one path interval to the next must be less than the maximum speed of the vehicle and less than the route imposed limitation. Also, v_i should be sufficiently low to enable breaking the vehicle in time for the next obstacle.

For movement of a vehicle between two points on the land surface, the MOBTANK model seems to provide a good subroutine for calculating the speed, which could be fed into the more complex AMC '71 and AMC '74 Mobility Models. MOBTANK is now being validated with the aid of actual experimental trials in the United Kingdom.

22-4.2 PATHS FOR VEHICLES AND VEHICULAR TARGETS

We next give a very brief description of probable paths for vehicles crossing terrain.

In order to lay a foundation for both the statistical measurements of mobility and models of tactics for evaluation studies, Peterson in 1957 (Ref. 6) developed a mathematical model of how routes may be chosen in crossing terrain. It is based on the premise that a route between two points is chosen so as to minimize some measure associated with the path—such as the total time required to traverse the routes, or exposure to enemy fire, or a measure which is a combination of these two factors.

In a later study involving a statistical model for describing and comparing routes over terrain, Peterson (Ref. 7) developed the idea of a single basic parameter which is the mean square change in direction per unit of path length. The distribution of sample statistics of this measure was developed and suitable tests of significance for determining differences in vehicle capabilities studied. An application is given to a map study by Peterson in Ref. 7.

Shear in 1973 (Ref. 8) gives an analytical method of describing the motion of a vehicle as it proceeds across the terrain. Velocities and accelerations are taken into account in building up a model, and the

generation of a polygonal path on which the speed is assumed to be continuous and bounded is carried out. The choice of parameters and slopes along the path is made in accordance with reasonable algorithms, and figures indicate the types of paths taken by the vehicle, showing the usefulness of his model for generating such paths.

Shear (Ref. 9) extended this work to a technique for the computation of a random path of a moving vehicle. Each generated path consists of a collection of circular arcs and straight line segments, and has a continuous first derivative at the join points. Also, each path is a random function depending upon piece-wise constant functions of three random variables: slope, radius of curvature, and acceleration. Thus, the paths generated may be very useful in analyses of the trade-off between maneuverability and survivability, or in the computation of lead error of gun systems, or for many other related studies.

22-4.3 WATERBORNE MOBILITY

Weapon system mobility in water generally is accomplished by fording or swimming. In the fording operation, the vehicle crosses a water obstacle by propelling itself through contact with the bed or ground beneath the water, by the same means used for propulsion on dry land. Key factors to be considered in assessing this type of mobility are the water depth and current; and the soil conditions on the bottom, which may vary from fairly firm sand to soft mud, and may be strewn with rock; the entrance and exit slopes of the bank; and vehicle geometry. The buoyancy effect which reduces the ground pressure must be considered also. These factors must be examined vis-a-vis the design characteristics of the vehicle in order to ascertain its suitability for fording operations.

In order to swim, the vehicle must possess sufficient buoyancy to float itself (or carry a flotation kit) when fully loaded, and a means of water propulsion and steering. These latter functions may be accomplished by the vehicle wheels or tracks with fin attachments, or by special screw-propellers or hydrojets. In either type of water operation, special problems are encountered in steering, transitions between land and water environments, climbing embankments, waterproofing, and underwater operation of the power plant. The AMC '71 Vehicle Mobility Model—which has subroutines to examine fording, swimming, and water entrance and exit—may be used to assess a wheeled vehicle waterborne mobility and related problems.

22-4.4 AIRBORNE PLATFORM CONSIDERATIONS

The detailed interrelations of the air-to-aircraft interface—as related to mobility, maneuverability, and agility—are considered somewhat beyond the scope of this handbook. However, environmental aspects do have a direct bearing on these characteristics and must be considered in any assessment of them. Weather is one of the most obvious of these factors affecting the ability of an air vehicle to operate, both as a weapon system platform or as a weapon system carrier. In addition to affecting flight operations, inclement weather also degrades parachute drop capabilities. Other environmental factors affecting airborne mobility, maneuverability, agility, and transportability—for both fixed and rotary wing aircraft, and which must be considered when attempting to quantify these characteristics for a specific system—include climatic conditions, topographic conditions, vegetation, the availability of landing or drop zones, and the soil conditions within these zones.

An excellent discussion concerning where we are headed in Army aviation research and development is given by Yaggy (Ref. 10), and this status report paper is recommended reading for the analyst who will work in the area of air mobility. As already indicated, the helicopter has provided the Army with an unprecedented mobility, maneuverability, and agility capability that has removed many of the communication and supply restrictions imposed by hazards and barriers of ground routes. Yaggy

(Ref. 10) points out that as helicopter and other VTOL technology is refined and improved, it is expected that these developments will be translated directly into greater air mobility and quick-action capability. Current projections indicate that a comprehensive research and development program can produce aircraft that have higher payloads with increased maneuverability; also, that fly faster over greater distances and under practically all conditions of weather. The current Army program in aviation research and development has the objective of developing the technology for superior aircraft to provide the aerial fire support, supply, surveillance, command and control, and communications that are essential to intra-theater air mobility and support to ground forces.

There are a multitude of models available to estimate the flight handling, maneuver, stability, and performance of helicopter, fixed wing, and tilt rotor aircraft. The Army Materiel Systems Analysis Activity (AMSAA) has a number of models which provide good first order estimates of these factors in a manner somewhat similar to the AMC '71 and '74 Mobility Models for surface vehicles. A listing of various AMSAA computer programs to evaluate air vehicles is given in Table 22-2. A few of these have been documented in reports, but in general the computer programs exist only as card decks and printout routines, which are available for use as required.

22-5 QUANTIFICATION OF MOBILITY, VEHICLE-TERRAIN, AND TERRAMECHANICS PROBLEMS

The problems of quantifying mobility, maneuverability, and agility for vehicles are rather involved theoretically. Therefore, studies are of a continuing nature in order that some valid and useful models may be developed eventually. Mobility often has been defined by pure vehicle parameters such as speed, acceleration, and turning rate, or variations of these. Also, parameters such as horsepower per ton or ground resistance also have been used. Dunetz and Masaitis (Ref. 11) apparently define mobility and agility in terms of the parameters which either increase or decrease the likelihood of a tank being hit by a projectile fired from some attacking weapon, or in other words they appear to consider the survivability of the vehicle in crossing terrain to carry out its mission.

Mobility also has been defined as a "steady state" condition, which is characterized by the velocity or time between two points, or in some instances merely by a "go" or "no-go" situation. At the tactical support level the definition of mobility would apply to the mass movement of vehicles, and hence mobility would be affected more by command and control problems than actual capability of the vehicles.

Agility has been defined as more or less a transient state, generally characterized by the vector change in motion of the vehicle. It may be measured by acceleration in a straight line, or deceleration, or as a change in direction as well. High speed or a rapid change in motion may be implied frequently, but also the ability to move in tight quarters (between trees or on narrow streets) would be important. Some claim that the probability of being hit by an antitank weapon or a missile system is inversely proportional to the agility of the vehicle. For example, Dunetz and Masaitis (Ref. 11) plot the probability of survival of a vehicle versus a "combat agility index" which is the standard deviation of acceleration in mils/second^2.

In a tactical sense, nevertheless, the ease of movement—through mobility, maneuverability, and agility—will buy survival, battlefield surprise, reaction to enemy surprise, and increased time to fight a given mission. Ease of movement may also buy endurance, reduced logistic load, and time as well. Thus, the importance of battlefield mobility.

The problem of quantifying and developing good, useful models for mobility depends very much on the interface between the vehicle and terrain features, as we have stated previously. Salisbury outlines some of the problems of the feasibility and development of terrain mobility prediction models in Ref.

TABLE 22-2. AMSAA AIR VEHICLE COMPUTER PROGRAMS

A. Power Required, Specific Range Analysis, and Rate of Climb
  1. Tanner Table Program — 40 Twist
  2. Tanner Table Program — 80 Twist
  3. Boeing Table Program — 80 Twist
  4. General Rotor Program Gives Maneuvering (AMSAA Rotor Equations)
  5. Initial Accelerations

B. Rotor Analysis
  6. General Rotor Program (Sikorsky)
  7. AMSAA Rigid Rotor Program — Also Tilt Rotor Version
  8. Flapping Analysis — Computes Stability Derivatives

C. Slipstream Effects
  9. Longitudinal Stability Includes Tilt Wings
  10. Propeller Aircraft Takeoff
  11. External Blowing Turbo-Fan Aircraft Takeoff

D. Stability/Control
  12. Helicopter Stability Derivatives
  13. Solution of Longitudinal Equations
  14. Longitudinal Motions
  15. Nap of Earth Flight
  16. Pop Ups and Side Step
  17. Pull Ups/Pushover
  18. Solution of Lateral Equations
  19. Lateral Motions
  20. Lateral/Longitudinal Turbulent Air
  21. Riccati Analysis (Preliminary)
  22. Solution of Polynomial Simultaneous Equations
  23. Longitudinal Loads (Preliminary) Load Controlled in Pitch
  24. Response to Gun Firing

E. Helicopter Parametric Design Studies
  25. Helicopter Program Costs (Can be used with any of design programs)
  26. Base Design Program (Tanner Rotor Data)
  27. Expanded Base Program (With Ferry Analysis)
  28. Expanded Base Program (With Ferry Analysis)
    a. Compound provisions use Boeing rotor data for expanded missions/multimissions
  29. AMSAA Abbreviated Program
    a. General rotor analysis, maneuverability, and any number of step mission capabilities
    b. Tilt rotor variation: aircraft mode, transition mode

F. Drone Parametric Design Studies
  30. Fixed Wing Drone Design
  31. Helicopter Drone Design

G. Systems Analysis
  32. Helicopter Resupply Problem (Armored Division Combat ASH-AAH) With or Without Breakthrough

H. Correlation Studies
  33. Two-Variable Regression Analysis
  34. Simultaneous Equations
  35. Regression Analysis
  36. Least Squares Poly Curve Fit
  37. Multivariable Regression Analysis

I. Helicopter Cost
  38. Helicopter Program Cost

12. He points out the existence of a wide range of types of environment over the earth, the problem of predicting probabilities of occurrence of terrain conditions at the "large scale" where future battles may be fought, and the selection of variables made on the basis of current vehicle parameters. He also gives many reviews of pertinent publications relating to terrain, mapping, mobility, soil descriptions,

and terrain obstacles in his report on the general subject (Ref. 12), which the analyst might well profit from reading.

As indicated already, the speed or velocity of vehicles represents one of the parameters which is used to describe mobility. Also, the speed of moving target vehicles is one of the more important characteristics that must enter into location or prediction errors, which affect the performance of weapon systems engaging such targets. A recent report by Chernick and Chernick (Ref. 13) gives an analysis of data on movements of a mixed convoy of tank and armored personnel carriers in a test conducted at Ft. Hood. The data consist of clock readings recorded every 1/20th of a mile and were used to predict target speeds. Five forcasting methods—including the autocorrelation function, exponential smoothing, projecting the most recent value, the N-period moving average, and a straight line average—were used to predict velocities of the vehicles over the hard surfaced tank trails at Ft. Hood. For the data studied, the variation of speeds about the average speed turned out to be random, so that a straight average gave the best forecast. Nevertheless, this does not mean that the straight average will be generally useful, and possibly the autocorrelation function or exponential smoothing might be advantageous for many types of data on speed prediction problems.

Thus, direct quantification of mobility, maneuverability, and agility characteristics, *per se*, of a weapon system is considered involved and rather difficult. Unlike other system characteristics such as rate of fire or reliability, there seems to be no widely agreed upon quantitative measures of mobility, maneuverability, and agility that possess a mathematical definition and accompanying units of measure. Instead, these characteristics must be inferred from other indices of system performance, either singly or in combination, depending upon the criteria selected for the specific weapon system and the specific conditions for which it is being analyzed. The indices of primary interest to the weapons systems analyst when assessing maneuverability, mobility, and agility are:

1. Speed
2. Time
3. Distance
4. Acceleration
5. Braking
6. Turning speeds and turning radius relations
7. Area or volume of coverage.

These indices are determined by the AMC Vehicle Mobility Models, and by the AMSAA Vehicle Models.

One of the basic problems in mobility of vehicles in a cross-country situation is that of developing suitably accurate terrain models that describe obstacles physically and the relative frequencies of vehicle contact with such obstacles. Eilers (Ref. 14) indicates a good approach to this problem by characterizing typical obstacles as step-ups or step-downs, bumps, ditches, ridge-transverse movement, ridge-longitudinal movement, and turning radii, and also deriving expressions for the approximate probabilities of a vehicle being able to negotiate such obstacles. Basic studies of this character will lead to many improvements in modeling the vehicle-terrain interface problem, and could provide needed input data to compute simulation of mobility.

A fairly extensive survey paper on agility has been prepared by AMSAA (Ref. 15), and some plans made for studying agility on a continuing basis. A point of interest is that of finding the target agility required to meet the threat of certain antitank weapons (including the COPPERHEAD), i.e., a countermeasure study. The chance of survival is given as

$$P(\text{Survival}) = 1 - p(h) \cdot p(k|h) \cdot \mathrm{P}_{LOS} \cdot P_{ACQ|LOS} \tag{22-17}$$

where

$p(h)$ = chance of a hit

$p(k|h)$ = chance of a kill given a hit

P_{LOS} = chance of an uninterrupted line of sight during missile flight

$P_{ACQ|LOS}$ = chance of acquisition given a line of sight.

Line of sight curves may be found in Ref. 16.

Bridgford, Heber, and Brandi (Ref. 17) have studied the effect of tank agility on survival in computer simulated combat situations. Vehicle speed and "evasive maneuver" are used to assist is describing the agility of a vehicle, and the increase in dispersion of a weapon firing at a moving tank is predicted by the EVASIV model which is a routine developed by AMSAA and modified by the Rodman Laboratory at Rock Island Arsenal to estimate the impaired accuracy of fire. The EVASIV model uses input data consisting of target speed, time of flight of round, target approach angle, range, clockwise/counterclockwise movements, and an "evasive" factor level in acceleration from zero to 6.9 m/s^2. The output gives the evasive bias, and the lead and the lay errors of the antitank weapon for the horizontal and vertical directions. This increased dispersion for the moving tank target is fed into a hit probability calculation called PHCALC, and in this way some quantification of survivability may be made. The reduction in hit probability due to increasing the level of evasiveness is very pronounced indeed. Graphs are given of hit probability levels versus speed and amounts of evasiveness for both the simple and the more complex fire control systems. Speeds under about 5 m/s appear to have negligible effect on hit probability but, at the higher speeds, both vehicle speed and evasiveness substantially reduce hit probability for simple fire control systems. For the complex fire control systems, speeds above 5 m/s appear to have little effect on hit probability, but evasive maneuvering, if it can be done by tanks, will degrade hit probability substantially. The authors also report the results of computer simulations based on DYNTACS, a high resolution, battalion level, armored combat model. This supports the importance of evasive maneuvers to improve survivability of the tank.

What about mobility for artillery and surface-to-surface missile systems? In this connection, Spears (Ref. 18) presents an interesting analytical argument for quantifying this type of problem. He advances the recommendation that the firepower of artillery and the mobility of artillery on the battlefield should be considered jointly. On this basis, Spears proposes a ratio, the number of enemy targets multiplied by the average target area (or total target area) an artillery fire unit can defeat within a given time divided by the total area such a mobile unit can threaten in that same time period. Thus, the ratio represents what might be called the "efficiency" with which artillery can successfully cover some ground area of tactical interest.

A comment on transportability of military vehicles is of interest here. When considering transportability, size and weight parameters must be considered also. Operational and environmental factors are included in assessing a specific weapon system mobility, maneuverability, or transportability through the degree they affect the inherent performance capabilities. These inherent parameter values and their relationship to the design properties of the vehicle being analyzed—e.g., the relationships between speed and horsepower, and suspension for a wheeled vehicle—are established through the analysis of prior test data or the application of prediction equations. These inherent values are then translated into operational values reflecting the operational and environmental conditions through the application of the applicable mobility, maneuverability, or transportability model—e.g., the AMC Vehicle Mobility Models. In this way, candidate designs can be assessed and their mobility and maneuverability, and transportability indices compared, and trade-offs can be established between these indices and design factors, cost, or other system measures such as survivability.

The example that follows illustrates this process. Assume the inherent performance (speed) parameter of a given weapon system vehicle under study is known (through results or mathematical relationships evaluations) for operation over various road, terrain, and weather conditions. Further assume that the operational mobility parameter of interest in the analysis is the time required to traverse the distance from a specified staging area to the forward edge of the battle area (FEBA). (This measure of mobility may be of interest to the weapon systems analyst since it determines when the weapon system can be committed to an engagement.) The means of determining this time measure depends upon the nature and detail of the scenario within which the weapon system is being analyzed. If the scenario is described by detailed maps with the staging area and FEBA identified, then specific routes and their associated terrain and road conditions also can be identified. The routes then can be divided into segments corresponding to the given speed/terrain and road condition data, and the expected time to traverse each segment ascertained such as in Refs. 8 and 9. The total time for each route would be the sum of the individual segment times. This time estimate may be further refined by weighting it by the effects of expected weather conditions and also the vehicle expected reliability and availability. Where the scenario is not specified by maps, another approach defines the percentage of each type of terrain/road condition and the total transit distance. The desired time measure for this situation would be generated using the given distance, percentage of each condition, and tabulated speed values for each condition.

When the scenario specifies the terrain, road, and weather conditions in terms of their probability distributions, then Monte Carlo simulation techniques can be used to determine the distribution of times for the system to reach the FEBA and their expected values.

Maneuverability characteristics can be developed in terms of operational parameters in a manner similar to that just illustrated for mobility. In this case, the parameter of interest would reflect both mobility and agility considerations. For example, mobility could be expressed in terms of the percentage of the combat area in which the weapon system can move at a specified speed, and agility expressed by the average acceleration that the vehicle can achieve within the combat area. The measure again would serve as an input to the overall weapon systems analysis, e.g., by bounding the area in which the vehicle can maneuver during the analysis of the engagement or by providing a probabilistic data base for the vehicle movement during the engagement in a Monte Carlo simulation. This parameter can be quantified again using the inherent parameter data and analysis of the terrain, weather, etc., available from the scenario within a model such as the AMC '71 and AMC '74 Vehicle Mobility Models.

Evaluation of the transportability characteristics of a weapon system is achieved again through consideration of inherent parameters. However, in this case, the resultant measure used in the overall weapon systems analysis is obtained less directly than for mobility or maneuverability. For example, the size and weight of the weapon system will determine which types of vehicles, within the time frame of the analysis, are capable of transporting the weapon system. Once the vehicles meeting the constraints have been identified, the transport vehicles themselves must be analyzed vis-a-vis the terrain, weather conditions, etc., and their inherent performance parameters—as was illustrated earlier for mobility—to determine the resultant impact of the transportability characteristics upon the input data, e.g., arrival time for the weapon system in the combat area, to the overall weapon systems analysis.

22-6 AN IMPORTANT APPLICATION OF MOBILITY MODELS

The problems of quantifying mobility often can turn out to have very important interfaces with other features of a weapon system. In fact, an old and continuing problem is that of optimizing or trying to

trade-off firepower, mobility, and armor protection for tanks in combat. This is a very involved problem which has been on the books for many years without any clear-cut solution, although recently some significant progress has been made. Masaitis and Woodward (Ref. 19) considered the problem of trading off the mobility of a tank versus armor protection in a quantitative way. Reed (Ref. 20) carried out an informative analytical study of mobility versus vulnerability for tanks, and later (Ref. 21) extended his ideas to some analytical aspects of maneuver and survivability. Francis and Masaitis (Ref. 22) also studied some models on mobility and survivability of a tank. Grubbs and Zeller (Ref. 23) apply Lanchester type combat theory and simulations to study trade-offs between armor protection, firepower, and mobility for tanks, looking particularly into whether it might be advisable to consider firing multiple rounds offensively as compared to increasing armor protection and mobility. For a terrain analysis of some tactical situations, see Ref. 16. The analyst who will be engaged in evaluations in these areas should find such studies of considerable interest, aside from building up a background for expanded investigations.

22-7 RECENT STUDIES

More recently, Battelle Institute (Frankfurt, Germany) has been engaged in the problem of modeling the mobility of military ground vehicles, and Melzer (Ref. 24) describes some of the current studies. Melzer's paper (Ref. 24) discusses modeling approaches aimed at deriving mobility information in a mission-oriented context, for example, in the simulation and gaming of land vehicle systems, like minimum and average time requirements for vehicle and unit movements and combat and/or logistic support missions. Melzer points out, as is well known, that mobility depends not only on the mechanical capabilities of the vehicle but also greatly on the environmental conditions under which it is operating. Moreover, mobility methodology is of interest to (1) the vehicle development community, (2) the vehicle procurement community, and (3) the user; accordingly, it may be desirable to develop a common, unified approach for all three.

Battelle Institute (Frankfurt) has been engaged in the mobility study since about 1974 and currently is running validation tests for mobility models to include the following operational steps:

1. Collecting vehicle data
2. Establishing basic vehicle characteristics and relations
3. Selecting terrain traverses (length: about 5 to 13 mi) and collecting terrain data
4. Carrying out speed tests with the corresponding vehicle (the driver has the order to drive as fast as he possibly can)
5. Simulating the test runs using the latest version of the Army Mobility Model
6. Evaluating simulated and actual test runs (comparison of simulated and measured speeds and times for specific terrain units and the total traverse).

These validation tests are being conducted during regular military trials. More recently their studies have been directed towards comparative mobility evaluation methods of different vehicles.

The mobility of a unit of several vehicles in a mission-oriented context depends highly on the overall efficiency of the terrain-driver-vehicle system. The main system variables considered are:

1. Vehicle type
2. Number of vehicles in the unit
3. Type of terrain
4. Type of mission.

The basic tools to demonstrate some of the mobility approaches are speed maps and speed profiles, and will involve the performance of individual vehicles and many vehicles in a unit.

For logistic type vehicles of the same type, such as 2½-ton trucks, the following relation is given (Ref. 24) for travel time T_c along some length ℓ of terrain

$$T_c = (\ell/v_s) + [N/(Dv_s)], \text{ h} \tag{22-18}$$

where

T_c = travel time for vehicle column, h

ℓ = length of terrain, mi

N = number of vehicles in column, vehicles

D = vehicle density per mi (depending on vehicle length and speed), vehicles/mi

v_s = speed of single vehicle, mph .

The relation between the individual vehicle speed v_s and that of the column is given as a function of the number of vehicles in Fig. 8 of Ref. 24. Also, Fig. 9 of Ref. 24 gives the speed for "stiff", "medium", and "soft" soil consistencies as a function of the number of vehicles.

Ref. 24 indicates that, for combat vehicles, the problem of modeling is much more difficult mainly due to the larger variety of missions combat vehicles have to fulfill. (This report, available from the author, may be of interest to the systems analyst.)

22-8 SUMMARY

Much work remains to be carried out on the development of widely accepted analytical models for describing mobility, maneuverability, and agility—especially for land vehicles under the terrain conditions of tactical interest in combat. Nevertheless, the characteristics or parameters involving speed, time, distance, acceleration, turning speed and turning radius, braking, evasive maneuvers, and standard deviations of such quantities may be involved in one way or the other, and much can be learned through computer simulations to find the effect of the basic physical characteristics in sensitivity analyses. Perhaps mobility is associated mostly with vehicle speed, the time required to travel between two points on the battlefield, or change of position. Agility, as currently considered, seems to deal with acceleration (or the standard deviation of acceleration), quick turning capability, and small turning radius. The concept of survivability of a vehicle on the battlefield also offers hope as an overall measure of mobility and agility. Maneuverability, on the other hand, may require some of the characteristics of both mobility and agility. Nevertheless, a widely accepted approach of study is to conduct computer simulations which take account of the major characteristics and variables of the vehicle-terrain interface problem in order to evaluate elements affecting mobility, maneuverability, and agility. One of the important problems of the systems analyst is to develop improved analytical models for mobility, maneuverability, and agility in order to perform better evaluations of weapon systems in expected combat environments.

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CHAPTER 23

LOGISTIC PLANNING AND SUPPORT

The design, development, production, and deployment of weapon systems must take into account the problems of logistic planning and support; therefore, the evaluation of weapons should involve the quantification of logistical factors. Some of the considerations for logistic planning and support type factors for the systems analyst are covered in this chapter, indicating the need for the analysis of a complex stochastic area of endeavor.

23-0 LIST OF SYMBOLS

A_i = expected asset level for item i
a = cost of first article procured or item produced
AYD = average yearly demand
B_i = expected backorders for item i
B_i = units short and to be backordered for the ith item
b = slope of progress curve
D_i = average yearly demand for item i
E_i = essentiality of item i
EOQ = Q = economic order quantity
$F(Y;t)$ = chance of Y or fewer demands in time t
$f(Y)$ = probability density function of Y
H = holding cost
h = holding cost rate
HC = total annual holding cost
I = holding cost rate
L = procurement lead time demand
L_i = expected lead time demand for item i
MAD = mean absolute deviation
N = number of items in the inventory system
OC = total annual order cost
OH_i = onhand inventory for item i
P = variable procurement setup cost
P' = cost to order
p = administrative cost of procurement
$PCER$ = percentage error
$PCERL$ = percentage error for lead time
PLT = procurement lead time, yr
Q_i = economic order quantity for item i
Q_W = order quantity parameter (Eq. 23-20)
Q' = reorder quantity = $b\sigma$
R_i = reorder warning point for item i
RS = requisitions short
S = safety level = $a\sigma$
S_i = requisition size for ith item

SL_i = safety level for item i
t = time
TVC = total variable cost
U_i = unit price of item i
u = unit price (Eq. 23-1)
V = variance-to-mean ratio
UP = unit price (Eq. 23-20)
x = variable of integration
x_i = cumulative unit number
y_i = cost of ith unit procured = ax_i^b
Z = $p/(huLV)$ = a convenient parameter
α = level of protection
β = constraint on requisitions short RS
γ = factor for relating σ and MAD
λ = forecast of average annual demand
ρ = parameter (Eq. 23-20)
σ = standard deviation of demand during lead time = $\sqrt{\lambda LV}$
σ = standard deviation of demand = $\gamma(PCERL)L = \gamma(PCER)L/[12(PLT)/9]^{1/2}$

23-1 INTRODUCTION

The Dictionary of US Army Terms (Ref. 1) defines logistic support as the "Provision of adequate materiel and services to a military force to assure successful accomplishment of assigned missions." Ref. 1 also defines logistic doctrine as "The creed derived from the body of principles applicable to the determination of requirements for, and the acquisition, distribution, maintenance, and disposal of logistic resources and services integral to a military capability."

Eccles (Ref. 2) points out that there seem to be two views of the term "logistics", the comprehensive view and the narrow view, and we quote:

"There seem to be two views as to just what is meant by the term 'logistics'.

"The comprehensive view is that logistics covers the creation and sustained support of combat forces. This view is based on Duncan Ballentine's statement (Ref. 3) and on the Joint Chiefs of Staff's official definition (Ref. 4).

Duncan Ballantine wrote:

> '. . . As the link between the war front and the home front the logistic process is at once the military element in the nation's economy and the economic element in its military operations. And upon the coherence that exists within the process itself depends the successful articulation of the productive and military efforts of a nation at war.' (Ref. 3)

"The Joint Chiefs of Staff Dictionary says:

> 'Logistics — The science of planning and carrying out the movement and maintenance of forces. In its most comprehensive sense, those aspects of military operations which deal with: a. design and development, acquisition, storage, movement, distribution, maintenance, evacuation, and disposition of materiel; b. movement, evacuation, and hospitalization of personnel; c. acquisition or construction, maintenance, operation, and disposition of facilities; and d. acquisition or furnishing of services.' (Ref. 4)

"The narrow view of logistics is nowhere explicitly expressed but nevertheless seems to be quite widely held. This view is that logistics deals with the functions of supply and maintenance in the field of actual operations, plus the "nuts and bolts" of irksome administration that no one seems to want.

"Between the recognizable views, there lie many other generally vague concepts of logistics—none of which can form the basis for a consistent and coherent use in either military terminology, organization, or education.

"When the narrow view of logistics is taken, those activities dealing with the producer phase of comprehensive logistics are classed under the general title of management."

An Army in the field may be divided for our purposes here into two component parts, i.e., the combat troops and the service forces. The combat troops cannot accomplish their mission without the support of the service forces which help them to move around the battlefield, and furnish them the supplies necessary to live and fight. The term "supplies" for military operations includes everything needed to equip, maintain, and operate armed forces in the field. It therefore includes food, clothing, medical supplies, arms, ammunition, fuel, gasoline and lubricants, construction and maintenance materials, vehicles, and varied types of machinery.

Until about the 17th century, armies lived off of the country they fought in; however, it was recognized that, other things being equal, the side which best supplied its combat troops would win the war. Thus, military planning on either a campaign or a grade scale requires constant application of the science of logistics—procurement, supply, and maintenance of all materiel for the combat forces. In fact, logistics now involves more or less the manpower and resources of a whole nation, at least for the large-scale conflicts.

A critical part of the logistic problem involves the transportation system. Thus, a nation must have a very efficient transportation system, whether for water, highway, rail, or air. This guarantees a steady flow of supplies from the rear to the front, and the concept of "Day of Supply" is much involved.

Finally, it can be seen that logistic processes involve stochastic problems. That is to say, equipments fail at random, the patterns of demand vary, and there are stochastically occurring combat losses in a complex environment. Hence, it should be expected that simple models of logistics are difficult to arrive at and that many logistic studies must be made using simulation procedures. Also, logistic problems often may involve life-testing and the replacement of parts due to wearout or failure.

23-2 SOME GENERAL CONSIDERATIONS

Since this Handbook is oriented primarily toward weapon systems analysis, and since support and logistic factors constitute a field in their own right, we will consider such problems only in the general context of requirements for analyzing weapon systems. The references cited at the end of this chapter can provide the reader with more detail should it be necessary to broaden the scope of any study to various areas of support and logistics.

All weapon systems depend on the support organizations of the US Army to maintain the capabilities inherent in the system. The organization for combat support and combat service support was described in Chapter 8. Weapon systems analysts should have an awareness of this support capability for two primary reasons:

1. Existing or planned support and logistic capabilities, including costs, may offer the analyst a range of alternatives which are important but otherwise not apparent. For example, is the relative value of the amphibious characteristic of a combat vehicle less when the existing engineer support includes only bridging capabilities? This clearly may affect the evaluation.

2. Support and logistic factors are necessary considerations in defining a realistic scenario for the study and in the quantifying of input parameters. These factors may not directly affect the primary mission capability of the weapon system but become important in determining the length of time a system may be effective, the cyclic nature of its sustained capabilities, and its costs. As such, these factors may therefore be important considerations in the weapon systems analysis process.

Principal support resources to be considered should include: (1) trained personnel, (both operators and maintainers); (2) special tools, test, and support equipment; (3) equipment publications; (4) repair parts; and (5) facilities. In fact, logistic support must be considered as a design parameter during the development of an item or system so that the final product is maintainable and supportable, and appropriate systems analysis type evaluations are needed to determine the effectiveness of support systems backing up the fielded weapon systems because of complexity and costs.

We now will discuss briefly some of the types of support.

23-3 INTELLIGENCE SUPPORT

Intelligence was introduced and defined in Chapter 9. The functions of intelligence which are of greatest interest to the weapon systems analyst are those of collecting, evaluating, analyzing, and interpreting information concerning the enemy (Ref. 5). These functions may be performed both in the prestrike phase, to identify possible targets, and in the poststrike phase, to assess target damage and mission accomplishment. Intelligence is important in the weapon engagement phase also.

Since all weapon systems require targeting intelligence for effective use, the analyst will be concerned with intelligence efforts to acquire targets and to assess target damage. Alternatives available for gaining that intelligence range from complete reliance on the organic or inherent capabilities of the weapon system under study to dependence on a separate intelligence organization for the information, or through any mixture of these two sources. There is a continuous range of alternatives available for consideration. It is conceivable that more than one discretely defined alternative from this range would be necessary or desirable. For example, a field artillery system might use its organic forward observation capability (Ref. 6) for targeting information, while also using intelligence information generated by other battlefield surveillance sources which are not part of the field artillery organization.

The analysis of a weapon system which depends on external intelligence sources for essential elements of information must include, as a minimum, assumptions concerning the timeliness, accuracy, precision, and reliability of the information furnished. Determination of the degree of adequacy of these elements is a joint responsibility involving the system developer, the system user, and the systems analyst. If all elements are judged to be adequate to support the weapon system, a statement to this effect should be made in the study. Should the elements appear to be inadequate in one or more areas, these areas must be identified and the analysis must address the probable effect of these weaknesses.

23-4 COMBAT SUPPORT

The Army elements providing combat support were identified in Chapter 8. Two of these elements, engineer and signal, are not primary weapon system users, but provide timely and necessary support for the systems employed by other Army elements. When new weapon systems are introduced into the Army, the combat support user agencies analyze their capabilities to cope with the added requirements and make appropriate changes in organization and procedure.

23-4.1 ENGINEER SUPPORT

There are several engineer support functions the weapon systems analyst should be aware of:

1. Survey of the weapon system emplacement site to establish its location on the ground. The analyst should learn the probable errors of such a survey and, if emplacement time is important, he should obtain estimates of the time required to perform the survey.

2. Site preparation or construction to accommodate the weapon system. As before, time may be important, as may be errors in smoothness, rigidity, or level of the weapon base.

3. Construction of fortifications to facilitate weapon system defense, including the employment of mines in the fortification of the site.

Weapon systems which depend on mobility to enhance their effectiveness may use the engineer capabilities to bridge rivers or chasms and to breach obstacles, including minefields. These engineer functions may offer the analyst several alternatives worthy of assessment in his study.

The analyst should be aware, however, that engineer functions do not normally include the defense of weapon system sites, nor do they include continuing direct support, other than water and map supply. The organization and capabilities of engineer units is given in Ref. 7. Specific topics of interest to the analyst desiring more detailed information are available in FM 5-XX series of Field Manuals or directly from the engineer user agency.

23-4.2 SIGNAL SUPPORT

The importance of communication in the exercise of command and control was presented in Chapter 8 and applies to all weapon systems. Studies concerned with weapon systems must recognize the methods of communication required, both within the weapon system and linking the system to other elements of the Army. Relatively simple weapon systems, such as small arms, may fit easily into small tactical units and require only the communication means already organic to these units. The signal requirements become more complex, however, as the weapon system increases in size or complexity; requires cryptographic, surveillance, or data processing capabilities; or attains a nuclear delivery capability.

The wide range of communication methods within the modern field Army (Ref. 7) offers the analyst an array of alternatives to provide necessary signal support. The system may use wire, radio, or a combination of these operating with voice, continuous wave, teletype, or digital information. The communication terminals may be operated by organic weapon system personnel or by signal support personnel. Electronic countermeasures should be taken into account, of course. Detailed information on signal support can be found in the FM 11-XX series of Field Manuals.

In devising the scenario for the weapon systems analysis, the primary signal support considerations would be the adequacy and timeliness of communications. The communication capability of large weapon systems having specially designed organic signal capability would be studied primarily by the agency developing this equipment. If, however, the communication system is at all complex and if sophisticated data transmission requirements are imposed as with some computer target selection systems, the analyst should be aware that waiting times (queues) may be critical. Capabilities of existing Army signal equipments are documented in current field and technical manuals. These factors should be used to provide the necessary inputs to the study scenario.

23-5 COMBAT SERVICE SUPPORT

The three primary combat service support sustaining functions of interest to the weapon systems analyst are transportation, supply, and maintenance. While these functions are both important and necessary to any weapon system, weapon systems analyses are generally concerned only with their overall adequacy. If the level of support required is within the normal capabilities of the combat service support organization, the adequacy of support is assumed and the study proceeds without further concern in this area. However, adequate never means perfect. The normal residual inadequacies must be known and considered. If the support required exceeds that normally available, the constraints are identified and their effect on the weapon system is analyzed to determine the significance of the constraint. This may be accomplished in conjunction with the appropriate user which has the mission of recommending and implementing changes to the applicable combat service support organization. The

user agency also will be an invaluable source of information concerning alternatives which may be available. A detailed description of combat service support is contained in Ref. 8.

23-5.1 TRANSPORTATION SUPPORT

Weapon systems movement requirements may be segmented into two principal categories:

1. Movement of the system to the deployment area. This may involve the use of land, sea, or air transportation with little regard for the system organic mobility capability.
2. Movement of the system to respond to the tactical situation. Here such factors as portability, mobility, and transportability become important (Chapter 22).

These considerations are first expressed in the materiel requirements documents which initiate the acquisition process. The weapon systems analyst often will find that means and modes of movement have already been considered by the user agency and the system developer. Accordingly, TRADOC and DARCOM become good data sources for any analysis involving the movement of a weapon system. Additional data on the capabilities of transportation support organizations can be found in the FM 55-XX and TM 55-XX series of manuals. Organizational composition of transportation units is given in the unit type table of the organization and equipment (TOE), extracts of which can be found in Ref. 9.

The three most frequently asked questions concerning transportation support for a system are, "How can it (the system) be moved?", "How long will it take to move?", and "What will it cost?" The first question requires knowledge of the physical characteristics of the system (including size, weight, and packaging), the means of movement available (self-propelled, organic transportation, support transportation), and the features of the terrain involved. The second question depends on the selections made in answer to the first question. Generally, the time required will consist of the preparation time, the loading time, the movement time, the unloading time, and the activation time. Movement time may be dependent on the speed of the carrier or on the speed of the slowest element in the convoy to which the carrier has been assigned. The answer to the third question is obvious. The answers to these questions become inputs to the study, while the factors influencing the answers—such as enemy capabilities and system vulnerability—become part of the scenario for the study.

23-5.2 SUPPLY SUPPORT

Supply support concerns the distribution of goods and materials to elements of the US Army. The ten classes of supply, as categorized by the US Army (Ref. 10), are listed in Table 23-1. While all classes might be of importance to a logistic analyst, the weapon systems analyst generally is interested

TABLE 23-1. ARMY CLASSES OF SUPPLY

CLASS	SUPPLY CATEGORY
I	Subsistence
II	Secondary Items and Expendables
III	Petroleum, Oils, and Lubricants (POL)
IV	Construction and Barrier Materials
V	Ammunition and Explosives
VI	Welfare and Personal Comfort
VII	Major Items
VIII	Medical
IX	Repair Parts
X	Agricultural and Economic, Nonmilitary

only in that supply support necessary to sustain and replenish the weapon system being studied. Normally, this would only include replenishment of that ammunition (Class V) and fuel (Class III) required to complete the mission postulated in the scenario. However, if the system mission time approaches the Mean Time Between Failure (MTBF) or Mean Time Between Overhauls (MTBO), repair parts (Class IX) may become significant and require consideration.

The weapon systems analyst may approach the problem of supply support in one of two possible ways. Selection of the approach will depend on the amount of detail in the scenario and the type of model used to represent the situation. First, he may determine the maximum quantities of supply to be consumed during a replenishment period and check these deterministic values against the capabilities of the supply support organization. If satisfied that supply support is adequate, infinite supply is assumed and supply support is dropped from further consideration. This approach would suffice for most studies considering the weapon system *in isolation*. The second approach would be to accumulate consumption data as the model is exercised. This consumption is then replenished from supply support sources in accordance with an algorithm established within the model. While considerably more complex and requiring more data items, this approach may be necessary in situations where several weapons may need replenishment from finite or limited resources. That is, priority of need between and among weapon systems must be entered into the evaluation algorithm.

Supply support is detailed in Ref. 8, and consumption factors are given in Ref. 7. Supply of repair parts (Class IX) is an integral part of maintenance support and is discussed in par. 23-5.3.

23-5.3 MAINTENANCE SUPPORT

Maintenance support consists of all personnel, procedures, facilities, tools, documents, repair parts, and other materials necessary to restore an item of equipment to operating status or to declare it unrepairable. It also includes recovery of damaged items and evacuation of unserviceable assets. This support may be applied at any of the following four levels:

1. Organizational maintenance support is performed at the operator or user level and generally is limited to preventive maintenance or servicing tasks.
2. Direct support maintenance is provided directly to using organizations by maintenance units consisting of trained combat service support personnel. It may be considered as a retail support activity.
3. General support maintenance provides backup to the direct support maintenance units and may be considered a wholesale combat service support activity.
4. Depot maintenance provides rebuild and major overhaul support.

Additional details of maintenance support activities are contained in Ref. 8. Information requisite to the planning and implementation of effective maintenance engineering programs is contained in Ref. 36.

The primary maintenance support concern in a weapon systems analysis is the determination of the minimum downtime required to restore a system to an operationally ready status along with the associated costs. For weapon systems still in development, the system or commodity developing agency should furnish system, subsystem, and component maintainability data. Generally, these data will represent inherent maintainability characteristics achievable under ideal conditions. The National Maintenance Point in Lexington, KY, can furnish actual field experience repair time data from The Army Maintenance Management System (TAMMS) on fielded systems of interest. These data also may be useful if taken from already fielded systems similar to the weapon system being analyzed, particularly when the system under study is still in the development stage. As a minimum, it will allow the

analyst to adjust the developer's idealized estimates to a more realistic operating environment characteristic. Quantification of system maintainability is discussed in Chapter 21.

23-6 INTERACTION AMONG LOGISTIC ACTIVITIES

As pointed out by Mirasol (Ref. 11), logistics, as used by the military, deals with the functions and activities associated with the procurement and operational support of materiel systems. Normally, therefore, such functions would include the supply, maintenance, transportation, reliability improvement, and availability of equipment systems. Often supply agencies are separated from the maintenance activities, and many times the research and development activities for a weapon system do not take into proper account the logistic support or maintenance requirements for a fielded system. For these reasons, the analysis of logistical problems in such a piecemeal manner often can lead to some rather serious consequences, since logistic activities are related and dependent in some way or other. For example, supply and transportation should not be separated since the stocking of items in forward echelons may be reduced by faster means of transportation which would reduce the time to fill a requisitioned item. Availability of items is affected by maintenance, and maintenance depends on the reliability of items in the pipeline and on the battlefield. The downtime caused by maintenance may be reduced if more additional manpower in the form of skilled mechanics were available, or more sophisticated diagnostic equipment could be provided, but both of these affect overall costs of fielded systems. Repair parts planning and the replacement of failed items must also constitute an efficient process. Thus, logistic support activites are interchangeable in some ways, and there are different ways of accomplishing the desired goals. Hence it is easy to see that the whole logistic support role needs the application of systems analysis techniques, just as that for the evaluation of weapon systems. The basic question, therefore, is how should overall costs be allocated in the logistic process? Mirasol (Ref. 11) discusses these problems in terms of a model involving a multistage, circular queuing system process, thereby getting into the problem of quantifying logistic support.

23-7 INTEGRATED LOGISTIC SUPPORT (ILS)

In view of the fact that much coordination and integration are required for effective logistic systems, the concept of integrated logistic support (ILS) is very important and is covered in the TM 38-703-series of Technical Manuals. These manuals present guidance and procedures for the application of integrated logistic support to Army materiel; the series consists of five manuals (Refs. 12-16). Ref. 15 might be of some special interest to the systems analyst since it provides methods for the collection, storage, manipulation, and retrieval of data for engineering and logistic analysis, and also for insuring integration of the required maintenance support elements as outlined in AR 750-1. Thus, it provides support for the management of maintenance engineering data.

A very good, informative discussion of the management of a maintenance engineering data system is given by Craddock (Ref. 17), who points out that logistic support must be considered as a design parameter during the development of an item so that the final product will be maintainable. The existing data system is the Logistics Support Analysis Record (LSAR) system. LSAR is a triservice system (see MIL-STD-1388 and MIL-STD-1388-1).

In dealing with support and logistic considerations, there should be ever-increasing concern for the survivability of the proposed support systems (involving detection, acquisition, damage, and ease of repair). Thus, a rather important area of systems analysis type activities is implied in the integrated logistic support system.

23-8 OUTLINE OF ELEMENTS FOR LOGISTIC SUPPORT

For purposes of record, it is of interest here to outline some of the important factors to be considered during the R&D phase of new materiel. Hence, the following elements should be taken into account when examining the logistic support implications of new materiel:

1. Support and Test Equipment (Built-in Fault Isolation) Special Tools, Peculiar Equipment, etc.
2. Repair Parts and Other Supply Support:
   a. Concept for replacement of components (modular, piece-part)
   b. Initial stockage levels (e.g., Organization, Direct Support, General Support, Depot)
   c. Stockage lists in consonance with SALS (Standard Army Logistics System)
   d. Direct exchange (DX) items identified
3. Transportation and Handling:
   a. Air Transportability — AR 70-39
   b. Ground Transportability — AR 70-44
   c. Shock and Vibration Transportability Criteria — TB 55-100
   d. Sling Eyes and Tie Downs — MIL-STD-209
   e. Towing Pintles — Need and Availability
4. Technical Data for Operation, Maintenance, Overhaul, and Supply — How and when will this information be provided?
5. Facilities — Special requirements for overhaul, rebuild, special maintenance or production base
6. Maintenance Plans Concept:
   a. User or Crew Functions — Specific tasks and logical separation points
   b. Direct Support Maintenance — Specific tasks and logical separation points
   c. General Support — Specific tasks and logical separation points
7. Logistic Support Resource Funds:
   a. Logistic support funding for each logistic element — Are these costs included or recognized in life cycle cost projections?
   b. Life cycle cost forecasts should be updated to each key decision point.
   c. Maintenance float (repair cycle and operational readiness float) — Will be included in logistic support portion of life cycle costs
   d. Initial provision cost of single item will not exceed . . . (Specify, e.g., 25%) of acquisition cost. (Include provisioning, documentation, technical instruction, concurrent repair parts, and maintenance float.)
   e. Logistic support costs ≥ (e.g., 10%) of acquisition cost of a single item per year after fielding.
8. Logistics Support Management Information — What is the minimum of operating data that should be maintained in order to keep a watch on the field performance of the system? (Sometimes this can be provided by sample data collection).
9. Personnel and Training:
   a. Skills needed
   b. Amount of time required to train
   c. Potential to use existing MOS personnel, etc.
10. Performance Characteristics for Reliability and Maintainability:
    a. Reliability — MTBF (define failure), e.g., corrective action cannot be deferred:
       (1) Until next scheduled maintenance
       (2) For remainder of life before overhaul, replacement, rebuild, or salvage if DS or GS is prescribed

(Corrective action is not deferrable if the item cannot commence to perform its function.)

b. Maintainability — Organization, DS, GS manhours per operational hour, e.g., 0.15 or 0.25. MTTR (Mean time to repair) — probability of: 0.95 to be diagnosed, repaired, and verified in 1 h at Organizational Level; 0.90 for 3 h at DS, 0.95 within 48 h at GS.

c. Durability — e.g., capable of completing 1200-1500 h before major replacements or rebuild

d. Time between overhaul (TBO)

e. Accessibility — Ease of maintenance. Test in user environment.

f. Built-in testing capability for fault isolation

g. After open storage for one year item capable of being restored to fully operational condition within, e.g., 48 h. (System storage life)

11. Human Factors Engineering — MIL-STD-1479 and MIL-H-46855
12. Vulnerability Characteristics and Survivability Objectives, e.g., nuclear hardening
13. Priority of Characteristics.

The given outline list should be helpful in studies of the required logistic support of an army in the field, as well as in the acquisition of a continuing data bank in order to model support requirements and hence make accurate predictions of logistic needs. In fact, let us reflect a bit further on support and especially the stochastic nature of the logistic problem.

A very brief look at Table 23-1 for the Army Classes of Supply should give the analyst a bird's eye view of the enormity of the logistic problem. Food and subsistence items have to be supplied in enormous quantities, as do medical supplies and secondary items, and many expendables. In addition, the quantities of ammunition and explosives reach very high tonnage; and petroleum, oils, and lubricants (POL) bring about a huge logistic burden. Repair parts for equipment must be supplied on a predictive and timely basis to keep the materiel in order and in proper service. In addition, much in the way of construction equipment is required for an army in the field. Thus, it appears to be of crucial importance to be able to plan and predict the whole logistic burden, much of which occurs on a probabilistic basis. We cannot, of course, delve into this broad subject to any great extent in this handbook, but it can be seen easily that methods of systems analysis and operations research techniques represent the most promising procedures for predicting the logistician's load and requirements. In the remainder of this chapter, we will therefore, consider some of the state of the art in helping to judge and predict the logistic burden on a sound scientific basis.

23-9 AMMUNITION DAY OF SUPPLY RATES

A highly important part of the logistic burden is that of Class V supplies or ammunition items. The combat troops must have a sufficient amount of all types of needed ammunition in order to carry out their mission successfully, and plans must be made for the timely procurement of new ammunition items. Hence, the need for anticipated combat expenditures of ammunition under normal, intense, or limited conflict situations. Ammunition day of supply rates for nonnuclear items are given in Supply Bulletin SB-38-26 (Ref. 18), and are kept up to date. Thus, the figures quoted may be used for logistic planning purposes, and it is desired to be able to predict the most realistic ammunition expenditure rates possible. It is of interest here to point out that the prediction of ammunition day of supply rates is now made through the use of systems analysis or war game type studies. The original idea to predict day of supply rates by systems analysis procedures goes back to the late 1950's and evolved at the US Army Ballistic Research Laboratories. Then, MG Nellie Lynd, Chief of the Field Service Division of the Office of the Chief of Ordnance, was aware of the weapon systems analysis studies at the BRL and requested help on predicting more realistic ammunition expenditures rates. Therefore, at General Lynd's request, the BRL undertook such a study and for the first time approached the problem by applying weapon systems analysis methods to hypothetical combat situations. The original work on this

problem is reported in BRL Memo Report No. 1395, April 1962 (Ref. 19) and led to the continuing use of evaluation techniques for prediction purposes. Wargame procedures and other military operations research type techniques are being used at the US Army Concepts Analysis Agency to aid in the accurate prediction of ammunition expenditures under various hypothesized combat conditions.

23-10 BATTLE CASUALTIES AND THE MEDICAL WORKLOAD

One of the important considerations affecting the overall logistic burden is, of course, that of battle casualties. Thus, it becomes of considerable importance to predict the medical work load that will likely be encountered in combat operations. Gilbert Beebe and Michael de Bakey (Ref. 20) published in 1952 an informative book concerning the incidence, mortality, and logistic implications of battle casualties based on available World War II data. They point out that the predictions indicated in their book will apply with confidence to more or less the conventional type of war, but not to nuclear type conflict. Their findings are as well documented as possible, based on available medical records, and they were careful to check and indicate the limitations on their results and predictions. Beebe and de Bakey give the incidence of hits and wounds; the relative frequency of disease, nonbattle casualties, and wounded or killed in action; the percentage of deaths from wounding; the locations of hits and wounds on body areas; some comments on the effectiveness of weapons, including the relative number of casualties suffered by infantry, armored personnel, field artillery, that due to air operations, etc. The percentage distributions of weekly rates for wounded personnel in combat operations per 1000 men engaged are also predicted. These considerations, therefore, may be taken into account in helping to predict the medical workload and logistic burden resulting therefrom.

The analyst engaged in logistic studies may be required on occasion to include the evaluation of battle casualties in his overall analyses, similar to the way he accounts for logistics due to Army classes of supply listed in Table 23-1.

Laughlin, Scoles, and Eyler (Ref. 21) also studied the medical workload problem, and used a simulation technique or model to predict survivor curves. Their curves give percent surviving as a function of time in hours after being wounded.

From the brief account of predicting ammunition requirements and medical workload in combat, we can see they have a significant effect on the logistic burden. Other factors affecting the logistic burden are equipment reliability and life-time. Hence, we would expect that some logistic support predictions may be looked upon as a problem in reliability and life-testing (Chapter 21) and treated somewhat generally in this manner. Indeed, much of the theory of Chapter 21 could well be applied, and we discuss this briefly in par. 23-11.

23-11 RELIABILITY, LOGISTICS, AND THE RELIABILITY IMPROVEMENT WARRANT

As just indicated, it certainly would amount to a significant omission if we did not discuss the effect of reliability on the logistic burden or treat logistic requirements in this context. In fact, it should be unmistakedly clear that operation of vehicles and materiel in the field and combat endeavors generally, bring about the concept of life-time probability distributions for materiel, military personnel, and some classes of supply. Moreover, since reliability and life-testing fields are really one and the same, one should logically expect that attrition processes resulting from combat should be treated as stochastic, life-time probability distribution problems, or "birth-to-death" distributions, and certainly there is an advantage in analyzing combat engagements in this manner. Hence, as we have said earlier, Chapter 21 gives many methods for predicting the life-time of equipments et al. Moreover, the reliability of equipment in terms of mechanical, electrical, or other types of breakdowns—aside from that

due to combat—will have a major effect on the operation and maintenance of equipment in the field. Hence, system reliability has to be one of the major factors associated with logistic support and support costs since the more frequently an item fails, the greater the need to replace or maintain the equipment. Also, it can be seen that a manufacturer of equipment usually is motivated only to produce equipment, and ordinarily would have little interest in what happens to the materiel after it has been in service for a while. In fact, the manufacturers' goal is to product equipment that meets the reliability and maintainability requirements of his product when it is delivered, but not throughout its life in the field. Thus, the need on the part of the Army is to improve reliability and life-times of its materiel and, in particular, the need is to increase the mean-time-to-fail, or push the reliability distribution "to the right" or toward longer life-times.

It is for these reasons that the Reliability Improvement Warrant (RIW) has come into existence. As discussed by Mlinarchik (Ref. 22), the RIW is a recently developed concept currently being employed on a trial basis by the DOD to motivate and provide an incentive to contractors to design and produce equipments which will have a low failure rate and low maintenance rate in field or operational use. This, therefore, should lead to increased attention on the part of the manufacturer concerning the details of reliability of his product in field usage. The philosophy of the RIW is that a contractor can warrant the continuing operation of his manufactured equipment in the field for an extended period of time at a negotiated fixed cost. Thus, the incentive exists for the Army and contractor to work hand-in-hand on a continuing basis. The details of the RIW are fully covered by Mlinarchik in Ref. 22, and some of the potential benefits are:

1. Greater emphasis on life cycle cost approach
2. Minimal initial support investment
3. Incentive for reducing repair costs
4. Increased incentive for contractor to introduce design/production changes to increase MTBF and reliability growth
5. Possible reduction in requirements for skilled military maintenance and support manpower.

The contractor may derive benefits in the areas of:

1. Increased profit potential when field MTBF is improved above the pricing base
2. Multiyear guaranteed business
3. Opportunity to become more familiar with the operational reliability and maintainability characteristics of his equipment, which should help him in obtaining follow-on contracts.

23-12 MILITARY SUPPLY TABLES

One of the most important problems needing solution to guarantee an efficient pipeline relates to the supply of repair parts or units of equipment as they are needed in the field. There will be the usual or normal failures of equipment due to wear in the field as vehicles and weapons are used, and in addition there will be losses from combat and accidents as well. Thus, it becomes important to effect timely replacement of needed units, including procurement planning, and to guarantee proper and prompt maintenance of materiel, etc. Failures of equipment will occur at random points in time and it would clearly be of great use and advantage to have a suitable model which would predict the types and number of failures of parts as a function of time. This often can be done based on the analysis of a rather limited amount of typical data, and the law of prediction developed will be of sufficient accuracy for procurement and supply. Since it is expected that the frequency of failure will not normally be very high, then often the chances that one, two, three, etc., replacement items will be needed in any time t may be described adequately by the Poisson distribution, for example. Geisler and Karr (Ref. 23)

made a rather general study of the problem of supply of parts from the point of view of a stochastic process and recommended a method of constructing military supply tables such that one minimizes the expected number of shortages to be encountered during a given period of time or supply activity. The supply table then indicates the amount of preassembled groups of repair parts which would be needed for a specified period without outside support. The probability model suggested by Geisler and Karr is discussed rather fully in their paper (Ref. 23).

Sutherland (Ref. 24) develops a graphical method of predicting repair parts requirements, based on the theory of Geisler and Karr (Ref. 23), assuming that the Poisson distribution is of sufficient accuracy for the problem. Sutherland's graphical solution appears to have some generality and his methodology might be very useful for other military applications.

The systems analyst engaged in the analysis of logistic problems naturally will have considerable interest in the problem of trying to predict the number of repair parts needed as a function of time, for the better supplied and maintained army usually will be the victor. Hence, Refs. 23 and 24 may have some basic or preliminary interest for the young analyst. We cannot cover this extensive subject very fully here but will give some of the current methodology for secondary item replacement predictions since such methodology possibly could be used elsewhere.

23-13 PROCUREMENT CYCLES AND SAFETY LEVELS OF SUPPLY

The Army cannot afford to wait and order or procure supplies only at the time they are sorely needed. In fact, it is clear that much planning is required for the thousands of items the Army must procure and have ready for issue well in advance of actual needs. Such planning is aided greatly by the use of proper inventory models which will result in timely availability of repair parts or materiel items for the Army in the field and ease its logistic burden.

A good account of the methodology for implementation of advanced inventory control techniques at Army national inventory control points is given by Rosenman (Ref. 25), updated by Department of Defense Instruction (DODI) 4140.39 (Ref. 26), and by Deemer and Kruse (Ref. 27).

Briefly, the methodology for secondary items (Class II) which has been used for some years now to estimate the reorder quantity Q and the safety level S follows, and is based partly on some original work at the Massachusetts Institute of Technology also:

1. First, one calculates the variance-to-mean ratio V of the demand as

$$V = \exp[(11.3451 \ln\lambda)/(18.2619 + \ln u)] \tag{23-1}$$

where

λ = forecast of average annual demand

u = unit price .

2. Then calculate the standard deviation σ of demand during lead time for procurement from Eq. 23-2

$$\sigma = \sqrt{\lambda L V} \tag{23-2}$$

where

L = procurement lead time demand .

3. Then a quantity Z is calculated as

$$Z = p/(huLV) \tag{23-3}$$

where

p = administrative cost of procurement

h = holding cost rate .

The quantity Z is used to enter a nomograph based on various levels of protection α, such as α = 50%, 70%, 90%, 95%, or 99%, in order to find two quantities a and b, where

a = cost of the first article procured or item produced

b = slope of progress curve

The unit price/quantity procured is based on the usual "progress curve" or "experience" curve

$$y_i = ax_i^b \tag{23-4}$$

where

y_i = cost of ith unit procured

x_i = cumulative unit number .

Finally, for a chosen level of protection α, and a and b determined from the nomograph (or table), then

$$\text{Reorder quantity} = Q' = b\sigma \tag{23-5}$$

and

$$\text{Safety level} = S = a\sigma \; . \tag{23-6}$$

Rosenman (Ref. 25) in his 1963 paper also gives the calculation of the total operating cost of the inventory, although we will record here the more up-to-date methods of Refs. 26, 27, and 28.

DODI 4140.39 (Ref. 26) covers procurement cycles and safety levels of supply for nonrepairable secondary items, and applies to Military Departments, the Defense Logistic Agency, and the Defense Contract Audit Agency. DODI 4140.39 does not cover principal major end items such as missiles, tanks, vehicles, or helicopters, for example. The policy is that the Military Departments and the Defense Logistic Agency will minimize the total variable cost relative to ordering and holding inventory at Inventory Control Points (ICP's) and their stock points, subject to a constraint on the average number of days forecast for delay in the availability of materiel (in terms of requisitions) for release by item managers or by the Automatic Data Processing (ADP) systems supporting the item managers. Relative essentiality also may be used as an additional element of consideration after approval by the Office of the Assistant Secretary of Defense (Installations and Logistics). Thus the objective of this policy, concisely stated, is:

> "To minimize the total of variable order and holding costs subject to a constraint on time-weighted, essentiality-weighted requisitions short."

The mathematical equation calls for the use of a shortage parameter which can be set to control the safety level, not only in consonance with the general policy previously stated, but also to satisfy other constraints that become necessary as a result of policy decisions relative to national priorities, e.g., contraints on budgeting or funding for inventory levels. This shortage parameter acts as the cost of one requisition short for one year in the model; however, it is determined on the basis of the average number of days to be forecast for delay in the availability of materiel or to satisfy a budget or funding constraint and, thus, is only an implied short cost, the true cost of the shortage being unknown. The total variable costs consist of cost to order, cost to hold, and the implied shortage cost.

The implied shortage cost is a function of other management decisions, i.e.,

1. The average number of days to be forecast for delay in the availability of materiel, or
2. The funds available for inventory levels.

In order to understand the methodology, the following definitions are of importance:

1. *Time-Weighted, Essentiality-Weighted Requisitions Short.* Time-weighting is the consideration of the average number of days delay in the availability of materiel; essentiality-weighting is the consideration of the relative essentiality of each item; and requisitions short are requisitions on backorder. Thus to minimize time-weighted, essentiality-weighted requisitions short is to minimize the average number of days delay in the availability of materiel (including requisitions not backordered) considering the relative essentiality of each item in a given inventory and the size of the requisition.

2. *Shortage Parameter.* A control used to constrain number of days of time-weighted, essentiality-weighted requisitions short forecast to that established by DOD policy or to the funds available for inventory. This control acts as an implied shortage cost.

3. *Implied Shortage Cost.* The assumed cost of a shortage based upon other management decisions relative to the number of days to be forecast for delay in the availability of materiel or the funds available for inventory levels.

4. *Delay in Availability of Materiel.* The number of days that elapse between the receipt of the requisition by the ultimate supply source and the transmission of the materiel release/issue instruction (either document or punched card) to the depot/storage site. (It is recognized that the time period will include some administrative actions not related to availability.) The Military Supply Transportation Evaluation Procedure (MILSTEP) time segment *ICP Availability Determination* is compatible with this definition and is used as a measure of effectiveness.

5. *Variable Cost to Order.* Those costs associated with the determination of requirements, processing of a purchase request, and subsequent contract actions through receipt of the order into the ICP system which will vary significantly in relation to the number of orders processed. Costs are considered "fixed" if they would remain constant should 50% of the workload be eliminated.

6. *Variable Cost to Hold.* Those costs associated with the cost of capital, inventory losses, obsolescence, storage, and other variable costs of maintaining an inventory. The 50% rule relative to variability applied to variable cost to order also should be applied here.

7. *Total Variable Cost (TVC).* The sum of the variable cost to order, variable cost to hold, and implied shortage cost. Procurement cycles and safety levels are determined through minimization of these costs for any given group of items in an inventory.

8. *Procurement Cycle (Quantity).* A requirement which represents the forecast demands between procurement actions based upon Economic Order Quantity *EOQ* considerations in the Total Variable Cost *TVC* model.

9. *Order Quantity.* The amount of new procurement for which funds are to be obligated, i.e., procurement cycle quantity plus reorder point shortage.

10. *Order.* A request for procurement action in the form of a new contract or, in the case of call-type contracts, the placement of an order against an existing contract.

11. *Safety Level.* The quantity of materiel which is required to be on hand to permit continued operation in the event of minor interruption of normal replenishment or unpredictable fluctuation in demand. The safety level determined in accordance with this instruction is structured to minimize time-weighted, essentiality-weighted requisitions short for those demands treated as recurring.

The basic mathematical model of DODI 4140.39 is briefly as follows (Ref. 26). The instruction requires the minimization of total variable holding and ordering cost subject to a constraint on time-weighted, essentiality-weighted requisitions short.

The annual variable cost to order stock is the number of times an order is placed in a year times the cost of each order. Mathematically, for item i, this is represented by:

$$\left(\frac{D_i}{Q_i}\right)P = \text{annual variable cost to order} \tag{23-7}$$

where
D_i = average yearly demand for item i
Q_i = order quantity for item i
P = variable procurement setup cost.

The total annual order cost OC for an N-item type inventory is:

$$OC = \sum_{i=1}^{N} \left(\frac{D_i}{Q_i}\right)P \tag{23-8}$$

The annual variable cost of holding inventory is applied to expected inventory on hand. The expected on hand inventory is the sum of the expected value of asset position (on hand plus on order) and expected backorders minus the quantity on order. The quantity on order is simply the mean lead time demand. Hence, the expected on hand inventory OH_i for item i is

$$OH_i = R_i + \frac{Q_i}{2} + B_i - L_i \tag{23-9}$$

where
R_i = reorder warning point for item i
B_i = expected backorders for item i
L_i = expected lead time demand for item i.

The total annual variable holding cost HC for an N-item type inventory system is

$$HC = \sum_{i=1}^{N} (R_i + \frac{Q_i}{2} + B_i - L_i)IU_i \tag{23-10}$$

where
I = holding cost rate
U_i = unit price of item i.

The DODI points out that the expected backorders can be dropped with *little* effect on the optimal decision rule. Hence, the approximate expected on hand inventory becomes

$$OH_i = SL_i = \frac{Q_i}{2} \tag{23-11}$$

since

$$R_i = L_i + SL_i$$

where

SL_i = safety level for item i

Now the expected asset level A_i is

$$A_i = L_i = SL_i + \frac{Q_i}{2} \cdot \quad (23\text{-}12)$$

Thus, the difference between the expected on hand inventory, and the expected asset position, is merely a constant term L_i of Eq. 23-11 which has *no* effect on the optimal decision rule for ordering.

These two costs, ordering and holding, are to be minimized subject to a constraint β on the time-weighted, essentiality-weighted requisitions short. This constraint can be written $RS \leq \beta$. The expected requisitions short RS are calculated from the units short B divided by the average requisition size S, i.e., for item i

$$RS_i = \frac{B_i}{S_i} \cdot \quad (23\text{-}13)$$

The time-weighted, essentiality-weighted requisitions short for an N-item inventory system is

$$RS = \sum_{i=1}^{N} \frac{E_i}{S_i Q_i} \int_{R_i}^{\infty} (x - R_i)[F(x + Q_i;t_i) - F(x;t_i)]dx \quad (23\text{-}14)$$

where

E_i = essentiality of item i

$F(Y;t)$ = probability of having Y or fewer demands in time t .

The cumulative distribution function $F(Y;t)$, or its density $f(Y)$, may be chosen to fit the chance of demand for an item as a function of time. Distributions used to date include the normal, the pseudo-Laplace, and the negative binomial functions. DODI 4140.39 leaves the critical problem of selecting the best probability of demand distribution up to the user.

By method of the Lagrange optimization technique the following expression for Total Variable Cost TVC must be minimized:

$$TVC = \sum_{i=1}^{N} \left[\frac{D_i}{Q_i} P + (R_i + \frac{Q_i}{2} + B_i - L_i)IU_i - \lambda (\beta - RS_i) \right]. \quad (23\text{-}15)$$

For Army items, the DARCOM Inventory Research Office at Frankford Arsenal recommends the use of the Economic Order Quantity (EOQ, or simply Q) given by

$$Q = \sigma/(\sqrt{2}\rho) + [\sigma^2/(2\rho^2) + Q_W^2/\rho]^{1/2} \quad (23\text{-}16)$$

where

$$\sigma = \text{standard deviation of demand} = \gamma(PCERL)L$$
$$= \gamma(PCER)L/[12(PLT)/9]^{1/2} \quad (23\text{-}17)$$

γ = quantity depending on the percent error for lead time ($PCERL$) and is found from Table 23-2

$PCERL$ = $(PCER)/[12(PLT)/9]^{1/2}$ (23-18)

$PCER$ = percent error, which depends on the frequency with which the item is demanded and is found from Table 23-3

PLT = procurement lead time

L = procurement lead time demand

ρ = $[1 + \exp(-\sqrt{2}\,Q_W/\sigma)]/[1 - \exp(-\sqrt{2}\,Q_W/\sigma)]$ (23-19)

Q_W = $\{2P'(AYD)/[H(UP)]\}^{1/2}$ (23-20)

P' = cost to order

AYD = average yearly demand

H = holding cost

UP = unit price .

Eqs. 23-16 through 23-20 involve the use of the negative binomial distribution for the demand and a lead time demand less than 20.

An estimate of the variability of demand is necessary for calculation of the EOQ or safety levels. If the variability is large, then a larger safety level will be required to achieve or guarantee a given degree of supply performance. The Navy and the Defense Logistic Agency currently measure variability of demand by tracking mean absolute deviation MAD of each item, in which case the standard deviation σ may be determined from the relation

$$\sigma = 1.25(MAD)\,. \tag{23-21}$$

Further details are given for estimating σ and are covered rather fully by Kaplan (Ref. 28).

Clearly, we might say that perhaps some similar methodology should also be developed and implemented for theater operations since the stochastic processes there may be much more severe!

TABLE 23-2. TABLE OF γ FOR THE NEGATIVE BINOMIAL DISTRIBUTION

Range	γ
$0 \le PCERL \le 0.5$	1.27
$0.5 \le PCERL \le 0.8$	1.33
$0.8 < PCERL \le 1.00$	1.42
$1.0 < PCERL$	1.52

TABLE 23-3. PERCENT ERROR *PCER*

	Extended Price	
Frequency	≤ $200	> $200
less than 5	1.701	1.286
5-8	1.262	1.019
9-16	1.024	0.792
17-32	0.910	0.656
33-62	0.575	
63-122	0.469	
over 122	0.409	

23-14 LOGISTIC SIMULATIONS

We already have pointed out that the problem of logistics involves stochastic supply problems, and—due to the complex nature of logistic planning and support, and the many, many variables involved—it is then necessary to use simulations in order to gain some understanding of the many details or the effects of input parameters it is desired to cover. We therefore will highlight some examples of logistic type simulations.

Roush (Ref. 29) discusses the use of simulation as a tool in logistic planning and evaluation. Some twelve years ago, the Deputy Chief of Staff for Logistics, DA, initiated a study project on Simulation and Gaming Methods for the Analysis of Logistics (SIGMALOG). The first study, SIGMALOG I, was conducted to assist analysts in determining detailed Army logistic requirements within functional areas of supply, transportation, medical replacements, maintenance, and construction in support of contingency plans or Army studies. A follow on study, SIGMALOG II, was conducted to compare current and projected combat service support units, selected ammunition rounds and major items of equipment, and intertheater transportation assets with the previously computed requirements to indicate national capability to accommodate one, two, or three simultaneous operations. The analyst is referred to Roush's 1974 paper (Ref. 29) for details.

Colon and Calfapietra (Ref. 30) discuss a computerized model used in logistic support analyses. They focus attention on the performance of logistic support analysis since the process enables the consideration of logistic support as a principal design parameter. Their model is used for evaluating alternative design approaches and proposes engineering changes in terms of logistics in addition to optimizing support posture for a fixed design.

The US Army Concepts Analysis Agency's treatment of the Logistic Support Force Structure (LSFS) is covered by Hurley and Arnold in Ref. 31. The Logistic Support Force Structure has traditionally been keyed to doctrinal requirements, and such requirements normally are established for support of a fully developed, mature theater. For the most part, therefore, they are not expressed on the basis of time-phased workloads which are generated by the deployment of combat forces. The methodology for LSFS, therefore, addresses these points and provides a capability to develop a time-phased combat support force that is synchronized with the workload requirements generated by the activity on the deployed combat force. Hurley and Arnold's paper (Ref. 31) provides an overview of the Force Analysis Simulation of Theater Administrative and Logistic Support (FASTALS) Model, and how a time-phased deployment list (TPFDL) is produced, including the methodology developed by the Concepts Analysis Agency.

Wood, Groover, Hutton, and Quatrevaux (Ref. 32) describe a computer simulation methodology developed at the US Army Ordnance Center and School for the evaluation of the operational effectiveness of alternative maintenance support concepts for the army in the field.

"MAWLOGS", or Models of the US Army Worldwide Logistic System, is a unique model development capability which provides tailor made computerized simulation models containing integrated maintenance, supply, and transportation functions. A paper by McHale, Raffiani, and Williams (Ref. 33) focuses on the development methodology for MAWLOGS, the various models, data bases, and the primary simulation results.

Shulman and Smith (Ref. 34) and Shulman and Iaeger (Ref. 35) describe what they call an operational availability and reliability (OAR) model in order to predict failure, corrective maintenance, and repair acquisition probabilities for Army systems. They use the simulation model because the Weibull distribution assumed for failure times, corrective maintenance, and spare replacements does not lend itself well to analytical manipulation except for simple systems. The conditions of a finite set of repair parts replenished at a statistically independent rate even cause analytical

methods to be very complex. The model description, input parameters and variables, and model outputs are given in Ref. 34, along with an example. The technique, however, apparently can be used as an aid in helping to predict the logistic burden.

23-15 SUMMARY

Logistic planning and support efforts cover a huge, highly important endeavor for the Army, with many fruitful areas of evaluation for the analyst. In this chapter, we have introduced only some of the more obvious type of problems the analyst might encounter, although hopefully we have conveyed the idea that logistics requires accurate estimation for complex stochastic processes in order for the Army to perform its mission successfully. The estimation problems involve appropriate modeling of lifetime probability distributions for repair parts and failures of equipment. The whole problem of providing for all classes of supply requires highly competent evaluations. Indeed, as we see it, there is a need for systems analysis type applications for the logistic problem just as we now have for the evaluation of weapon systems. The young analyst will find many of the papers listed in the Bibliography of pertinent interest.

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CHAPTER 24

THE WSEIAC EVALUATION MODEL

An account is given of the Weapon Systems Effectiveness Industry Advisory Committee (WSEIAC) model or methodology for evaluating weapon systems. This study of methodology was performed for the US Air Force in the mid-1960's and attempts to evaluate weapon systems on the basis of three primary factors: (1) availability (readiness), (2) dependability (reliability), and (3) capability (performance). These three factors are converted to a single measure of effectiveness which characterizes the overall performance of a weapon system. Examples illustrating the methodology are given.

24-0 LIST OF SYMBOLS

$\bar{A}$ = availability (readiness) vector
$\bar{A}'$ = transpose of $\bar{A}$, i.e., $\bar{A}'$ is a row vector
a = constant depending on radar cross section of the target
a_i = chance that the system is in state i at the beginning of the mission
a_1 = probability that the system is operable at a random point in time
a_2 = probability that the system is in repair at a random point in time
$[C]$ = capability (performance) matrix
$\bar{C}$ = capability vector
$[C(p_k)]$ = $\bar{C}(p_k)$ = vector of kill probabilities
$[C_0]$ = $\bar{C}_0$ = capability vector for radar detection at maximum range (at zero time or start of mission)
$c_i(0)$ = ith component of $\bar{C}_0$
$c_i(15)$ = element of $\bar{C}_{15}$
c_{jk} = value of the kth figure of merit, conditional on the effective system state j
$[C_{15}]$ = $\bar{C}_{15}$ = capability vector for continuous tracking of radar during 15-min mission
$[D]$ = dependability (reliability) matrix
$[D(15)]$ = dependability matrix for 15-min mission
d_{ij} = probability that the effective state of the system during the mission was in state j, given that the mission was begun in state i
E = effectiveness of radar system
$\bar{E}'$ = $[e_1, e_2, \ldots, e_k, \ldots, e_n]$ = effectiveness vector
E(max range) = E_{max} = effectiveness of radar to detect and acquire target at maximum range
E(complete system) = overall effectiveness of complete surface-to-air gun system, including radar
E_T = E/E_{max} = effectiveness of radar in track, given detection at maximum range
e_k = value of kth figure of merit (FOM)
$MTBF$ = mean time between failures
$MTTR$ = mean time to repair
P_T = transmitter power
p = chance of radar detecting target

$p(k_i)$ = element of $\bar{C}(p_k)$
R_i = reliability of ith subsystem or component
R_r = reliability of remainder of range radar
R_t = reliability of a transmitter
r = range to target
T = system mission time
T = expected nonfailed time
z = root-mean-square noise amplitude
α_R = availability of the combination, consisting of antenna, receiver, display, and synchronizer
α_T = availability of each transmitter, i.e., the chance that a transmitter is operating
λ = system failure rate
λ_r = failure rate of a receiver, antenna, and synchronizer of range radar
λ_t = failure rate of a transmitter
μ = system repair rate

24-1 INTRODUCTION

As is well known, the design and development of military equipment or systems have always crowded the state of the art, technologically speaking, for it is often necessary to outstrip any potential enemy in military system effectiveness as, for example, in numbers of weapons produced. Moreover, in recent times the designers have been faced simultaneously with increasingly novel demands, very high reliability and, even more acutely, limited test data. Such requirements and considerations bring about the need for an integrated methodology of system program management, utilizing all available data and techniques to pinpoint problem areas and provide an accurate numerical estimate of the effectiveness of systems during all phases of the acquisition program of the weapons. The US Air Force, in order to cope with the general problem, convened a special weapon systems effectiveness industry advisory committee in September 1963 to study the problem and make recommendations to management. The committee's reports were published early in 1965 and are listed as Refs. 1-6.

The WSEIAC came forth with some novel methods of analyzing overall system performance or effectiveness—which no doubt have had a significant impact on military operations research—even though the exact methodology proposed has not been adopted widely for Army usage. A particular point of interest, however, is that the US Air Force's problems encompass some very complex systems, and the WSEIAC methodology includes the judgment of the study committee on this point.

With our account of availability and reliability of weapon systems (Chapter 21), the coverage of hit probabilities and target damage (Chapter 20), and vulnerability and lethality (Chapter 15), we are now in a position to discuss the WSEIAC model and its aims in assessing overall weapon system performance in this chapter.

The WSEIAC evaluation study recognizes that mathematical models employing analytical methods and machine simulation programs are an essential part of the integrated, overall methodology. In fact, it points out clearly that any effective analysis requires a combination of theory, procedures, and sufficient data. The committee considered that a system should be ready to operate properly when a demand was placed upon it at any random point in time. Furthermore, given the "availability" of the system, reliable operation during the mission becomes a necessity, i.e., "dependability" is an important requirement. Finally, the system must be capable of performing effectively as intended. Thus, the

idea of using an availability (readiness) vector to indicate the probable state of a system at the beginning of a mission, a dependability (reliability) matrix to describe system stochastic states during mission, and a capability (performance) vector or matrix to characterize effectiveness, given system availability and dependability, describes the general approach.

The WSEIAC definitions and objectives as given in Ref. 2, Vol. II, are:

1. *System Effectiveness* is a measure of the extent to which a system may be expected to achieve a set of specific mission requirements and is a function of availability, dependability, and capability.

2. *Availability* is a measure of the system condition at the start of a mission and is a function of the relationships among hardware, personnel, and procedures.

3. *Dependability* is a measure of the system condition at one or more points during the mission—given the system condition(s) at the start of the mission—and may be stated as the probability (or probabilities or other suitable mission oriented measure) that the system will (a) enter and/or occupy any one of its significant states during a specified mission, and (b) perform the functions associated with those states.

4. *Capability* is a measure of the ability of a system to achieve the mission objectives—given the system condition(s) during the mission—and specifically accounts for the performance spectrum of a system.

The objectives of system effectiveness evaluation are to:

1. Evaluate system designs and compare alternative configurations.
2. Provide numerical estimates for use in defense planning.
3. Provide management visibility at every phase of a system life cycle of the extent to which the system is expected to meet its specific operational requirements (SOR).
4. Provide timely indication of the necessity for corrective actions.
5. Compare the effect of alternative corrective actions.

We thus see that system readiness is a highly important measure of effectiveness. The tank gunner who shoots first has the advantage—see Chapter 17, for example.

The WSEIAC concept characterizes dependability as the reliability of the system during the mission, once an engagement occurs. It does not necessarily include in the concept of "reliability" the idea of capability in which the latter term is used to describe of characterize terminal effects more or less. Reliability, on the other hand is often, and sometimes in fact, widely used as an overall measure of system performance, Thus the precise division of factors or tasks may not be considered of vital importance, provided all of the important system characteristics are taken into account in the overall analysis of system performance.

24-2 GENERAL CONSIDERATIONS

The WSEIAC concept or model envisages four phases of system life: (1) conceptual, (2) definition, (3) acquisition, and (4) operational. It also describes and requires recognition and treatment of eight tasks used in evaluating the effectiveness of systems, as follows:

1. Mission definition
2. System description
3. Specification of figure(s) of merit
4. Identification of accountable factors
5. Model construction
6. Data acquisition
7. Parameter estimation
8. Model exercise.

A figure of merit (FOM) is an index which indicates the quality of the system; the FOM serves to indicate just what performance can be expected from the system. FOM's should be expressed in an operationally-oriented form that can be readily understood and used for evaluation and planning purposes. Of course, probabilities, or chances of occurrence, are most often and appropriately used as figures of merit although means, standard deviations, etc., may be appropriate also. The FOM will depend largely on the particular system under evaluation. For a rifle, it might be the kill probability averaged over ranges of engagement. For a radar, this may be the probability of target detection and/or the probability of successful (accurate) track. FOM's are more or less the same as measures of effectiveness (MOE's), and system effectiveness has often been defined as the vector of the most meaningful FOM's or MOE's.

The model is a technique for combining key information to estimate system effectiveness, and serves as a probabilistic representation of system performance. Most often, the model used will be a mathematical model, or perhaps a computer model for situations which require too complex a mathematical model or for cases where an appropriate mathematical model has not been developed.

Model construction and selection of the most appropriate models for system evaluation are obviously important steps in any evaluation process, including the WSEIAC model. The first step in construction of the model is to identify and describe the significantly different "*states*" in which the system might exist as the mission is initiated and carried out. The system "states" are distinguishable conditions of the system which result from events occurring prior to and during the mission. An air defense gun system may be "down" and hence unavailable to initiate a mission against an enemy aircraft when necessary. Thus the chances of the weapon being "down" or "up", and hence available for a mission, become important. (The idea of states for duels was described in Chapter 17.)

After describing the system states, the next step is to determine the probabilities of each of the sets of significant states which are appropriate at the beginning of the mission. This array of probabilities is called the *availability vector*.

For each succeeding time interval after start of the mission, an array of state probabilities is related to possible accountable factors—these probabilities being dependent or conditional on the effective state during the previous time interval. Thus, for example, a failure in any time interval predetermines the possible states in the succeeding intervals, especially if no repair is possible. These arrays of conditional probabilities are called the *dependability matrix*.

The next step is the construction of the *capability vector* or matrix, which is an array of probabilities giving measures of the ability of the system to achieve mission objectives, given system condition(s) during the mission. It can be said that the capability matrix represents the expected FOM's for the system, given availability and dependability.

Thus, and in summary, model construction can be described in four steps: (1) state description, (2) determination of the availability vector, (3) determination of the dependability matrix, and (4) determination of the capability vector or matrix. The overall MOE's are obtained finally by taking the product of the availability vector, the dependability matrix, and the capability matrix or vector. Data acquisition and parameter estimation form a very important part of the problem to estimate the elements used in the vectors and matrices.

The reader will note that the problem of costing weapon systems has hardly been mentioned; this is due to the fact that basically in the WSEIAC approach the first analysis is made of effectiveness properties. Then, costs to support various systems or alternative system arrangements may be made, and finally included in the decision process.

It can be said that model exercise or the extent of model exercise represents the final phase of a WSEIAC type study, and the amount of information derived depends naturally upon previous tasks in

the process of analysis. As indicated in Ref. 2, "Results of the model exercise by phases may vary from a single point estimation of effectiveness with low confidence (Conceptual Phase) to an elaborate read-out of information on systems, subsystems, equipment, etc., including estimates of effectiveness factors and their elements as well as parameter variation analysis and system change analysis (during the later portion of the Acquisition Phase)."

24-3 MATHEMATICAL MODEL CONSTRUCTION

24-3.1 ANALYTICAL EXPRESSIONS FOR EFFECTIVENESS

We now describe the WSEIAC model in some analytical detail in order to understand the model construction problem. Recall that the System Effectiveness E may be thought of or defined as a vector of FOM's for the given system. Further, the structure for establishing and evaluating this vector is based on an enumeration of a number n of significantly different system states, 1, 2, 3, ..., n. The overall structure is, of course, composed of the three different parts we have discussed—i.e., an availability vector $\bar{A}$, a dependability matrix $[D]$, and a capability matrix $[C]$ or vector $\bar{C}$—the elements of which are defined as follows:

e_k = value of the kth FOM

a_i = chance that the system is in state i at the beginning of the mission

d_{ij} = probability that the effective state of the system during the mission was in state j, given that the mission was begun in state i

c_{jk} = value of the kth figure of merit, conditional on the effective system state j.

Then, based on these definitions we may write that the system effectiveness vector $\bar{E}$ can be expressed as the product

$$\bar{E}' = \bar{A}'[D][C] \tag{24-1}$$

where $\bar{A}'$ is the transpose of $\bar{A}$, i.e., $\bar{A}'$ is a row vector.
We also note that

$$\bar{E}' = [e_1, e_2, \ldots, e_n] \tag{24-2}$$

and any element e_k is given by

$$e_k = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i d_{ij} c_{jk} \,. \tag{24-3}$$

24-3.2 THE AVAILABILITY VECTOR

The availability vector $\bar{A}'$ is a row vector $[a_1, a_2, \ldots, a_n]$ the elements of which are the probabilities of various system states at that particular point in time when the mission begins. As we see, and depending on the particular application, $\bar{A}'$ may be a multielement vector, but if the system is such that there are only two possible or significant system states, i.e., (1) "operable" or (2) "failed" (in repair), then the availability vector $\bar{A}'$ consists of only two components, a_1 and a_2; i.e.,

$$\bar{A}' = [a_1, a_2] \tag{24-4}$$

where

a_1 = probability that the system is operable at a random point in time

a_2 = probability that the system is in repair at a random point in time .

The chance that the system is operating (state 1) will be given by

$$\left.\begin{aligned} \text{Availability} = a_1 &= \frac{\text{(mean time to failure)}}{\text{(mean time to failure)} + \text{(mean time to repair)}} \\ &= \frac{1/\lambda}{1/\lambda + 1/\mu} \end{aligned}\right\} \quad (24\text{-}5)$$

where

λ = system failure rate

μ = system repair rate .

Moreover, the chance that the system is down or in repair (state 2) is given by

$$\left.\begin{aligned} \text{Unavailability} = a_2 &= \frac{\text{(mean time to repair)}}{\text{(mean time to repair)} + \text{(mean time to fail)}} \\ &= \frac{1/\mu}{1/\mu + 1/\lambda} \ . \end{aligned}\right\} \quad (24\text{-}6)$$

Another way of estimating the availability a_1 is to take the total successful operating time of a system and divide this by the sum of the total operating time and the total time in repair. If a system is checked periodically so that is is unattended and unmonitored for a constant period of time T, and the cycle of checkout and standby repeats itself indefinitely, then one may estimate the availability a_1 from the expected nonfailed time T divided by the sum of the duration of T and the time down in checkout and/or repair.

Availability can, of course, become a rather involved study for many complex systems. For example, in Chapter 21 we discussed the reliability of series, parallel, and more complex systems. It may be that the availability of the system would be the chance that the system is actually working as intended, in which case the availability calculation becomes that of determining the system reliability at the appropriate point in time. For a series system, for example, the availability a_1 might be estimated from the formula for the reliability of a series arrangement, i.e.,

$$a_1 = \prod_{i=1}^{n} R_i \quad (24\text{-}7)$$

where R_i is the reliability of the ith subsystem or component. Also, system reliability estimation procedure (Chapter 21) may apply during the mission or, that is, during the WSEIAC "dependability" phase of evaluation.

As Ref. 2 points out, "Models for computing the elements of the availability vector must take into account failure and repair time distributions, preventive maintenance and miscellaneous down-time schedules, checkout procedures, personnel deployment, spare parts, supply facilities as well as transportation and various administrative actions.

"The following list of possible system states should be referred to when establishing availability models:

(1) Assigned to alert/standby and nonfailed
(2) Assigned to alert/standby and failed in a manner detectable by field test
(3) Assigned to alert/standby and failed in a manner undetectable by field test
(4) On alert/standby and waiting for checkout/diagnosis
(5) Off alert/standby and in checkout/diagnosis
(6) Off alert/standby and waiting for spares."

Example 24-1:

A new range radar for a surface-to-air gun system with optical tracking consists of two transmitters in parallel to assure high reliability, an antenna, a receiver, and a display and synchronizer for the operator. Past experience indicated that a single transmitter would not guarantee the desired high reliability of obtaining range information. Each transmitter has an $MTBF$ (mean time between failure) of 10 h and the $MTTR$ (mean time to repair) a transmitter has averaged 1 h. The combination of the antenna, receiver, display, and synchronizer has demonstrated a $MTBF$ of 50 h; the $MTTR$ is only 30 min. Assume that the availability of the surface-to-air gun system depends only on the radar to detect, acquire, and give continuous range for an aerial target. What are the appropriate system states at the start of the mission? Also, calculate the availability vector for the system.

It is clear that the important radar system states at the start of the mission should be the following:

System State Definition	Definition of State
1	All units operating properly
2	One transmitter fails, but the other transmitter and all other units operate properly.
3	System fails because both transmitters fail or any one of the other units fail.

Let

α_T = availability of each transmitter, i.e., the chance that a transmitter is operating

α_R = availability of the combination consisting of antenna, receiver, display, and synchronizer

Then

$$\alpha_T = \frac{MTBF}{MTBF + MTTR} = \frac{10}{10 + 1} = 0.909$$

$$\alpha_R = \frac{MTBF}{MTBF + MTTR} = \frac{50}{50 + 0.5} = 0.990 .$$

Further, for the availability vector,

$$a_1 = \alpha_T^2\alpha_R = (0.909)^2(0.990) = 0.818$$

$$a_2 = [\alpha_T(1 - \alpha_T) + (1 - \alpha_T)\alpha_T]\alpha_R$$

$$= 2\alpha_T(1 - \alpha_T)\alpha_R = (2)(0.909)(0.091)(0.990) = 0.164$$

$$a_3 = 1 - a_1 - a_2 = 0.018 .$$

Hence, the availability vector $\bar{A}'$ is

$$\bar{A}' = [a_1 \quad a_2 \quad a_3]$$

$$= [0.818 \quad 0.164 \quad 0.018]$$

where

a_1 = 0.818 = chance all system components are operating

a_2 = 0.164 = chance the radar is operating with only one transmitter

a_3 = 0.018 = chance that the radar fails (is down).

24-3.3 THE DEPENDABILITY MATRIX

The availability vector for a system evaluation having been determined, the next step is that of establishing the dependability matrix. This requires a representation of the system attributes during the course of a mission and is conditional on its state of readiness at the beginning of the mission.

The dependability matrix is a square array of numbers (probabilities) involving the n significant system states as follows:

$$[D] = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix} . \tag{24-8}$$

As previously defined, any element d_{ij} in the dependability matrix represents the chance that the effective state of the system during the mission is j, given that the mission was begun in state i. More specifically, for Example 24-1 as a case in point, then d_{11} would be the chance that the radar system ends the mission with all units operating properly, given that it started in this same state or way.

Since for any given state i at the start of the mission we must have the system completing the mission in a single state of all the significant states, then the sum of the probabilities in a row must be unity, i.e.,

$$\sum_{j=1}^{n} d_{ij} = 1, \qquad i = 1, 2, \ldots, n \tag{24-9}$$

For many weapon systems, repair during the mission is not possible; accordingly, some of the elements of the dependability matrix will consequently be zero. For example, in Example 24-1, d_{21}, d_{32}, and d_{31} will be zero since if only one transmitter is operating properly at the start of the mission, then both cannot be operating at the end of the mission (unless quite accidentally) since no repair is possible, or for d_{32} and d_{31} a failed system at mission start cannot become operational. If states are numbered in increasing order of degradation as in Example 24-1, then the dependability matrix is triangular and we see for such cases that

$$[D] = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ 0 & d_{22} & \dots & d_{2n} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ 0 & 0 & \dots & d_{nn} \end{bmatrix}. \tag{24-10}$$

We further remark that since no repair is possible during the mission for many weapon systems requiring quick response, or otherwise, then the system state at the end of the mission is really the only important mission criterion.

As may be seen from our Example 24-1, the establishment of the categories or elements for the availability matrix could be rather critical in the WSEIAC evaluation procedure. In Example 24-1, we more or less pinpointed the importance of using two transmitters in parallel, even though the range radar system could operate when only a single transmitter functions properly. Nevertheless, we could well (and in many applications it would be more meaningful to) use only two states for the system, i.e., (1) an operable state, or (2) a failed state. Moreover, final judgment on the availability vector "locks the analyst in" on the dependability matrix, while simple categories such as "operable" or "failed" might be passed on to the dependability matrix with advantage and facility since it may be sufficient to know just that the system is in a failed state without any need to know which particular component failed. For the case where the system is judged to be merely "operable" or "failed", and the availability vector so constructed, then the dependability matrix is simple indeed, consisting of only four elements:

$$[D] = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}. \tag{24-11}$$

The d_{ij} in Eq. 24-11 are defined as follows:

d_{11} = probability that the system is operable at the end of the mission, given that it was operable at the start of the mission

d_{12} = probability that the system is failed at the end of the mission, given that it was operable at the start of the mission

d_{21} = probability that the system is operable at the end of the mission, given that it was failed at the start of the mission

d_{22} = probability that the system is failed at the end of the mission, given that it was failed at the start of the mission.

Furthermore, if no repair is possible during the mission and an exponential failure law for the system can be applied, then

$$[D] = \begin{bmatrix} \exp(-\lambda T) & 1 - \exp(-\lambda T) \\ 0 & 1 \end{bmatrix} \tag{24-12}$$

where

λ = system failure rate

T = duration of mission.

For some systems, sufficient repair during a mission is possible. For example, repair might be possible for a ground control approach system for an airstrip, or even a truck or tank in transit to or from front areas, etc. When repair is possible during mission time, then exponential failure-time and repair-time laws apply to many systems, and if we put

λ = system failure rate

μ = system repair rate

T = mission time

then Ref. 2 shows that for the "operable" or "failed" (two element) case, then the 2×2 dependability matrix elements are

$$d_{11} = \frac{\mu}{\lambda + \mu} + \left(\frac{\lambda}{\lambda + \mu}\right) \exp[-(\lambda + \mu)T] \tag{24-13}$$

$$d_{12} = \frac{\lambda}{\lambda + \mu} \left\{1 - \exp[-(\lambda + \mu)T]\right\} \tag{24-14}$$

$$d_{21} = \frac{\mu}{\lambda + \mu} \left\{1 - \exp[-(\lambda + \mu)T]\right\} \tag{24-15}$$

$$d_{22} = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} \exp[-(\lambda + \mu)T] . \tag{24-16}$$

Example 24-2:

Assume the data given in Example 24-1, except that no repair of the radar is possible during a mission time of 15 min. Calculate the dependability matrix for the mission.

Since each transmitter has an $MTBF$ = 10 h, the failure rate λ_t of each transmitter is

$$\lambda_t = 1/10 = 0.1 \text{ failure per hour} .$$

Likewise, for the combination of receiver, antenna, synchronizer et al., the failure rate λ_r is

$$\lambda_r = 1/50 = 0.02 \text{ failure per hour} .$$

Hence, assuming exponentially distributed failure times for the components of the radar, we have that the reliability of each transmitter during mission time is

$$R_t = \exp(-\lambda_t T) = \exp[-0.1(0.25)] = 0.975 .$$

Likewise, the reliability for the combination receiver et al. is

$$R_r = \exp(-\lambda_r T) = \exp[-0.02(0.25)] = 0.995 .$$

Now for the dependability matrix, d_{11} represents the chance that all components begin the mission operating and continue to operate throughout the mission. Hence, both transmitters must operate throughout, as must the combination, the chance of this being

$$d_{11} = R_t R_t R_r = \exp[-(2\lambda_t + \lambda_r)T] \tag{24-17}$$

$$= (0.975)^2(0.995) = 0.946 .$$

Continuing, d_{12} is the chance that all radar components operate at start of the mission, but one transmitter fails during the mission time, i.e.,

$$\left.\begin{aligned} d_{12} &= R_t(1 - R_t)R_r + (1 - R_t)R_t R_r \\ &= 2\exp[-(\lambda_t + \lambda_r)T] - 2\exp[-(2\lambda_t + \lambda_r)T] \end{aligned}\right\} \tag{24-18}$$

$$= 2(0.970) - 2(0.946) = 0.048 .$$

For d_{13}, which is the chance that as the mission ends the radar fails even though all units started the mission operating properly, we calculate

$$d_{13} = 1 - d_{11} - d_{12} = 0.006 .$$

For the remaining elements, d_{22} is the chance that the radar starts and completes the mission with only one transmitter operating properly, i.e.,

$$d_{22} = R_t R_r = \exp[-(\lambda_t + \lambda_r)T] \tag{24-19}$$

$$= (0.975)(0.995) = 0.970 .$$

Finally, the reader easily can see that

$$d_{21} = d_{31} = d_{32} = 0$$

$$d_{23} = 1 - d_{22} = 1 - 0.970 = 0.03$$

and d_{33}, the chance that the rader starts and ends the mission failed is

$$d_{33} = 1 .$$

Hence, the complete dependability (reliability) matrix is

$$[D(15)] = \begin{bmatrix} 0.946 & 0.048 & 0.006 \\ 0 & 0.970 & 0.030 \\ 0 & 0 & 1 \end{bmatrix} .$$

24-3.4 THE CAPABILITY MATRIX OR VECTOR

The establishment of the capability matrix or vector is the final step in setting up the WSEIAC model for evaluation purposes. Of some particular interest is the fact that there is some leeway, it might be said, in setting up and using the capability matrix since it depends only on the final possible states j (the initial states i having been "multiplied out" in the product of the availability vector and dependability matrix) and the FOM's or terminal effectiveness k. Thus, the element c_{jk} of the capability matrix is the kth FOM, k being a different subscript, associated with system performance in effective system state j. The element c_{jk} depends very much on the system being evaluated, and hence it is advisable to set up the capability matrix for and in terms of a particular application. Usually, the c_{jk} will be in terms of probabilities, although they could represent other indices or FOM's.

For capability FOM's, we might be interested in the capability of the radar to detect a target at maximum range and/or its capability to detect, acquire, and track the target throughout the mission giving continuous and accurate range data, etc. Ref. 2 (p. 70) and Ref. 7 show that if p is the chance of the radar detecting a target, then

$$\ln(1 - p) = - aP_T/(2z^2r^4) \tag{24-20}$$

where

a = constant depending on radar cross section of the target
P_T = transmitter power
z = root-mean-square noise amplitude
r = range to target.

For illustrative purposes, a computation using Eq. 24-20 for the power of both transmitters operating might give a probability of detection equal to 0.90, say, at a maximum range of 20 mi. Then, if one of the transmitters is not operating (state 2), it may be calculated from Eq. 24-20 that the chance of detection is reduced to 0.683 (Ref. 2). Hence, the capability vector for detecting a typical target at maximum range would be

$$[C_0] = \bar{C}_0 = \begin{bmatrix} c_1(0) \\ c_2(0) \\ c_3(0) \end{bmatrix} = \begin{bmatrix} 0.900 \\ 0.683 \\ 0.000 \end{bmatrix} \tag{24-21}$$

Hence, with such a calculation, we would have immediately that at the start of the mission, the effectiveness of the radar in detecting and acquiring the target at maximum range would be measured by

$$E(\text{max range}) = \bar{A}'\bar{C}_0 = [a_1 \ a_2 \ a_3] \begin{bmatrix} c_1(0) \\ c_2(0) \\ c_3(0) \end{bmatrix}$$

$$= [0.818 \ 0.164 \ 0.018] \begin{bmatrix} 0.900 \\ 0.683 \\ 0.000 \end{bmatrix} \quad (24\text{-}22)$$

$$= 0.848 \; .$$

Hence, the overall chance that the radar detects and acquires the target at maximum range, no matter what its state, is 0.85. (Target detection at a closer range would give a higher probability, of course.)

Note that we have not used the dependability matrix in this particular calculation since the availability of the system and the capability to detect and acquire the target at maximum range occur instantaneously under the assumptions, i.e., with zero mission time. (Of course, it might be argued that the dependability matrix reduces to the identity matrix in this particular case.)

Now that we have evaluated target detection and acquisition at near-maximum range, we are very much interested in continuous tracking and accurate range measurement during the mission of 15 min. For this case, the elements of the capability vector must represent the probability of tracking with required accuracy during the mission once the target has been detected properly.

These calculations depend on the particular radar, its continuous tracking capability, the target and its characteristics, the target path, etc. It is certainly unnecessary to go into such details here in view of our major interest in basic WSEIAC evaluation methodology; therefore, for the states $j = 1, 2, 3$, which the mission encompasses, we will facilitate our quantitative solution by assuming that for the mission of 15 min, we have

$$[C]_{15} = \bar{C}_{15} = \begin{bmatrix} c_1(15) \\ c_2(15) \\ c_3(15) \end{bmatrix} = \begin{bmatrix} 0.97 \\ 0.88 \\ 0.00 \end{bmatrix} \quad (24\text{-}23)$$

for the states 1, 2, and 3 in this example. In other words, the chance of proper tracking is 0.97 for both transmitters operating properly, and 0.88 for only a single transmitter, and zero otherwise.

24-3.5 MODEL EXERCISE

Having indicated the modeling approach for the WSEIAC type evaluation, and the calculation of the elements of the availability vector, the dependability matrix, and the capability vector (which could be a matrix also), we are ready to exercise the model by carrying out the indicated overall calculations. This is given by the radar system effectiveness measure E.

$$E = \bar{A}'[C_0][D(15)]\bar{C}_{15} \quad (24\text{-}24)$$

where $[C_0]$ is treated as a diagonal matrix, and is

$$[C_0] = \begin{bmatrix} 0.900 & 0 & 0 \\ 0 & 0.683 & 0 \\ 0 & 0 & 0 \end{bmatrix} .$$

Example 24-3:

Use the data of Examples 24-1 and 24-2, par. 24-3.4, and the WSEIAC methodology developed herein. Calculate the overall effectiveness figure for the radar subsystem for the surface-to-air gun system.

Perform the multiplication indicated in Eq. 24-24:

$$E = [0.818 \ 0.164 \ 0.018] \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.683 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.946 & 0.048 & 0.006 \\ 0 & 0.970 & 0.030 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.97 \\ 0.88 \\ 0 \end{bmatrix}$$

$$= 0.802$$

which indicates the relative effectiveness of the radar only. Thus, we see that the chance of successful detection, track, and obtaining continuous radar range during the mission for the radar subsystem is 0.802. The effectiveness in track E_T given target detection and acquisition is

$$E_T = E/E(\text{max range}) = 0.802/0.848 = 0.95 .$$

24-4 ADDITIONAL CONSIDERATIONS

The discussion so far in this chapter indicates the WSEIAC approach to determination of the effectiveness of the radar only; this should, no doubt, be of much interest to both the analyst and the decision maker. Indeed, for the decision maker we have been able to use the WSEIAC technique and obtain a single figure of 80.2% as a measure of effectiveness for the radar subsystem, even though there exist different possible states for the system and many different probabilities are involved. Thus, the advantage and usefulness of such an analysis is obvious. Nevertheless, the curious reader and the manager easily will see that we did not evaluate the entire surface-to-air gun system, including the radar. We wish to point out that this too can be done, either by starting the process from scratch with the entire gun system effectiveness in mind, or reasoning as follows at this stage.

Consider that the entire system in this case is a surface-to-air gun system with range only radar and optical fire control tracking for aiming the gun(s) at the target aircraft. Furthermore, consider that if both radar transmitters operate properly during the mission, then some 15 rounds can be fired at the target with engagement hit (kill) probability of 0.85 determined from the methods of Chapter 20; however, if only a single transmitter operates, then there is time for only 10 rounds with kill chance of 0.65. Hence, we can say that given the capability of the radar for range data, the conditional kill probability vector for optical track and aiming of the guns is

$$[C(p_k)] = \bar{C}(p_k) = \begin{bmatrix} p(k_1) \\ p(k_2) \\ p(k_3) \end{bmatrix} = \begin{bmatrix} 0.85 \\ 0.65 \\ 0.00 \end{bmatrix} . \qquad (24\text{-}25)$$

Moreover, the overall or complete gun system engagement kill probability against the enemy aircraft or the effectiveness measure would be found from the augmented equation:

$$E(\text{complete system}) = \bar{A}'[C_0][D(15)][C_{15}]\bar{C}(p_k) \qquad (24\text{-}26)$$

where $[C_{15}]$ is now the diagonal matrix

$$[C_{15}] = \begin{bmatrix} c_1(15) & 0 & 0 \\ 0 & c_2(15) & 0 \\ 0 & 0 & c_3(15) \end{bmatrix} = \begin{bmatrix} 0.97 & 0 & 0 \\ 0 & 0.88 & 0 \\ 0 & 0 & 0 \end{bmatrix} . \qquad (24\text{-}27)$$

Perform the multiplication indicated by Eq. 24-26:

$$E(\text{complete system}) = [0.818 \;\; 0.164 \;\; 0.018] \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.683 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} 0.946 & 0.048 & 0.006 \\ 0 & 0.970 & 0.030 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.97 & 0 & 0 \\ 0 & 0.88 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.85 \\ 0.65 \\ 0 \end{bmatrix}$$

$$= 0.64 .$$

which indicates the complete surface-to-air gun system kill probability against the target aircraft. If this kill chance for a single surface-to-air gun system is considered too low, then several independent gun systems of this type must be deployed in the defended area, or the system redesigned, or a different (perhaps guided missile) system used for this particular role.

This example further indicates how the analyst can build on the basic WSEIAC methodology in order to make more complete, overall evaluations. Thus, some variation in the WSEIAC approach could possibly lead to more generality of application.

24-5 SUMMARY

The WSEIAC approach to the evaluation of weapon systems represents a very clever analytical idea which was developed by a competent study group faced with the problem of determining the effectiveness of complex systems. Although the basic approach consists of using an availability (readiness) vector, a dependability (reliability) matrix, and a capability (performance) matrix or vector, some variation from the central approach leads to more generality. We have indicated in some detail just

how to conduct a WSEIAC evaluation and shown that a very attractive characteristic of such a weapon systems analysis procedure is the final, *single* measure of effectiveness (MOE). In any event, the WSEIAC methodology is deserving of much further study, although the precise formulation and application of the technique may be perhaps a bit restrictive for the analysis of some weapon systems. The analysts, nevertheless, would do well to keep the WSEIAC type methodology in mind.

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INDEX

INDEX (cont'd)

E

INDEX (cont'd)

INDEX (cont'd)

INDEX (cont'd)

INDEX (cont'd)

(DRCDE-L)

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159 (S)†	*Dynamics of Ballistic Impact, Part Two (U)
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179	Explosive Trains
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197	Development Guide for Reliability, Part Three, Reliability Prediction
198	Development Guide for Reliability, Part Four, Reliability Measurement
199	*Development Guide for Reliability, Part Five, Contracting for Reliability
200	Development Guide for Reliability, Part Six, Mathematical Appendix and Glossary
201	Helicopter Engineering, Part One, Preliminary Design
202	Helicopter Engineering, Part Two, Detail Design
203	Helicopter Engineering, Part Three, Qualification Assurance
204	Helicopter Performance Testing
205	Timing Systems and Components
210	Fuzes
211	Fuzes, Proximity, Electrical, Part One
212 (S)	Fuzes, Proximity, Electrical, Part Two (U)
213 (S)	Fuzes, Proximity, Electrical, Part Three (U)
214 (S)	Fuzes, Proximity, Electrical, Part Four (U)
215	Fuzes, Proximity, Electrical, Part Five
235	Hardening Weapon Systems Against RF Energy
238	Recoilless Rifle Weapon Systems
240 (C)	Grenades (U)
242	Design for Control of Projectile Flight Characteristics
244	Ammunition, Section 1, Artillery Ammunition--General, with Table of Contents, Glossary, and Index for Series
245	Ammunition, Section 2, Design for Terminal Effects
247	Ammunition, Section 4, Design for Projection
248	Ammunition, Section 5, Inspection Aspects of Artillery Ammunition Design
249	Ammunition, Section 6, Manufacture of Metallic Components of Artillery Ammunition
250	Guns--General
251	Muzzle Devices
252	**Gun Tubes
253†	*Breech Mechanism Design
255	Spectral Characteristics of Muzzle Flash
260	Automatic Weapons
270	Propellant Actuated Devices
280	Design of Aerodynamically Stabilized Free Rockets
281 (SRD)	Weapon System Effectiveness (U)
283	Aerodynamics
284	Trajectories
285	Elements of Aircraft and Missile Propulsion
286	Structures
290	Warheads--General
300	Fabric Design
312	Rotational Molding of Plastic Powders
313	Short Fiber Plastic Base Composites
327	Fire Control Systems--General
329	Fire Control Computing Systems
331	Compensating Elements
340	Carriages and Mounts--General
341	Cradles
342	Recoil Systems
343	Top Carriages
344	Bottom Carriages
345	Equilibrators
346	Elevating Mechanisms
347	Traversing Mechanisms
350	Wheeled Amphibians
355	The Automotive Assembly
356	Automotive Suspensions
357	Automotive Bodies and Hulls
358†	Analysis and Design of Automotive Brake Systems
360	Military Vehicle Electrical Systems
361	Military Vehicle Power Plant Cooling
410†	Electromagnetic Compatibility (EMC)
411 (S)	Vulnerability of Communication-Electronic and Electro-Optical Systems (Except Guided Missiles) to Electronic Warfare, Part One, Introduction and General Approach to Electronic Warfare Vulnerability (U)
412 (C)	Part Two, Electronic Warfare Vulnerability of Tactical Communications (U)
413 (S)	Part Three, Electronic Warfare Vulnerability of Ground-Based and Airborne Surveillance and Target Acquisition Radars (U)
414 (S)	Part Four, Electronic Warfare Vulnerability of Avionics (U)
415 (S)	Part Five, Optical/Electronic Warfare Vulnerability of Electro-Optic Systems (U)
416 (S)	Part Six, Electronic Warfare Vulnerability of Satellite Communications (U)
417 (S)†	Vulnerability of Guided Missile Systems to Electronic Warfare (U)
445	Sabot Technology Engineering
470†	Metric Conversion Guide

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