Người soạn:
Nguyễn Nhật Quân
Nguyễn Thành Liêm

a)$T_1\left[x\ \ n\ \ \right] = \sum_{k=0}^{n} x\ \ k$

Unstable:

Choosing $n = \infty$, we obtain $\sum_{k=0}^{\infty} x\ \ k\ \ = \infty$

Thus, the system is unstable.

Causal:

The output depends only on the present input.

Linear:

We find the response of the system to two sepate input signals $x_1\ \ n$ and $x_2\ \ n$. The result is

$$y_1 = \sum_{k=0}^{n} x_1\ \ k$$

$$y_2 = \sum_{k=0}^{n} x_2\ \ k$$

The output of the system to a linear combination of $x_1\ \ n$ and $x_2\ \ n$ is

$$y_3\ \ n\ \ = T\left[a_1 x_1\ \ n\ \ + a_2 x_2\ \ n\ \ \right] = \sum_{k=0}^{n}\left[a_1 x_1\ \ k\ \ + a_2 x_2\ \ k\ \ \right] = a_1 \sum_{k=0}^{n} x_1\ \ k\ \ + a_2 \sum_{k=0}^{n} x_2\ \ k \quad (1)$$

Finally, a linear combination of the two outputs in yields

$$a_1 y_1\ \ n\ \ + a_2 y_2\ \ n\ \ = a_1 \sum_{k=0}^{n} x_1\ \ k\ \ + a_2 \sum_{k=0}^{n} x_2\ \ k \quad (2)$$

By comparing (1) with (2), we conclude that the system is linear.

Time – Variant:

We have:

$$y\ \ n - n_0\ \ = \sum_{k=0}^{n-n_0} x\ \ n - n_0\ \ = \sum_{k=0}^{t} x\ \ t \quad (1)$$

$$T\ \ x\ \ n - n_0\ \ \ = \sum_{k=0}^{n} x\ \ n - n_0\ \ = \sum_{k=0}^{t} x\ \ t - t_0 \quad (2)$$

By comparing (1) and (2), we obtain the system is time – variant.

b) $T_2\left[x(n)\right] = \sum_{k=n-10}^{n+10} x(k)$

Stable:

$$T_2\{x(n)\} = \left|\sum_{k=n-10}^{n+10} x(k)\right| \le \sum_{k=n-10}^{n+10} |x(k)| \le \left[\left|n+10-(n-10)\right|+1\right]M \le 21M$$

$$\lim_{n\to\infty} T_2\{x(n)\} = 21M < \infty$$

Thus, $T_2\{x(n)\}$ is stable.

Causal:

The output depends only on the present input, thus $T_2\{x(n)\}$ is causal.

Linear:

We find the response of the system to two sepate input signals $x_1(n)$ and $x_2(n)$.

The result is

$$y_1 = \sum_{k=n-10}^{n+10} x_1(k)$$

$$y_2 = \sum_{k=n-10}^{n+10} x_2(k)$$

The output of the system to a linear combination of $x_1(n)$ and $x_2(n)$ is

$$y_3(n) = T\left[a_1x_1(n) + a_2x_2(n)\right] = \sum_{k=n-10}^{n+10} \left[a_1x_1(k) + a_2x_2(k)\right] = a_1\sum_{k=n-10}^{n+10} x_1(k) + a_2\sum_{k=n-10}^{n+10} x_2(k) \quad (1)$$

Finally, a linear combination of the two outputs in yields

$$a_1y_1(n) + a_2y_2(n) = a_1\sum_{k=n-10}^{n+10} x_1(k) + a_2\sum_{k=n-10}^{n+10} x_2(k) \quad (2)$$

By comparing (1) with (2), we conclude that the system is linear.

Time – Variant:

We have,

$$y(n-n_0) = \sum_{k=n-10-n_0}^{n+10-n_0} x(n-n_0) = \sum_{k=t-10}^{t+10} x(t) \quad (1)$$

$$T\{x(n-n_0)\} = \sum_{k=n-10}^{n+10} x(n-n_0) = \sum_{k=0}^{t} x(t-t_0) \quad (2)$$

From (1) and (2), we have the system is time – variant.

c) $T_3\left[x \;\; n\right] = x \;\; -n$

Unstable:

We have,

$$\left|T_3 \;\; x \;\; n \;\;\right| = \left|x \;\; -n\;\;\right| \le x\left[\left|-n\right|+1\right] \le \left|-n\right| M$$

On the other hand,

$$\lim_{n\to\infty} \;\; \left|-n\right|+1 \;\; = \infty$$

Thus, we conclude the system is unstable.

Noncausal:

For example 1, $n=-1$, which yields $y \;\; -1 \;\; = x \;\; 1$. Thus the output at $n=-1$ depends on the input at $n=1$, which is two unit of time into the future.

Linear:

We find the response of the system to two separate input signals $x_1 \;\; n$ and $x_2 \;\; n$.

The result is

$$y_1 = x_1 \;\; -n$$

$$y_2 = x_2 \;\; n$$

The output of the system to a linear combination of $x_1 \;\; n$ and $x_2 \;\; n$ is

$$y_3 \;\; n \;\; = T\left[a_1 x_1 \;\; n \;\; + a_2 x_2 \;\; n \;\;\right] = a_1 x_1 \;\; -n \;\; + a_2 x_2 \;\; -n$$

Finally, a linear combination of the two outputs in yields

$$a_1 y_1 \;\; n \;\; + a_2 y_2 \;\; n \;\; = a_1 x_1 \;\; -n \;\; + a_2 x_2 \;\; -n$$

Time – Invariant:

We have,

$$y \;\; n-n_0 \;\; = x \;\; -n+n_0 \qquad (1)$$

$$T_3 \;\; x \;\; n-n_0 \;\;\; = x \;\; -n-n_0 \quad (2)$$

Comparing (1) and (2), we conclude the system is time – variant.